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The feasible constant speed helical trajectories for propeller driven airplanes

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Abstract. The motion of propeller driven airplanes, flying at constant speed on ascending or descending helical trajectories is analyzed. The dynamical abilities of the airplane are shown to result in restrictions on the ranges of the geometrical parameters of the helical path. The physical quantities taken into account are the variation of air density with altitude, the airplane mass change due to fuel consumption, its load factor, its lift coefficient, and the thrust its engine can produce. Formulas are provided for determining all the airplane dynamical parameters on the trajectory. A procedure is proposed for the construction of tables from which the flyability of trajectories at a given angle of inclination and radius can be read, with the corresponding minimum and maximum speeds allowed, the final altitude reached and the amount of fuel burned. Sample calculations are shown for the Cessna 182, a Silver Fox like unmanned aerial vehicle, and the C-130 Hercules.

Keywords: airplane helical trajectory; banked turn; airplane equation of motion; circular arc connection; automatic trajectory planning

1. Introduction

This work constitutes a contribution to the enterprise of endowing unmanned aerial vehicles (UAVs) with complete autonomy, i.e., the ability to conduct their mission without human intervention. A fundamental task they then have to be able to perform consists in automatically re-planning their trajectory when unforeseen circumstances require them to modify their flight plan. Essential tools to perform this task are formulas or tables which indicate what trajectories are flyable, according to the airplane dynamics, and provide basic information such as the amount of fuel required, the time of flight, etc.

It is important to remark that for producing optimal or very good trajectories, many factors have to be analyzed, not only properties of the mathematical curve itself but very much also those of the vehicle involved. Whereas the authors of the first studies on trajectory planning in 3D were mainly interested in finding the shortest curve between two points with departure and arrival directions (for example Dubins (1957), present studies now define optimality by taking into account many more factors than the length. Firstly, of course, the vehicle considered should have

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Nomenclature

a	speed of sound in air. At altitude h, a(h)= $\sqrt{\gamma R T(h)}$. At sea level, a(0)=340.3029 m/s
a_1	absolute value of the slope of the temperature as a function of altitude, below 11 km, $a_1=6.5\times10^{-3}$ K/m
AFR	air fuel ratio (about 14.7)
AR	aspect ratio=b^2/S
b	wingspan
c	specific fuel consumption in Newton per Watt-second, that is in m^{-1}
C_D	global drag coefficient for the aircraft= $C_{D0}+\frac{C_L^2}{\pi e AR}$ (Drag polar)
C_{D0}	global drag coefficient at zero lift
C_L	global lift coefficient for the aircraft
D	drag $=\frac{1}{2}\rho_\infty S C_D V_\infty^2$
e	Oswald's efficiency factor
g	gravitational constant=9.8 m/s^2
h	altitude of airplane
h_c	service ceiling
L	lift=$\frac{1}{2}\rho_\infty S C_L V_\infty^2$
P	power of the engine in Watt
R	specific gas constant for air=287.058 J/(kg K)
S	wing area
t	time variable
T_s	temperature at sea level=288.16 K
T(h)	temperature at altitude h
v_3	vertical component of airplane velocity
V_∞	airplane speed with respect to the undisturbed air in front of it
V_{NE}	speed never to be exceeded
W	weight of the airplane
W_1	weight of the empty airplane
W_f	maximum weight of fuel
W_0	maximum take-off weight (MTOW)
γ	ratio of the constant pressure specific heat to the constant volume specific heat=c_P/c_V=1.4 for air
η	propeller efficiency
ρ_s	air density at sea level=1.225 kg/m^3
ρ_∞(h)	density of undisturbed air in front of airplane, at altitude h, $\rho_s\left[\frac{T(h)}{T_s}\right]^{4,2433}$