Finite element elastoplastic homogenization model of a corrugated-core sandwich structure

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Abstract. This study aimed to develop an elastoplastic homogenization model to accurately predict the elastoplastic static behavior of a corrugated-core sandwich structure. A panel composed of two planar layers and one corrugated layer is modeled by a homogeneous orthotropic single-layer plate. A plane stress elastoplastic model is adopted to describe the behavior of each layer. Homogenization is achieved by local integration across the thickness of each layer. The proposed homogenization model is implemented in the ABAQUS finite element software using UGENS user subroutine. The results obtained by our model are compared to those obtained by full 3D simulations under different loading conditions. The comparisons show the efficiency and the accuracy of the proposed elastoplastic homogenization model.

Keywords: composite structure; corrugated cardboard; elastoplastic behavior; FEM simulation; homogenization

1. Introduction


To reduce the calculation time, a corrugated-core sandwich panel can be replaced by an orthotropic plate and its equivalent rigidities can be obtained by analytical or finite element methods using homogenization procedures. Aboura et al. (2004) developed an analytical homogenization model based on the theory of laminates and compared successfully their model with numerical and experimental results. Buannic et al. (2003) presented a homogenization technique based on an asymptotic expansion method in which the overall behavior of a corrugated structure is determined from the constitutive laws of each constituent. Biancolini (2005) evaluated the equivalent stiffness properties of corrugated board using a numerical approach by the finite element method. Carlsson et al. 2001 and Nordstrand et al. (1994) obtained analytically the effective mechanical properties of a corrugated structure including transverse shear. Nordstrand (1995) calculated the critical buckling force of a corrugated structure simply supported and subjected to compression by the finite element method. Nordstrand (2004a) also analyzed the buckling and post-buckling of an orthotropic plate by including the effect of transverse shear. In another work, Nordstrand (2004b) included transverse shear in his analysis to predict the critical buckling force. To consider the transverse shear in laminated composite plates, high-order shear deformation theories can be used (Adim et al. 2016, Hassaine Daouadji et al. 2019). Different theories were extensively developed and successfully used to study functionally graded material (FGM) sandwich plates (Adim and Hassaine Daouadji (2016), Rabahi et al. (2018)), and hybrid laminated composite plates (Adim et al. 2018, Benhenni et al. 2019).

Most of these techniques are based on the analytical or semi-analytical analysis of the specific geometric pattern of the structure. They are therefore limited for general applications involving geometric shapes or arbitrary microstructures. More general multi-scale modeling techniques were developed to consider the geometrical or microstructural specificity of the structure (Nezamabadi et al. 2009, Cong et al. 2015, Dayyani et al. 2015).

For many years, the analysis and design of a corrugated-core sandwich structures have been based on the linear theory of elasticity, but it is well known that this approach is
limited because it does not take advantage of the ability of many materials to withstand stresses greater than their elastic limits. The main difficulty of plastic analysis is its mathematical complexity, which has been overcome by the advent of high-speed computers and the efficient use of numerical methods.

Paper and paperboard are mainly made of wood fibers. Due to the continuous papermaking process, the wood fibers are oriented mainly in the machine direction. This process generates three characteristic directions: machine direction (MD), transverse direction (CD) and thickness direction (ZD). This makes paperboard an anisotropic material: it is 2 to 4 times more rigid in the MD direction than CD direction, while it can undergo greater deformation in CD direction. The in-plane mechanical properties of a paperboard are relatively easy to determine by standard tests; however, the out-of-plane properties are more difficult to obtain. Stenberg (2003) has shown that the Young's modulus following ZD is about 200 times weaker than that of MD. Stenberg et al. (2001) observed that the deformation in the plane is negligible during compression along the thickness. Therefore, plastic modelling of paperboard must account for orthotropy. The common plasticity models used for paperboard modelling are Hill (1948), Hoffman (1967), Tsai and Wu (1971), Xia et al. (2002), Mäkelä and Östlund (2003). Harrysson and Ristimaa (2008) presented an anisotropic elastoplastic constitutive model of paper material and a yield surface inspired by the Tsai–Wu failure criterion was introduced. The model was implemented into a finite element software and a creasing operation of a 3D corrugated unhomogenized board panel was investigated. Rabczuk et al. (2004) developed a homogenization method for sandwich structures considering material nonlinearities including buckling of the core based on the equivalence of the continuum stored energy density function and a discrete energy associated to a representative core cell. Chang et al. (2006) developed an elastic perfectly plastic analysis of a corrugated-core sandwich plate. In their analysis, only the yielding at central point was considered. Shokrollahi et al. (2017) developed the EP-HDQm approach for analyzing sandwich cylindrical shell panels undergoing elastoplastic deformation using harmonic differential quadrature method. In their approach, the faces of the sandwich shell panel were considered as isotropic materials with linear work hardening behavior while the core is assumed to be an isotropic elastic material.

The present paper deals with a homogenization procedure which considers plastic nonlinear effects in corrugated-core cardboard. The effective elastoplastic parameters for shell model that replaces corrugated-core sandwich structure is modeled by combining numerical and experimental approaches. An orthotropic elasticity is combined with the isotropic plasticity equivalent model (IPE). Standard experimental tests are used to calibrate the parameters of the corrugated cardboard constituents. The novelty of this work lies in considering an orthotropic plasticity behavior of the faces and the fluting of a corrugated cardboard panel, and the elastoplastic response of the homogenized plate is achieved by integration across the thickness of each layer providing the local equivalent plastic deformation. The proposed homogenization model is implemented in the ABAQUS (2019) finite element software. The results obtained by our model are compared to those obtained by full 3D simulations under different loading conditions.

2. Homogenization model

A corrugated-core sandwich plate consists of two facing plates and a corrugation core shown in Fig. 1.

The fluting shape is described with a sine function

\[ h(x) = \frac{h_c - t_2}{2} \sin \left( \frac{2\pi x}{P} \right) \]  

(1)

where: \( h_c \) is a distance between liners, \( t_2 \) is the flute thickness and \( P \) is the fluting period, and the rotation angle is given by

\[ \theta(x) = \tan^{-1} \left( \frac{dh(x)}{dx} \right) \]  

(2)

The three-dimensional (3D) corrugated-core sandwich panel will be reduced to an equivalent two-dimensional (2D) structurally orthotropic thick plate continuum. The role of homogenization is to obtain a homogeneous panel of equivalent stiffness of the heterogeneous corrugated-core sandwich panel. In case of this kind of structure, this can be done by using the Classical Laminate Plate Theory (CLPT) presented in detail in the book of Reddy (2003).

Firstly, we recall the elastic homogenization then we will extend it to the elastoplastic case.

2.1 Elastic homogenization

According to Kirchhoff’s assumptions strains in plate and shell elements can be decomposed into membrane strains \( \{e^m\} \) and curvature effects \( \{κ\} \)

\[ \{ε\} = \{ε^m\} + z\{κ\} \]  

(3)

In the state of plane stress, the stress can be expressed in the matrix form Reddy (2003)

\[ \{σ\} = [Q]\left( \begin{bmatrix} ε_x^m & ε_y^m & ε_{xy}^m \\ ε_x^m & ε_y^m & ε_{xy}^m \\ ε_{xy}^m & ε_{xy}^m & ε_{yy}^m \end{bmatrix} + z \begin{bmatrix} κ_x \\ κ_y \\ κ_{xy} \end{bmatrix} \right) \]  

(4)

where \([Q]\) is the orthotropic elasticity matrix in plane stresses defined as

\[ [Q] = \begin{bmatrix} E_x/(1−ν_{xy}ν_{yx}) & ν_{yx}E_x/(1−ν_{xy}ν_{yx}) & 0 \\ ν_{xy}E_y/(1−ν_{xy}ν_{yx}) & E_y/(1−ν_{xy}ν_{yx}) & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \]  

(5)
Membrane resultant forces and bending-torsion resultant moments are obtained by integrating stresses through the thickness Reddy (2003)

\[
\begin{align*}
N_x &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \, dz = [A] [e^m] + [B] [\kappa] \\
N_y &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y \, dz \\
N_{xy} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} \, dz
\end{align*}
\]

(6)

\[
\begin{align*}
M_x &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \, dz = [B] [e^m] + [D] [\kappa] \\
M_y &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y \, dz \\
M_{xy} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} \, dz
\end{align*}
\]

(7)

The coefficients of the [A], [B] and [D] matrices are given as follows (Talbi et al. 2009, Luong et al. 2020)

\[
\begin{align*}
[A(x)] &= [Q_1] t_1 + [Q_2(\theta(x))] t'_2 + [Q_3] t_3 \\
[B(x)] &= [Q_1] z_1 t_1 + [Q_2(\theta(x))] z_2 t_2' + [Q_3] z_3 t_3 \\
[D(x)] &= [Q_1] \left( z_1^2 t_1 + \frac{t_1^2}{12} \right) \\
&\quad + [Q_2(\theta(x))] \left( z_2^2 t_2' + \frac{t_2'^2}{12} \right) \\
&\quad + [Q_3] \left( z_3^2 t_3 + \frac{t_3^2}{12} \right)
\end{align*}
\]

(8)

(9)

(10)

where subscripts 1, 2 and 3 denote outer liner, fluting and inner liner respectively, and \( t'_2 = t_2 / \cos \theta \).

Where \( z_1, z_2 \) and \( z_3 \) are defined as

\[
\begin{align*}
z_1 &= \frac{1}{2} t_1 + \frac{1}{2} h_c \\
z_2 &= \frac{1}{2} t'_2 + h(x) \\
z_3 &= -\frac{1}{2} t_2 - \frac{1}{2} h_c
\end{align*}
\]

(11)

The transverse shear matrix [F] is defined as follows (Hammou et al. 2012)

\[
[F(x)] = k \left[ [C_1] t_1 + [C_2(\theta(x))] t'_2 + [C_3] t_3 \right]
\]

(12)

with \( k = 5/6 \) and the matrix \([C]\) is defined as

\[
[C] = \begin{bmatrix} G_{xx} & 0 \\ 0 & G_{yy} \end{bmatrix}
\]

(13)

Equivalent global rigidities are obtained by homogenization over a fluting period \( P \):

\[
[M] = \frac{1}{P} \int_0^P [M(x)] \, dx
\]

(14)

where \([M]\) is generic matrix representing \([A]\), \([B]\), \([D]\) and \([F]\) respectively.

However, corrugated-core cardboard is more complex than a laminated plate because of the fluting core and the cavities between the two liners. Consequently, some global effective stiffnesses obtained by the theory of laminated plates (Eq. (14)) should be modified as detailed in our previous works (Abbès and Guo 2010, Hammou et al. 2012).

### 2.2 Plastic homogenization

In this part, we take the example of a corrugated cardboard structure to illustrate the homogenization process. This method can be extended to other types of sandwich corrugated-core structures using the appropriate behavior law.

In this work, we use the elastoplastic model proposed by Mäkelä and Östlund (2003). The anisotropic material behavior is introduced into the model by an isotropic plasticity equivalent (IPE) transformation tensor introduced by Karafillis and Boyce (1993). The IPE-material can be considered as a fictitious isotropic material, subjected to a stress state that equals the corresponding stress state in the real anisotropic material. The yield criterion was formulated by Mäkelä and Östlund (2003) as:

\[
f = \sigma_{eq} - Y = \left( \frac{3}{2} \langle s \rangle \langle s \rangle \right)^{1/2} - E_0 (\epsilon_0 + \epsilon_{eq}) \leq 0
\]

where \( Y \) is the yield stress, \( \epsilon_{eq} \) is the equivalent plastic strain, \( E_0, \epsilon_0 \) and \( n \) are model parameters, \( \langle s \rangle \) is the deviatoric stress row vector defined in Eq. (16), and \( \langle s \rangle \) is its transpose \( \langle s \rangle = \langle s \rangle^T \).

\[
\{s\} = \begin{bmatrix} s_x \\ s_y \\ s_{xy} \end{bmatrix} = [L]\{\sigma\}
\]

(16)

where the parameters \( a, b, c, d \) describe the anisotropy of the material.

The integration plasticity algorithm is applied at each layer of the corrugated cardboard (liners and flutes) to calculate the stress state and the elastoplastic matrix in the local coordinate system using three integration points for each layer. It should be noted that the local coordinate system of the flutes is defined by the angle \( \theta(x) \) (Eq. (2)).

For each integration point, the starting point of the calculation of the stress state, is an assumption of pure elastic behavior leading to a trial stress state \( \{\sigma_{trial}\} \). The value of the yield criterion \( f \) is evaluated using Eq. (15): if \( f < 0 \) a pure elastic deformation is occurring during the increment and the evaluated stress state is the correct stress state, if \( f > 0 \) the deformation is partly plastic and the elastic trial stress state must be corrected for plastic deformation such as the Direct Scalar Algorithm using the algorithm detailed in Li et al. (2007).

Finally, the matrices \([A]\), \([B]\) and \([D]\) in Eqs. (8)-(10) are replaced by the corresponding elastoplastic matrices \([A^p]\), \([B^p]\) and \([D^p]\).

### 2.3 Finite element implementation

The corrugated cardboard is modelled by shell elements,
the resultant forces and moments are calculated incrementally as

\[
\begin{bmatrix}
[dN] \\
[dM]
\end{bmatrix} = \begin{bmatrix}
[A^P] & [B^P] \\
[B^P] & [D^P]
\end{bmatrix} \begin{bmatrix}
[d\varepsilon_m] \\
[d\kappa]
\end{bmatrix}
\]

(17)

where the matrices \([A^P], [B^P]\) and \([D^P]\) are determined by the plastic homogenization model implemented in ABAQUS (2019) using a user subroutine UGENS.

A 4-nodal shell element S4R with 6 degrees of freedom per node (3 displacements and 3 rotations) is used. In the UGENS subroutine, the known generalized strain and curvature increments \(D\varepsilon\) and \(D\kappa\) are given at the beginning of the increment. The generalized forces \(F(\varepsilon, t_i)\) and \(F_{\text{exp}}(t_i)\) are updated at the end of the increment. The elastoplastic matrices are calculated and stored in the array \(DDNDDE(6,6)\) at the end of the increment. This array will be used by ABAQUS (2019) to define the element stiffness matrix in the FE solution. The transverse shear stiffness is directly introduced in the input file of ABAQUS (2019).

### 3. Model calibration

In order to test and evaluate the proposed model, the model parameters were firstly determined for the outer and inner liners and the fluting whose dimensions are given in Table 1.

#### Table 1 Dimensions of corrugated cardboard constituents

<table>
<thead>
<tr>
<th>t₁ (mm)</th>
<th>t₂ (mm)</th>
<th>t₃ (mm)</th>
<th>h (mm)</th>
<th>P (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.208</td>
<td>0.144</td>
<td>0.208</td>
<td>3.584</td>
<td>8.0</td>
</tr>
</tbody>
</table>

#### Table 2 Elastic parameters

<table>
<thead>
<tr>
<th>Eₓ (MPa)</th>
<th>Eᵧ (MPa)</th>
<th>νₓᵧ</th>
<th>νᵧₓ</th>
<th>Gₓᵧ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2433</td>
<td>860</td>
<td>0.234</td>
<td>0.083</td>
<td>1077</td>
</tr>
<tr>
<td>1130</td>
<td>626</td>
<td>0.130</td>
<td>0.072</td>
<td>303</td>
</tr>
</tbody>
</table>

#### Table 3 Plastic parameters

<table>
<thead>
<tr>
<th>E₀ (MPa)</th>
<th>ε₀</th>
<th>n</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.5</td>
<td>4.97E-3</td>
<td>1.0</td>
<td>2.498</td>
<td>2.498</td>
<td>1.622</td>
<td></td>
</tr>
<tr>
<td>87.3</td>
<td>4.25E-3</td>
<td>1.0</td>
<td>2.178</td>
<td>2.178</td>
<td>1.871</td>
<td></td>
</tr>
</tbody>
</table>

For each corrugated cardboard constituent, ten experimental tensile tests were carried out for three directions (MD, CD, and 45°) at a crosshead speed of 10 mm/min under standard conditions (23°C and 50%RH). The material behavior in each direction was taken as the average of these ten tests.

To determine the plastic parameters the obtained load vs elongation curves are fitted by minimizing the least square error defined in Eq. (18):

\[
\delta F = \frac{1}{N} \sum_{i=1}^{N} \left( F(P, t_i) - F_{\text{exp}}(t_i) \right)^2
\]

(18)

where \(F(P, t_i)\) and \(F_{\text{exp}}(t_i)\) are the model and experimental forces at \(t_i\) sampling point, respectively, \(P = [E_0, \varepsilon_0, n, a, b, c, d]\) is the unknown parameter vector and \(N\) is the number of sampling points.

The determined material parameters for the elastoplastic model of each constituent are summarized in Tables 2 and 3.

The tensile tests in each direction were simulated using finite element analysis and the numerically obtained solutions were compared to the experimental data. Fig. 2 shows a comparison between the experimental results and the numerically obtained results for the liners and the fluting using the elastoplastic model. The constitutive model gives accurate predictions of the experimental behavior in the three loading directions.

### 4. Homogenization model validations

The developments in the above sections have been firstly validated with the available results in the literature, and then applied to different examples and comparisons are performed with full 3D simulations.

#### 4.1 Cylindrical sandwich shell panel subjected to a uniform lateral pressure

A cylindrical sandwich shell panel with clamped edges,
subjected to a uniform lateral pressure $q_l$ presented by Shokrollahi et al. (2017) in considered for the validation of the present model (See Fig. 3). In this case study, the faces are made of an elastoplastic steel material ($E = 210 \text{ GPa}$, $v = 0.3$, $E_t = 30 \text{ GPa}$, and $\sigma_y = 240 \text{ MPa}$) and the core is entirely filled with an elastic foam material ($E_c = 180 \text{ MPa}$ and $G_c = 65.69 \text{ MPa}$).

Fig. 4 presents lateral uniform pressure versus maximum lateral deflection curves showing a very good agreement between our results and the results obtained by Shokrollahi et al. (2017).

Moreover, Fig. 5 shows the transverse deflection for longitudinal and circumferential directions of the shell panel undergoing elastic ($q_l = 300 \text{ kPa}$) and plastic ($q_l = 500 \text{ kPa}$) deformations. The longitudinal and circumferential in-plane normal stress $\sigma_x$ at the top surface and in-plane $\sigma_\theta$ at the bottom surface of the cylindrical shell plate are plotted in Figs. 6 and 7, respectively. A good agreement between our results those obtained by Shokrollahi et al. (2017) using their EP-HDQM method. The results near to the boundaries cannot be trusted according to the Saint-Venant’s principle.

### 4.2 Simulation of tensile test

Simulations of tensile tests in the MD-$x$, CD-$y$ and $45^\circ$ directions are performed using the proposed homogenization model and the 3D full model (Fig. 8). The geometric dimensions of the corrugated cardboard and the elastoplastic properties of liners and fluting are given in Tables 1-3. The 3D structure and the 2D homogenized plate are meshed with rectangular reduced integration shells elements (S4R) with a mesh size of 0.5 mm (Fig. 8).

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**Fig. 3** Geometry of the sandwich cylindrical shell panel

**Fig. 4** Lateral uniform pressure vs. maximum lateral deflection of the cylindrical shell panel.

**Fig. 5** Transverse deflection for (a) longitudinal ($x$) and (b) circumferential ($\theta$) directions.

**Fig. 6** In-plane normal stress $\sigma_x$ at the top surface of the cylindrical shell plate for (a) longitudinal ($x$) and (b) circumferential ($\theta$) directions.
Fig. 7 In-plane normal stress $\sigma_\theta$ at the bottom surface of the cylindrical shell plate for (a) longitudinal ($x$) and (b) circumferential ($\theta$) directions.

Fig. 8 Corrugated cardboard 3D structure and 2D homogenized plate.

Fig. 9 Full 3D and homogenized tensile tests comparison.

Fig. 10 CPU time (in seconds) comparison.

Fig. 11 Full 3D and homogenized cantilever beam comparison.

Fig. 9 compares predictions of 3D structure and 2D homogenized plate FEM simulations. The presented results show a very good agreement between the full 3D and homogenized 2D FEM solutions. The relative difference between the two models is less than 2%. The CPU time is given in Fig. 10 showing a reduction by a factor of 2 to 4 with our model.

4.3 Simulation of cantilever beam

Simulations of cantilever beam with vertical displacement at the end in the MD-$x$, CD-$y$ and 45° directions are performed using the proposed homogenization model and the 3D full model. Dimensions, meshing, and material parameters are the same as previous example (Tables 1-3 and Fig. 8).

Fig. 11 shows the deflection-load curves in MD-$x$, CD-$y$ and 45° for the 3D structure and the homogenized plate. The maximum difference between the two models is less than 2.6%. The CPU time is given in Fig. 10 showing a reduction by a factor of 2 to 4 with our model.
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The von Mises stresses are shown in Fig. 12 for the 3D fluting and the corresponding integration point in the homogenized plate for the CD-oriented cantilever beam. These results show a very good comparison of the stress distribution between the 2 models.

4.4 Simulation of double-edge-notched (DENT) tensile test

A tensile test on a double-edge-notched (DENT) specimen implies a more complex stress state. Simulations of this test in the CD-y direction are performed using the proposed homogenization model and the 3D full model. The geometry and dimensions of the specimen are given in Fig. 13. The specimen is fixed at one end and stretched by 4 mm at the other end. The 3D structure and the 2D homogenized plate are meshed with rectangular reduced integration shells elements (S4R) with a mesh size of 0.5 mm. The material parameters are the same as previous examples (Tables 1-3).

The load-elongation curve predictions of 3D structure and 2D homogenized plate FEM simulations are shown in Fig. 14. The presented results show a very good agreement between the full 3D and homogenized 2D FEM solutions. The CPU time is 808 s for the full-3D simulation, while it is only 309 s for H-2D simulation.

The von Mises stresses are shown in Fig. 15 for the 3D fluting and the corresponding integration point in the homogenized plate of the DENT specimen. These results show a very good comparison of the stress distribution between the two models, even around the notches where there is a high concentration of the stresses. To go further in the comparison, the stress vs strain curves are plotted in Fig. 16 at the stress concentration point in the notch. In this figure, it is shown that the homogenized plate and the full 3D structure give close results across the range of deformation.

Furthermore, the stress distribution along a path between the two notches is plotted in Fig. 17. Both models give the same level of stress. The curve of the H-model is smooth while that of the Full-3D model seems to follow the undulation of the fluting. The results near to the two notches cannot be trusted according to the Saint-Venant’s principle.
5. Conclusions

A numerical elastoplastic homogenization model to accurately predict the elastoplastic static behavior of a corrugated-core sandwich structure has been proposed in this study. An orthotropic elasticity is combined with the isotropic plasticity equivalent model (IPE). Standard experimental tests were used to calibrate the parameters of the corrugated cardboard constituents. Homogenization was achieved by local integration across the thickness of each layer. The proposed homogenization model was implemented in the ABAQUS finite element software. The results show a very good agreement between the simulations of the 3D composite structures and the 2D homogenized plate. The proposed model allows CPU time saving and a reduction in the preparation of the geometries and the meshes.

References


A multilevel computational strategy for microscopic compos.


