Advances in Nano Research, *Vol. 4, No. 1 (2016) 31-44*
DOI: http://dx.doi.org/10.12989/anr.2016.4.1.031

Free vibration analysis of chiral double-walled carbon nanotube using non-local elasticity theory

Kaddour Rakrak[1,4], Mohamed Zidour[*2,3], Houari Heireche[1], Abdelmoumen Anis Bousahla[1] and Awda Chemi[1]

[1]*Laboratoire de Modélisation et Simulation Multi-échelle, Université de Sidi Bel Abbés, Algerie*
[2]*Laboratoire des Matériaux et Hydrologie, Université de Sidi Bel Abbés, BP 89 Cité Ben M'hidi, 22000 Sidi Bel Abbés, Algerie*
[3]*Université Ibn Khaldoun, BP 78 Zaaroura, 14000 Tiaret, Algerie*
[4]*Département de Physique, Faculté des Sciences, Université Hassiba benbouali, Chlef, Algerie*

(Received December 24, 2015, Revised February 25, 2016, Accepted March 2, 2016)

Abstract. This article is concerned with the free vibration problem for chiral double-walled carbon nanotube (DWCNTs) modelled using the non-local elasticity theory and Euler Bernoulli beam model. According to the governing equations of non-local Euler Bernoulli beam theory and the boundary conditions, the analytical solution is derived and two branches of transverse wave propagating are obtained. The numerical results obtained provide better representations of the vibration behaviour of double-walled carbon nanotube, where the aspect ratio of the (DWCNTs), the vibrational mode number, the small-scale coefficient and chirality of double-walled carbon nanotube on the frequency ratio (χN) of the (DWCNTs) are significant. In this work, the numerical results obtained can be used to predict and prevent the phenomenon of resonance for the forced vibration analyses of double -walled carbon nanotubes.

Keywords: free vibration; double-walled carbon nanotube; frequency ratio; chirality; non-local

1. Introduction

Since the multi-walled carbon nanotube (MWCNT) and single-walled carbon nanotube (SWCNT) are discovered by Iijima (1991), Iijima *et al.* (1993). Carbon nanotubes are cylindrical macromolecules composed of carbon atoms which have received tremendous attention from various branches of science. Varieties of experimental, theoretical, and computer simulation approaches indicate that carbon nanotubes (CNTs) possess mechanical (Wong *et al*. 1997, Zidour *et al*. 2015) and physical properties leading to many potential applications (Qian *et al*. 2002, Baghdadi *et al*. 2015). In particular, CNTs hold substantial promise as superfibres for composite materials (Lau *et al*. 2002). Others studies have showed that they have good properties so they can be used for nanoelectronics, nanodevices, nanocomposites and nanostructures (Gafour *et al*. 2013, Besseghier *et al*. 2015, Tagrara *et al*. 2015). Bounouara *et al*. (2016) have studied *A* nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates

∗Corresponding author, Ph.D., E-mail: zidour.mohamed@yahoo.fr

 ISSN: 2287-237X (Print), 2287-2388 (Online)

resting on elastic foundation. Ould Youcef *et al*. (2015) studied the bending and stability of nanowire using various HSDTs.

Since CNTs are extremely small, the experiments methods to predict the responses of nanostructures under different loading conditions are quite difficult. Therefore, computational simulations have been regarded as a powerful tool for the study of the properties of CNTs. There are two major categories for simulating the mechanical properties of CNTs: molecular dynamics (MD) simulation and continuum mechanics.

The molecular dynamics (MD) simulations are used. This approach represents the dynamics of atoms or molecules of the materials by a discrete solution of Newton's classical equations of motion. But the computational problem here is that the time steps involved in the (MD) simulations are limited by the vibration modes of the atoms to be of the order of femto-seconds (10-15 s) (Ranjbartoreh *et al*. 2007). Cornwell and Wille (1997) used the (MD) to obtain the Young's modulus of (SWCNTs) about 0.8 TPa. Jin and Yuan (2003) used (MD) and force-constant approach and reported the Young's modulus of (SWCNTs) to be about 1236±7 GPa. In this study the Young's modulus of (SWCNTs) using (MD) simulation obtained by Bao Wen Xing *et al.* (2004) is used in the formulations, when the Young's moduli of (SWCNTs) are in the range of 929.87±11.5 GPa. These results are in good agreement with the existing experimental results.

The continuum mechanics methods have been effectively used to study mechanical behaviors of not only single-walled carbon nanotubes (SWCNTs) but also MWCNTs (Tounsi *et al*. 2013a, Bouazza *et al*. 2014a). Recently, the continuum mechanics approach has been widely and successfully used to study the responses of micro and nanostructures, such as the static (Bourada *et al*. 2015, Hebali *et al*. 2014, Belabed *et al*. 2014), the buckling (Ait Amar Meziane *et al*. 2014, Berrabah *et al*. 2013, Amara *et al*. 2010, Chemi *et al.* 2015), free vibration (Bousahla *et al*. 2014, Maachou *et al*. 2011, Benzair *et al*. 2008), wave propagation (Ait Yahia *et al*. 2014, Naceri *et al*. 2011, Zidour *et al*. 2012) and thermo-mechanical analysis of (CNTs) (Bouderba *et al*. 2013, Tounsi *et al*. 2013b). More recently, utilize a continuum shell model to predict the mechanical behavior of single and multi-walled carbon nanotubes embedded in a polymer or metal matrix and their results are compared with molecular dynamics simulations. Yoon *et al*. (2003) have studied the vibration of multi-walled carbon nanotubes embedded in an elastic medium by using Euler beam theory. Fu *et al*. (2006) have studied the nonlinear vibration analysis of embedded carbon nanotubes.

The vast majority of structural theories are derived using the nonlocal elasticity theory, based on the hypothesis that the stress at a point is a function of strains at all points in the continuum. Eringen (1972), Boumia *et al*. (2014), Zhang *et al*. (2005), Zidour *et al*. (2014), Benguediab *et al*. (2014), Lu *et al.* (2006), Heireche *et al*. (2008) have used the nonlocal elasticity constitutive equations to study vibration of CNTs. There are some studies for vibration of (CNTs), which assumes (CNT) as a cylindrical shell. The sound wave propagation in (CNTs) by means of a simplified shell is investigated by Natsuki *et al*. (2005). Murmu and Adhikari (2010) have analyzed the longitudinal vibration of double nanorod systems using the non-local elasticity. The nonlocal elasticity theory is applied in various types of nanostructures (nano FGM structures, nanotube.) such as the static (Zemri *et al*. 2015, Aissani *et al* 2015), the buckling (Larbi Chaht *et al*. 2015), free vibration (Belkorissat *et al*. 2015).

The present study is concerned with the use of the non-local Euler Bernoulli elastic beam model to analyse the effects of chirality of double-walled carbon nanotubes (DWCNTs) on the wave propagation. The characteristic of transverse wave propagating in (CNTs) is investigated and the effects of chirality, the vibrational mode number and aspect ratio of the (DWCNTs) are studied and discussed.

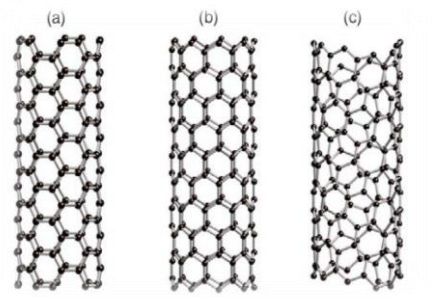

Fig. 1 single-walled carbon nanotube, (a) armchair, (b) zigzag, (c) chiral

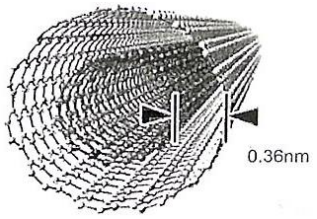

Fig. 2 double-walled carbon nanotube

2. Single and double-walled carbon nanotube

A single-walled carbon nanotube (SWCNT) is theoretically assumed to be made by rolling a graphene sheet. Therefore, the double-walled carbon nanotubes (DWCNTs) are considered as two nanotube shells coupled through the van der Waals interaction between them (Fig. 2). The fundamental structure of carbon nanotubes can be classified into three categories as zigzag, armchair and chiral shown in (Fig. 1).

The diameter of (SWCNTs) and (DWCNTs) can be expressed in terms of integers (*n*, *m*) (Tokio 1995)

$$d_{in} = a\sqrt{3(n^2 + m^2 + nm)}/\pi, \tag{1}$$

$$d_{out} = d_1 + 2h \tag{2}$$

where the integer pair (*n*, *m*) are the indices of translation and *h*, *a* are layer distance and length of the carbon-carbon bond which is (1.42*A*°).

3. Nonlocal theory of double- walled carbon nanotube (DWCNTs)

Based on Eringen nonlocal elasticity model (Eringen 1983) the stress at a reference point is considered to be a functional of the strain field at every point in the body. In the limit when the effects of strains at points other than *x* are neglected, one obtains classical or local theory of elasticity.

The equations of motion for transversely vibrating Euler beam can be obtained as (Doyle 1997)

$$\frac{\partial T}{\partial x} + p(x) = \rho A \frac{\partial^2 w}{\partial t^2} \tag{3}$$

where $p(x)$ is the distributed transverse force along axis x, w is the transverse deflection, ρ is the density, A is the area of the cross section of the nanotube, and T is the resultant shear force on the cross section, which

The weakness of this work is that the effects of the transverse shear deformations and stretching thickness are neglected. Recently, the Euler Bernoulli beam model has been widely and successfully used to study the vibration analysis of functionally graded plates using a simple shear deformation theory, such as the sandwich plates (Bennoun *et al.* 2016, Hamidi *et al.* 2015) and the concept of the neutral surface position (Bellifa *et al.* 2016, Al-Basyouni *et al.* 2015).

The one-dimensional nonlocal constitutive relation for the nanotube can be approximated to

$$\sigma_x - e0a^2 \frac{\partial^2 \sigma_x}{\partial x^2} = E_{SWNT}\left(-z\frac{\partial^2 w}{\partial x^2}\right) \tag{4}$$

where E_{SWNT} is the Young's modulus of single-walled nanotubes, w is the transverse displacement. Thus, the scale coefficient ($e0a$) in the modelling will lead to small-scale effect on the response of structures at nano size. In addition, $e0$ is a constant appropriate to each material, and a is an internal characteristic length of the material (e.g., length of C-C bond, lattice spacing, granular distance).

Tu and Ou-Yang **(**2002) indicated that the relation between Young's modulus of multi-walled carbon nanotubes (MWCNTs) and the layer number N' can be expressed as

$$E_{MWNT} = \frac{N'}{N' - 1 + t/h}\frac{t}{h}E_{SWNT} \tag{18}$$

where E_{MWNT}, E_{SWNT}, t, N' and h are Young's modulus of multi-walled nanotubes, Young's modulus of single-walled nanotubes, effective wall thickness of single-walled nanotubes, number of layers and layer distance. In the case of single-walled carbon nanotubes, N'=1 and E_{MWNT}= E_{SWNT}.

The Young's moduli used in this study of three types of double-walled carbon nanotubes (DWCNTs), armchair, zigzag and chiral tubules, are calculated by Xing *et al*. (2004) based on molecular dynamics (MD) simulation. They numeric results are in good agreement with the existing experimental ones (Liu *et al*. 2001, Tombler *et al*. 2000).

The resultant bending moment M and shear force can be defined by

$$M = \int_A z\sigma_x dA \quad , \quad T = \frac{dM}{dx} \tag{5}$$

According to Eq. (5) and Eq. (4) thus can be expressed as

$$\left[1 - (e_O a)^2 \frac{\partial^2}{\partial x^2}\right] M = -EI\frac{\partial^2 w}{\partial x^2} \tag{6}$$

where ($I = \int_A z^2 dA$) is the moment of inertia.

By substituting Eqs. (3) and (5) into Eq. (6), the bending moment *M* and the shear force *T* for the non-local model can be expressed as

$$M = -EI\frac{\partial^2 w}{\partial x^2} + (e_O a)^2\left[\rho A\frac{\partial^2 w}{\partial t^2} - p(x)\right] \tag{7}$$

$$V = -EI\frac{\partial^3 w}{\partial x^3} + (e_O a)^2\left[\rho A\frac{\partial^3 w}{\partial x \partial t^2} - \frac{\partial p(x)}{\partial x}\right] \tag{8}$$

Substituting Eq. (8) into Eq. (3) the following differential equation of a non-local Euler Bernoulli beam theory.

$$EI\frac{\partial^4 w}{\partial x^4} + \left(1 - e_O a^2\frac{\partial^2}{\partial x^2}\right)\left(\rho A\frac{\partial^2 w}{\partial t^2} - p(x)\right) = 0 \tag{9}$$

The double-walled carbon nanotubes are distinguished from traditional elastic beam by their hollow two-layer structures and associated intertube van der Waals forces. Assuming that the inner and outer tubes have the same thickness and effective material constants, the Eq. (9) can be used to each of the inner and outer tubes of the double-walled carbon nanotubes.

$$\begin{cases} EI_1\dfrac{\partial^4 w_1}{\partial x^4} + \left(1 - e_O a^2\dfrac{\partial^2}{\partial x^2}\right)\left(\rho A_1\dfrac{\partial^2 w_1}{\partial t^2}_1 - p_{12}\right) = 0 \\ EI_2\dfrac{\partial^4 w_2}{\partial x^4} + \left(1 - e_O a^2\dfrac{\partial^2}{\partial x^2}\right)\left(\rho A_2\dfrac{\partial^2 w_2}{\partial t^2} - p_{21}\right) = 0 \end{cases} \tag{10}$$

where subscripts 1 and 2 are used to denote the inner and outer tubes, respectively, p_{12}, p_{21} denotes the van der Waals pressure per unit axial length exerted on the inner tube by the outer tube and the outer tube by the inner tube respectively. The van der Waals pressure should be a linear function of the difference of the deflections of two adjacent layers at the point as follows

$$\begin{cases} p_{12} = c(w_2 - w_1) \\ p_{21} = -p_{12} = -c(w_2 - w_1) \end{cases} \tag{11}$$

where *c* is the intertube interaction coefficient per unit length between two tubes, which can be estimated by Gafour *et al*. (2013)

$$c = \frac{320(d_{in})erg/cm^2}{0.16a^2} \tag{12}$$

where d_{in} is the radius of the inner tube.

Let us consider a double-walled carbon nanotube of length *L* in which the two ends are simply supported, so vibrational modes of the (DWCNT) are of the form (Heireche *et al*. 2008)

$$w_1 = \overline{W}_1 e^{i\omega t}\sin\lambda x,\ w_2 = \overline{W}_2 e^{i\omega t}\sin\lambda x \text{ et } \lambda = \frac{k\pi}{L},\ (k = 1,2,.....) \tag{13}$$

where $\overline{W_1}$ and $\overline{W_2}$ are the amplitudes of deflections of the inner and outer tubes.

Substituting Eqs. (11)-(13) into Eq. (10), one can easily obtain the homogeneous system

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \overline{W_1} \\ \overline{W_2} \end{Bmatrix} = 0 \tag{16}$$

Where

$$K_{11} = EI_1\lambda^4 + \left(1 + e_O a^2\lambda^2\right)\left(-\rho A_1\omega^2 + c\right)$$

$$K_{12} = K_{21} = -c\left(1 + e_O a^2\lambda^2\right)$$

$$K_{22} = EI_2\lambda^4 + \left(1 + e_O a^2\lambda^2\right)\left(-\rho A_2\omega^2 + c\right)$$

Solving Eq. (16) the lower and higher natural frequency, of the DWCNT in which the effects of different parameters are shown.

$$\omega^2 = \frac{1}{2}\left(\alpha \pm \sqrt{\alpha^2 - 4\beta}\right) \tag{17}$$

Where α and β in equations yields are defined as

$$\alpha = \frac{c\left(A_1 + A_2\right)}{\rho A_1 A_2} + \frac{E\lambda^4\left(A_1 I_2 + A_2 I_1\right)}{\rho A_1 A_2\left(1 + \left(e_0 a\right)^2\lambda^2\right)}$$

$$\beta = c\lambda^4 \frac{\left(EI_1 + EI_2\right)}{\rho^2 A_1 A_2\left(1 + \left(e_0 a\right)^2\lambda^2\right)} + \lambda^8 \frac{E^2 I_1 I_2}{\rho^2 A_1 A_2\left(1 + \left(e_0 a\right)^2\lambda^2\right)^2}$$

4. Results and discussions

Based on the formulations obtained above with the nonlocal Euler-Bernoulli beam theory, the effect of aspect ratio of (DWCNTs), vibrational mode number, small-scale coefficient and chirality of double-walled carbon nanotube on vibration properties of double-walled nanotubes are discussed here. The parameters used in calculations for the zigzag DWCNTs are given as follows: the effective thickness of (CNTs) taken to be 0.258 nm, the mass density ρ=2.3 g/cm^3, layer distance h=0.34 nm and poisson ratio υ=0.19. (Naceri *et al*. 2011, Boumia *et al*. 2014).

The Young's modulus of (SWCNTs) and (DWCNTs) employed in this study (Table 1), are calculated by Xing *et al*. (2004), Tu and Ou-Yang (2002) respectively. The results show the decreasing of Young's modulus (DWCNTs) for some chirality nanotube. The reason for this phenomenon is attributed to the weak van der Waals forces between the inner and outer tube.

To investigate the effect of scale parameter on vibrations of double-walled nanotubes, the results including and excluding the nonlocal parameter are compared. It follows that the ratios of the results are respectively given by

$$\chi = \frac{\omega_{NE}}{\omega_{LE}} \tag{18}$$

Table 1 Lists the values of Young's modulus of single and double carbon nanotube for different chirality's

(*n*, *m*)	Young's modulus (SWNT) (GPa) Xing *et al*. (2004)	Young's modulus (DWNT) (GPa) Tu and Ou-Yang (2002)
Armchair		
(8,8) (12,12)	934.960	806.755
(10,10) (15,15)	935.470	807.195
(12,12) (17,17)	935.462	807.188
(14,14) (19,19)	935.454	807.181
(16,16) (21,21)	939.515	810.685
(18,18) (23,23)	934.727	806.554
(20,20) (25,25)	935.048	806.831
Zigzag		
(14,0) (23,0)	939.032	810.268
(17,0) (26,0)	938.553	809.855
(21,0) (30,0)	936.936	808.460
(24,0) (33,0)	934.201	806.100
(28,0) (37,0)	932.626	804.741
(31,0) (40,0)	932.598	804.717
(35,0) (44,0)	933.061	805.116
Chiral		
(12,6) (18,10)	927.671	800.465
(14,6) (20,10)	921.616	795.241
(16,8) (22,12)	928.013	800.760
(18,9) (24,13)	927.113	799.984
(20,12) (26,16)	904.353	780.345
(24,11) (30,15)	910.605	785.739
(30,8) (36,12)	908.792	784.175

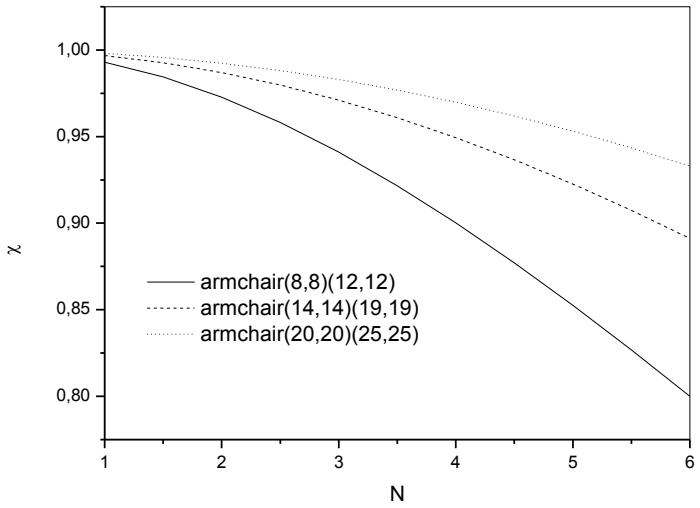

Fig. 3 Relationship between the lower frequency ratio of DWCNT, chirality of armchair carbon nanotube and the mode number; The values of (*L*/*d*) is 30 and (*e*0*a*=2 nm)

Where (ω_{LE}, ω_{NE}) are the frequency based on the local and nonlocal Euler Bernoulli beam respectively.

In the present study, the (Figs. 3-5) illustrate the dependence of the lower frequency ratio on the chirality of three double-walled carbon nanotubes (DWCNTs), armchair, zigzag and chiral for different values of mode numbers. The ratio of the length to the diameter, *L/d*, is 30 and small scale coefficient *e*0*a*=2 nm. The frequency ratio (χ) serves as an index to assess quantitatively the scale effect on CNT vibration solution. This means that the application of the local Euler Bernoulli beam model for CNT analysis would lead to an overprediction of the frequency if the scale effect between the individual carbon atoms in CNTs is neglected.

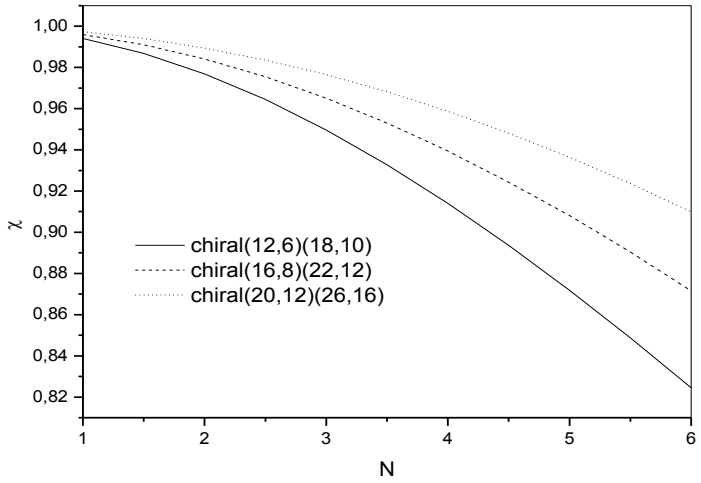

Fig. 4 Relationship between the lower frequency ratio of DWCNT, chirality of chiral carbon nanotube and the mode number; The values of (*L/d*) is 30 and (*e*0*a*=2 nm)

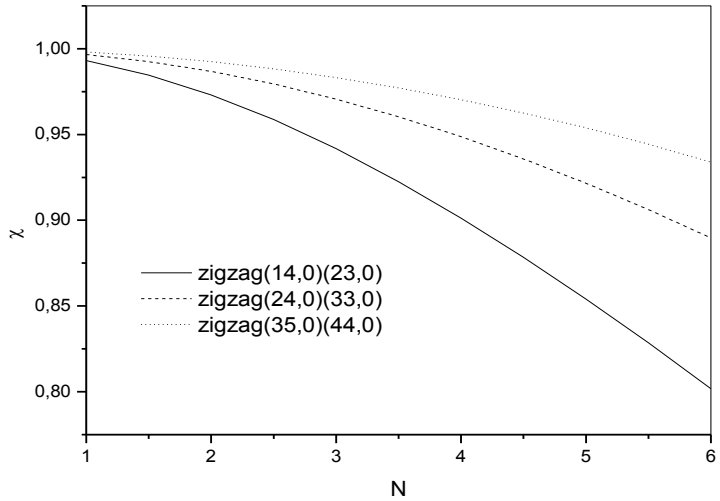

Fig. 5 Relationship between the lower frequency ratio of DWCNT, chirality of zigzag carbon nanotube and the mode number; The values of (*L/d*) is 30 and (*e*0*a*=2 nm)

It is clearly seen from (Fig. 3) that the ranges of frequency ratio for these chirality of double-walled carbon nanotubes (DWCNTs) are quite different, the range is the away then unity for armchair (8,8) (12,12), but the range is the near for armchair (20,20) (25,25). Same variation of frequency ratio is clearly seen from (Figs. 4 and 5) for chiral and zigzag nanotubes. The scale effect diminishes with increasing the index of translation (*n*, *m*) and becomes more significant with the increase of the vibrational mode *N*.

The reason for this difference perhaps is attributed to the increasing or decreasing of carbon nanotube diameter.

The effect of mode number (*N*) on the frequency ratios (χ) for various chirality of double-carbon nanotube is demonstrated in (Fig. 6) with aspect ratio (*L/d*=30) and small-scale coefficient (*e*0*a*=2 nm). In this figure, it is observed that as the mode number increase, the scale effect on the frequency ratios (χN) increase and decrease with increasing the index of translation (*n*, *m*). It is clearly that the ranges of the frequency ratios (χ) are quite different, the range is the smallest for armchair (20,20) (25,25), but the range is the largest for zigzag (14,0) (23,0). The reason for this difference perhaps is attributed to the increasing or decreasing of carbon nanotube diameter.

The scale effect is becomes obvious for the higher vibration mode number. This significance in higher modes is attributed to the diameter and the influence of small wavelength. For smaller wavelengths, the interactions between atoms are increasing and these loads to an increase in the nonlocal effects.

The (Fig. 7) show the effect of aspect ratio (*L/d*) on the frequency ratios (χ) for various chirality of double-carbon nanotube (DWCNTs), armchair, zigzag and chiral with small-scale coefficient (*e*0*a*=2 nm) and *N*=2. In these figure, we present the effect of long and diameter of double-walled carbon nanotubes (DWCNTs) on the frequency ratios (χ). The scale effect increase as one transit from the armchair (20,20) to the chiral (16,8) and then, zigzag (14,0) chirality, when the diameter of nanotube is decreasing. This increases in the scale effect is most pronounced when the nanotube is short. However, it is observed, that the frequency ratios (χ) is more affected by the long of

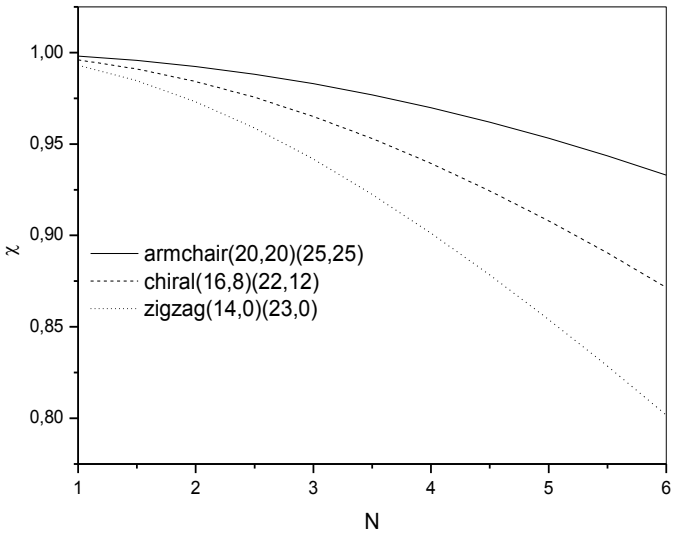

Fig. 6 Relationship between the lower frequency ratio of DWCNT and the mode number for differences chirality of carbon nanotube; The values of (*L/d*) is 30 and (*e*0*a*=2 nm)

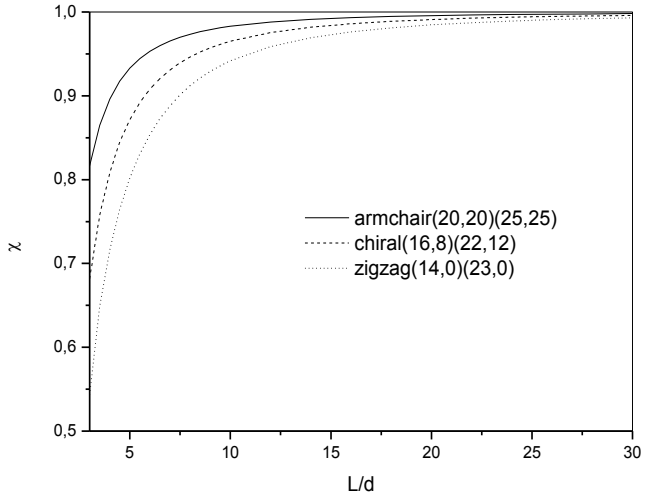

Fig. 7 Effect of aspect ratio and chirality of double carbon nanotube on the lower frequency ratio of DWCNT with ($e0a$=2 nm, N=2).

Table 2 Lists the values of the lower frequency ratio for different armchair chirality's, mode number (N) and aspect ratios (L/d) of carbon nanotube, when the value of scale coefficients ($e0a$) is 2 nm

Armchair	L/d=0		L/d=15	
	N=1	N=2	N=1	N=2
(8,8) (12,12)	0.94101	0.80002	0.97275	0.90018
(10,10) (15,15)	0.95472	0.83938	0.97932	0.92228
(12,12) (17,17)	0.96421	0.86877	0.98378	0.93796
(14,14) (19,19)	0.97104	0.89113	0.98695	0.94943
(16,16) (21,21)	0.97610	0.90840	0.98928	0.95804
(18,18) (23,23)	0.97996	0.92205	0.99104	0.96466
(20,20) (25,25)	0.98296	0.93292	0.99239	0.96985

Table 3 Lists the values of the lower frequency ratio for different Chiral chirality's, mode number (N) and aspect ratios (L/d) of carbon nanotube, when the value of scale coefficients ($e0a$) is 2 nm

Chiral	L/d=10		L/d=15	
	N=1	N=2	N=1	N=2
(12,6) (18,10)	0.94964	0.82448	0.97690	0.91403
(14,6) (20,10)	0.95618	0.84391	0.98000	0.92469
(16,8) (22,12)	0.96508	0.87161	0.98419	0.93942
(18,9) (24,13)	0.97029	0.88869	0.98661	0.94818
(20,12) (26,16)	0.97648	0.90990	0.98945	0.95870
(24,11) (30,15)	0.97979	0.92157	0.99096	0.96439
(30,8) (36,12)	0.98301	0.93321	0.99242	0.96995

nanotube. The reason for this phenomenon is that a carbon nanotube with larger long or smaller diameter has a larger curvature, which results in a more significant distortion of (C-C) bonds.

Table 4 Lists the values of the lower frequency ratio for different Zigzag chirality's, mode number (*N*) and aspect ratios (*L/d*) of carbon nanotube, when the value of scale coefficients (*e*0*a*) is 2 nm

Zigzag	*L/d*=10		*L/d*=15	
	N=1	*N*=2	*N*=1	*N*=2
(14,0) (23,0)	0.94169	0.80184	0.97308	0.90126
(17,0) (26,0)	0.95365	0.83617	0.97882	0.92054
(21,0) (30,0)	0.96470	0.87033	0.98401	0.93878
(24,0) (33,0)	0.97062	0.88972	0.98676	0.94872
(28,0) (37,0)	0.97647	0.90970	0.98944	0.95866
(31,0) (40,0)	0.97979	0.92144	0.99096	0.96436
(35,0) (44,0)	0.98323	0.93393	0.99252	0.97032

The effects of chirality, mode number and aspect ratio presented in previous figures are detailed in the Tables 2-4. For various chirality nanotube armchair, chiral and zigzag for the first and second modes with different length-to-diameter ratios based on the non-local Euler Bernoulli beam model are listed in (Tables 2-4). The results show the dependence of the different chirality's of carbon nanotube, Aspect Ratio and, vibrational mode number on the frequency ratios (χ).

5. Conclusions

This paper studies the vibration of double-carbon nanotube (DWCNTs) based on Eringen's nonlocal elasticity theory and the Euler-Bernoulli beam theory. Influence of the small-scale coefficient, the vibrational mode number, the aspect ratio and the chirality on the lower frequency of the armchair, zigzag and chiral CNTs is shown. According to the study, the results showed the dependence of the vibration characteristics on the chirality of DWCNTs and the nonlocal parameter. With the results, the dynamic properties of the DWCNT beam have been discussed in detail; they are shown to be very different from those predicted by classic elasticity when nonlocal effects become considerable.

According to the study, it is observed that as the mode number increase, small-scale effects increases. The reason of these increases is attributed to the influence of small wavelength when the interactions between atoms are increasing. In addition, the lower frequency also is affected by the increasing or decreasing of long or diameter of (DWCNTs). This affection is most pronounced for short nanotube. The reason for this phenomenon is that a carbon nanotube with higher long has a larger curvature, which results in a more significant distortion of (C-C) bonds.

The investigation presented may be helpful in the application of DWCNTs, such as ultrahigh-frequency resonators, electron emission devices, high-frequency oscillators and mechanical sensors.

Acknowledgments

This research was supported by the Algerian national agency for development of university research (ANDRU) and university of sidi bel abbes (UDL SBA) in Algeria.

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