Structural Monitoring and Maintenance, *Vol. 3, No. 4 (2016) 395-406*
DOI: http://dx.doi.org/10.12989/smm.2016.3.4.395

Output only structural modal identification using matrix pencil method

Satish Nagarajaiah*[1] and Bilei Chen[2]

[1]*Department of Civil Eng. (Joint Appt.-MECH, MSNE),*
213 Ryon, MS 318, CEE, Rice Univ, 6100 Main St., Houston, TX 77005, USA
[2]*Structural Engineer, Shell Westhollow Technology Center, 3333 Hwy 6, Houston, TX 77082, USA*

(Received November 3, 2016, Revised December 8, 2016, Accepted December 10, 2016)

Abstract. Modal parameter identification has received much attention recently for their usefulness in earthquake engineering, damage detection and structural health monitoring. The identification method based on Matrix Pencil technique is adopted in this paper to identify structural modal parameters, such as natural frequencies, damping ratios and modal shapes using impulse vibration responses. This method can also be applied to dynamic responses induced by stationary and white-noise inputs since the auto- and cross-correlation function of the two outputs has the same form as the impulse response dynamic functions. Matrix Pencil method is very robust to noise contained in the measurement data. It has a lower variance of estimates of the parameters of interest than the Polynomial Method, and is also computationally more efficient. The numerical simulation results show that this technique can identify modal parameters accurately even if the noise level is high.

Keywords: modal analysis; identification; natural frequency

1. Introduction

Existence of structural damage in civil engineering infrastructures (Nagarajaiah and Basu 2009, Nagarajaiah and Kalil 2016, Nagarajaiah and Yang 2016), such as buildings, bridges, etc., may greatly influence the overall performance of the system or even lead to disastrous failures. Therefore, detecting the acute damage caused by earthquakes, impacts, or explosions immediately after the event or monitoring long-term deterioration due to the environmental and human use is necessary and can then be used to assess and plan future use and repairs (Johnson, Lam *et al*. 2004).

The presence of damage in a structure will change vibration modes, such as modal shapes, natural frequencies and damping ratios. Changes in the modal parameters may not be the same for each mode since the changes depends on the nature, location and severity of the damage (Salawu 1997). Modal parameter identification thus has a great potential in earthquake engineering, structural identification, damage detection and structural health monitoring. Over the past twenty years, many structural modal parameter identification methods have been proposed. Detailed

*Corresponding author, Professor, E-mail: satish.nagarajaiah@rice.edu

http://www.techno-press.org/?journal=smm&subpage=7 ISSN: 2288-6605 (Print), 2288-6613 (Online)

literature reviews have been provided by Doebling, Farrar *et al*. (1996).

Modal parameters can be identified from measured vibration responses, which are excited by artificial external forces or ambient forces in the service environment. Usually, the easy way to obtain the vibration parameters is to use structural free responses. The free vibration is generated by suddenly terminating external excitations on the structure. The corresponding dynamic response is the linear combination of sinusoidal functions with different structural damped natural frequencies. The concept of free vibration has recently been extended to the responses induced by ambient vibrations, where the external forces are assumed to be stationary and white-noise (James, Carne *et al*. 1995, Caicedo, Dyke *et al*. 2004). In this case, the auto- or cross-correlation function of two outputs has the same form as the free vibration of a linear system. The same phenomenon happens to the response of an object due to a burst of electromagnetic energy (Sarkar and Pereira 1995).

Many methods have been developed to extract the dynamic characteristics of free vibration structures, such as Matrix Pencil Method (Sarkar and Pereira 1995), Eigenvalue Realization Analysis (ERA method) (Juang and Pappa 1985), Polynomial Method (or the Prony-type Method) (Hua and Sarkar 1989), Hilbert-Huang Transformation Method (Yang, Lei *et al*. 2003) and so on. In general, the Matrix Pencil method is more robust to noise contained in the measurement data. It has a lower variance of estimates of the parameters of interest than the Polynomial Method, and is also computationally more efficient. It has been shown that for the BPMP (band-pass matrix pencil method), the variance of the estimates can come close to the Cramer-Rao bound, when the signal-to-noise ratio is greater than 12 dB (Sarkar and Pereira 1995).

In this paper, the Matrix Pencil technique is used to extract modal parameters of the linear structures. The main contribution of this paper is to identify structural natural frequencies and modal shapes (if a full set of measurements is available) using the free vibration responses or under ambient vibration conditions. The structure is assumed to vibrate freely or subjected to ambient forces. Measured data can be displacement, velocity or acceleration responses. These data are arranged orderly to form a matrix and SVD decomposition is utilized to filter noisy data. The generalized eigenvalue problem can then be solved to obtain the system natural frequencies and damping ratios. If dynamic responses of all DOF are measured, the corresponding mode shapes can be obtained by the least-square method. The numerical examples showed that this method is accurate even if the sampled data are heavily polluted by noise, and its potential for application in structural identification is remarkable.

2. Formulation of free decay vibration of structures

The dynamic response of free vibration of multi-degree-of-freedom (MDOF) systems can be expressed as follows

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = 0 \tag{1}$$

where M, C and K are the system mass matrix, damping matrix and stiffness matrix, respectively. They are all $n \times n$ matrix. $x(t) \subseteq \mathbf{R}^n$ is the system displacement vector. Moreover, when the system is subjected to stationary and white-noise excitations, the dynamic equation of the structure is (James, Carne *et al*. 1995, Caicedo, Dyke *et al*. 2004)

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \tag{2}$$

where $F(t)$ is the stationary and white-noise input vector. Assume the structural parameter matrices M, C and K are deterministic. Post-multiplying Eq. (2) by a reference scalar response process $x_i(s)$ and taking the expected value of each side yield

$$MR_{\ddot{X}X_i}(\tau)+CR_{\dot{X}X_i}(\tau)+KR_{XX_i}(\tau)=R_{FX_i}(\tau) \tag{3}$$

where $\tau = t\text{-}s$, $R(\cdot)$ denotes a vector of correlation function. Notice the following relations

$$R_{\dot{A}B}(\tau)=\frac{\partial}{\partial t}R_{AB}(\tau)\text{; and }\ R_{\ddot{A}B}(\tau)=\frac{\partial^2}{\partial t^2}R_{AB}(\tau) \tag{4}$$

Recognize that the responses of the system are uncorrelated to the stochastic inputs for $\tau > 0$, and assuming that the random vector process $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ are weakly stationary, considering only the homogenous part, we can write Eq. (3) as

$$M\ddot{R}_{XX_i}(\tau)+C\dot{R}_{XX_i}(\tau)+KR_{XX_i}(\tau)=0 \tag{5}$$

It has the same form as the free response of a linear structure. Therefore, the following analysis is based on Eq. (1), but the results are the same for both Eqs. (1) and (5). Eq. (1) can be converted to the state-space model

$$\dot{X}=AX \tag{6}$$

where

$$X=\begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix}\ \text{ and }\ A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{7}$$

The solution of Eq. (6) is

$$X=\sum_{j=1}^{2n}\psi_j e^{\lambda_j t} \tag{8}$$

where $\psi_j \subseteq \mathbf{R}^{2n}$ and λ_j are eigenvectors and eigenvalues of the matrix A, respectively. Since X composes of the system displacement $x(t)$ and velocity $\dot{x}(t)$, where the latter vector is the derivative of the former one, ψ_j can be split into two parts

$$\psi_j=\begin{pmatrix} \phi_j \\ \lambda_j\phi_j \end{pmatrix} \tag{9}$$

where ϕ_j is an $n\times1$ complex vectors, then the displacement response of the system can be described as

$$x(t)=\sum_{j=1}^{2n}\phi_j e^{\lambda_j t} \tag{10}$$

Based on Eq. (10), the acceleration response of the system can be expressed as

$$\ddot{x}(t)=\sum_{j=1}^{2n}\lambda_j^2 e^{\lambda_j t}\phi_j \tag{11}$$

Each scalar element of the acceleration vector $\ddot{x}(t)$ is the linear combination of exponential functions. Once the eigenvalues λ_j of the system is identified, the natural frequencies and damping ratios of the system are

$$p_j = \sqrt{\left(\mathrm{Real}\left(\lambda_j\right)\right)^2 + \left(\mathrm{Imag}\left(\lambda_j\right)\right)^2} \quad \text{and} \quad \zeta_j = \frac{abs\left(\mathrm{Real}\left(\lambda_j\right)\right)}{\sqrt{\left(\mathrm{Real}\left(\lambda_j\right)\right)^2 + \left(\mathrm{Imag}\left(\lambda_j\right)\right)^2}} \tag{12}$$

where p_j and ζ_j are the natural circular frequency and damping ratio of the system, respectively.

After sampling, the time variable, t, is replaced by $k\Delta t$, where Δt is the sample period. Set $z_j = e^{\lambda_j \Delta t}$, Eq. (11) can be rewritten as

$$\ddot{x}\left(k\Delta t\right) = \sum_{j=1}^{2n} \lambda_j^2 \phi_j z_j^k \tag{13}$$

In practice, the measured data are always polluted by noise of some level. So we must add noise term into the above equation

$$\ddot{y}\left(k\Delta t\right) = \ddot{x}\left(k\Delta t\right) + n\left(k\Delta t\right) = \sum_{j=1}^{2n} \lambda_j^2 \phi_j z_j^k + n\left(k\Delta t\right) \tag{14}$$

The technique based on Matrix-Pencil-Method (Sarkar and Pereira 1995) can be used to extract exponential components z_j, even if only one-point measurement is known provided that this measurement contains all system information (provided the measurement is not at the node of a mode, in which case, that particular pole cannot be extracted). Moreover, it is more powerful than Ibrahim Method since it is not much sensitive to noise. After obtaining the values of λ_j, we can compute the natural circular frequencies and damping ratios of the system using Eq. (12). This method is first introduced by Sarkar and Pereira (1995) to estimate the parameters of a sum of complex exponentials. In the following section, we will briefly illustrate this method and then apply it to identify modal parameters.

3. Application of matrix pencil method to modal parameter identification

In Eq. (14), every component of the vector $\ddot{x}(t)$ is the linear combination of complex exponentials. Suppose $\ddot{x}_l$ and $\ddot{y}_l$ are the lth element of the vector $\ddot{x}$ and $\ddot{y}$, respectively. We obtain

$$\ddot{y}_l\left(k\Delta t\right) = \ddot{x}_l(k\Delta t) + n_l\left(k\Delta t\right) = \sum_{j=1}^{M} R_{jl} z_j^k + n_l\left(k\Delta t\right) \tag{15}$$

where $M = 2n$, $R_{jl} = \lambda_j^2 \phi_{jl}$, ϕ_{jl} is the lth element of the vector ϕ_j.

In order to extract these frequencies and damping ratios, the following two matrices: Y_1 and Y_2 are defined as (Sarkar and Pereira 1995)

$$\left[Y_1\right] = \begin{bmatrix} \ddot{x}_l(0) & \ddot{x}_l(1) & \cdots & \ddot{x}_l(L-1) \\ \ddot{x}_l(1) & \ddot{x}_l(2) & \cdots & \ddot{x}_l(L) \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_l(N-L-1) & \ddot{x}_l(N-L) & \cdots & \ddot{x}_l(N-2) \end{bmatrix}_{(N-L)\times L} \tag{16}$$

$$[Y_2]=\begin{bmatrix} \ddot{x}_l(1) & \ddot{x}_l(2) & \cdots & \ddot{x}_l(L) \\ \ddot{x}_l(2) & \ddot{x}_l(3) & \cdots & \ddot{x}_l(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_l(N-L) & \ddot{x}_l(N-L+1) & \cdots & \ddot{x}_l(N-1) \end{bmatrix}_{(N-L)\times L} \tag{17}$$

where L is referred as the pencil parameter. The pencil parameter L is very useful in eliminating noise effects contained in the data set. If the noise term in Eq. (15) is ignored temporarily, we have the following relationships

$$[Y_2]=[Z_1][R][Z_0][Z_2] \tag{18}$$

$$[Y_1]=[Z_1][R][Z_2] \tag{19}$$

where

$$[Z_1]=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \cdots & z_M^{N-L-1} \end{bmatrix}_{(N-L)\times M} \tag{20}$$

$$[Z_2]=\begin{bmatrix} 1 & z_1 & \cdots & z_1^{L-1} \\ 1 & z_2 & \cdots & z_2^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_M & \cdots & z_M^{L-1} \end{bmatrix}_{M\times L} \tag{21}$$

$$[Z_0]=diag[z_1,z_2,\cdots,z_M] \tag{22}$$

$$[R]=diag[R_{1l},R_{2l},\cdots,R_{Ml}] \tag{23}$$

where diag[·] denotes an $M\times M$ diagonal matrix.

Based on Eqs. (18) and (19), we obtain

$$[Y_2]-\lambda[Y_1]=[Z_1][R]\{[Z_0]-\lambda[I]\}[Z_2] \tag{24}$$

where $[I]$is the $M\times M$ identity matrix. When $\lambda=z_i$, $i=1,2,\ldots,M$, the ith row of $\{[Z_0]$- $\lambda[I]\}$ is zero, and this matrix will be rank deficient. Hence, the parameters z_i can be treated as the generalized eigenvalue of the matrix pair $\{[Y_2]; [Y_1]\}$. Equivalently, the problem of solving z_i can be cast as the following eigenvalue problem

$$[Y_1]^+[Y_2]x=\lambda x \tag{25}$$

where $[Y_1]^+$ is the Moore-Penrose pseudoinverse of $[Y_1]$. It is defined as

$$[Y_1]^+=\{[Y_1]^H[Y_1]\}^{-1}[Y_1]^H \tag{26}$$

where the superscript "H" denotes the conjugate transpose. Here the above method is the same as some version of Ibrahim time domain method (Ibrahim and Mikulcik 1977) if $L=M$. However, if the data are polluted by noise, L should be much larger than M for data "purification".

In the presence of noise, the total-lease-squares Matrix Pencil has been used to combat noise (Sarkar and Pereira 1995). In this implementation, the data matrix [Y] is formed from the noise-contaminated data $y(t)$ by combining [Y_1] and [Y_2] as

$$[Y]=\begin{bmatrix} \ddot{y}_l(0) & \ddot{y}_l(1) & \cdots & \ddot{y}_l(L) \\ \ddot{y}_l(1) & \ddot{y}_l(2) & \cdots & \ddot{y}_l(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{y}_l(N-L-1) & \ddot{y}_l(N-L) & \cdots & \ddot{y}_l(N-1) \end{bmatrix}_{(N-L)\times(L+1)} \tag{27}$$

Notice that [Y_1] is obtained from [Y] by deleting the last column, and [Y_2] is obtained from [Y] be deleting the first column. The singular value decomposition (SVD) of matrix [Y] is performed as

$$[Y]=[U][\Sigma][V]^H \tag{28}$$

where $[U]^H[U]=[I]$, $[V]^H[V]=[I]$, and [$\sum$] is a diagonal matrix containing the singular value of [Y]. Tapan and Pereira (1995) proposed that the parameter L is chosen between N/3 to N/2 for efficient noise filtering. For these values of L, the variance in the parameters z_i, due to noise, has been found to be minimal (Hua and Sarkar 1990a, b). Since L and N are very large, the dimension of matrix [Y] is very big. But it is not necessary to operate such big matrix. The system order is assumed to be $M = 2n$, thus the "filtered" matrix $[V']$ is constructed so that it contains only M dominant right singular vectors of [V]

$$[V']=[v_1 \quad v_2 \quad \cdots \quad v_M] \tag{29}$$

Based on the definition of [Y_1], [Y_2] and [Y], we have

$$[Y_1]=[U][\Sigma'][V_1]^H \tag{30}$$

$$[Y_2]=[U][\Sigma'][V_2]^H \tag{31}$$

where [V_1] is obtained from $[V']$ with the last row of $[V']$ deleted; [V_2] is obtained by removing the first row of $[V']$; and $[\Sigma']$ is obtained from the M columns of [$\sum$] corresponding to the M dominant singular values. Substitute Eqs. (30) and (31) into Eq. (25) and solve the eigenvalue problem for λ_j or z_j (j = 1,2,…,2n). Note: With the availability of a single floor acceleration measurement, only the poles z_j (j = 1,2,…,2n) can be obtained, but not all mode shapes (as it becomes an under-determined problem Yang and Nagarajaiah 2013); to determine all mode shapes all measurements are necessary. If the measurements of all DOF are given and z_j (j = 1,2,…,2n) have already been identified, Eq. (14) will leads to

$$[\ddot{y}(0) \quad \ddot{y}(1) \quad \cdots \quad \ddot{y}(M)]=[\phi_1 \quad \phi_2 \quad \cdots \quad \phi_{2n}]\begin{bmatrix} \lambda_1^2 & \lambda_1^2 e^{\lambda_1 \Delta t} & \cdots & \lambda_1^2 e^{\lambda_1 (M-1)\Delta t} \\ \lambda_2^2 & \lambda_2^2 e^{\lambda_1 \Delta t} & \cdots & \lambda_2^2 e^{\lambda_1 (M-1)\Delta t} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{2n}^2 & \lambda_{2n}^2 e^{\lambda_1 \Delta t} & \cdots & \lambda_{2n}^2 e^{\lambda_1 (M-1)\Delta t} \end{bmatrix} \tag{32}$$

Therefore, the modal shapes ϕ_1, ϕ_2, …, ϕ_{2n} can be determined by least-square method.

4. Numerical simulations

4.1 Example 1

This numerical example is the same as that used by Yang, Lei *et al*. (2003). The detailed illustration of this model is listed as follows. A three-story shear-type building model is shown in Fig. 1. The mass, stiffness and viscous damping of each storey unit are identical with m_j=1000kg, k_j=980 kN/m, c_j=2.814 kN·s/m, respectively, for j=1,2,3. Suppose that an impact loading is applied to the second floor and its magnitude is 1 kN. Hence, the initial velocity is excited. Based on the identification method proposed above, all natural frequencies ω_j and the damping ratio ξ_j (j=1,2,3) can be identified using any one of the measured signals. Here we use the acceleration data of the third floor.

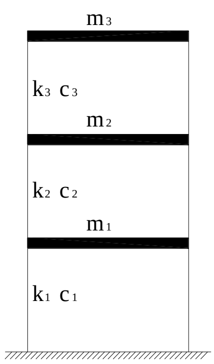

Fig. 1 The three-story shear type building model

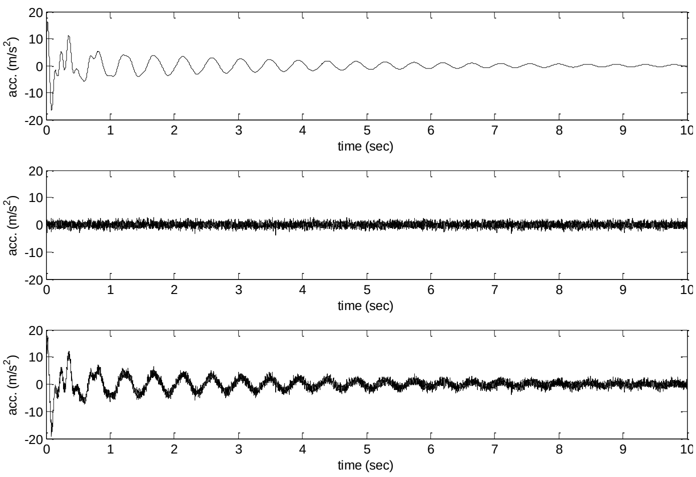

Fig. 2 The simulated signal of the third floor acceleration response with 5% noise level (a) Acceleration without noise (b) noise (c) Acceleration with noise

Table 1 The identified natural frequencies and damping ratios (L=800, N=1600)

Mode	Theoretic Values		Identified Values (R_{pi}=0%)		Identified Values (R_{pi}=5%)	
	Frequency (Hz)	Damping Ratio (%)	Frequency (Hz)	Damping Ratio (%)	Frequency (Hz)	Damping Ratio (%)
1	2.22	2.0	2.22	2.0	2.21	2.0
2	6.21	5.6	6.21	5.6	6.21	5.4
3	8.98	8.1	8.98	8.1	8.97	7.9

Table 2 The identified modal shapes (M=2000)

DOF	Theoretic Values			Identified Values (R_{pi}=0%)			Identified Values (R_{pi}=5%)		
1	1	1	1	1	1	1	1	1	1
2	1.80	0.45	-1.25	1.80	0.45	-1.25	1.81	0.47	-1.245
3	2.25	-0.80	0.56	2.25	-0.80	0.55	2.25	-0.82	0.56

The simulated time history $\ddot{x}_3(t)$ of the third floor acceleration response without noises is shown in Fig. 2(a). The noise level associated with each measurement $\ddot{x}_j(t)$ is expressed by $R_{pj} = \sigma_j / \max|\ddot{x}_j(t)|$ in which σ_j is the root mean square value of the noise $\upsilon_j(t)$ associated with the measurement $\ddot{x}_j(t)$. In other words, R_{pj} is the ratio of the noise root mean square value to the peak signal, which is different from the environmental noise to signal noise. A sample function of the noise process $v_3(t)$ associated with the measured acceleration $\ddot{x}_3(t)$ of the third floor is shown in Fig. 2(b), in which the noise level is R_{p3} = 5% with a bandwidth of 500 Hz. The measured acceleration response $\ddot{y}_3(t)$, which is the sum of Figs. 2(a) and 2(b), is shown in Fig. 2(c). The simulation result is listed in Table 1. Here, we choose L=800 and N=1600. Compared to the exact result, we can find that the result based on Matrix Pencil Method is very accurate. It seems that the Matrix Pencil Method is more robust to noise and the identified results are more close to exact values. The amazing thing is that the identified modal parameters will be closer to the exact values with the increase of L and N.

If the acceleration measurement of all 3-DOF are given, Eq. (32) can be used to find the modal shapes. They are shown in Table 2. Compared to the exact modal shapes which are directly computed from the general eigenvalues problem $Kx = \lambda Mx$, the modal shapes identified here are very accurate. Once the modal parameters are identified, the stiffness matrix can be obtained provided that the mass matrix remains the same (Yang, Lei *et al.* 2003). Thus structural health monitoring can be achieved by observing the identified stiffness matrix compared with the initial one.

Fig. 3 Picture of benchmark structure

4.2 Example 2

The second example is applied to Phase I of the IASC-ASCE Benchmark problem and the simulation results are compared to those obtained using NExT-ERA method (Caicedo, Dyke *et al.* 2004). The photography of benchmark structure built in UBC is shown in Fig. 3. It is a four-story, two-bay by two-bay building. Each bay is 1.25 m × 1.25 m in plan and 0.9 m high. Slabs are placed at each floor level to simulate the mass of a structure. In some cases, the floor slabs on the roof are placed to produce an asymmetric mass distribution. Two finite element models based on this structure are developed to generate the simulated response data. The first is a 12 degree of freedom (DOF) shear-building model that constraints all motion except two horizontal translations and one rotation per floor. The second is a 120-DOF model that only requires that floor nodes have the same horizontal translation and in-plane rotation. The columns and beams are modeled as Euler-Bernoulli beams in both finite element models. The braces are bars with no bending stiffness. In this paper, only 12 DOF model is considered. There are 6 damage patterns considered for the benchmark problem. Only damage pattern 2 (in which, all of the braces in the first and third floors are removed) is used to verify the proposed method in this paper.

Table 3 The identified results (L=800, N=1600, noise level=10%)

Mode	Theoretic Values		Identified Values (Matrix Pencil)		Identified Values (ERA)	
	Frequency (Hz)	Damping Ratio (%)	Frequency (Hz)	Damping Ratio (%)	Frequency (Hz)	Damping Ratio (%)
1	9.40	1.0	9.39	1.0	9.39	1.1
2	25.6	1.0	25.4	0.98	25.5	0.95
3	38.7	1.0	38.5	1.0	38.7	1.1
4	48.0	1.0	47.9	0.91	47.9	0.89

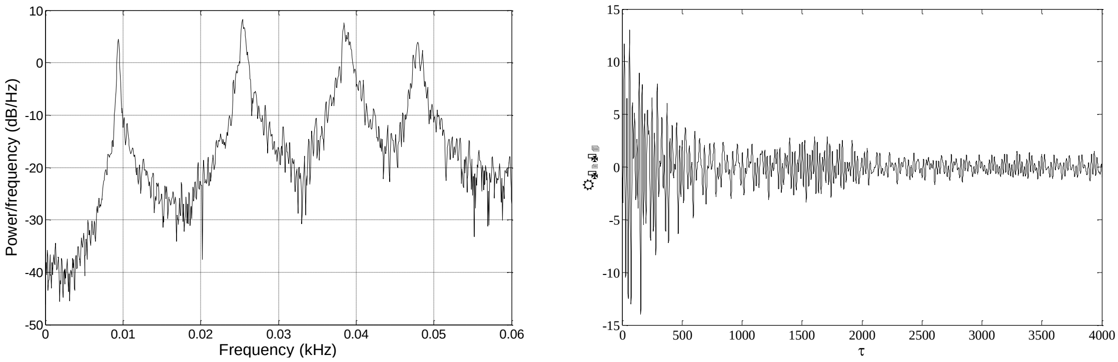

Fig. 4 Representative cross-spectral density function and cross-correlation function (Case 1, undamaged)

Table 4 The identified modal shapes (*M*=2000)

Mode No.	1	2	3	4
Theoretical Mode Shapes ϕ_j	1.0	1.0	1.0	1.0
	1.82	0.69	-1.0	-2.63
	2.40	-0.31	-0.69	3.08
	2.64	-1.0	1.21	-2.16
Identified Mode Shapes $\bar{\phi}_j$	1.0	1.0	1.0	1.0
	1.79	0.70	-1.0	-2.56
	2.36	-0.30	-0.72	2.91
	2.65	-1.01	1.21	-2.0
Accuracy indicator γ_j	0.99995	0.99996	0.9999	0.99977

Matlab programs discussed in Johnson, Lam *et al*. (2003) (see also IASC-ASCE 2003) are used to generate simulation data, in which, the sampling time interval is 0.001s, damping ratio for each mode is 1% and noise level is 10%. A representative cross-spectral density function and cross-correlation function (Case 1, undamaged) are shown in Fig. 4. The acceleration response of the fourth floor in y-direction is used to compute natural frequencies. The identified frequencies and damping ratios are listed in Table 3. Four frequencies in Table 3 are all in y-direction. Here, we choose L = 800 and N = 1600. The coherence parameter γ_j indicates whether the identified jth mode shape is close to the true shape. The definition of γ_j is

$$\gamma_j = \frac{\left|\phi_j^T \bar{\phi}_j\right|}{\left[\left(\phi_j^T \phi_j\right)\left(\bar{\phi}_j^T \bar{\phi}_j\right)\right]^{1/2}} \tag{33}$$

where the parameter ϕ_j is the jth true mode shape, $\bar{\phi}_j$ is the jth identified mode shape. γ_j can have only the values between 0 and 1. $\gamma_j \rightarrow 1$ as $\bar{\phi}_j \rightarrow \phi_j$ indicates that the realized jth mode shape of the system is very close to the true values of the system. Table 4 shows that the identified mode shapes using matrix pencil method are very close to the true mode shapes. Matrix Pencil Method leads to accurate results and is robust to noise.

5. Conclusions

The identification method based on Matrix Pencil Method is very powerful and robust to noisy measurement data. In this paper output only modal identification with Matric Pencil Method is presented. The method is implemented on a three degree of freedom system and the IASC-ASCE Benchmark problem. The identified modal parameters (poles) of the linear system using the Matrix Pencil Method have good accuracy. Meanwhile, these parameters can be identified even if there is only one DOF measurement provided that this measurement contains all the system information; however, to estimate all the mode shapes one would require measurements at all DOF. When all DOF measurements are available, all system mode shapes can also be identified. Two numerical simulation results demonstrated the effectiveness of the proposed method. The Matrix Pencil Method is robust to noise as well.

Acknowledgments

The authors wish to acknowledge the support of the *Texas Institute for the Intelligent Bio-Nano Materials and Structure for Aerospace Vehicles*, funded by NASA Cooperative Agreement no. NCC-1-02038.

References

Caicedo, J.M., Dyke, S.J. and Johnson, E.A. (2004), "Natural excitation technique and eigensystem realization algorithm for phase I of the IASC-ASCE benchmark problem: Simulated data", *J. Eng. Mech. - ASCE*, **130**(1), 49-60.

Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1996), "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review", Research Rep. No. LA-13070-MS, ESA-EA, Los Alamos National Laboratory, N.M..

Hua, Y. and Sarkar, T.K. (1989), "Generalized pencil-of-function method for extracting poles of an EM system from its transient response", *IEEE T. Antenn. Propag.*, **37**(2), 229-234.

Hua, Y. and Sarkar, T.K. (1990), "A perturbation property of the TLS-LP method", *IEEE T. Acoustics, Speech Signal Pr.*, ASSP-38, 2004-2005.

Hua, Y. and Sarkar, T.K. (1990), "On the total least squares linear prediction method for frequency estimation", *IEEE T. Acoustics, Speech Signal Pr.*, ASSP-38, 2186-2189.

Ibrahim, S.R. and Mikulcik, E.C. (1977), "A Method for the direct identification of vibration parameters from the free response", *Shock Vib. Bulletin*, **47**(4), 183-198.

James, G.H., Carne, T.G. and Lauffer, J.P. (1995), "The Natural Excitation Technique (NExT) for modal parameter extraction form operating structures", *Modal Analysis: the Int. J. Anal. Exp. Modal Anal.*, **10**(4), 260-277.

Johnson, E.A., Lam, H.F., Katafygiotis, L.S. and Beck, J. (2004), "Phase I IASC-ASCE structural health monitoring benchmark problem using simulated data", *J. Eng. Mech. - ASCE*, **130**(1), 3-15.

Juang, J.N. and Pappa, R.S. (1985), "An eigensystem realization algorithm for modal parameter identification and model reduction", *J. Guid. Control Dynam.*, **8**, 620-627.

Nagarajaiah, S. and Erazo, K. (2016), "Structural monitoring and identification of civil infrastructure in the United States", *Struct. Monit. Maint*., **3**(1), 51-69.

Nagarajaiah, S. and Yang, Y. (2016), "Modeling and harnessing sparse and low-rank data structure: A new paradigm for structural dynamics, identification, damage detection, and health monitoring", *Struct. Control Health Monit*., DOI: 10.1002/stc.1851.

Nagarjaiah, S. and Basu, B. (2009), "Output only modal identification and structural damage detection using time frequency & wavelet techniques", *Earthq. Eng. Eng. Vib*., **8**(4), 583-605, doi:10.1007/s11803-009-9120-6

Salawu, O.S. (1997), "Dectection of structural damage through changes in frequency: a review", *Eng. Struct.*, **19**(9), 718-723.

Sarkar, T.K. and Pereira, O. (1995), "Using the matrix pencil method to estimate the parameters of a sum of complex exponentials", *IEEE Antenn. Propag. M.*, **37**(1), 48-55.

Yang, J.N., Lei, Y., Pan, S. and Huang, N. (2003), "System identification of linear structures based on Hilbert-Huang spectral analysis. Part 1: normal modes", *Earthq. Eng. Struct. D.*, **32**, 1443-1467.

Yang, Y. and Nagarajaiah, S. (2013), "Output-only modal identification with limited sensors using sparse component analysis", *J. Sound Vib*., **332**(19), 4741-4765. DOI: 10.1016/j.jsv.2013.04.004