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A new nonlocal HSDT for analysis of stability of single layer graphene sheet

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Abstract. A new nonlocal higher order shear deformation theory (HSDT) is developed for buckling properties of single graphene sheet. The proposed nonlocal HSDT contains a new displacement field which incorporates undetermined integral terms and contains only two variables. The length scale parameter is considered in the present formulation by employing the nonlocal differential constitutive relations of Eringen. Closed-form solutions for critical buckling forces of the graphene sheets are obtained. Nonlocal elasticity theories are used to bring out the small scale influence on the critical buckling force of graphene sheets. Influences of length scale parameter, length, thickness of the graphene sheets and shear deformation on the critical buckling force have been examined.

Keywords: buckling; graphene sheets; nonlocal elasticity; HSDT

1. Introduction

Nanotechnology is an emerging technology considering the characterization, design, production and application of materials, structures and systems within the control of matter on the nanometer length scale, that is, at the level of atoms and molecules. Mechanical analysis of nanostructures and nanocomposite has been reported by many researchers (Belkorissat *et al*. 2015, Kolahchi and Moniri Bidgoli 2016, Kolahchi *et al*. 2016a, b, 2017a, b, c, Madani *et al*. 2016, Ahouel *et al*. 2016, Bilouei *et al*. 2016, Arani and Kolahchi 2016, Besseghier *et al*. 2017, Bouafia *et al*. 2017, Bellifa *et al*. 2017a, Zamanian *et al*. 2017, Kolahchi and Cheraghbak 2017, Hajmohammad *et al*. 2017, Kolahchi 2017, Zarei *et al*. 2017, Shokravi 2017a, b, c, d, Karami *et al*. 2018a). In recent years, the single layered graphene sheets (SLGSs) was successfully prepared and it has triggered a novel wave of carbon materials investigation (Novoselov *et al*. 2004).

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Experimental simulations on nanostructures are difficult, so most researchers employed theoretical formulation or numerical methods to investigate the nanomaterials (Eringen 1983). The classical continuum elasticity model was often employed in a very long time, which is a scale-independent model and cannot capture the size influence of nanomaterials. But the small scale influence is very important in nanoscale structures, so several size-dependent continuum models were developed. However, the most employed continuum theory for investigating small scale structures is the nonlocal elasticity model proposed by Eringen (1983). In nonlocal elasticity model, the small scale influence is considered by supposing that the stress components at a point x is dependent not only on the strain components at the same point x but also on all other points in the domain. The nonlocal elasticity model has been widely employed in small scale materials, a number of investigations have been carried out on bending, vibration and buckling analysis of micro/nanostructures (Reddy and Pang 2008, Hashemi and Samaei 2011, Wang 2009, Mustapha and Zhong 2010, Eltaher *et al*. 2012, Berrabah *et al*. 2013, Benguediab *et al*. 2014, Besseghier *et al*. 2015, Zemri *et al*. 2015, Ebrahimi and Salari 2015, Chemi *et al*. 2015, Janghorban and Zare 2011, Behravan Rad 2015, Larbi Chaht *et al*. 2015, Ghadiri and Jafari 2016, Ghorbanpour Arani *et al*. 2016, Jandaghian and Rahmani 2016, Akbaş 2016, Bennoun *et al*. 2016, Eltaher *et al*. 2016, Mehar *et al*. 2016, Bounouara *et al*. 2016, Janghorban 2016, Rakrak *et al*. 2016, Khetir *et al*. 2017, Mouffoki *et al*. 2017, Shahsavari *et al*. 2018, Karami *et al*. 2018b, c, Youcef *et al*. 2018).

The Eringen's nonlocal elasticity model is widely employed when investigating the nonlocal effect of SLGSs. Murmu and Pradhan (2009a) studied the dynamic response of SLGSs embedded in elastic medium based on nonlocal elasticity model. Babaei and Shahidi (2010) utilized the Galerkin procedure to analyse the nonlocal effect on the stability of quadrilateral nanoplates based on nonlocal elasticity theory. Murmu and Pradhan (2009b) discussed the stability of bi-axially compressed orthotropic nanoplates. Pradhan (2009a) examined the stability of SLGSs based on nonlocal elasticity and HSDT. Samaei *et al*. (2011) studied the buckling analysis of SLGSs embedded in elastic medium based on nonlocal Mindlin plate theory. Ghorbanpour Arani *et al*. (2012) presented a buckling analysis and smart control of SLGS using elastically coupled PVDF nanoplate based on the nonlocal Mindlin plate theory. Samaei *et al*. (2015) investigated the vibration of a graphene sheet embedded in an elastic medium with consideration of small scale effect. Alipour Ghassabi *et al*. (2017) presented for the first time a free vibration analysis of functionally graded rectangular nanoplates considering spatial variation of the nonlocal parameter. Mokhtar *et al*. (2018) presented a novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory. Sobhy (2014) presented a generalized two-variable plate theory for multi-layered graphene sheets with arbitrary boundary conditions. Yazid *et al*. (2018) proposed a novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium. It is noted that in recent years various plate/shell theories (Rastgaar *et al*. 2006, Matsunaga 2008, Pradyumna and Bandyopadhyay 2008, Reddy 2011, Xiang *et al*. 2011, Shahrjerdi *et al*. 2011, Ait Amar Meziane *et al*. 2014, Yaghoobi *et al*. 2014, Ahmed 2014, Belabed *et al*. 2014, Hebali *et al*. 2014, Swaminathan and Naveenkumar 2014, Ait Yahia *et al*. 2015, Al-Basyouni *et al*. 2015, Hamidi *et al*. 2015, Hadji *et al*. 2015, Bellifa *et al*. 2016, Houari *et al*. 2016, Draiche *et al*. 2016, Bouderba *et al*. 2016, Akavci 2016, Bousahla *et al*. 2016, Aldousari 2017, Rahmani *et al*. 2017, Kar *et al*. 2018, Bouhadra *et al*. 2018, Attia *et al*. 2018) are developed and the need of proposing a new nonlocal HSDT is desired. Moreover, in many of the above mentioned HSDTs as in the CPT or the simple FSDT proposed by Thai and Choi (2013), the expression $\partial w/\partial x$ or $\partial \theta/\partial x$ are present in the displacement field. Consequently, the numerical computation is harder to handle. Normally C^{-1}-

FEM is required. However, this can be changed if the displacement field is composed with undetermined integral terms as in this paper.

In this work, the buckling of a single-layer graphene sheet in the presence of length scale is investigated based on a new nonlocal HSDT. Higher order plate model presents the kinematics better, does not use shear correction coefficient and yield more accurate interlaminar stress variations (Pradhan 2009b). In principle, it is possible to expand the displacement field in terms of the thickness coordinate up to any desired degree. However, because of the algebraic complexity and computational effort involved with higher order models in return for gain in accuracy, theories with higher number of variables have not been attempted. The consideration of the integral term in the displacement field led to a reducing in the number of unknowns and governing equations, thus saving computational time. Nonlocal elasticity theory by Eringen (1983) has been incorporated in the investigation. The effect of nonlocal parameter on the critical buckling load of the graphene sheets with different parameters such as length and thickness of the graphene sheets are examined in detail.

2. Mathematical formulation

The coordinate system employed for the graphene sheet is indicated in Fig. 1.

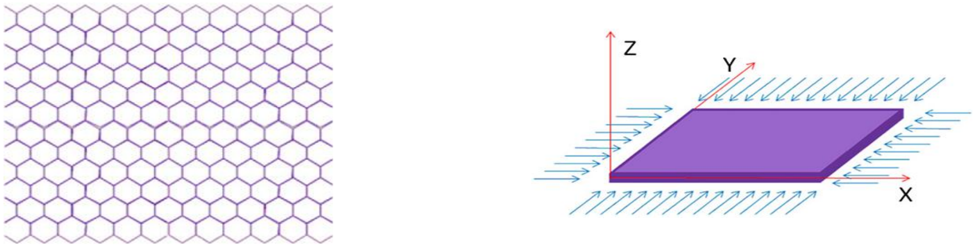
Origin is selected at one corner of the middle surface of the graphene sheet. The x, y coordinates of the axes are considered along the length and width of the graphene sheet. $z$coordinate is considered along the thickness of the graphene sheet. The kinematic of the proposed theory is expressed as follows (Bellifa *et al*. 2017b, Chikh *et al*. 2017, El-Haina *et al*. 2017, Menasria *et al*. 2017, Fourn *et al*. 2018, Younsi *et al*. 2018)

$$u(x,y,z) = -z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{1a}$$

$$v(x,y,z) = -z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \tag{1b}$$

$$w(x,y,z) = w_0(x,y) \tag{1c}$$

where $P^{(n)}$ is the effective material characteristic of FGM of layer n. P_m and P_c present the



(a) (b)

Fig. 1 Model of single layered graphene sheet: (a) discrete model; (b) continuum model property of the bottom and top faces of layer 1 ($h_0 \le z \le h_1$), respectively, and vice versa for layer 3 ($h_2 \le z \le h_3$) depending on the volume fraction $V^{(n)}$ (n = 1, 2, 3). Note that P_m and P_c are, respectively, the corresponding properties of the metal and ceramic of the FGM sandwich plate. The volume fraction $V^{(n)}$ of the FGMs is assumed to obey a power-law function along the thickness direction (Taibi *et al*. 2015)

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{2}$$

In this work, the shape function in Eq. (2) is expressed by a cubic function and assures an accurate distribution of shear deformation through the nanoplate thickness and allows to transverse shear stresses vary as parabolic across the thickness as satisfying shear stress free surface conditions without using shear correction factors. Indeed, it should be mentioned that contrary to the first shear deformation theory (FSDT), the proposed theory does not require shear correction factors. The kinematic relations can be expressed as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \qquad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{3}$$

where

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y} \int \theta \, dx + k_2 \dfrac{\partial}{\partial x} \int \theta \, dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1 \int \theta \, dy \\ k_2 \int \theta \, dx \end{Bmatrix} \tag{4a}$$

and

$$g(z) = \frac{df(z)}{dz} \tag{4b}$$

The integrals defined in the above equations shall be resolved by a Navier type technique and can be expressed as follows

$$\frac{\partial}{\partial y} \int \theta \, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x} \int \theta \, dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \int \theta \, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta \, dy = B' \frac{\partial \theta}{\partial y} \tag{5}$$

where the coefficients A' and B' are given according to the type of solution employed, in this case using Navier method. Therefore, A' and B' are written as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{6}$$

where α and β are defined in expression (18).

2.1 Governing equations

Principle of virtual work can be applied to determine the governing equations of the graphene sheet. Using the principle of virtual displacements (Bousahla *et al*. 2014, Bourada *et al*. 2015, Mahi *et al*. 2015, Beldjelili *et al*. 2016, Abdelaziz *et al*. 2017, Belabed *et al*. 2018, Kaci *et al*. 2018), the following governing equations can be obtained

$$\frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + \overline{N} = 0 \tag{7a}$$

$$\begin{aligned} &-k_1 M_x^s - k_2 M_y^s - \left(k_1 A' + k_2 B'\right)\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \\ &k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = 0 \end{aligned} \tag{7b}$$

where the stress resultants M and S are defined by

$$\left(M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (z, f)\sigma_i dz, \quad (i = x, y, xy) \text{ and } \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{8}$$

With

$$\overline{N} = \left[N_x^0 \frac{\partial^2 w}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w}{\partial x \partial y} + N_y^0 \frac{\partial^2 w}{\partial y^2} \right] \tag{9}$$

2.2 Nonlocal theory and constitutive relations

Unlike the local theory, the nonlocal theory of elasticity assumes that the stress at a point is related not only to the strain at that point but also to strains at all other points of the body. Based on work proposed by Eringen (1983), the nonlocal stress tensor σ at point x is expressed by

$$\sigma - \mu \nabla^2 \sigma = \tau \tag{10}$$

where τ is local stress tensor at a point x defined versus the strain by the Hooke’s law; μ is the scale parameter which introduces the small scale effect.

2.3 Stress resultants

For a graphene sheet, the nonlocal constitutive relation in Eq. (10) takes the following forms

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} - \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{11}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively.

By substituting Eq. (3) into Eq. (11) and the subsequent results into Eq. (8), the stress resultants are obtained as

$$C_{11} = C_{22} = \frac{E}{1-\nu^2}, \qquad C_{12} = \frac{\nu E}{1-\nu^2}, \qquad C_{44} = C_{55} = C_{66} = \frac{E}{2(1+\nu)}, \tag{12}$$

By substituting Eq. (3) into Eq. (11) and the subsequent results into Eq. (8), the stress resultants are obtained as

$$\begin{Bmatrix} M^b \\ M^s \end{Bmatrix} - \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix} M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} D & D^s \\ D^s & H^s \end{bmatrix}\begin{Bmatrix} k^b \\ k^s \end{Bmatrix}, \qquad S^s - \mu\left(\frac{\partial^2 S^s}{\partial x^2} + \frac{\partial^2 S^s}{\partial y^2}\right) = A^s\gamma, \tag{13}$$

where

$$M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \qquad M^s = \{M_x^s, M_y^s, M_{xy}^s\}^t \tag{14a}$$

$$k^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \qquad k^s = \{k_x^s, k_y^s, k_{xy}^s\}^t \tag{14b}$$

$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \qquad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \qquad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{14c}$$

$$S^s = \{S_{xz}^s, S_{yz}^s\}^t, \qquad \gamma = \{\gamma_{xz}, \gamma_{yz}\}^t, \qquad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{14d}$$

where D_{ij} , D_{ij}^s etc., are the plate stiffness, defined by

$$\left(D_{ij}, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(z^2, z\, f(z), f^2(z)\right)dz, \quad (i, j = 1,2,6) \tag{15a}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} C_{ij}\left[g(z)\right]^2 dz, \quad (i, j = 4,5) \tag{15b}$$

2.4 Governing equations in terms of displacements

The nonlocal governing equations of the proposed plate theory can be expressed in terms of displacements (w_0, θ) by substituting stress resultants in Eq. (13) into Eq. (7) as

$$
\begin{aligned}
&-D_{11}d_{1111}w_0-2(D_{12}+2D_{66})d_{1122}w_0-D_{22}d_{2222}w_0\\
&+\left(k_1\,D_{11}^s+k_2\,D_{12}^s\right)d_{11}\theta+2\left(k_1A'+k_2B'\right)D_{66}^s d_{1122}\theta\\
&+\left(k_2\,D_{22}^s+k_1\,D_{12}^s\right)d_{22}\theta+\overline{N}-\mu\nabla^2\overline{N}=0
\end{aligned}
\tag{16a}
$$

$$
\begin{aligned}
&\left(k_1\,D_{11}^s+k_2\,D_{12}^s\right)d_{11}w_0+2\left(k_1A'+k_2B'\right)D_{66}^s d_{1122}w_0\\
&+\left(k_2\,D_{22}^s+k_1\,D_{12}^s\right)d_{22}w_0+\left(k_1^2\,A'^{\,2}A_{55}^s+k_1\,A'A_{55}^s\right)d_{11}\theta\\
&+\left(k_2^2\,B'^{\,2}A_{44}^s+k_2\,\,B'A_{55}^s\right)d_{11}\theta-\left(k_1A'+k_2B'\right)^2H_{66}^s\,d_{1122}\theta\\
&-\left(k_1\left(k_1\,H_{11}^s+k_2\,H_{12}^s\right)+k_2\left(k_1\,H_{12}^s+k_2\,H_{22}^s\right)\right)\theta=0
\end{aligned}
\tag{16b}
$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$
d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\quad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},\quad d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},\quad d_i=\frac{\partial}{\partial x_i},\qquad (i,j,l,m=1,2).
\tag{17}
$$

3. Analytical solutions

A simply supported rectangular nanoplate with length a and width b is considered here. Based on Navier method, the following expansions of generalized displacements are chosen to automatically satisfy the simply supported boundary conditions

$$
\begin{Bmatrix} w_0\\ \theta \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} W_{mn}\sin(\alpha\,x)\sin(\beta\,y)\\ \mathrm{X}_{mn}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix}
\tag{18}
$$

where W_{mn} and X_{mn} are arbitrary coefficients to be determined,

and

$$
\alpha=m\pi/a\qquad\text{and}\qquad\beta=n\pi/b
\tag{19}
$$

By substituting Eq. (17) into Eq. (16) we obtain some results that concern the buckling of nanoplate subjected to a system of uniform in-plane compressive loads N_x^0 and N_y^0 (N_{xy}^0 = 0).

Assuming that there is a given ratio between these forces such that $N_x^0=-N_0$ and $N_y^0=-\gamma N_0;\ \gamma=N_y^0/N_x^0$ (here γ is non-dimensional load parameter), we get

$$
([K])\{\Delta\}=\{0\}
\tag{20}
$$

where $\{\Delta\}$ denotes the column

$$\{\Delta\}^T = \{W_{mn}, X_{mn}\}, \tag{21}$$

with

$$\begin{bmatrix} K_{11}+\overline{N}_{cr} & K_{12} \\ K_{12} & K_{22} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{22}$$

in witch

$$\begin{gathered} K_{11} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right), \\ K_{12} = -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1A' + k_2B')\,D_{66}^s\alpha^2\beta^2 - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right), \\ K_{22} = -H_{11}^s k_1^2 - 2H_{12}^s k_1k_2 - H_{22}^s k_2^2 - (k_1A' + k_2B')^2 H_{66}^s\alpha^2\beta^2 - \left(k_1A'\right)^2 A_{55}^s\alpha^2 - \left(k_2B'\right)^2 A_{44}^s\beta^2, \\ \overline{N}_{cr} = N_{cr}\left(\alpha^2 + \gamma\beta^2\right) \end{gathered} \tag{23}$$

The critical buckling loads (N_{cr}) can be obtained from the stability problem $|K| = 0$.

4. Results and discussion

First, it should be noted that the present theory is already applied to other type of structures such as laminated composite plates (Merdaci *et al*. 2016) and functionally graded plates (Hebali *et al*. 2016) where the accuracy and usability of the proposed plate theory is clearly demonstrated. Thus, in this work, this theory is extended to study the buckling response of nanoplate.

It can be observed from Eq. (21) that the percentage difference in buckling forces computed via local and nonlocal elasticity theory will depend on (i) size of the graphene sheet, (ii) mode of buckling and (iii) scale parameter. In the present study the graphene sheet is assumed to be a square graphene sheet. Buckling load ratio is defined as the ratio of the buckling load determined by employing nonlocal elasticity to that computed using local elasticity theory (μ = 0). For various scale parameters and lengths of the graphene sheet the buckling load ratios are presented in Fig. 2.

The obtained results are compared with those given using the nonlocal HSDT developed by Pradhan (2009a, b). It can be seen from the examination of this figure that the present HSDT

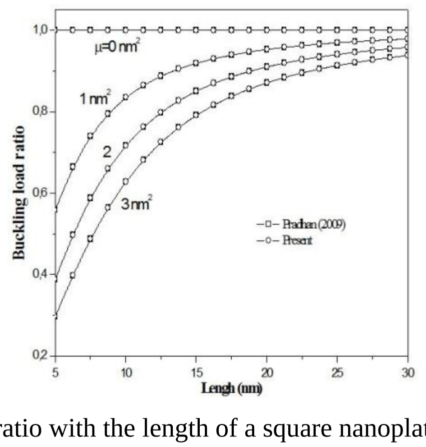
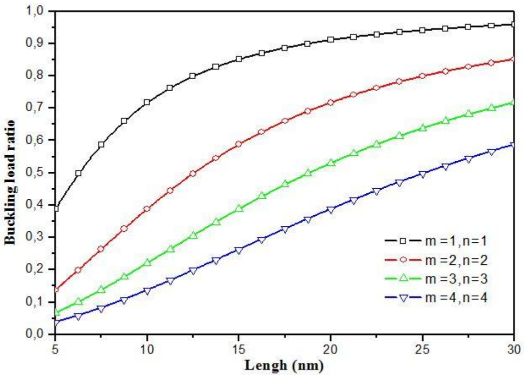


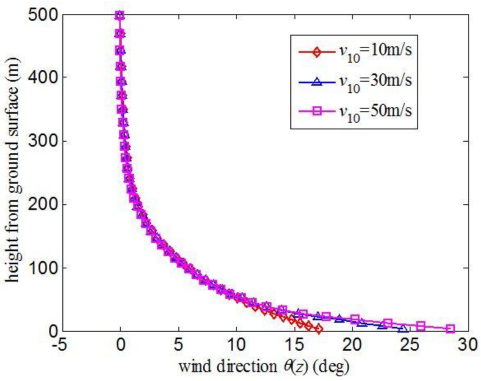
Fig. 2 Variation of buckling load ratio with the length of a square nanoplate for various nonlocal parameters

with only two unknowns give the same results to those predicted by HSDT developed by Pradhan (2009a, b) involving three unknowns. Further, from this example it can be seen that lower buckling load ratio is determined at higher values of scale parameter. In other words, classical elasticity model (μ = 0) over predicts buckling loads. So if classical elasticity results are employed for experimental prediction of Young's modulus, the value will be under determined. Further it can be seen that as length increases, buckling load ratio increases. This remark is attributed to the fact that scale effect is more important in the case of small nanolengths.

Fig. 3 presents buckling load ratio for different lengths of the graphene sheet and different modes of buckling. The value of scale parameter (μ) is considered to be 2 nm^2. It can be observed that the buckling load ratios diminish with increasing the buckling modes. This reveals that scale parameter is more important in higher buckling modes. Values of elastic modulus E = 1.02 TPa, thickness of graphene sheet h = 0.34 nm and Poisson's ratio ν = 0.3 have been employed in the above investigation. However, the increase of the length of the graphene sheet leads to a reduction of the buckling load ratio.



(a)



(b)

Fig. 3 Variation of buckling load ratio with length of a square graphene sheet for various modes of buckling for (a) $m = n$; and (b) $m \neq n$

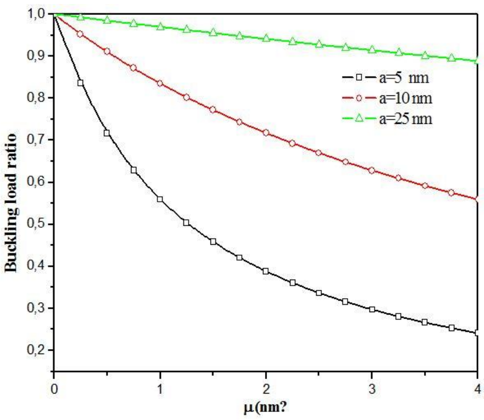


Fig. 4 Variation of buckling load ratio with scale parameter for various length of a square graphene sheet

5. Conclusions

Equations of stability of a refined two variable plate theory are obtained based on Eringen's differential constitutive equations of nonlocal elasticity. By considering further simplifying suppositions to the existing HSDTs and with the incorporation of an undetermined integral term, the number of variables and governing equations of the developed HSDT are diminished by one, and hence, make this model simple and efficient to utilize. The governing equations are then analytically solved to determine analytical solution for buckling loads of all edges simply supported graphene sheets. Influences of scale parameter and length of the graphene sheet on buckling load ratio based on the nonlocal elasticity model are examined. Buckling load ratio diminishes with increasing buckling mode number. As the size of the graphene sheet diminishes the influence of nonlocal elasticity becomes more important and predicts smaller buckling load ratio. This is more considerable for higher modes. The practical utilities of this theory are: (1) there is no need to use a shear correction factor; (2) the finite element model based on this model will be free from shear locking since the classical plate theory comes out as a special case of the proposed theory; and (3) the theory is simple and time efficient due to involving only four unknowns as opposed to five numbers in the case of FSDT or other HSDTs. In conclusion, it can be said that the proposed model is accurate and efficient in predicting the buckling response of the graphene sheet.

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