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Dynamic analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT

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Abstract. In the present work the dynamic analysis of the functionally graded rectangular nanoplates is studied. The theory of nonlocal elasticity based on the quasi 3D high shear deformation theory (quasi 3D HSDT) has been employed to determine the natural frequencies of the nanosize FG plate. In HSDT a cubic function is employed in terms of thickness coordinate to introduce the influence of transverse shear deformation and stretching thickness. The theory of nonlocal elasticity is utilized to examine the impact of the small scale on the natural frequency of the FG rectangular nanoplate. The equations of motion are deduced by implementing Hamilton's principle. To demonstrate the accuracy of the proposed method, the calculated results in specific cases are compared and examined with available results in the literature and a good agreement is observed. Finally, the influence of the various parameters such as the nonlocal coefficient, the material indexes, the aspect ratio, and the thickness to length ratio on the dynamic properties of the FG nanoplates is illustrated and discussed in detail.

Keywords: nonlocal elasticity theory; FG nanoplate; free vibration; refined theory; elastic foundation

1. Introduction

The discovery of carbon nanotubes (CNTs) introduced a novel era in the nano scientific world (Iijima 1991). Since then, several investigations have been realized in the topic of the physical, electrical, mechanical and chemical behaviors of the nanostructures. The primary works demonstrate that the mechanic properties of the nano-structures are different from other well-employed materials (Miller and Shenoy 2000, Bellifa *et al*. 2017a, Bensaid 2017, Ehyaei *et al*. 2017, Karami *et al*. 2017, Bouadi *et al*. 2018, Bensaid *et al*. 2018, Mehar and Panda 2018, Bakhadda *et al*. 2018, Akbas 2018, Tang and Liu 2018, Yazid *et al*. 2018, Youcef *et al*. 2018, Mokhtar *et al*. 2018, Kadari *et al*. 2018, Karami *et al*. 2018a, b, c, d, Cherif *et al*. 2018, Draoui *et al*. 2019, Adda Bedia *et al*. 2019, Karami *et al*. 2019a, b, Semmah *et al*. 2019). The important properties of such structures have favored their applications in several fields such as nanodevices, nano-bearings, nanooscillators, hydrogen storage, and electrical batteries.

The plate-as nanostructures like nanoplates or nano-scale sheets are very important kinds of the nanostructures with 2D shapes (Shahadat *et al*. 2018). They contain important mechanic properties (Iijima 1991, Miller and Shenoy 2000, Shen and Zhang 2010, Pradhan and Phadikar 2009, Eltaher *et al*. 2012, 2016, Ebrahimi and Salari 2015, Khorshidi *et al*. 2015, Chemi *et al*. 2015, Akbaş 2016, Ghorbanpour Arani *et al*. 2012, Janghorban 2016, Wu *et al*. 2018) and with these unique characteristics they become ideal candidates for multifarious field of nanotechnology industry incorporating energy storage (Ma *et al*. 2008), nano electrome-chanical systems, strain, mass and pressure sensors (Sakhaee-Pour *et al*. 2008a, b), solar cells (Aagesen and Sorensen 2008), photo-catalytic degradation of organic dye (Ye *et al*. 2006), composite materials (Rafiee *et al*. 2010) and ect. The size-dependent continuum modeling of the nanostructures has taken a wide attention by the scientific community because the controlled experimentations in nanosize are difficult and molecular dynamic simulations are highly expensive computationally. We can found in the literature various size dependent continuum models such as modified couple stress theory (Koiter 1969, Mindlin and Tiersten 1962, Toupin 1962), strain gradient elasticity theory (Nix and Gao 1998, Lam *et al*. 2003, Aifantis 1999, Li *et al*. 2016) and nonlocal elasticity theory (Eringen 1972). Among these models, the

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theory of nonlocal elasticity has been widely employed (Peddieson *et al*. 2003, Reddy 2007, Reddy and Pang 2008, Heireche *et al*. 2008, Murmu and Pradhan 2009a, b, Wang 2009). To overcome the shortcomings of the conventional elasticity theory, Eringen and Edelen (1972) proposed the nonlocal elasticity model in 1972. They modified the conventional continuum mechanics to consider the small scale influences. It should be noted that in the nonlocal elasticity theory, the tensor of stress at an arbitrary point in the continuum of nano-material is related not only on the tensor of strain at that point but also on the tensor of strain at all other points in the continuum. Both the atomistic simulation data and the experimental studies on phonon dispersion indicated the accuracy of this remark (Eringen 1983, Chen *et al*. 2004).

The functionally graded materials (FGMs) are the novel generation of new composite materials in the family of engineering composites, whose characteristics are changed smoothly between two surfaces and the benefits of this combination lead to new structures which can withstand in important mechanical loads under high temperature environments (Ebrahimi and Rastgoo 2008a, b). Presenting new characteristics, FGMs have also attracted considerable research interests, which were principally focused on their bending, buckling and dynamic properties of FG structures (Ebrahimi *et al*. 2009a, b, Bouderba *et al*. 2013, 2016, Hebali *et al*. 2014, Meziane *et al*. 2014, Houari *et al*. 2016, Boukhari *et al*. 2016, Bennoun *et al*. 2016, Bousahla *et al*. 2016, Bellifa *et al*. 2017b, Sekkal *et al*. 2017a, b, Benahmed *et al*. 2017, Atmane *et al*. 2017, Shahsavari *et al*. 2018, Benchohra *et al*. 2018, Younsi *et al*. 2018, Faleh *et al*. 2018a, b, Bouazza *et al*. 2018, Zine *et al*. 2018, Bouhadra *et al*. 2018, Bourada *et al*. 2018, Boukhlif *et al*. 2019, Khiloun *et al*. 2019, Bourada *et al*. 2019, Zaoui *et al*. 2019).

In addition, structural complements such as plates, beams and membranes in micro or nano-length size are often employed as elements in micro/nano electromechanical systems (MEMS/NEMS). Thus understanding the mechanics and physics characteristics of nanostructures is necessary for its practical uses. In past decades, the dynamic of FGMs has been employed extensively. Malekzadeh and Heydarpour (2012) studied the dynamic behavior of rotating FG cylindrical shells under thermal environment by using the first-order shear deformation theory (FSDT) of shells. Ungbhakorn and wattanasakulpong (2013) examined the thermo-elastic dynamic response of FG plates carrying distributed patch mass based on HSDT. Kumar and Lal (2013) examined the first three natural frequencies of the free axisymmetric vibration of the 2D FG annular plates resting on Winkler foundation by employing differential quadrature technique and Chabyshev collocation method. Based on the 3D theory of elasticity and considering that the mechanical characteristics of the materials changed continuously in the direction of thickness, the 3D free and forced vibration investigation of FG circular plate with various boundary conditions was established by Nie and Zhong (2007). 3D elasticity theory was utilized, and novel sets of admissible functions for the kinematics were developed to improve the effectiveness of the Ritz technique in modeling the behavior of the cracked plates. Matsunaga (2008) analyzed the buckling stresses and the natural frequencies of FG plates by considering the influences of transverse shear and normal deformations. Ke *et al*. (2013) proposed a non-conventional micro-plate model for the axisymmetric nonlinear dynamic analysis of annular FG micro-plates by using the modified couple stress theory, FSDT and von-Karman geometric nonlinearity theory. Ke *et al*. (2012) also investigated the bending, stability and dynamic of annular FG micro-plates based on the modified couple stress theory and FSDT. Asghari and Taati (2013) employed a size-dependent approach for mechanical investigations of FG micro-plates based on the modified theory of couple stress. Kocaturk and Akbas (2012) examined the thermal influence on post-buckling response of FGM beams based on Timoshenko beam theory and by employing finite element formulation. The vibration characteristics of beam with power law properties graduation in the transversal or the axial directions was reported by Alshorbagy *et al*. (2011). Recently, Eltaher *et al*. (2012, 2013a) used a finite element approach for dynamic investigation of FG nanoscale beams based on nonlocal Euler-Bernoulli beam theory. They also discussed the size-dependent bending-buckling response of FG nanobeams by using the nonlocal continuum theory (Eltaher *et al*. 2013b). Dynamic behavior of simply supported Timoshenko FG nanoscale beams were studied by Rahmani and Pedram (2014). Zemri *et al*. (2015) investigated the mechanical response of FG nanoscale beam using a refined nonlocal shear deformation theory beam theory. Belkorissat *et al*. (2015) examined the dynamic properties of FG nano-plate using a new nonlocal refined four variable theory. Ahouel *et al*. (2016) studied the size-dependent mechanical behavior of FG trigonometric shear deformable nanobeams including neutral surface position concept. Bounouara *et al*. (2016) presented a nonlocal zeroth-order shear deformation theory for free vibration of FG nanoscale plates resting on elastic foundation. Khetir *et al*. (2017) developed a novel nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates. Bouafia *et al*. (2017) proposed a nonlocal quasi-3D theory for bending and free flexural vibration behaviors of FG nanobeams. Besseghier *et al*. (2017) analyzed the dynamic response of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory. Mouffoki *et al*. (2017) examined the dynamic response of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory. Karami *et al*. (2019c) investigated the wave propagation of FG anisotropic nanoplates resting on Winkler-Pasternak foundation. Recently, several authors proposed advanced plate/beam theories to study the mechanical behavior of nano- or macro-structures (Belabed *et al*. 2014, Hamidi *et al*. 2015, Kar and Panda 2016a, b, Bousahla *et al*. 2014, Beldjelili *et al*. 2016, Sahoo *et al*. 2016, Draiche *et al*. 2016, Bouazza *et al*. 2016, Mehar and Panda 2016, Becheri *et al*. 2016, Katariya *et al*. 2017a, b, c, El-Haina *et al*. 2017, Fahsi *et al*. 2017, Mehar *et al*. 2017, Ebrahimi *et al*. 2017, Chikh *et al*. 2017, Sahoo *et al*. 2017, Abdelaziz *et al*. 2017, Singh and Panda 2017, Hirwani *et al*. 2017, Katariya and Panda

2018, Ellali *et al*. 2018, Mehar *et al*. 2018a, b, Katariya *et al*. 2018a, b, Kaci *et al*. 2018, Attia *et al*. 2018, Dash *et al*. 2018, Belabed *et al*. 2018, Katariya and Panda 2019, Katariya *et al*. 2019).

In the current work, the dynamic of FG nanoscale plates is studied based on the cubic quasi 3D high shear deformation theory in the conjunction with the nonlocal elasticity model. By considering the integral term in the kinematic led to a reduction in the number of variables and equations of motion. The Navier solution is employed to investigate the dynamic behavior of the FG nanoplates. It is considered that the material characteristics are varying within the thickness according to the power law variation. Numerical results are provided to be utilized as benchmarks for the application and the design of nanoelectronic and nano-drive devices, nano-oscillators, and nanosensors, in which nanoplates act as basic elements. They can also be useful as valuable sources for validating other approximate methods and formulations.

2. Theory and formulation

2.1 Nonlocal power-law FG nanoplate equations

Consider a rectangular nanoscale plate of length *a*, width *b*, and total thickness *h* and composed of FGMs within the thickness as demonstrated in Fig. 1.

$$E(z) = \left(E_c - E_m\right) V_f(z) + E_m \tag{1}$$

$$\rho(z) = \left(\rho_c - \rho_m\right) V_f(z) + \rho_m \tag{2}$$

where the subscripts *c* and *m* denote the ceramic and metallic constituents, respectively, and V_f is the volume fraction that is given by the following expression

$$V_f(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{3}$$

where *n* is the gradient index and takes only positive values. Poisson's ratio *ν* is the same for all the ceramic/ metal materials that are employed here, so it is considered to be constant and is assumed to be equal to 0.3 throughout the investigation (Reddy 2011). The typical values for metals

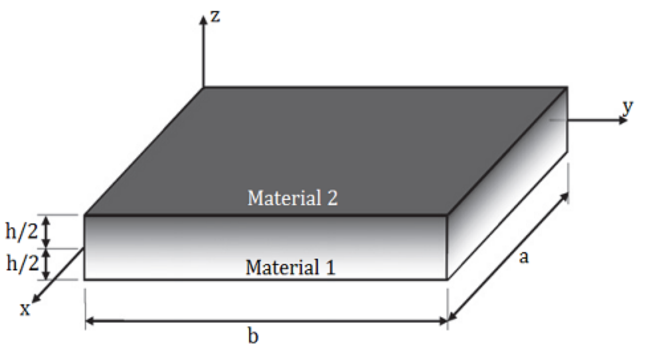

Fig. 1 The geometry of a FGM plate

Table 1 The material properties of the employed FG plate

Material	Properties		
	E (GPa)	*ν*	*ρ* (kg/m^3)
Aluminum (Al)	70	0.3	2702
Alumina (Al_2O_3)	380	0.3	3800
Zirconia (ZrO_2)	200	0.3	5700
Si_3N_4	348.43	0.3	2370
SUS304	201.04	0.3	8166

and employed in the FG nanoscale plate are reported in Table 1.

2.2 The nonlocal elasticity theory

In nonlocal theory, the field of stress at each point body is a function of the field of strain. So stress plays a considerable role in the model which is presented by the following expression (Khorshidi *et al*. 2015)

$$t_{ij} = \int_V \alpha\left(\left|X' - X\right|\right) \sigma_{ij}\left(X'\right) dV' \tag{4}$$

where *X* is a point on the body that the tensor of stress on its efficacy, *X*' can be any point else in the body, *V* is the volume of a region of the body that integral is considered on it, σ_{ij} is the tensor of classical stress, $\alpha(|X' - X|)$ is the nonlocal kernel function related to the internal characteristic length. With respect to characteristics of nonlocal kernel function $\alpha(|X' - X|)$ that are presented by Eringen (1983), taking in a Greens function of a linear differential operator, $\Im$, can be defined as following

$$\Im\alpha\left(\left|X' - X\right|\right) = \delta\alpha\left(\left|X' - X\right|\right) \tag{5}$$

Substituting Eq. (5) into Eq. (4), the primary expression (1) form of the following differential equation is determined as

$$\Im\, t_{ij} = \sigma_{ij} \tag{6}$$

For the nonlocal linear elastic solids, the equations of motion have the following form (Narendar 2011)

$$t_{ij,j} + f_i = \rho(z)\ddot{u}_i \tag{7}$$

where *ρ* is the mass density, f_i body loads and u_i is the vector of displacement. Substituting Eq. (7) into Eq. (6) yields to the following relation

$$\sigma_{ij} + \Im\left(f_i - \rho(z)\ddot{u}_i\right) = 0 \tag{8}$$

The nonlocal theory with the linear differential operator for the 3D case is presented by the following expression (Sakhaee-Pour *et al*. 2008a)

$$\Im = 1 - \mu^2 \nabla^2 \tag{9}$$

where ∇^2 is the Laplace operator, which in Cartesian coordinates is defined by $\nabla^2 = \partial^2 / x^2 + \partial^2 / y^2 + \partial^2 / z^2$ and $\mu = e_0 a$, a is the internal property length and e_0 is the material constant which is predicted by the experiment. The value of the nonlocal parameter is related to the boundary condition, the chirality, the mode shapes, the number of walls, and the nature of motions (Hosseini-Hashemi *et al*. 2013a). There is no accurate way to compute this parameter, but it is considered that the factor be obtained by conducting a comparison of dispersion curves from nonlocal elasticity and lattice dynamics of nano-material crystal structure (Hosseini-Hashemi *et al*. 2013a).

2.3 The assumptions made in the present theory

(1) The components of displacement u and v are the axial displacements of the middle plane in x and y directions respectively, and w is the vertical displacement of the middle plane in z direction. The magnitude of the vertical displacement w is not of the same order as the thickness h of the plate and is small with respect to the plate thickness.
(2) The axial displacements, u and v incorporate three parts:

- A displacement part equivalent to the displacement used in the classical plate theory (CPT).
- A displacement component owing to the shear deformation which is included via undetermined integral.
- The shear strains in z direction are zero in the bottom and top faces of the plates.

(1) The vertical displacement w in z direction is considered to be a function of y and x coordinates.
(2) The nanoplate is subjected to the vertical load only.

The displacement field of the cubic shear deformation model is expressed as below (Abualnour *et al*. 2018)

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \quad (10a)$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \quad (10b)$$

$$w(x,y,z) = w_0(x,y) + g(z)\varphi_z(x,y) \quad (10c)$$

The coefficients k_1 and k_2 depends on the geometry. In this work, the shape function is considered based on the cubic function given by

$$f(z) = \frac{5}{4}\left(z - \frac{4\,z^3}{3h^2}\right) \quad (11)$$

and u_0 (x, y), v_0 (x, y), w_0 (x, y), θ (x, y) and φ_z (x, y) are the five variables displacement functions of middle surface of the plate.

With the linear supposition of von-Karman strain, the displacement strain field will be as what follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},$$
$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}, \qquad \varepsilon_z = g'(z)\,\varepsilon_z^0 \quad (12)$$

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \qquad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x \partial y}\end{Bmatrix}, \quad (13a)$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int \theta\,dx + k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix},$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}k_2\int\theta\,dy + \dfrac{\partial \varphi_z}{\partial y}\\ k_1\int\theta\,dx + \dfrac{\partial\varphi_z}{\partial x}\end{Bmatrix}, \qquad \varepsilon_z^0 = \varphi_z \quad (13b)$$

The integrals presented in the above equations shall be resolved by a Navier type solution and can be expressed as follows

$$\frac{\partial}{\partial y}\int\theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \qquad \frac{\partial}{\partial x}\int\theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y},$$
$$\int\theta\,dx = A'\frac{\partial\theta}{\partial x}, \qquad \int\theta\,dy = B'\frac{\partial\theta}{\partial y} \quad (14)$$

where the coefficients A' and B' are expressed according to the type of solution employed, in this case by using Navier. Therefore, A' and $B'$are written as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \quad (15)$$

where α and β are defined in expression (29).

The Hamilton's principle is utilized to determine the equation of motion. The Hamilton's principle in case of local form is obtained as what follows (Al-Basyouni *et al*. 2015, Bourada *et al*. 2015, Attia *et al*. 2015, Yahia *et al*. 2015, Bellifa *et al*. 2016, Benadouda *et al*. 2017, Zidi *et al*. 2017, Klouche *et al*. 2017, Hachemi *et al*. 2017, Fourn *et al*. 2018)

$$0 = \int_0^t \delta\left(U - K\right)dt \quad (16)$$

where δ is the variation operator, U is the strain energy, and K is the kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned}\delta U &= \int_V \begin{pmatrix} \sigma_x \delta\, \varepsilon_x + \sigma_y \delta\, \varepsilon_y + \sigma_z \delta\, \varepsilon_z \\ + \tau_{xy} \delta\, \gamma_{xy} + \tau_{yz} \delta\, \gamma_{yz} + \tau_{xz} \delta\, \gamma_{xz} \end{pmatrix} dA\, dz \\ &= \int_A \Big\{ N_x \delta\, \varepsilon_x^0 + N_y \delta\, \varepsilon_y^0 + N_z \delta\, \varepsilon_z^0 + N_{xy} \delta\, \gamma_{xy}^0 \\ &\quad + M_x^b \delta\, k_x^b + M_y^b \delta\, k_y^b + M_{xy}^b \delta\, k_{xy}^b + M_x^s \delta\, k_x^s \\ &\quad + M_y^s \delta\, k_y^s + M_{xy}^s \delta\, k_{xy}^s + S_{yz}^s \delta\, \gamma_{yz}^s + S_{xz}^s \delta\, \gamma_{xz}^0 \Big\} dA \end{aligned} \tag{17}$$

where A is the top surface and the stress resultants N, M, and S are expressed by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\, \sigma_i\, dz \qquad (i = x, y, xy);$$

$$N_z = \int_{-h/2}^{h/2} g'(z)\, \sigma_z\, dz \tag{18a}$$

and

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g(z) \left(\tau_{xz}, \tau_{yz}\right) \tag{18b}$$

The variation of kinetic energy is expressed as

$$\begin{aligned}\delta K &= \int_V \left[\dot u\, \delta\, \dot u + \dot v\, \delta\, \dot v + \dot w\, \delta\, \dot w\right] \rho(z)\, dV \\ &= \int_A \Big\{ I_0 \left[\dot u_0 \delta \dot u_0 + \dot v_0 \delta \dot v_0 + \dot w_0 \delta \dot w_0\right] \\ &\quad - I_1 \left(\dot u_0 \frac{\partial \delta \dot w_0}{\partial x} + \frac{\partial \dot w_0}{\partial x} \delta\, \dot u_0 + \dot v_0 \frac{\partial \delta \dot w_0}{\partial y} + \frac{\partial \dot w_0}{\partial y} \delta\, \dot v_0 \right) \\ &\quad + J_1 \left(\left(k_1 A'\right) \left(\dot u_0 \frac{\partial \delta\, \dot\theta}{\partial x} + \frac{\partial \dot\theta}{\partial x} \delta\, \dot u_0 \right) \right. \\ &\quad \left. + \left(k_2 B'\right) \left(\dot v_0 \frac{\partial \delta \dot\theta}{\partial y} + \frac{\partial \dot\theta}{\partial y} \delta\, \dot v_0 \right) \right) \\ &\quad + I_2 \left(\frac{\partial \dot w_0}{\partial x} \frac{\partial \delta\, \dot w_0}{\partial x} + \frac{\partial \dot w_0}{\partial y} \frac{\partial \delta\, \dot w_0}{\partial y} \right) \\ &\quad + K_2 \left(\left(k_1 A'\right)^2 \left(\frac{\partial \dot\theta}{\partial x} \frac{\partial \delta\, \dot\theta}{\partial x} \right) + \left(k_2 B'\right)^2 \left(\frac{\partial \dot\theta}{\partial y} \frac{\partial \delta\, \dot\theta}{\partial y} \right) \right) \\ &\quad - J_2 \left(\left(k_1 A'\right) \left(\frac{\partial \dot w_0}{\partial x} \frac{\partial \delta\, \dot\theta}{\partial x} + \frac{\partial \dot\theta}{\partial x} \frac{\partial \delta\, \dot w_0}{\partial x} \right) \right. \\ &\quad \left. + \left(k_2 B'\right) \left(\frac{\partial \dot w_0}{\partial y} \frac{\partial \delta\, \dot\theta}{\partial y} + \frac{\partial \dot\theta}{\partial y} \frac{\partial \delta\, \dot w_0}{\partial y} \right) \right) \\ &\quad + J_0 \left(\dot\varphi_z\, \delta\, \dot w_0\right) + K_3 \left(\dot\varphi_z \delta\, \dot\varphi_z\right) \Big\} dA \end{aligned} \tag{19}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; ρ (z) is the mass density; and (I_0, J_0, I_1, I_2, J_1, J_2, K_2, K_3) are mass inertias expressed as

$$\left(I_0, J_0, I_1, I_2\right) = \int_{-h/2}^{h/2} \left(1, g(z), z, z^2\right) \rho(z)\, dz \tag{20a}$$

$$\left(J_1, J_2, K_2, K_3\right) = \int_{-h/2}^{h/2} \left(f(z), z\, f(z), f^2(z), g^2(z)\right) \rho(z)\, dz \tag{20b}$$

Substituting the expressions for δU and δK from Eqs (18) and (19) into Eq. (20) and integrating by parts and collecting the coefficients of δu_0, δv_0, δw_0, $\delta\theta$, and $\delta\varphi_z$, the following equations of motion of the plate are obtained as

$$\begin{aligned} &\delta\, u_0 : \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot u_0 - I_1 \frac{\partial \ddot w_0}{\partial x} + k_1 A' J_1 \frac{\partial \ddot\theta}{\partial x} \\ &\delta\, v_0 : \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot v_0 - I_1 \frac{\partial \ddot w_0}{\partial y} + k_2 B' J_1 \frac{\partial \ddot\theta}{\partial y} \\ &\delta\, w_0 : \ \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} \\ &\qquad = I_0 \ddot w_0 + I_1 \left(\frac{\partial \ddot u_0}{\partial x} + \frac{\partial \ddot v_0}{\partial y} \right) - I_2 \nabla^2 \ddot w_0 \\ &\qquad + J_2 \left(k_1 A' \frac{\partial^2 \ddot\theta}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot\theta}{\partial y^2} \right) + J_0\, \ddot\varphi_z \\ &\delta\, \theta : -k_1 M_x^s - k_2 M_y^s - \left(k_1 A' + k_2 B'\right) \frac{\partial^2 M_{xy}^s}{\partial x \partial y} \\ &\qquad + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} \\ &\qquad = -J_1 \left(k_1 A' \frac{\partial \ddot u_0}{\partial x} + k_2 B' \frac{\partial \ddot v_0}{\partial y} \right) \\ &\qquad - K_2 \left(\left(k_1 A'\right)^2 \frac{\partial^2 \ddot\theta}{\partial x^2} + \left(k_2 B'\right)^2 \frac{\partial^2 \ddot\theta}{\partial y^2} \right) \\ &\qquad + J_2 \left(k_1 A' \frac{\partial^2 \ddot w_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot w_0}{\partial y^2} \right) \\ &\delta\, \varphi_z : \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z = J_0\, \ddot w_0 + K_3\, \ddot\varphi_z \end{aligned} \tag{21}$$

2.4 The nonlocal elasticity model for FG nano-plate

The constitutive relations of nonlocal theory for a FG nano-plate using Eq. (6) can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} - \mu \nabla^2 \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{22}$$

where

$$C_{11}=C_{22}=C_{33}=\frac{E(z)(1-\nu)}{(1-2\nu)(1+\nu)},$$
$$C_{12}=C_{13}=C_{23}=\frac{E(z)\nu}{(1-2\nu)(1+\nu)}, \tag{23}$$
$$C_{44}=C_{55}=C_{66}=\frac{E(z)}{2(1+\nu)},$$

Integrating Eq. (20) over the plate’s cross-section area yields the force–strain and the moment–strain of the nonlocal refined FG nano-plates as follows

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \\ N_z \end{Bmatrix} - \mu\nabla^2 \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \\ N_z \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{11} & 0 & B_{11}^s & B_{12}^s & 0 & X_{13} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 & X_{23} \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 & Y_{13} \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 & Y_{23} \\ 0 & 0 & B_{66} & 0 & 0 & D_{11} & 0 & 0 & D_{66}^s & 0 \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 & Y_{13}^s \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 & Y_{23}^s \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s & 0 \\ X_{13} & X_{23} & 0 & Y_{13} & Y_{23} & 0 & Y_{13}^s & Y_{23}^s & 0 & Z_{33} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x} \\ -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial x^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \\ k_1\theta \\ k_2\theta \\ \left(k_1A' + k_2B'\right)\frac{\partial^2\theta}{\partial x\partial y} \\ \varphi_z \end{Bmatrix} \tag{24a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} - \mu\nabla^2 \begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} k_2B' \frac{\partial\theta}{\partial y} + \frac{\partial\theta_z}{\partial y} \\ k_2A' \frac{\partial\theta}{\partial x} + \frac{\partial\theta_z}{\partial x} \end{Bmatrix} \tag{24b}$$

Where the cross-sectional rigidities are defined as follows

$$\left(A_{ij}, A_{ij}^s, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(1, g^2(z), z, z^2, f(z), z\, f(z), f^2(z)\right)dz \tag{25a}$$

$$\left(X_{ij}, Y_{ij}, Y_{ij}^s, Z_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z, f(z), g'(z)\right) g'(z) C_{ij}\, dz \tag{25b}$$

The nonlocal equations of motion of FG nano-plates in terms of the displacement can be obtained by substituting Eqs. (24a) and (24b), into Eq. (21) as follows

$$\begin{aligned} & A_{22}\, d_{22}v_0 + A_{66}\, d_{11}v_0 + \left(A_{12}+A_{66}\right) d_{12}u_0 + X_{23}d_2\theta_z \\ & - B_{22}\, d_{222}w_0 - \left(B_{12}+2B_{66}\right) d_{112}w_0 \\ & + \left(B_{66}^s \left(k_1A' + k_2B'\right)\right) d_{112}\theta + \left(B_{22}^s k_2 + B_{12}^s k_1\right) d_2\theta \\ & = \left(1-\mu\nabla^2\right) \left(I_0\ddot{v}_0 - I_1\, d_2\ddot{w}_0 + J_1 B' k_2\, d_2\ddot{\theta}\right), \end{aligned} \tag{26a}$$

$$\begin{aligned} & A_{22}\, d_{22}v_0 + A_{66}\, d_{11}v_0 + \left(A_{12}+A_{66}\right) d_{12}u_0 + X_{23}d_2\theta_z \\ & - B_{22}\, d_{222}w_0 - \left(B_{12}+2B_{66}\right) d_{112}w_0 \\ & + \left(B_{66}^s \left(k_1A' + k_2B'\right)\right) d_{112}\theta + \left(B_{22}^s k_2 + B_{12}^s k_1\right) d_2\theta \\ & = \left(1-\mu\nabla^2\right) \left(I_0\ddot{v}_0 - I_1\, d_2\ddot{w}_0 + J_1 B' k_2\, d_2\ddot{\theta}\right), \end{aligned} \tag{26b}$$

$$\begin{aligned} & B_{11}\, d_{111}u_0 + (B_{12}+2B_{66}) d_{122}u_0 + \left(B_{12}+2B_{66}\right) d_{112}v_0 \\ & B_{22}d_{222}v_0 + Y_{13}d_{11}\varphi_z - D_{11}\, d_{1111}w_0 \\ & - 2\left(D_{12}+2D_{66}\right) d_{1112}w_0 - D_{22}d_{1112}w_0 \\ & + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{12}\theta + 2\left(D_{66}^s (k_1A' + k_2B')\right) d_{1122}\theta \\ & + \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22}\theta \\ & = \left(1-\mu\nabla^2\right) \begin{pmatrix} I_0\ddot{w}_0 + I_1 (d_1\ddot{u}_0 + d_2\ddot{v}_0) \\ - I_2 (d_{11}\ddot{w}_0 + d_{22}\ddot{w}_0) \\ + J_2 ((k_1A'd_{11}\ddot{\theta} + k_2B'd_{22}\ddot{\theta})) + J_0\ddot{\varphi}_2 \end{pmatrix} \end{aligned} \tag{26c}$$

$$\begin{aligned} & - (B_{11}^s k_1 + B_{12}^s k_2) d_1u_0 - \left(B_{66}^s (k_1A' + k_2B')\right) d_{122}u_0 \\ & - \left(B_{66}^s (k_1A' + k_2B')\right) d_{112}v_0 - (B_{12}^s k_1 + B_{22}^s k_2) d_2u_0 \\ & - k_1Y_{13}^s\varphi_z - k_2Y_{23}^s\varphi_z + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11}w_0 \\ & + 2\left(D_{66}^s (k_1A' + k_2B')\right) d_{1112}w_0 + \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22}w_0 \\ & + H_{11}^s k_1^2\theta - H_{22}^s k_2^2\theta - 2H_{12}^s k_1k_2\theta \\ & - \left((k_1A' + k_2B')^2 H_{66}^s\right) d_{1112}\theta \\ & + A_{44}^s (k_2B')^2 d_{22}\theta + A_{55}^s (k_1A')^2 d_{11}\theta \\ & + A_{44}^s (k_2B')^2 d_{22}\varphi_z + A_{55}^s (k_2A')^2 d_{11}\varphi_z \\ & = \left(1-\mu\nabla^2\right) \begin{pmatrix} - J_1 (k_1A'd_1\ddot{u}_0 + k_2B'd_2\ddot{v}_0) \\ + J_2 (k_1A'd_{11}\ddot{w}_0 + k_2B'd_{22}\ddot{w}_0) \\ - K_2 ((k_1A')^2 d_{11}\ddot{\theta} + (k_2B')^2 d_{22}\ddot{\theta}) \end{pmatrix} \end{aligned} \tag{26d}$$

$$\begin{aligned} & X_{13}d_1u_0 + X_{23}d_2u_0 + Z_{33}\varphi_z + Y_{13}d_{11}w_0 + Y_{23}d_{22}w_0 \\ & A_{44}^s (k_2B')d_{22}\theta + A_{55}^s (k_1A')d_{11}\theta + A_{44}^s d_{22}\varphi_z + A_{55}^s d_{11}\varphi_z \\ & = (1-\mu\nabla^2)(J_0\ddot{\varphi}_z + K_3\ddot{w}_0), \end{aligned} \tag{26e}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \qquad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},$$
$$d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1, 2). \tag{27}$$

3. Solution procedures

Here, based on the Navier type procedure, an analytical solution of the governing equations for dynamic of a simply supported FG nanoplate is presented. The displacement functions are written as product of undetermined coefficients and known trigonometric functions to respect the governing equations and the conditions at x = 0, a and y = 0, b. The following displacement fields are assumed to be of the form

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \\ \varphi_z \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\; e^{i\omega t}\cos(\alpha x)\sin(\beta y) \\ V_{mn}\; e^{i\omega t}\sin(\alpha x)\cos(\beta y) \\ W_{mn}\; e^{i\omega t}\sin(\alpha x)\sin(\beta y) \\ X_{mn}\; e^{i\omega t}\sin(\alpha x)\sin(\beta y) \\ Y_{mn}\; e^{i\omega t}\sin(\alpha x)\sin(\beta y) \end{Bmatrix} \tag{28}$$

where (U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}) are the unknown Fourier coefficients.
with

$$\alpha = m\pi / a \qquad \text{and} \qquad \beta = n\pi / b \tag{29}$$

Inserting Eq. (28) into Eqs. (26), leads to

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} \end{bmatrix} - \lambda\, \omega^2 \begin{bmatrix} M_{11} & 0 & M_{13} & M_{14} & 0 \\ 0 & M_{22} & M_{23} & M_{24} & 0 \\ M_{13} & M_{23} & M_{33} & M_{34} & M_{35} \\ M_{14} & M_{24} & M_{34} & M_{44} & 0 \\ 0 & 0 & M_{35} & 0 & M_{55} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{30}$$

$$\begin{aligned}
S_{11} &= -(A_{11}\alpha^2 + A_{66}\beta^2), \\
S_{12} &= -\alpha\beta\,(A_{12} + A_{66}), \\
S_{13} &= \alpha(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2), \\
S_{11} &= -(A_{11}\alpha^2 + A_{66}\beta^2), \\
S_{14} &= \alpha(k_1 B_{11}^s + k_2 B_{12}^s - (k_1 A^{'} + k_2 B^{'})B_{66}^s\beta^2), \\
S_{15} &= X_{13}\alpha \;\; S_{11} = -(A_{11}\alpha^2 + A_{66}\beta^2), \\
S_{22} &= -(A_{66}\alpha^2 + A_{22}\beta^2), \\
S_{23} &= \beta(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2), \\
S_{24} &= \beta(k_2 B_{22}^s + k_1 B_{12}^s - (k_1 A^{'} + k_2 B^{'})B_{66}^s\alpha^2) \\
S_{25} &= X_{23}\beta \\
S_{33} &= -(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4), \\
S_{34} &= -k_1(D_{11}^s\alpha^2 + D_{12}^s\beta^2) \\
&\quad + 2(k_1 A^{'} + k_2 B^{'})\, D_{66}^s\alpha^2\beta^2 \\
&\quad - k_2(D_{22}^s\beta^2 + D_{12}^s\alpha^2) \\
S_{35} &= -(Y_{13})\alpha^2 - (Y_{23})\beta^2 \\
S_{44} &= -k_1(H_{11}^s k_1 + H_{12}^s k_2) - (k_1 A^{'} + k_2 B^{'})^2 H_{66}^s\alpha^2\beta^2 \\
&\quad - k_2(H_{12}^s k_1 + H_{22}^s k_2) - (k_1 A^{'})^2 A_{55}^s\alpha^2 \\
&\quad - (k_2 B^{'})^2 A_{44}^s\beta^2 \\
S_{45} &= -(k_1 A^{'})A_{55}^s\alpha^2 - (k_2 B^{'})A_{44}^s\beta^2 + k_1 Y_{13}^s + k_2 Y_{23}^s
\end{aligned} \tag{31}$$

$$\begin{aligned}
&S_{55} = -(A_{55}^s)\alpha^2 - (A_{44}^s)\beta^2 - Z_{33} \\
&M_{11} = -I_0, \quad M_{13} = \alpha I_1, \quad M_{14} = -k_1 J_1 A^{'}\alpha, \\
&M_{15} = 0, \quad M_{22} = -I_0, \quad M_{23} = \beta I_1, \\
&M_{24} = -k_2 B^{'}\,\beta J_1, \quad M_{25} = 0 \\
&M_{33} = -I_0 - I_2(\alpha^2 + \beta^2), \\
&M_{34} = J_2(k_1 A^{'}\alpha^2 + k_2 B^{'}\beta^2), \quad M_{35} = -J_0, \\
&M_{44} = -K_2((k_1 A^{'})^2\alpha^2 + (k_2 B^{'})^2\beta^2), \quad M_{45} = 0 \\
&M_{55} = -K_3, \quad \lambda = (1 + \mu(\alpha^2 + \beta^2))
\end{aligned} \tag{31}$$

4. Numerical results and discussions

In this work, two separate parts are considered; in the first part, have been examined and validated isotropic rectangular nano-plate, and in the second part, it does for FG one.

4.1 Isotropic rectangular nano-plate

Only homogeneous plate (n = 0) is employed in this part for the verification.

Tables 2-4 provide the first three non-dimensional frequency and Frequency Ratios (FR) for simply supported boundary condition with different values of aspect ratio ($\eta = b/a$), specified values of non-dimensional scale parameter ($\zeta = \mu/a$) and the thickness to length ratio h/a = 0.1 on rectangular nano-plates. The natural frequency parameters written in non-dimensional form $\beta = \omega a^2\sqrt{\rho h/D}$, $D = Eh^3 / 12(1 - v^2)$ are the flexural rigidity. The nano-plate is made of the following material properties: E = 210 GPa, v = 0.3 and ρ = 7800 (kg/m^3). The computed frequencies based on the proposed nonlocal cubic shear deformation theory are compared with those given by Hosseini-Hashemi *et al.* (2013b) based on Mindlin Plate Theory (MPT) and those reported by Khorshidi *et al.* (2015) based on exponential shear deformation theory. Also, the Frequency Ratio (FR) expression between the nonlocal and local non-dimensional frequencies is given as what follows

$$FR = \frac{\beta^{NL}}{\beta^{L}} \tag{32}$$

where β^{NL} is the non-dimensional nonlocal frequency parameter, and β^{L} is the non-dimensional local frequency parameter.

It can be seen from Tables 2-4, that the obtained values for non-dimensional nonlocal frequency parameter β^{NL} are in good agreement with those provided by Khorshidi *et al.* (2015) and Hosseini-Hashemi *et al.* (2013b). The introduction of stretching thickness effect makes the nanoplate more stiffness.

4.2 FGM plate

Table 5 presents a comparison of the frequency parameters $\bar{\beta} = \omega h\sqrt{\rho_c/E_c}$ for AL/AL_2O_3 square moderately thick plates with those provided by Hosseini-

Table 2 The variations of the non-dimensional frequency ($\beta = \omega a^2\sqrt{\rho h/D}$) and the frequency ratio (FR) for the nonlocal plate (m = 1, n = 1)

	Method		$\zeta = 0$	$\zeta = 0.2$	$\zeta = 0.4$	$\zeta = 0.6$	$\zeta = 0.8$
		β^{NL}	FR	FR	FR	FR	FR
$\eta = 0.6$	Present ($\varepsilon_z \neq 0$)	35.0858	1.0000	0.6335	0.3789	0.2633	0.2005
	Present ($\varepsilon_z = 0$)	35.0045	1.0000	0.6335	0.3789	0.2633	0.2005
	Khorshidi *et al*. (2015)	35.015	1.0000	0.6335	0.3789	0.2633	0.2005
	Hosseini-Hashemi *et al*. (2013b)	35.0643	1.0000	0.6335	0.3789	0.2633	0.2005
$\eta = 0.8$	Present ($\varepsilon_z \neq 0$)	24.2431	1.0000	0.7051	0.4451	0.3146	0.2412
	Present ($\varepsilon_z = 0$)	24.2034	1.0000	0.7051	0.4451	0.3146	0.2412
	Khorshidi *et al*. (2015)	24.2084	1.0000	0.7051	0.4451	0.3146	0.2412
	Hosseini-Hashemi *et al*. (2013b)	24.2330	1.0000	0.7050	0.4451	0.3146	0.2412
$\eta = 1$	Present ($\varepsilon_z \neq 0$)	19.0902	1.0000	0.7475	0.4904	0.3512	0.2708
	Present ($\varepsilon_z = 0$)	19.0653	1.0000	0.7475	0.4904	0.3512	0.2708
	Khorshidi *et al*. (2015)	19.0684	1.0000	0.7475	0.4904	0.3512	0.2708
	Hosseini-Hashemi *et al*. (2013b)	19.0840	1.0000	0.7475	0.4904	0.3512	0.2708

Table 3 The variations of the non-dimensional frequency ($\beta = \omega a^2\sqrt{\rho h/D}$) and the frequency ratio (FR) for the nonlocal plate (m = 2, n = 1)

	Method		$\zeta = 0$	$\zeta = 0.2$	$\zeta = 0.4$	$\zeta = 0.6$	$\zeta = 0.8$
		β^{NL}	FR	FR	FR	FR	FR
$\eta = 0.6$	Present ($\varepsilon_z \neq 0$)	60.3530	1.0000	0.5216	0.2923	0.1997	0.1511
	Present ($\varepsilon_z = 0$)	60.1243	1.0000	0.5216	0.2923	0.1997	0.1511
	Khorshidi *et al*. (2015)	60.1556	1.0000	0.5216	0.2923	0.1997	0.1511
	Hosseini-Hashemi *et al*. (2013b)	60.2869	1.0000	0.5216	0.2923	0.1997	0.1511
$\eta = 0.8$	Present ($\varepsilon_z \neq 0$)	50.3554	1.0000	0.5594	0.3197	0.2194	0.1663
	Present ($\varepsilon_z = 0$)	50.1930	1.0000	0.5594	0.3197	0.2194	0.1663
	Khorshidi *et al*. (2015)	50.2147	1.0000	0.5594	0.3197	0.2194	0.1663
	Hosseini-Hashemi *et al*. (2013b)	50.3100	1.0000	0.5594	0.3197	0.2194	0.1664
$\eta = 1$	Present ($\varepsilon_z \neq 0$)	45.6216	1.0000	0.5799	0.3353	0.2308	0.1752
	Present ($\varepsilon_z = 0$)	45.4869	1.0000	0.5799	0.3353	0.2308	0.1752
	Khorshidi *et al*. (2015)	45.5048	1.0000	0.5799	0.3353	0.2308	0.1752
	Hosseini-Hashemi *et al*. (2013b)	45.5845	1.0000	0.5799	0.3353	0.2308	0.1752

Table 4 The variations of the non-dimensional frequency ($\beta = \omega a^2\sqrt{\rho h/D}$) and the frequency ratio (FR) for the nonlocal plate (m = 2, n = 2)

	Method		$\zeta = 0$	$\zeta = 0.2$	$\zeta = 0.4$	$\zeta = 0.6$	$\zeta = 0.8$
		β^{NL}	FR	FR	FR	FR	FR
$\eta = 0.6$	Present ($\varepsilon_z \neq 0$)	122.0595	1.0000	0.3789	0.2005	0.1352	0.1018
	Present ($\varepsilon_z = 0$)	121.2246	1.0000	0.3789	0.2005	0.1352	0.1018
	Khorshidi *et al*. (2015)	121.356	1.0000	0.3789	0.2005	0.1352	0.1018
	Hosseini-Hashemi *et al*. (2013b)	121.7770	1.0000	0.3789	0.2006	0.1352	0.1018
$\eta = 0.8$	Present ($\varepsilon_z \neq 0$)	87.3788	1.0000	0.4451	0.2412	0.1635	0.1233
	Present ($\varepsilon_z = 0$)	86.9235	1.0000	0.4451	0.2412	0.1635	0.1233
	Khorshidi *et al*. (2015)	86.9898	1.0000	0.4451	0.2412	0.1635	0.1233
	Hosseini-Hashemi *et al*. (2013b)	87.2357	1.0000	0.4451	0.2412	0.1635	0.1233

Table 4 Continued

	Method		$\zeta = 0$	$\zeta = 0.2$	$\zeta = 0.4$	$\zeta = 0.6$	$\zeta = 0.8$
		β^{NL}	FR	FR	FR	FR	FR
$\eta = 1$	Present ($\varepsilon_z \neq 0$)	70.1122	1.0000	0.4904	0.2708	0.1843	0.1393
	Present ($\varepsilon_z = 0$)	69.8093	1.0000	0.4904	0.2708	0.1843	0.1393
	Khorshidi *et al*. (2015)	69.8517	1.0000	0.4904	0.2708	0.1843	0.1393
	Hosseini-Hashemi *et al*. (2013b)	70.0219	1.0000	0.4904	0.2708	0.1844	0.1393

Table 5 The comparison of the natural frequency parameter ($\bar{\beta} = \omega h\sqrt{\rho_c h/E_c}$) for AL/AL_2O_3 square plates ($\eta = 1$)

h/a	(m,n)	Method	n = 0	0.5	1	4	10
0.05	(1, 1)	Present ($\varepsilon_z \neq 0$)	0.0148	0.0126	0.0115	0.0100	0.0095
		Present ($\varepsilon_z = 0$)	0.0148	0.0125	0.0113	0.0098	0.0094
		Khorshidi *et al*. (2015)	0.0148	0.0125	0.0113	0.0098	0.0094
		Hosseini-Hashemi *et al*. (2010)	0.0148	0.0128	0.0115	0.0101	0.0096
		Zhao *et al*. (2009)	0.0146	0.0124	0.0112	0.0097	0.0093
0.1	(1, 1)	Present ($\varepsilon_z \neq 0$)	0.0578	0.0494	0.0449	0.0389	0.0368
		Present ($\varepsilon_z = 0$)	0.0577	0.0490	0.0442	0.0381	0.0364
		Khorshidi *et al*. (2015)	0.0577	0.0490	0.0442	0.0381	0.0364
		Matsunaga (2008)	0.0577	0.0492	0.0443	0.0381	0.0364
		Hosseini-Hashemi *et al*. (2010)	0.0577	0.0492	0.0445	0.0383	0.0363
		Zhao *et al*. (2009)	0.0568	0.0482	0.0435	0.0376	0.3592
	(1, 2)	Present ($\varepsilon_z \neq 0$)	0.1381	0.1184	0.1077	0.0923	0.0868
		Present ($\varepsilon_z = 0$)	0.1376	0.1174	0.1059	0.0903	0.0856
		Khorshidi *et al*. (2015)	0.1377	0.1174	0.1059	0.0902	0.0856
		Matsunaga (2008)	0.1381	0.1180	0.1063	0.0904	0.0859
		Zhao *et al*. (2009)	0.1354	0.1154	0.1042	-	0.085
	(2, 2)	Present ($\varepsilon_z \neq 0$)	0.2122	0.1825	0.1660	0.1409	0.1318
		Present ($\varepsilon_z = 0$)	0.2113	0.1807	0.1631	0.1378	0.1301
		Khorshidi *et al*. (2015)	0.2114	0.1808	0.1632	0.1377	0.1300
		Matsunaga (2008)	0.2121	0.1819	0.1640	0.1383	0.1306
		Zhao *et al*. (2009)	0.2063	0.1764	0.1594	-	0.1289
0.2	(1, 1)	Present ($\varepsilon_z \neq 0$)	0.2122	0.1825	0.1660	0.1409	0.1318
		Present ($\varepsilon_z = 0$)	0.2113	0.1807	0.1631	0.1378	0.1301
		Khorshidi *et al*. (2015)	0.2114	0.1808	0.1632	0.1377	0.1300
		Matsunaga (2008)	0.2121	0.1819	0.1640	0.1383	0.1306
		Hosseini-Hashemi *et al*. (2010)	0.2112	0.1806	0.1650	0.1371	0.1304
		Zhao *et al*. (2009)	0.2055	0.1757	0.1587	0.1356	0.1284
	(1, 2)	Present ($\varepsilon_z \neq 0$)	0.4660	0.4042	0.3677	0.3047	0.2812
		Present ($\varepsilon_z = 0$)	0.4623	0.3987	0.3607	0.2980	0.2771
		Khorshidi *et al*. (2015)	0.4629	0.3993	0.3611	0.2976	0.2772
		Matsunaga (2008)	0.4658	0.4040	0.3644	0.3000	0.2790
	(2, 2)	Present ($\varepsilon_z \neq 0$)	0.6760	0.5893	0.5365	0.4381	0.4009
		Present ($\varepsilon_z = 0$)	0.6691	0.5807	0.5254	0.4284	0.3948
		Khorshidi *et al*. (2015)	0.6691	0.5807	0.5254	0.4280	0.3947
		Matsunaga (2008)	0.6753	0.5891	0.5444	0.4362	0.3981

Table 6 The comparison of the fundamental frequency parameter ($\bar{\beta} = \omega h\sqrt{\rho_c h/E_c}$) for AL/$ZrO_2$ square plates (η = 1)

Method	$n=0$		$n=1$			$\delta=0.2$		
	$\delta=\frac{1}{\sqrt{10}}$	$\delta=0.1$	$\delta=0.05$	$\delta=0.1$	$\delta=0.2$	$n=2$	$n=3$	$n=5$
Present $\varepsilon_z \neq 0$	0.5424	0.0672	0.0160	0.0624	0.2300	0.2285	0.2290	0.2295
Present $\varepsilon_z = 0$	0.5380	0.0671	0.0158	0.0619	0.2277	0.2257	0.2263	0.2272
Khorshidi *et al*. (2015)	0.4629	0.0577	0.0158	0.0619	0.2278	0.2288	0.2301	0.2327
Matsunaga (2008)	0.4658	0.0577	0.0158	0.0619	0.2285	0.2264	0.2270	0.2281
Vel and Batra (2004)	0.4658	0.0577	0.0153	0.0596	0.2192	0.2197	0.2211	0.2225
HSDT[(a)]	0.4658	0.0578	0.0157	0.0613	0.2257	0.2237	0.2243	0.2253
FSDT[(a)]	0.4619	0.0577	0.0162	0.0633	0.2333	0.2325	0.2334	0.2334
Hosseini-Hashemi *et al*. (2010)	0.4618	0.0576	0.0158	0.0611	0.2270	0.2249	0.2254	0.2265

(a) Pradyumna and Bandyopadhyay (2008)

Table 7 The frequency parameter ($\beta = \omega a^2\sqrt{\rho_c h/E_c}$) for AL/$ZrO_2$ plates (δ = 0.2, n = 1)

$\frac{a}{b}$	2	1.5	1	2/3	0.5
Present $\varepsilon_z \neq 0$	3.2091	3.6702	4.9411	7.5878	10.9096
Present $\varepsilon_z = 0$	3.1796	3.6354	4.8909	7.5005	10.7682
Khorshidi *et al*. (2015)	3.1198	3.3720	4.9325	6.9551	9.9853

Hashemi *et al*. (2010), Zhao *et al*. (2009), Khorshidi *et al*. (2015) and Matsunaga (2008) when n = 0, 0.5, 1, 4 and 10. In addition, the corresponding mode shapes m and n, representing the number of half-waves in the x and y directions, respectively, are given for any of the frequency parameters $\bar{\beta}$.

In Table 6, a comparison of the results ($\bar{\beta} = \omega h\sqrt{\rho_m/E_m}$) for AL/$ZrO_2$ square plates with those of 2D HSDT (Matsunaga 2008), 3D theory by using the power series procedure (Vel and Batra 2004), finite element HSDT method (Pradyumna and Bandyopadhyay 2008), finite element FSDT method (Hosseini-Hashemi *et al*. 2008), an analytical FSDT solution (Hosseini-Hashemi *et al*. 2010) and HSDT solution Khorshidi *et al*. (2015) is demonstrated. From Tables 5 and 6, it can be confirmed that there is a very good agreement among the results confirming the high accuracy of the proposed analytical formulation. The effect of the geometric ratio $\eta = b/a$ on the frequency parameters $\beta = \omega a^2\sqrt{\rho_c h/E_c}$ of a rectangular Al/ZrO_2 plate ($\delta = h/a$ = 0.2, n = 1) is shown in Table 7.

From Table 7, it can be deduced that with a reduction in the aspect ratio, the frequency parameter increases due to the increase of the stiffness of the plate. It can be also observed that the stretching effect increases the frequency parameter.

In Table 8, the influences of different parameters on the non-dimensional frequencies of the rectangular FG nanoplate are presented. From these results, it is found that by increasing the scale parameter, the rate of variation of non-dimensional frequencies diminishes, because by

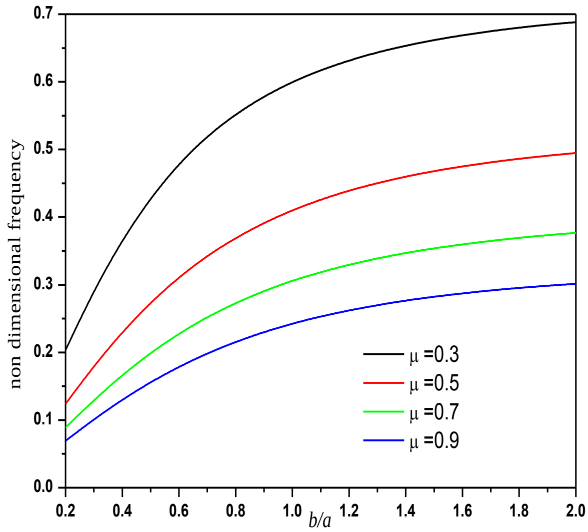

Fig. 2 The influences of the aspect ratio and the scale parameter on the non-dimensional frequency

increasing the scale parameter, the strain energy diminishes, and it causes a reduction of the rigidity of the plates.

In Fig. 2, the influences of the aspect ratio and the scale parameter on the non-dimensional frequency of the rectangular nanoscale plates are illustrated. It is demonstrated that with an increase in the ratio b/a, the non-dimensional frequency increases. It is observed that for the lower ratios of b/a, the effect of the scale parameters diminishes.

In Fig. 3, the influences of the scale parameter on the frequency ratio of the nano-plates are demonstrated for different modes of vibration. From these results, it seems that the frequency ratios for the lower modes are more than those for the higher modes.

Fig. 4 demonstrates the influence of the gradient index on the dimensionless two first frequencies of FG nano-plate (SUS304/ Si_3N_4) with a/h = 10 for different values of the small scale parameter. It can be observed that the dimensionless frequency diminishes as the gradient index increases. This is due to the fact that an increase in the

Table 8 The effect of the non-dimensional nonlocal parameter ζ and the gradient index n on the non dimensional frequencies $\bar{\beta} = \omega h\sqrt{\rho_c h/E_c}$ of the rectangular FG nanoplate (AL/AL_2O_3)

ζ	$\frac{a}{b}$	$\frac{h}{a}$	Method	Gradient index		
				0	5	10
0.0	0.5	0.2	Present $\varepsilon_z \neq 0$	0.1381	0.0909	0.0868
			Present $\varepsilon_z = 0$	0.1376	0.0891	0.0856
			Khorshidi *et al.* (2015)	0.2114	0.1357	0.0856
		0.1	Present $\varepsilon_z \neq 0$	0.0365	0.0244	0.0234
			Present $\varepsilon_z = 0$	0.0365	0.0239	0.0231
			Khorshidi *et al.* (2015)	0.0365	0.0239	0.0231
	1.0	0.2	Present $\varepsilon_z \neq 0$	0.2122	0.1386	0.1318
			Present $\varepsilon_z = 0$	0.2113	0.1358	0.1301
			Khorshidi *et al.* (2015)	0.2310	0.1356	0.1300
		0.1	Present $\varepsilon_z \neq 0$	0.0578	0.0384	0.0368
			Present $\varepsilon_z = 0$	0.0577	0.0377	0.0364
			Khorshidi *et al.* (2015)	0.0577	0.0377	0.0364
0.1	0.5	0.2	Present $\varepsilon_z \neq 0$	0.1306	0.0858	0.0819
			Present $\varepsilon_z = 0$	0.1299	0.0841	0.0808
			Khorshidi *et al.* (2015)	0.1299	0.1239	0.0808
		0.1	Present $\varepsilon_z \neq 0$	0.0345	0.0230	0.0221
			Present $\varepsilon_z = 0$	0.0345	0.0226	0.0218
			Khorshidi *et al.* (2015)	0.0345	0.0226	0.0218
	1.0	0.2	Present $\varepsilon_z \neq 0$	0.1939	0.1266	0.1205
			Present $\varepsilon_z = 0$	0.1931	0.1241	0.1189
			Khorshidi *et al.* (2015)	0.1932	0.1239	0.1188
		0.1	Present $\varepsilon_z \neq 0$	0.0528	0.0351	0.0337
			Present $\varepsilon_z = 0$	0.0527	0.0344	0.0332
			Khorshidi *et al.* (2015)	0.0527	0.0344	0.0332
0.2	0.5	0.2	Present $\varepsilon_z \neq 0$	0.1130	0.0744	0.0710
			Present $\varepsilon_z = 0$	0.1126	0.0730	0.0701
			Khorshidi *et al.* (2015)	0.1127	0.0728	0.0700
		0.1	Present $\varepsilon_z \neq 0$	0.0299	0.0199	0.0191
			Present $\varepsilon_z = 0$	0.0299	0.0196	0.0189
			Khorshidi *et al.* (2015)	0.0299	0.0196	0.0189
	1.0	0.2	Present $\varepsilon_z \neq 0$	0.1586	0.1036	0.0985
			Present $\varepsilon_z = 0$	0.1579	0.1015	0.0972
			Khorshidi *et al.* (2015)	0.1580	0.1014	0.0972
		0.1	Present $\varepsilon_z \neq 0$	0.0432	0.0287	0.0275
			Present $\varepsilon_z = 0$	0.0431	0.0282	0.0272
			Khorshidi *et al.* (2015)	0.0431	0.0282	0.0272
0.3	0.5	0.2	Present $\varepsilon_z \neq 0$	0.0950	0.0626	0.0597
			Present $\varepsilon_z = 0$	0.0948	0.0613	0.0589
			Khorshidi *et al.* (2015)	0.0948	0.0613	0.0589
		0.1	Present $\varepsilon_z \neq 0$	0.0252	0.0168	0.0161
			Present $\varepsilon_z = 0$	0.0251	0.0165	0.0159
			Khorshidi *et al.* (2015)	0.0251	0.0165	0.0159

Table 8 Continued

ζ	$\frac{a}{b}$	$\frac{h}{a}$	Method	Gradient index		
				0	5	10
			Present $\varepsilon_z \neq 0$	0.1273	0.0831	0.0791
		0.2	Present $\varepsilon_z = 0$	0.1268	0.0815	0.0781
			Khorshidi *et al.* (2015)	0.1269	0.0814	0.0780
0.3	1.0		Present $\varepsilon_z \neq 0$	0.0347	0.0231	0.0221
		0.1	Present $\varepsilon_z = 0$	0.0346	0.0226	0.0218
			Khorshidi *et al.* (2015)	0.0346	0.0226	0.0218
			Present $\varepsilon_z \neq 0$	0.0801	0.0527	0.0503
		0.2	Present $\varepsilon_z = 0$	0.0798	0.0517	0.0497
			Khorshidi *et al.* (2015)	0.0798	0.0516	0.0496
	0.5		Present $\varepsilon_z \neq 0$	0.0212	0.0142	0.0136
		0.1	Present $\varepsilon_z = 0$	0.0212	0.0139	0.0134
			Khorshidi *et al.* (2015)	0.0212	0.0139	0.0134
0.4			Present $\varepsilon_z \neq 0$	0.1040	0.0679	0.0646
		0.2	Present $\varepsilon_z = 0$	0.1036	0.0666	0.0638
			Khorshidi *et al.* (2015)	0.1037	0.0665	0.0638
	1.0		Present $\varepsilon_z \neq 0$	0.0283	0.0189	0.0181
		0.1	Present $\varepsilon_z = 0$	0.0283	0.0185	0.0178
			Khorshidi *et al.* (2015)	0.0283	0.0185	0.0178

gradient index leads to a decrease in the stiffness of the FG nano-plate.

5. Conclusions

The size-dependent dynamic properties of FG nano-plate are analytically studied by using a simple cubic refined plate model based on the nonlocal differential constitutive relations of Eringen. The kinematic of the present theory is modified by considering undetermined integral terms in in-plane displacements which results in a reduced number of variables compared with other HSDT of the same order. Comparing the obtained results with those found in the literature for FG nano-plates demonstrates a high stability and accuracy of the present solution. What presented herein

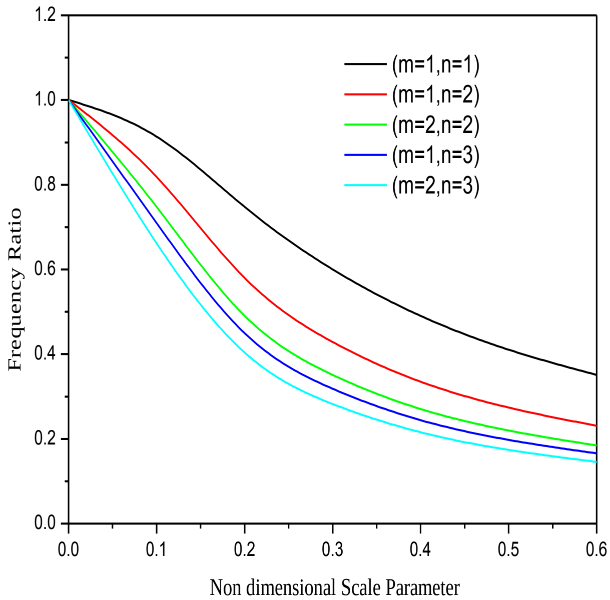

Fig. 3 The effects of the aspect ratio and the nonlocal parameter on the non-dimensional frequency

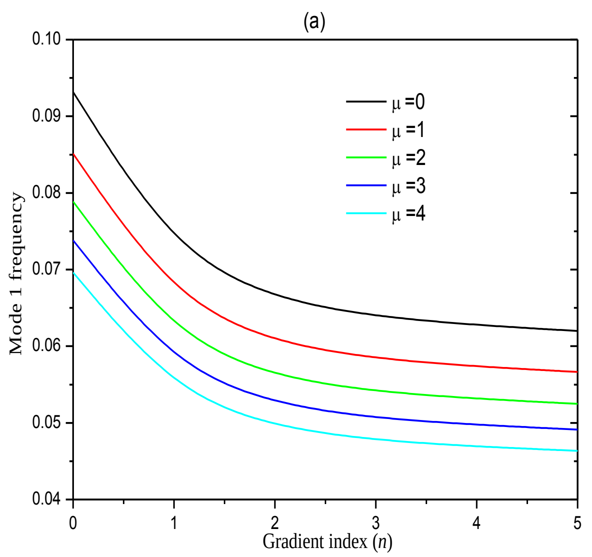

Fig. 4 Influence of the gradient index (n) and the scale parameter (μ) on dimensionless frequency for a simply supported square FG plate with $a / h = 10$: (a) first frequency; (b) second frequency

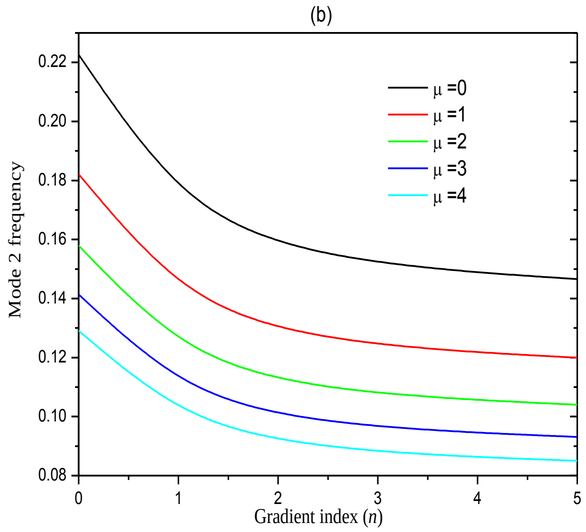

Fig. 4 Continued

demonstrates the influences of the variations of the scale parameter, the ratio of the thickness to the length, the gradient indexes and the aspect ratio on the frequency values of a FG nano-plate. It is demonstrated that the frequency ratio diminishes with increasing the mode number and the value of the scale parameter, and also increasing the gradient index causes the non-dimensional frequencies to decrease.

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