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# Mode shape identification using response spectrum in experimental modal analysis

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**Abstract.** The set of processes performed to determine the dynamic characteristics of the constructed structures is named experimental modal analysis. Using experimental modal analysis and interpreting its results, structural failure can be assessed and then it would be possible to plan for their repair and maintenance. The purpose of the experimental modal analysis is to determine the resonance frequencies, mode shapes and Mode damping for the structure. Diverse methods for determining the shape of the mode by various researchers have been presented. There are pros and cons for each of these methods. This paper presents a method for determining the mode shape of the structures using the response spectrum in the experimental modal analysis. In the first part, the principles of the proposed method are described. Then, to check the accuracy of the results obtained from the proposed method, single and multiple degrees of freedom models were numerically and experimentally investigated.

**Keywords:** experimental modal analysis; mode shape; response spectrum; structural dynamics

## 1. Introduction

In the past two decades, Experimental modal analysis (EMA) has become a universal knowledge to define, improve and optimize the dynamic characteristics of engineering structures. Modal analysis is not only applied in mechanical engineering and aeronautics but also has broad applications in building structures, biomechanical issues, space structures, acoustics equipment, transportation and nuclear power plants. The modal analysis of structures is divided into two major parts. In the first part, which is known as a direct method, the mass and stiffness matrix of the structure is determined and the dynamic properties of the structure are investigated. The second part is called the experimental modal analysis (EMA), which actually is a reverse method for determining the dynamic characteristics of the structure. In the EMA, responses of the structures under the input excitations are recorded using the field experiments and the dynamic characteristics of the structure are calculated by the output response. In the late nineteenth century, EMA methods have been developed. In the early studies, the proposed methods determined the

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system's dynamic characteristics by recording the input excitation and output response. One of these methods is the peak picking in the frequency response function (FRF). On this basis, in a frequency response function, a part of a peak response is extracted as the response of the structure at a resonance frequency, and the characteristics of the structure with this mode are calculated by this pattern. The paradigm of this method is also used in many of the following modal analysis methods. The Nyquist circle (1932) is another one of the earliest EMA methods. In this method, the FRF of the structure is described, based on the real and imaginary response and the dynamic characteristics of the system is calculated using the Nyquist circle properties. By the development of devices for measuring and monitoring of the health of structures, researches on experimental modal analysis have been growing rapidly. In many of the structures, recording of the input excitation into the structure is so difficult. For this reason, newly developed EMA methods do not require the recording of the input excitation to the structure, which is commonly known as operational modal analysis (OMA). By the advancement of these methods, research on the health monitoring of large structures has grown significantly. Operational modal analysis can be divided into two fields, in time and frequency domains. Each asset category has advantages and disadvantages. Brincker *et al.* (2001) presented the Frequency Domain Decomposition (FDD), which is one of the methods in the frequency domain. This method extracts the frequency of the resonance and the mode shape of the structure using the random vibration principles. In this method, a relationship is established between Input and output power system density matrix, using random vibration relations. Given that near the natural frequency of the system, one mode usually produces the highest response of the system, so the response in this range is very similar to the mode shape of the resonance frequency. If the power response spectral density matrix at each frequency is split into its singular values and vectors, the frequencies, shapes, and damping modes of each mode are obtained. In another subset of the experimental modal analysis, the system characteristics have directly extracted from the recorded response of the structure in the time domain. The advantages of this method are to reduce errors and numerical problems in converting the response to the frequency domain. In some of these methods, it is assumed that the recorded response of the structure can be expressed by harmonic and exponential terms, and then, these harmonic terms can be calculated by the iterative methods. Using harmonic terms, the modal properties of the system is extracted. The method presented by Ibrahim (1973) is one of the examples of these methods. In some such time domain methods, the dynamical characteristics of the system are obtained using the inverse Laplace transform of the frequency response. Exponential Complex Squares Least (LSCE) is another example of these methods. Many of the existing methods have high computational cost. In recent years, due to improvements in optimization techniques, some of these methods are combined with optimization algorithms to reduce the computational costs. In this paper, considering the concepts of the acceleration response spectrum recorded from an experimental modal test, a method is presented to obtain the mode shape. On this basis, the proposed method is described. Then, to verify the accuracy of the obtained results, numerical and experimental models are developed to evaluate the performance of the proposed method to extract the mode shape. The proposed approach is an operational modal analysis method in the time domain and has simple mathematical and engineering concepts. Considering that the proposed method does not require an iterative process to achieve an optimal solution, the computation costs are significantly reduced. The basis of this method is the combination of the concept of the response spectrum in earthquake engineering and experimental modal analysis.

## 2. Theoretical considerations of the proposed method

The process of determining the intrinsic dynamical properties of a system is called modal analysis. The mathematical model where the dynamic behavior of the system is extracted is called the modal model of the system. In the experimental modal analysis, the aim is to achieve the dynamic characteristics of the system using the mathematical relations contained in the modal model. According to structural dynamics, the dynamic equilibrium equation of a multiple degree freedom systems can be shown as below: Eq. (1)

$$[M]\ddot{x} + [c]\dot{x} + [k]x = F(t) \tag{1}$$

Using the Laplace transform and the above equation, the Frequency Response Function (FRF) can be expressed as polynomial fractions. Many experimental modal analysis methods in the frequency domain use Eq. (2) in their computational process.

$$[H(\omega)] = \sum_{k=1}^{n} \frac{A(k)}{(j\omega - \omega_k)} + \frac{A(k^*)}{(j\omega - \omega^*{}_k)} \tag{2}$$

In the time domain methods, the dynamic characteristics of the system are generally computed by the equalization of x(t) response and exponential or harmonic response. If a system with one degree of freedom under is subjected to a harmonic sinusoidal load, and the modal excitation frequency of a system is being equal to its one-degree frequency, the system will be subjected to resonance and the system response will be more than normal. This response is calculated for a low-damping system under the harmonic sinusoidal load by Eq. (3), and if the system is linear, the velocity and acceleration of the system in resonance state is calculated by the derivation of Eq. (3).

$$u(t) = (u_{st})\frac{1}{2\xi}[e^{-\xi\omega_n t}\left(\cos(\omega_D t) + \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_D t)\right) - \cos(\omega_n t)] \tag{3}$$

A random acceleration, which dynamically applied to a system, can be represented as a sum of its initial harmonic waves. Now, if there is a harmonic in the early harmonics, which its frequency is equal to the natural frequency of the structure, and the magnitude of this frequency is such that can excite the structure, the resonance will occur in some parts of the structural response under this random vibration, and the amplitude of the response increases significantly. While drawing this random excitation response spectrum, we observe this pattern as peaks in the response spectrum. When a system is subjected to external forces, it is induced to vibrate. The equation of motion of a system with n degrees of freedom is paired with n normal second-order differential equations. As can be seen in Eq. (4), the displacement-velocity-acceleration equation of each degree of freedom, can be expressed as a linear combination of the natural modes of the structure.

$$u = \sum_{r=1}^{N} \varphi_r \, q_r = \varphi_1 q_1 + \cdots + \varphi_n q_n \tag{4}$$

In Eq. (4), $\varphi_n$ is the mode shape and $q$ is the modal contribution coefficient for each mode. By inserting Eq. (4) in Eq. (1), the multi-degree system's response can be converted to a set of n second-order unpaired differential equation. In Eq. (5), by solving generalized single-degree of freedom system and combining them together, the multiple-degree system response can be determined.

$$\ddot{x}_n + 2\xi_n \omega_n \dot{x}_n + \omega_n^2 x_n = f(t) \tag{5}$$

Now, if we assume that the structure has n degrees of freedom and is subjected to an excitation with a vibrating frequency of one of the structure modes, resonance occurs at some periods of time in this generalized one-degree of freedom structure, and the displacement in this period of time shows the temporal pattern of the mode shape of the main structure.

Finding the maximum response of all single degree of freedom (SDOF) systems which undergo acceleration record is called a spectrum. In the experimental modal analysis, the response of different points of the structure (usually the acceleration) is recorded. Collecting these data, the dynamic characteristics of the system (resonance frequency, mode shape, and damping) are calculated. Due to the excitation of the structure by external loading, each point of the structure would be accelerated. If the input excitation frequency is identical to one of the natural frequencies of the structure, the structure resonates in some intervals of time. The structure's response in these times can be a good representative of the structure's modes shape. Consider the case that virtually, structures with SDOF that have one of the frequencies of the main structure are placed on the points where the accelerometers are connected. When the excitation frequency to the system is same as the resonance frequency of that mode, according to the response of those virtual single degree of freedom structures (acceleration velocity displacement) the mode shape of that frequency can be obtained. Figs. 1 and 2 shows a schematic view of the extraction process of mode shape for a structure with 3 degrees of freedom. Basically, the proposed method comprises three parts.

The first step: At this step, the spectrum of displacement for the recorded accelerations of the structure is plotted and according to the peak value of the graph, the resonance frequency is obtained.

The second step: There were points of the structure where the acceleration was recorded. The response of the structures with SDOF, with respect to the resonance frequencies of the first step, is plotted.

The third step: For each resonance frequency, responses of SDOF structures mentioned in the second step to be plotted on the same graph. Then at the maximum response value points, the magnitude and sign of recorded response represent the mode shape at that point of the structure.

## 3. Simulations and experiments

In Section4, a three degree of freedom structure, lump mass, and spring are subjected to different excitations on its different degrees of freedom to simulate the experimental modal analysis of structures in this section, finite element method is used. Structures are excited and then the acceleration of each of the three degrees of freedom is derived using the finite element method. Afterward, the proposed method examines the results. In Section 5 a laboratory structure with two degrees of freedom is made and is subjected to free vibration. Then by the proposed method, its mode shapes are extracted. In sections 6 and 7 a two-dimensional frame and a cantilever beam are examined. In these two sections, the acceleration response of points is extracted using the finite element method and then by the proposed method, the mode shapes are captured.

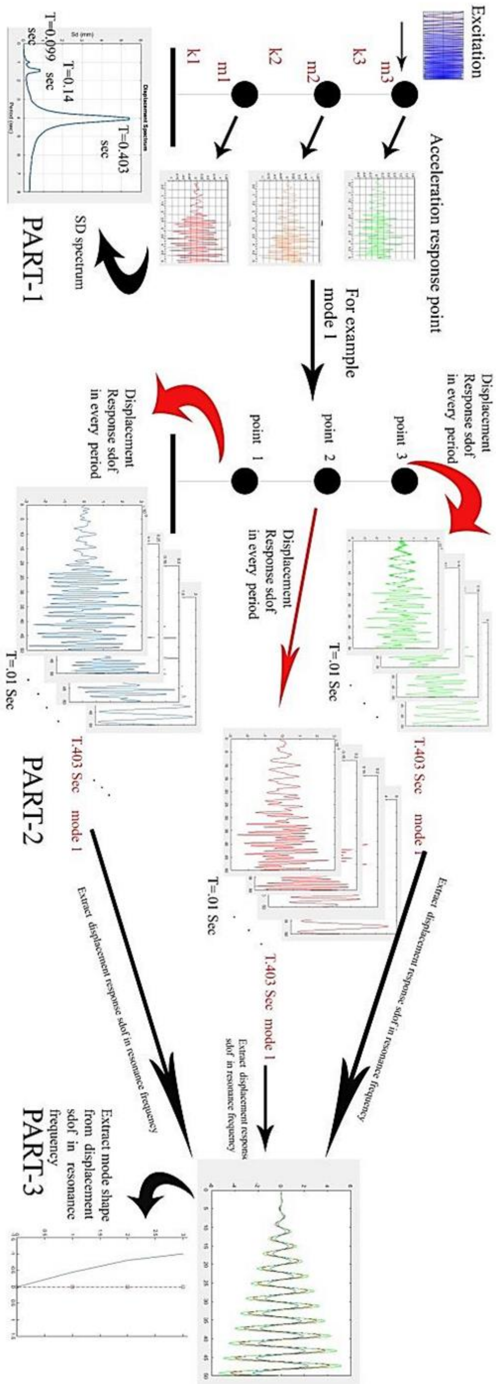



Fig. 1 Schematic view of the mode shape extracted by the proposed method

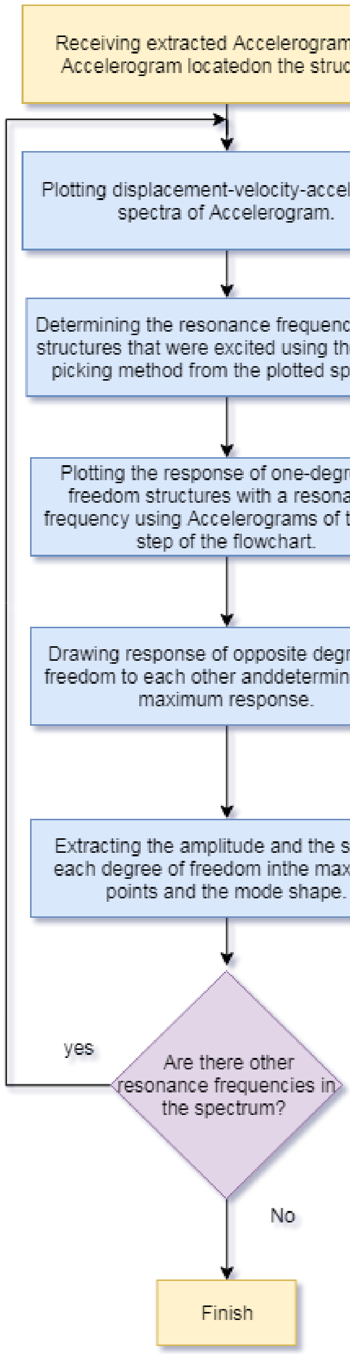



Fig. 2 Flowchart of the proposed method

## 4. Mass-spring structure with three degrees of freedom

In order to verify the proposed method, three degrees of freedom structure lump of mass and spring is considered, as it is shown in Fig. 3. The mass of each floor of the structure is 80 ton and its lateral stiffness is 980 kN/cm. The periods of the three modes of this structure are respectively 0.403, 0.143 and 0.1 seconds. In addition, the mode shapes associated with the first three modes are shown in Fig. 3. The damping of the structure is considered to be Rayleigh damping. The structure’s damping is assumed to be 5% for the first and third modes and 4.7% for the second mode.

In order to evaluate the effect of the type and location of excitation, the structures subjected to different excitations on their different degrees of freedom are studied. In the first case, the structure is excited by a sine wave with increasing amplitude and frequency on the third degree of freedom. In many large structures, excitation of the structure is accomplished by mechanical shakers or unbalanced mass. In these shakers, with increasing speed of rotation, frequency, and the force exerted on the system increases. In Fig. 4(a), a view of the load and its frequency band can be observed.

In the second excitation, the load on the first degree of freedom is subjected to the same excitation as the impact hammer in modal analysis. In this excitation, the first degree of freedom in the first 58 milliseconds is subjected to a constant load, and this load decreases to zero. Then the structure vibrates freely for 5 seconds. In Fig. 4(b) this loading is shown.

In the third excitation, the second degree of freedom is excited randomly. In Fig 4(c), this random loading is displayed. In many structures, for various reasons, structures cannot be subject to forced vibration and should be subject to ambient excitation for modal analysis. Generally, these ambient excitations have random behavior. The structure would be excited for 5 seconds and after loading it would undergo free vibration.

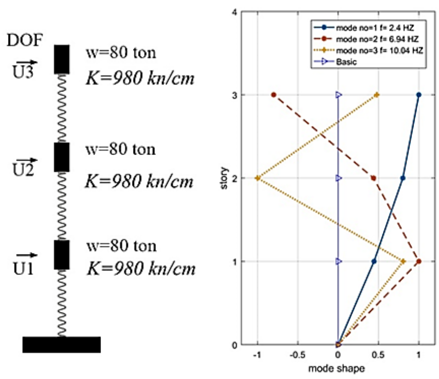



Fig. 3 Characteristics of a mathematical model of the structure lump mass and spring, using finite element method

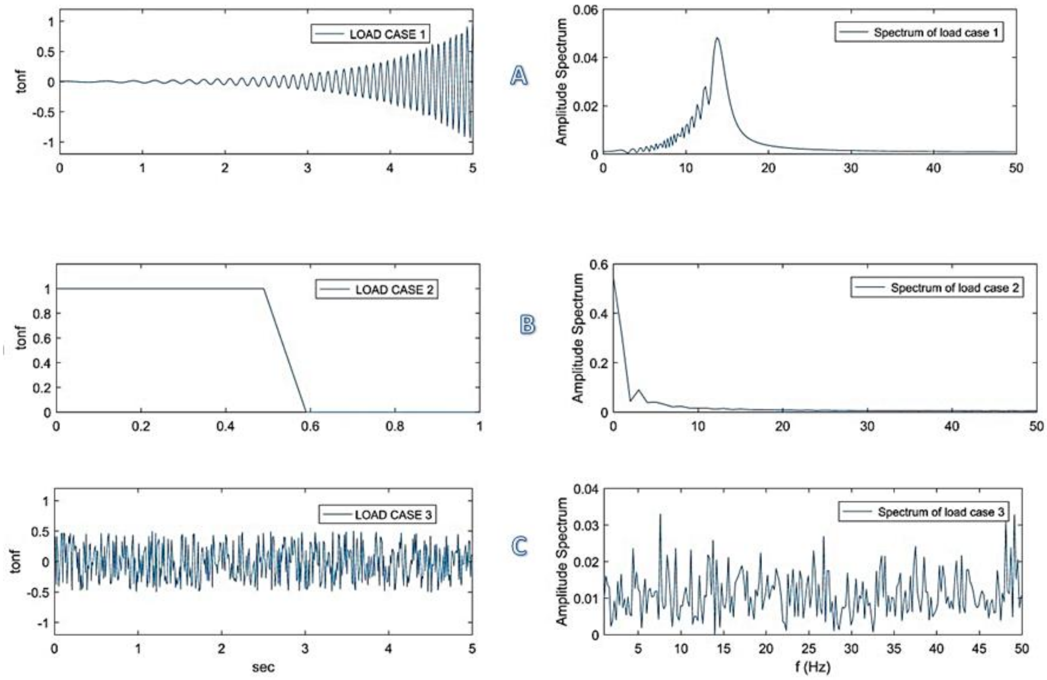



Fig. 4 Fourier spectrum and load pattern applied to three degrees of freedom structure

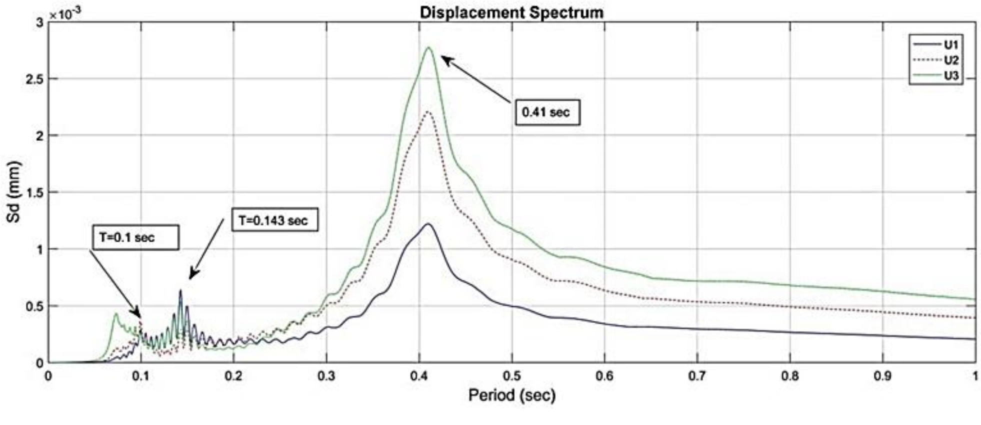



Fig. 5 Displacement spectrum associated with three degrees of freedom of the mass-spring structure

After receiving the acceleration response from all degrees of freedom of the structure, the displacement spectrum of these points with zero damping is plotted. Considering the peak points of the diagram, the resonant frequencies of the structure is determined. In this section, if the degrees of freedom are located on the nodes of the mode shape of the structure, the resonance frequency of that mode of the structure may not be determined on the response spectrum of that point. The presented proposed method for the first mode loading is described as an example. In the first section, by plotting the displacement spectrum with zero damping, we can extract the resonance frequencies of the structure. Fig. 5 displays a view of this spectrum.

In Fig. 5, we can see the periods of the first three modes of the structure. After determining the resonance frequencies, the responses of a single degree of freedom structure with the period of 0.41 seconds, subjected to accelerations recorded at points with 1, 2 and 3 degrees of freedom are plotted on the same graph. When we go through this step, we observe that the responses of single degree of freedom structures have resonant behavior. In this case, part of the diagram that corresponds to maximum magnitudes is drawn, and according to the responses, for each degree of freedom, the amplitude proportion and sign is determined. This output is a representation of the mode shape of the structure at a certain frequency. Fig. 6 schematically shows how the shape mode is extracted in case the load pattern Fig. 4(a) is applied. To derive the mode shape, we can also use the response of structure in the form of its displacement, velocity, and acceleration.

To investigate the effect of different loading, three different loading patterns have been exerted on the structure. The results of the mode shape obtained by the proposed method are compared with the ones received from the finite element method.

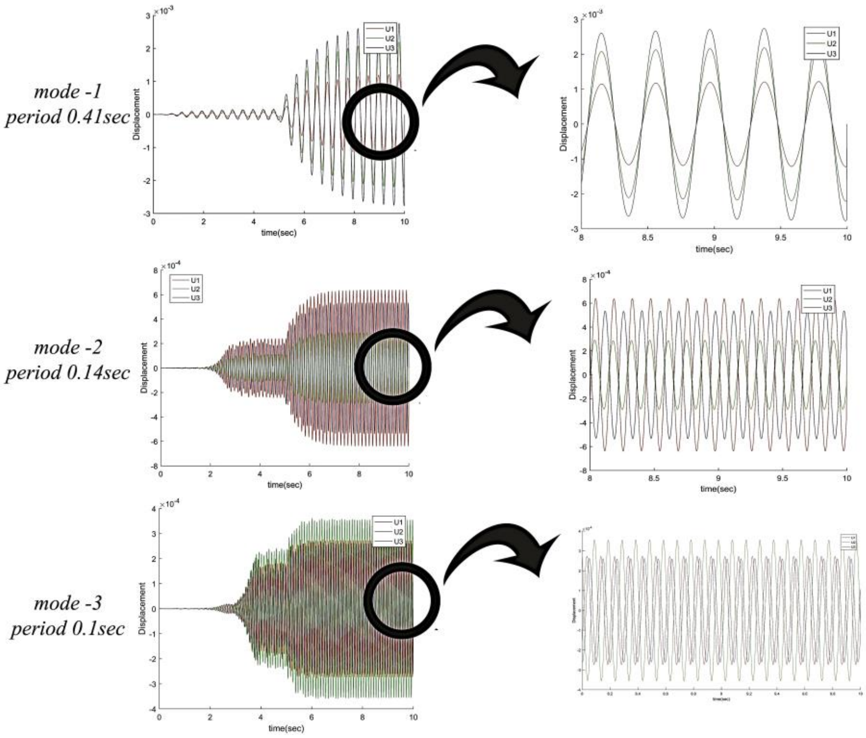



Fig. 6 Derivation of the mode shape from the response of single degree of freedom structures

Table 1 The mode shape results of the three degrees of freedom structure

| Mode-1 | DOF U1 | DOF U2 | DOF U3 |
|---|---|---|---|
| FEM | 0.439 | 0.792 | 1 |
| Present method case 1 | 0.44 | 0.796 | 1 |
| Present method case 2 | 0.44 | 0.8 | 1 |
| Present method case 3 | 0.44 | 0.8 | 1 |
| Mode-2 | DOF U1 | DOF U2 | DOF U3 |
| FEM | -1 | -0.439 | 0.792 |
| Present method case 1 | -1 | -0.43 | 0.83 |
| Present method case 2 | -1 | -0.479 | 0.793 |
| Present method case 3 | -1 | -0.44 | 0.77 |
| Mode-3 | DOF U1 | DOF U2 | DOF U3 |
| FEM | -0.792 | 1 | -0.439 |
| Present method case 1 | -0.81 | 1 | -0.73 |
| Present method case 2 | -0.83 | 1 | -0.4397 |
| Present method case 3 | -0.768 | 1 | -.043 |

## 5. Laboratory model of the two degrees of freedom structure

For further evaluation, using the proposed method, a laboratory structure with two degrees of freedom is examined. The structure is composed of a rod with a diameter of 6 Millimeter and two concentrated masses weighing 200 gr. The mechanical properties of consumed materials in the test are shown in Table 2. MPU6050 accelerometers are attached to each mass to record the acceleration. The sampling rate in this test is 625 Hz, and in order to excite this structure, the masses are subjected to initial displacement in opposite directions, and then released, which results in free vibration for the structure. The height of the rod is 84 cm, a mass is located at the free end and another is placed at a distance of 34 cm from the rod support.

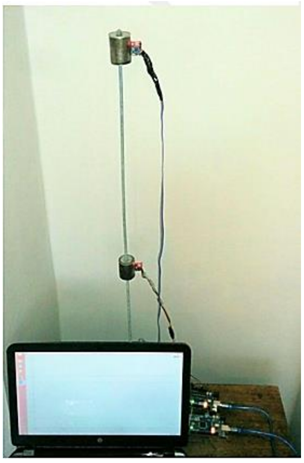
To verify the test results, the structure is also modeled using the SAP2000 as finite element software, and its two resonant frequencies and mode shape are extracted. Fig 7 shows a profile of this structure along with its dynamic characteristics.

We plot the displacement, velocity and acceleration spectrum for the recorded accelerations of the test. These three diagrams are shown in Fig. 8. Regarding the peaks of the diagram, the two first- resonance frequencies of the structure in the test were recorded as 2.8 and 20.87 Hz, respectively.

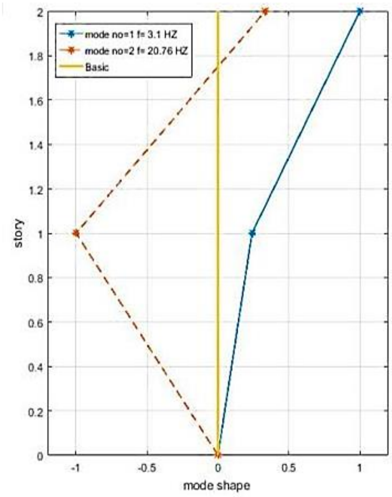
The first resonant frequency has 2.8% discrepancy with the result of the finite element method, and the second mode frequency is similar to the response of the finite element method. Also looking into the first part of the graph of the acceleration spectrum in Fig. 8, two peak frequencies of 184 and 388 Hz are also visible. These are the two frequencies of the third and fourth modes of the structure and are consistent with finite element results. In studying the mode shape of this two degree of freedom structure, only two primary modes are addressed. In the next step, the responses of single degree of freedom structures with frequencies of the first two structure modes, 2.87 and 20.87 Hz, are plotted on the same graph.

Table 2 Properties of the consumed material in two degrees of freedom structure

| Material | $\gamma(\mathrm{kg}/m^3)$ | $E(\frac{N}{m^2})$ | $\nu$ |
|---|---|---|---|
| Steel | 5824 | 2.45E11 | 0.3 |



(a) Laboratory structure




(b) Mode shape obtained from the finite element model

Fig. 7 Laboratory Model of the two degrees of freedom structure

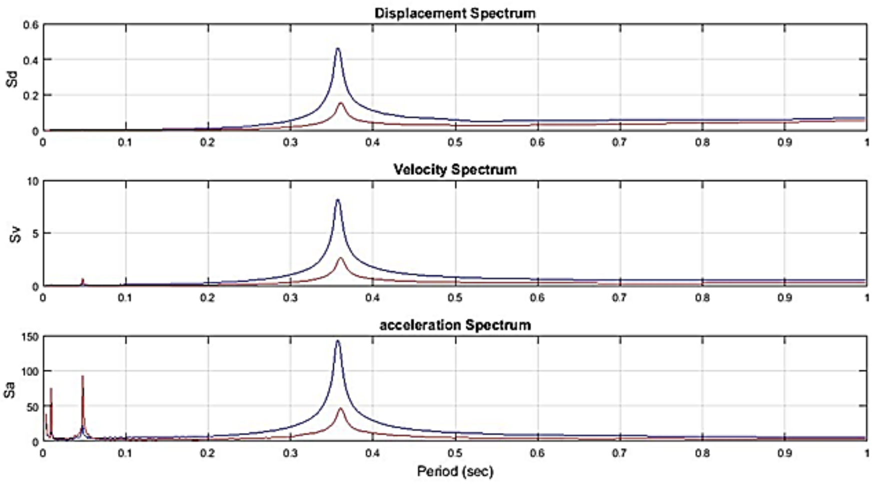
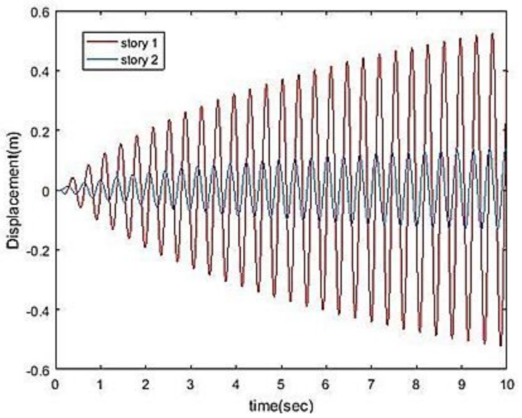
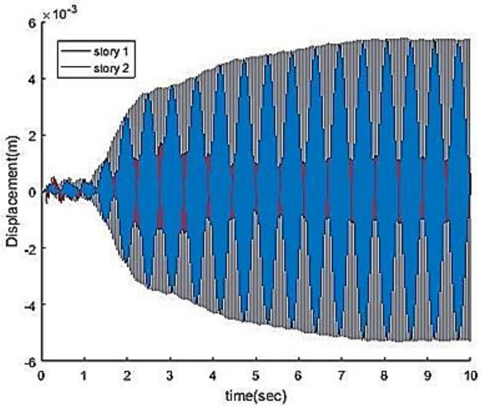



Fig. 8 Displacement - acceleration and velocity spectrum, associated with acceleration records for two degrees of freedom structure




(a) Response of single degree of freedom structures for the frequency of 2.8 Hz




(b) Response of single degree of freedom structures for the frequency of 20.87 Hz

Fig. 9 Response of Laboratory Model of the two degrees of freedom structure

Table 3 Results extracted from the proposed method, laboratory model, and finite element model

| Mode-1 | Story 1 | Story 2 |
|---|---|---|
| FEM | 0.24 | 1 |
| Present method Experiment with | 0.26 | 1 |
| Mode-2 | Story 1 | Story 2 |
| FEM | -1 | 0.332 |
| Present method Experiment with | -1 | 0.31 |

Then, in order to extract the mode shape, in Fig. 9 in each graph, we refer to the parts corresponding to the highest response. In these parts, the magnitudes and signs of each degree of freedom are extracted and would be scaled according to the maximum response. In Table 3, the mode shape results extracted from the test can be seen.

## 6. Two-dimensional frame

For further evaluation of the proposed method, a two-dimensional three-stories frame is considered. The height and frame widths are 3 and 4 m respectively and the column's and beam's sections are IPE200 and IPE160, respectively. In Fig. 10, we can see a profile of the geometry and the dynamic properties of the structure.

To excite the two-dimensional frame, a horizontal randomized load is applied on the columns. The reason for choosing this loading pattern is its similarity to wind pressure on the structure. Furthermore, on frame beams, a gravity dead load of 18 KN/m magnitude is applied. The system's damping is assumed to be 5% for the first and third modes and 4.7% for the second mode.

First, we plot the three spectrums of displacement-velocity and acceleration for the selected degrees of freedom in each story. These three spectrums are shown in Fig. 11. By using the peak points of the graph in Fig. 11, the resonance frequencies in the structure are obtained. When the responses of single degree of freedom structures corresponding to the resonance frequencies are plotted on the same graph and exciting points are investigated, the mode shape of each mode is derived.

Since the frame is two-dimensional, the acceleration recorded in each node of the structure comprises two components. To capture the pattern of mode shape of the structure in both dimensions, it is possible to separately derive the mode shape in each direction and the resultant vector would be the final form of the mode shape. As an example, the mode shape in the lateral direction, for three points of the frame in each floor is extracted. To extract the Mode shape in this frame, the acceleration spectrum is used. Considering the peak in Fig. 11, the periods of 1.95, 0.55 and 0.29 seconds are obtained as the first two frequencies of the structure. In Fig. 12, the acceleration response of single degree of freedom structures with periods associated with the first two modes of the structure are plotted on the same graph.

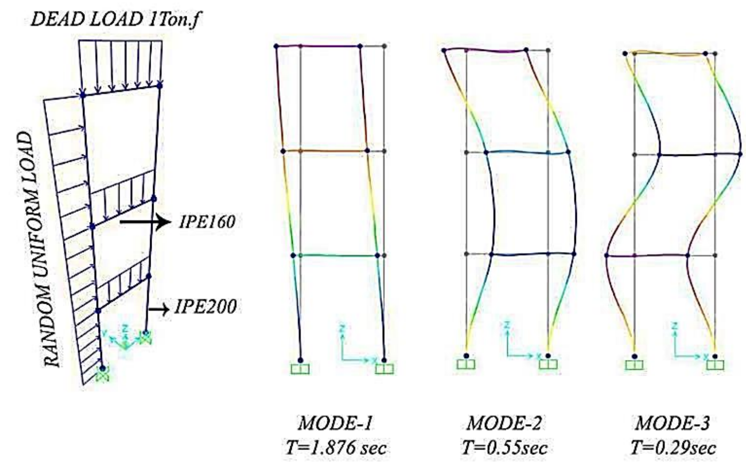



Fig. 10 Dynamic characteristics of the 2D frame, using finite element method

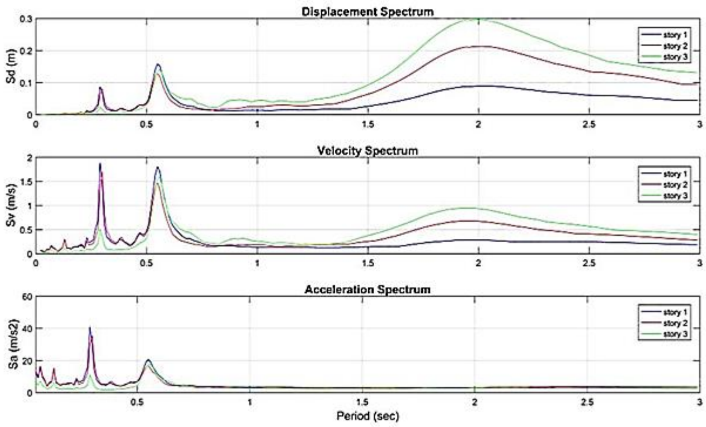
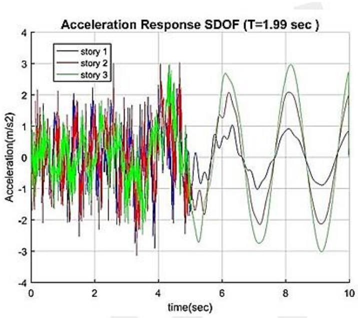


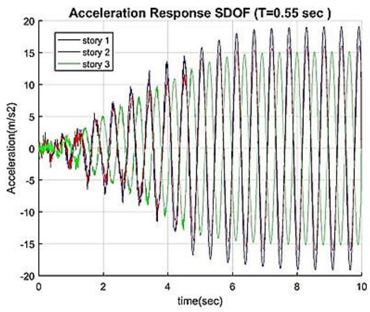

Fig. 11 spectrum of displacement, velocity, acceleration of a three-story two-dimensional frame

Table 4 The mode shape results for the two-dimensional frame in the first three structure modes

| Mode-1 | Story 1 | Story 2 | Story 3 |
|---|---|---|---|
| FEM | 0.293 | 0.71 | 1 |
| Present method case 4 | 0.299 | 0.717 | 1 |
| Mode-2 | Story 1 | Story 2 | Story 3 |
| FEM | -1 | -.787 | 0.8745 |
| Present method case 4 | -1 | -.8 | 0.86 |
| Mode-3 | Story 1 | Story 2 | Story 3 |
| FEM | 1 | -0.89 | 0.34 |
| Present method case 4 | 1 | -0.87 | 0.27 |




(a) first mode




(b) second mode

Fig. 12 Acceleration response of single degree of freedom structures in resonance frequencies in three stories of the two-dimensional frame

In Fig. 12(a), we can see the responses of single degree of freedom structures with a period of 1.99 seconds. Resonance occurs after 5 seconds in the response of structures. It is shown in Fig. 12(b) that from the very beginning, structures have resonance. Regarding the peaks in the graph, according to the magnitudes and the sign of response, the mode shape of the structure on the corresponding degree of freedom can be extracted. In Table 4, the mode shape results obtained from the proposed method and the finite element method are compared.

To examine the correlation between the results of the proposed method and the finite element method, the modal assurance criterion (MAC) is used, which is defined in the following Eq. (6).

$$MAC(i,j)=\frac{\left|\{\phi\}'_i*\{\phi\}_j\right|^2}{\left|\{\phi\}'_i*\{\phi\}_j\right|*\left|\{\phi\}'_j*\{\phi\}_i\right|} \tag{6}$$

In Eq. (6), $\{\phi\}_j$ and $\{\phi\}_i$ are mode vectors, resulted from the finite element method and the proposed method. In fact, the MAC index represents the size of the cousin of the angle between the mode shape vectors. If the mode shape vectors coincide, the value of this index is equal to one. In Fig. 13, MAC index is plotted to evaluate the correlation between the results of the finite element and the proposed method. Accordingly, this correlation is approved.

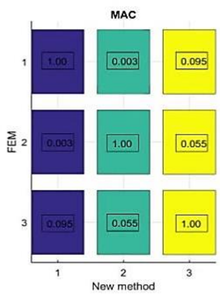



Fig. 13 Modal assurance criterion, results of the finite element and the proposed method

## 7. Cantilever beam

For further investigations, a cantilever beam with IPE160 section is studied. The length of the beam is 3 meters and is subjected to a random pattern loading. In Fig. 14, we can see a view of the mode shape and the frequencies of the first three modes of the beam. In this modeling, it is assumed that on the four points shown in Fig. 14, the accelerometers are attached and only the acceleration of these four points is available.

Table 5 Mode shape results of the cantilever beam in the first three modes of the structures

| Mode-1 | DOF U1 | DOF U2 | DOF U3 | DOF U4 |
|---|---|---|---|---|
| FEM | 0.033 | 0.25 | 0.608 | 1 |
| Present method case 5 | 0.033 | 0.259 | 0.608 | 1 |
| Mode-2 | DOF U1 | DOF U2 | DOF U3 | DOF U4 |
| FEM | -0.17 | -0.73 | 0.309 | 1 |
| Present method case 5 | -0.177 | -0.765 | 0.355 | 1 |
| Mode-3 | DOF U1 | DOF U2 | DOF U3 | DOF U4 |
| FEM | 0.44 | 0.86 | -0.74 | -1 |
| Present method case 5 | 0.15 | 0.93 | -0.32 | -1 |

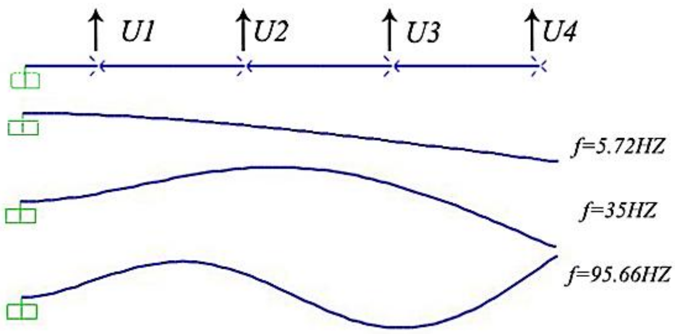



Fig. 14 Degrees of freedom of mode shape of the first three frequencies of the beam, using finite element method

Random loading on the beam causes well excitation of the first frequency of the structure, but the third resonance frequency of the beam is not fully excited. This caused the third Mode shape not to be captured accurately. Table 5 compares the results of the proposed method with the ones derived from the finite element method.

## 8. Conclusions

In the present article, a method for determining the mode shape of structures in the experimental modal analysis is suggested. This method in the time domain approximates the mode shape of the structure. In the proposed method, for extraction of the mode shape, it is not necessary to record the input excitation to the structure. Considering just the response of the input acceleration on the points where accelerometers are installed, the mode shape is extracted. To understand the philosophy of this method, consider the virtual single degree of freedom structures are attached to the points where accelerometers are installed. When the input excitation affects the resonant frequency of the main structure, the behavior of these SDOF structures leads to obtain the shape mode. Basically, this method comprises three steps. In the first step, the spectrum of the recorded accelerations is plotted and the resonance frequencies of the main structure are derived. In the second step, the responses of SDOF structures are plotted on the same graph. In the third stage, regarding the maximum points of the second stage diagrams, mode shape is extracted. To evaluate the accuracy of the proposed method, different structures analytically and experimentally were subjected to various excitations and their mode shapes were extracted. The results show that the proposed method properly provides mode shape responses. The accuracy of the method reduces by increasing the damping of the main structure. In the proposed method, to determine the mode shape of the structure, the displacement, velocity or acceleration responses of SDOF structures can be used. There is no difference between using any of these three options for extracting the mode shape. In order to obtain adequate precision, it is better for the responses of SDOF structures to be plotted with a damping close to zero.

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