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Comparison of black and gray box models of subspace identification under support excitations

Diptojit Datta[a] and Anjan Dutta*

Department of Civil Engineering, Indian Institute of Technology Guwahati, Guwahati-781039, India

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Abstract. This paper presents a comparison of the black-box and the physics based derived gray-box models for subspace identification for structures subjected to support-excitation. The study compares the damage detection capabilities of both these methods for linear time invariant (LTI) systems as well as linear time-varying (LTV) systems by extending the gray-box model for time-varying systems using short-time windows. The numerically simulated IASC-ASCE Phase-I benchmark building has been used to compare the two methods for different damage scenarios. The efficacy of the two methods for the identification of stiffness parameters has been studied in the presence of different levels of sensor noise to simulate on-field conditions. The proposed extension of the gray-box model for LTV systems has been shown to outperform the black-box model in capturing the variation in stiffness parameters for the benchmark building.

Keywords: subspace identification; support-excited structures; time-varying systems; damage-detection

1. Introduction

The growing concern for the deteriorating conditions of world's infrastructure has led to the emergence of structural health monitoring as a field of paramount research interest. In the civil engineering community, structural health monitoring (SHM) is used to determine the condition or health of the structure and determine its suitability to sustain its purpose in the future. Many major SHM technologies are being implemented all over the world for assessing the performance of large scale civil infrastructure (Nagarajaiah and Erazo 2016). System identification plays a fundamental role in structural health monitoring since it is the tool which is used to create a mathematical model of a structural system, using measured data from the same structure (Ljung 1999). This mathematical model of a monitored structure, can show notable differences during different stages of its lifespan. These differences can be used to quantify and localize the damage in the structure, which helps in devising a suitable plan for retrofitting.

Introduced by Overschee and Moor (1994), subspace system identification emerged as a promising technique in the control theory community. Since then, the method has been popularly used in system identification in the control theory community. The method has gained popularity in the civil engineering community mostly because of its ability to determine the mathematical

*Corresponding author, Professor, E-mail: adutta@iitg.ernet.in
[a] M.Tech., Email: dipto.datta@gmail.com

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model of a structural system directly from experimental data, without the requirement of an explicit canonical parameterization. The use of ambient vibrations for monitoring the health of structures is very popular in civil engineering and vibrations due to ground motion is one of the most commonly studied problems (Borsaikia *et al*. 2011). Subspace identification, being an input-output based technique, is particularly popular for support-excited problems because the input ground motion is easily measurable. Since subspace identification is essentially a black-box data-driven approach, numerous studies have been done to render a physical interpretation to the state-space solution for the extraction of stiffness parameters. Xiao *et al.* (2001) proposed a technique to extract physical system matrices from a minimal realization, with prior knowledge of the structure's mass. Lus *et al.* (2003) suggested a transformation that maps the arbitrary state-space solution to one consistent with the symmetric eigenvalue problem. Caicedo *et al.* (2004) proposed a technique to extract stiffness parameters from the natural frequencies and mode shapes of a structure by solving an overdetermined system of equations through the least-squares approach. All these methods, however, relied on an indirect approach to obtain the stiffness parameters from the state-space matrices. Kim and Lynch (2012) proposed a direct method of extracting the stiffness parameters of a structure from the arbitrary state-space solution through a novel observability canonical form conversion of the system matrices. Their method is not only computationally efficient, but also succeeds in giving a physical interpretation to the arbitrary state-space solution obtained from subspace identification. This method, termed as the gray-box model, extended subspace identification into the realm of SHM.

Although subspace identification was primarily developed for linear time-invariant (LTI) systems, engineers constantly looked for strategies to extend the method for linear time-varying (LTV) systems. Verhaegen and Yu (1995) presented some algorithms for the subspace identification of LTV systems from an ensemble set of input-output measurements. Their study utilized the multivariable output error state space (MOESP) class of subspace algorithms. Liu (1997) proposed a subspace-based method for the identification of varying transition matrices using an ensemble of response sequences. The varying transition matrix at each instant is estimated by the singular value decomposition (SVD) of two consecutive Hankel matrices. Verdult and Verhaegan (2002) addressed the major problem of subspace identification for parameter-varying systems concerned with the large dimensions of the data matrices involved.

They suggested the use of the most dominant rows of the data matrices for estimation of the model and thus made the technique more computationally efficient. Shi *et al.* (2007) proposed using a series of Hankel matrices constructed directly from the combined input-output measurements. The equivalent state-space system matrices are then estimated at each instant by consecutively performing SVD of the Hankel matrices. The method showed promising results for a two degrees-of-freedom spring-mass-damper system. Marchesiello *et al.* (2010) introduced the so called 'frozen technique' for LTV systems based on the concept of dividing the entire response time-history into many windows in which the system is assumed to be time-invariant. This method also referred to as the short-time stochastic subspace identification (ST-SSI) uses short time windows superimposed and shifted by the sampling period. They also introduced the concept of angle variation of subspaces to predict the behavior of the LTV system in the near future.

After delving deep into the literature, it was found that subspace identification has tremendous potential for the identification of LTI systems as well as LTV systems. In this paper, a comparative study of the damage detection capabilities of black-box and gray-box models of subspace identification has been done for time-invariant and time-varying systems for structures subjected to ground acceleration. Although the gray-box model introduced by Kim and Lynch (2012) was primarily for the identification of time-invariant systems, its extension for LTV systems using the concept of short-time windows has been shown to possess great potential. A numerically simulated model of the IASC-ASCE Phase-I benchmark building (Johnson *et al.* 2004) subjected to support-excitation has been used to compare the two models for different damage scenarios. The effect of varying noise levels in the sensors have been simulated to show how the two models perform when the data is corrupted by noise. One time-invariant damage case and two time-varying damage cases have been investigated using the black-box and gray-box models. The time-varying damage cases have been simulated in such a way that the damage occurs in the structural system at some instant during the period of application of ground motion, and the stiffness parameters change instantaneously.

This paper is organized as follows: Sections 2 and 3 describe the theoretical background of the black-box and gray-box models of subspace identification for LTI systems. Section 4 extends the black-box and gray-box models for LTV systems using the concept of short time windows. Section 5 presents the results obtained for different damage cases on the IASC-ASCE Phase-I benchmark building. Finally, section 6 presents our conclusions.

2. Theory of subspace identification and the black-box model

2.1 Theory of subspace identification

Subspace identification is based on the principles of linear algebra and geometric projections, wherein, the history of outputs from a system are projected onto a space perpendicular to the history of the inputs to the system (Shahmedr and Mussa-Ilvaldi 2012). For a system without noise, the discrete state-space formulation can be written as

$$x^{n+1} = Ax^n + Bu^n \,; \qquad y^n = Cx^n + Du^n \tag{1}$$

where *A, B, C, D* are the system matrices,{u} is sequence of inputs given to the system, {y} is the sequence of outputs obtained from the system, and *x* is the *n*-dimensional unknown discrete state vector. The inputs and outputs are arranged in *Hankel* matrices as follows

$$Y_{1|i} = \begin{bmatrix} y^{(1)} & y^{(2)} & \cdots & y^{(j)} \\ y^{(2)} & y^{(3)} & \cdots & y^{(j+1)} \\ \vdots & \vdots & \vdots & \vdots \\ y^{(i)} & y^{(i+1)} & \dots & y^{(i+j-1)} \end{bmatrix} \tag{2}$$

$$U_{1|i} = \begin{bmatrix} u^{(1)} & u^{(2)} & \cdots & u^{(j)} \\ u^{(2)} & u^{(3)} & \cdots & u^{(j+1)} \\ \vdots & \vdots & \vdots & \vdots \\ u^{(i)} & u^{(i+1)} & \cdots & u^{(i+j-1)} \end{bmatrix} \tag{3}$$

where $i<<j$, i.e., the number of rows is much lesser than the number of columns. The unknown state sequence matrix X_i is defined as

$$X_i \equiv [x^{(1)} \quad x^{(i+1)} \quad \dots \quad x^{(i+j-1)}] \tag{4}$$

The observability matrix Γ_i is defined as

$$\Gamma_i \equiv \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix} \tag{5}$$

The known matrix $W_{1|i}$ containing the history of the inputs and outputs is defined as

$$W_{1|i} \equiv \begin{bmatrix} U_{1|i} \\ Y_{1|i} \end{bmatrix} \tag{6}$$

Projecting the history of observations $Y_{i+1|2i}$ onto the subspace perpendicular to the history of inputs $U^{\perp}_{i+1|2i}$ gives

$$\Gamma_i X_{i+1} = [Y_{i+1|2i}/U^{\perp}_{i+1|2i}][W_{1|i}/U^{\perp}_{i+1|2i}]^{*} W_{1|i} = O_{i+1} \tag{7}$$

where operator '/' denotes the geometric projection of a matrix on to another matrix, and * is the pseudo-inverse. Performing singular value decomposition (SVD) of O_{i+1} gives

$$O_{i+1} = PSV = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_i \end{bmatrix} \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ 0 & s_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_n \end{bmatrix} [v^{(1)} \quad v^{(2)} \quad \ldots \quad v^{(j)}] \tag{8}$$

where P and V^T are orthogonal matrices and S is the diagonal matrix containing the singular values for a system of order *n*. For the singular value decomposition (SVD) of the O_{i+1} matrix, weighing matrices can be used as follows

$$W_1 O_{i+1} W_2 = PSV \tag{9}$$

where W_1 and W_2 weighing matrices. Based on the different weighing matrices in Eq. (9), there are different classes of subspace identification such as N4SID, CVA, MOESP. Throughout this study, N4SID has been used where both the weighing matrices are equal to the identity matrix of order equal to the order of O_{i+1}. The estimate of the unknown state vector is obtained as

$$\hat{X}_{i+1} = S^{1/2} V \tag{10}$$

From the estimate of the state vector $\hat{X}_{i+1}$, the parameters of the system (matrices *A*, *B*, *C*, *D*) can be obtained from Eq. (1) by the method of least-squares. The modal parameters are obtained from the eigen value decomposition (EVD) of the system matrix *A* by $A = \Phi\Lambda\Phi^{-1}$, where Φ is the eigenvector matrix and Λ is the diagonal eigenvalue matrix. The discrete-time eigen values are converted to continuous time eigen values by the equation

$$\lambda_{ci} = \ln(\lambda_{di}) / \Delta t \tag{11}$$

where λ_{ci} is the continuous-time eigen value, λ_{di} is the discrete-time eigen value and Δt is the time step of the digital data acquisition system. The natural frequencies ω_{ni} and damping ratios ξ_i are then obtained from the conjugate pair of complex-valued eigenvalues as $\lambda_{ci}, \lambda_{ci}^{*} = -\xi_i\omega_{ni} \mp i\omega_{ni}\sqrt{(1-\xi_i^2)}$. The mode shape vector for the i^{th} mode $\phi_i \in C^n$ is calculated as $C\phi$.

The complex mode shapes obtained from subspace identification are converted to normal modes (Ranieri and Fabbrocino 2014). The stiffness parameters of the structural system can be obtained from this black-box model by solving an over-determined system of equations through the least-squares approach as outlined by Caicedo *et al.* (2004).

2.2 Stiffness parameters from black-box model

The characteristic equation for an *n-DOF* system can be written as

$$(K - \lambda_i M)\begin{Bmatrix} \phi_i^1 \\ \phi_i^2 \\ \vdots \\ \phi_i^n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{12}$$

where K and M are the stiffness and mass matrix respectively. $\lambda_i = \omega_i^2$ is the i^{th} eigen value and ϕ_i is the i^{th} mode shape vector. Eq. (12) is expanded and rearranged as

$$\Delta_i k = \Lambda_i \tag{13}$$

where

$$\Delta_i = \begin{bmatrix} \phi_i^1 & \phi_i^1 - \phi_i^2 & 0 & \cdots & 0 \\ 0 & \phi_i^2 - \phi_i^1 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \phi_i^{n-1} - \phi_i^{n-2} & \phi_i^{n-1} - \phi_i^n \\ 0 & 0 & \cdots & 0 & \phi_i^n - \phi_i^{n-1} \end{bmatrix} \tag{14}$$

$$k = \begin{Bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{Bmatrix} \tag{15}$$

$$\Lambda_i = \begin{bmatrix} \phi_i^1 \lambda_i m_1 \\ \phi_i^2 \lambda_i m_2 \\ \vdots \\ \phi_i^n \lambda_i m_n \end{bmatrix} \tag{16}$$

Substituting Eqs. (14)-(16), Eq. (13) may be assembled for all *n*-degrees of freedom & written as

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix}\begin{Bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{Bmatrix} = \begin{Bmatrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_n \end{Bmatrix} \tag{17}$$

The above is an over-determined system and k can be calculated by computing a pseudo-inverse denoted by * as

$$k = \Delta^* \Lambda \tag{18}$$

The K_B matrix from the black-box model for a shear-building may then be constructed as

$$K_B = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & k_{n-1} + k_n & -k_n \\ 0 & 0 & \cdots & -k_n & k_n \end{bmatrix} \tag{19}$$

This model of subspace identification described above is termed as the black-box model since the system matrices *(A, B, C, D)* are of an arbitrary nature and represent only one of the infinitely possible solutions of Eq. (1). Although the system matrix *A* obtained from the black-box model preserves the modal properties of the dynamic system, it does not represent the physics-based system matrix. A novel approach for subspace identification by linking the physics-based white-box model to the data-driven black-box model, termed as the gray-box model, was proposed by Kim and Lynch (2012) and is described in the subsequent section.

3. Gray-box model for subspace identification

The gray-box model of subspace identification for support-excited structures, as the name suggests, is derived by explicitly linking the data-driven black-box model, through the observability canonical form conversion, to the physics-based white-box model. This conversion gives a physically interpretable state-space solution for the dynamic system and facilitates the direct extraction of stiffness parameters from the system matrix *A*.

3.1 Physics-based white-box model

The equation of motion for a multi-degree of freedom system subjected to support acceleration is given by

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = -M\{1\}\ddot{u}_g(t) \tag{20}$$

where M, C, K, represent the mass, damping force and stiffness matrices respectively. In addition, $u(t)$ and $\ddot{u}_g(t)$ represent the relative displacement vector and the support acceleration respectively.

The state vector is defined as

$$x(t) \equiv [\ddot{u}(t)^T \;\; \ddot{u}(t)^T]^T \tag{21}$$

Next, the state-space physics-based model is defined in continuous-time domain as:

$$\begin{aligned} \dot{x}(t) &= A_{cp}x(t) + B_{cp}\ddot{u}_g(t) \\ y(t) &= C_{cp}x(t) + D_{cp}\ddot{u}_g(t) \end{aligned} \tag{22}$$

where

$$A_{cp} = \begin{bmatrix} 0 & I \\ M^{-1}K & -M^{-1}C \end{bmatrix} \in R^{2nx2n} \tag{23}$$

$$B_{cp} = \begin{bmatrix} \{0\} \\ -\{1\} \end{bmatrix} \in R^{2n} \tag{24}$$

$$C_{cp} = [I \quad 0] \in R^{nx2n} \tag{25}$$

$$D_{cp} = \{1\} \in R^{n} \tag{26}$$

3.2 Gray-box model

The discrete system matrices obtained from the black-box model as A_d, B_d, C_d, D_d from Eq. (1) are converted to the continuous form as

$$A_c = \frac{1}{\Delta t}\ln(A_d) \tag{27}$$

$$B_c = \left(\int_0^{\Delta t} \exp(A_c\,\tau)\, d\tau)\right)^{-1} B_d \tag{28}$$

$$C_c = C_d\,; \;\; D_c = D_d \tag{29}$$

For any invertible and non-singular matrix T, the following linear transformation of the system matrices (A_c, B_c, C_c, D_c) is also a solution of Eq. (22).

$$A_c{}' = TA_cT^{-1}\,; \;\; B_c' = TB_c\,; \;\; C_c{}' = C_cT^{-1}\,; \;\; D_c{}' = D_c \tag{30}$$

The observability canonical form conversion requires that this invertible non-singular matrix be set equal to the observability matrix Γ i.e., (T= Γ). The observation matrix C_c is expressed as follows

$$C_c = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \tag{31}$$

where c_1 ... c_n are *n* row vectors. With the use of row-wise expressions for C_c, the observability matrix is composed as follows

$$\Gamma = \begin{bmatrix} c_1^T & \cdots & (c_1A_c{}^{\gamma-1})^T & c_2^T & \cdots & (c_2A_c{}^{\gamma-1})^T & \cdots & c_n{}^T & \cdots & (c_nA_c{}^{\gamma-1})^T \end{bmatrix} \tag{32}$$

where the observability indices γ in Eq. (32) are equal to 2.

After the observability canonical form conversion, the matrix A_{cc} is obtained in the form as shown

$$A_{cc} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ * & * & * & * & \cdots & \cdots & * & * \\ 0 & 0 & 0 & 1 & \cdots & \cdots & 0 & 0 \\ * & * & * & * & \cdots & \cdots & * & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 \\ * & * & * & * & \cdots & \cdots & * & * \end{bmatrix} \in R^{2nx2n} \tag{33}$$

where * corresponds to non-zero elements. Since the order of the states in the experimentally identified model is different from that of the pre-defined states in the physics-based model, reordering of the states in matrix A_{cc} is required. After suitably reordering the states in Eq. (33), the matrix A_{cg} of the gray-box model is obtained in the form shown below.

$$A_{cg} = \begin{bmatrix} 0 & I \\ X & Y \end{bmatrix} \in R^{2nx2n} \tag{34}$$

Comparing the gray-box system matrix A_{cg} in Eq. (34) with the physics-based system matrix A_{cp} in Eq. (23), the stiffness and damping matrices from the gray-box model are obtained as

$$\begin{aligned} K_G &= -MX \\ C_G &= -MY \end{aligned} \tag{35}$$

4. Subspace identification for linear time-varying systems

Time-invariant models are generally considered sufficient to evaluate the dynamic properties of structures subjected to service loads. However, under certain circumstances, when the loading conditions are extreme (such as during earthquakes), the structure may undergo significant damage such that the material or geometric properties change considerably with time. Changes in such properties leads to a modification in the dynamic characteristics of the system such as stiffness and damping. Modelling a system as time-invariant in such cases would lead to significant errors in the estimated stiffness and damping parameters of the system. In such cases, a linear time-varying (LTV) model will capture the transition in the system more effectively, and should be used to assess the condition of the system or to diagnose its failure. For time-invariant systems, the stiffness of the system doesn’t change during the analysis. However, for time-varying systems, the stiffness of the system changes during the analysis and this makes the system identification of time-varying systems a more challenging problem. In this study, subspace identification of time-varying systems is carried out using the technique of dividing the LTV system into a number of discrete LTI systems using short time-windows, with the assumption that the system remains time-invariant during the length of the chosen time-window (Moaveni and Asgarieh 2012).

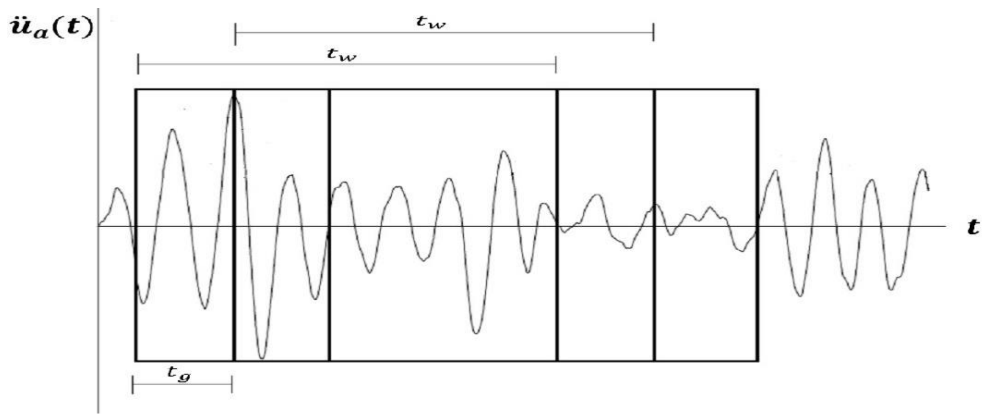


Fig. 1 Division of a sample response time-history into short time windows

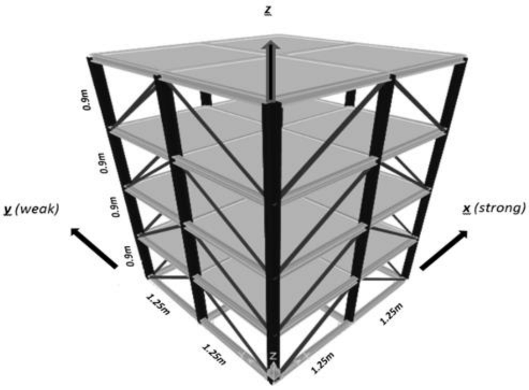


Fig. 2. IASC-ASCE Phase-I structural health monitoring benchmark building

4.1 Subspace identification using short-time windows

Let $\ddot{u}_a(t)$ be the absolute acceleration response measured from a certain sensor of an LTV system, as shown in Fig. 1. The time-history is divided into a number of segments using time windows as shown. The length of each time window (t_w) is a crucial parameter in the identification technique since the window is required to be small enough so that the assumption of the LTI system is valid. It is also required to be large enough such that the requirement of a large data set

for subspace identification is also met. Generally, a time window (t_w) of at least 300 time steps is found to be necessary to meet the requirement of data set length. The time window is generally kept the same throughout the process. The time windows overlap with each other and there is a time gap (t_g) between the start of one time window and the start of the consecutive time window.

The time gap (t_g) is generally kept at around 20-50 time steps. This time gap is also a very important parameter and determines how effectively the variations in stiffness is captured in the identification. After dividing the input and output time-histories into such windows, subspace identification is performed using both the gray-box and black-box models separately, for each window, using the data-set of the respective windows. The identified system parameters from each window are assigned to the central point of that window (Marchesiello *et al.* 2010).

5. Numerical example

The numerically simulated model of the IASC-ASCE Phase-I structural health monitoring benchmark building has been used for the comparison of damage detection capabilities of the the gray-box and black-box models. The structure, as shown in Fig. 2, is a 4-storey, 2-bay by 2-bay steel frame structure located in the Earthquake Engineering Research Laboratory at the University of British Columbia (UBC). It has a floor plan of 2.5 m × 2.5 m and is 3.6 m high. The members are made of hot rolled grade 300W steel with a nominal yield stress of 300 MPa. The columns are all oriented to be stronger in bending about the *y*-axis. The bracings are assumed to have no bending stiffness.

The masses of the first, second, third and fourth storeys are 3200 kg, 2400 kg, 2400 kg and 1600 kg respectively. The building is modelled as an 8-DOF system with one translational degree of freedom along each of the two axes (x and y) on each floor. A constant modal damping ratio of 5% is considered for all the modes. A 30-sec duration El Centro (1940) is applied along both the axes simultaneously. The peak ground accelerations are scaled to 0.053 g and 0.044 g along the x-axis and y-axis respectively. Eight translational acceleration responses and the two input ground accelerations are measured from the model, each sampled at 50 Hz. The output data channels are fed through a fourth order Butterworth filter with a cut-off frequency of 25 Hz. To assess the damage detection capabilities of the two subspace models, three different damage scenarios have been studied. One time-invariant and two time-varying damage cases have been investigated wherein, damage is incorporated by changing the stiffness parameters without any change in the damping properties of the structure. The mass of the system is assumed to be known and remains unchanged throughout the study. To simulate on-field conditions, a zero-mean gaussian white-noise is added to the data channels from each sensor. The damage cases are defined as follows:

Time-invariant damage case:

- Damage Case-I: No stiffness in the braces of the first storey (i.e., the braces contribute in mass, but provide no resistance).

Time-varying damage case:

(i) Linear variation

- Damage Case-II: The first storey stiffness linearly drops by 10% of its initial value from time instant $t=10s$ to $t=20s$.

(ii) Sudden variation

- Damage Case-III: The first storey stiffness suddenly drops by 20% of its initial value at time instant $t=10s$.

5.1 Results and discussion

5.1.1 Time-invariant case

Table 1 shows the percentage errors in damaged first storey stiffness obtained for different signal-to-noise ratios (SNR) for the gray-box and black-box models along the x-direction for damage case-I. The errors in estimated damaged storey stiffness e are calculated with respect to the actual damaged storey stiffness as

$$e = \frac{|K_a - K_i|}{K_a} \times 100 \tag{36}$$

Table 1 Percentage errors in damaged storey stiffness(case-I) for different noise levels along x-direction

Noise Level (SNR in dB)	Black-Box	Gray-Box
No Noise	0.62	0.56
120	0.87	0.90
100	2.54	1.29
80	5.35	3.58
60	9.66	5.76
40	12.38	7.93

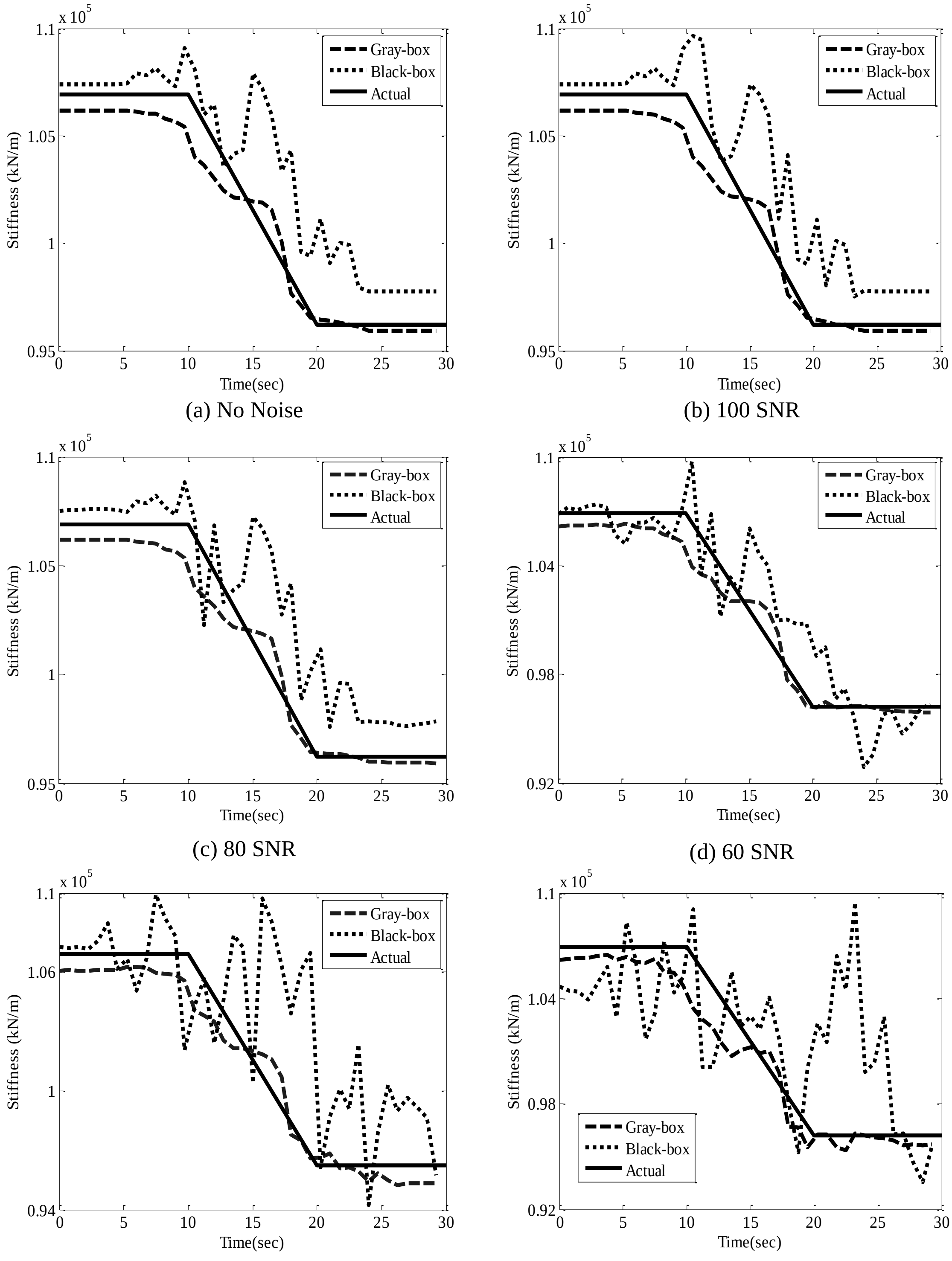
where K_a represents the actual damaged storey stiffness and K_i represents the identified damaged storey stiffness. The results obtained from the black-box and gray-box models at lower noise levels (upto 120 dB SNR) show similar accuracies. However, as the noise is increased (above 80 SNR), the gray-box models provides more accurate results than the black-box model. Similar results are obtained along the weaker (y-direction) as well.

5.1.2 Time-varying case

The identified variation in the damaged storey stiffness for damage case-II and III along the x-direction have been shown in Figs. 3 and 4. The sampling period for data acquisition is 0.02s and the data is obtained for the entire 30s of the ground motion and thus contains 1500 data points for each set of inputs and outputs. A time window length of $t_w = 300$ time steps, i.e., 6s and a time gap of $t_g = 30$ time steps i.e. 0.6s has been taken. Since the system is modelled as an 8-DOF system, a model order of 16 has been taken for both the black-box and gray-box models. Looking at the identified variations in the storey stiffness, it is observed that at lower noise levels both the models capture the variations in stiffness reasonably well. However, at higher noise levels, the gray-box model clearly gains an upper hand. At noise levels as high as 40 dB SNR, the black-box model yields unacceptable results, while the gray-box model still manages to capture the variation fairly well. It is also observed that for both the linearly time-variant and abrupt variation cases, the estimated stiffness starts to drop earlier than the actual time instant. This is expected since some of the time windows will contain data from the damaged as well as undamaged state of the structure. The stiffness parameters obtained from these intermediate time windows will naturally show a drop from the initial value because the stiffness from each time window is assigned to a time instant corresponding to the center of the window.

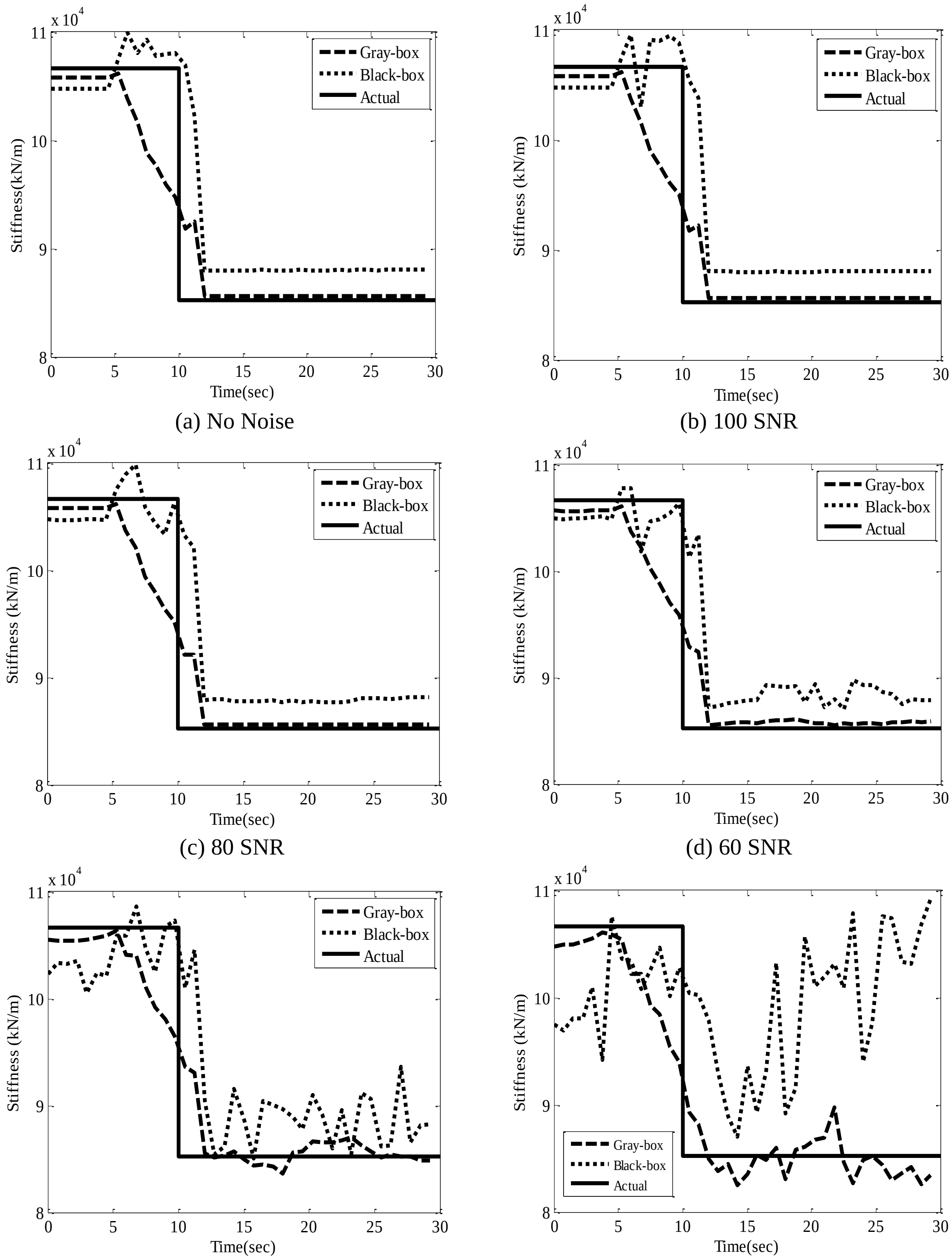
5.1.3 Effects of changing the time-gap parameter t_g

The estimated variations in stiffness with different time-gap (t_g) parameters for damage case-II at a constant noise level of 100dB SNR are shown in Fig. 5. Counterintuitively, a very small time-gap does not lead to the best possible results, which can be clearly seen in the black-box estimation (Fig. 5(a)). The smallest possible time-gap of a single time step, or the sampling period (Marchesiello *et al.* 2010) of *0.02s* produces instability at the instant when the stiffness starts to drop. It is also inefficient since it increases the computational effort. On the other hand, a very large time-gap of half the window size (Moaveni and Asgarieh 2012), i.e., *150* time steps, yields lesser number of stiffness data points and is thus not able to capture the variations properly. Hence, the black-box model appears to be very sensitive to the time-gap parameter. The gray-box model, however, yields impressively good results (Fig. 5(b)) even with shorter time-gap parameters. Similar results were obtained for damage case-III and different noise levels. In all, it is observed that the gray-box model is far less sensitive to the time-gap parameter as compared to the black-box model. One possible solution to the overlapping problem is to increase the sampling frequency, such that sufficient number of data points can be obtained without the requirement of any overlapping between successive time windows. For example, if the sampling frequency is increased to 300 Hz, a time window of $t_w = 1s$ will provide sufficient data points without any overlapping. In this study, however, practical conditions have been kept in mind and a reasonable on-field sampling frequency of 50Hz has been taken. Hence for such cases, a time-gap of more than 10% of the window size is recommended.



(a) No Noise
(b) 100 SNR
(c) 80 SNR
(d) 60 SNR
(e) 50 SNR
(f) 40 SNR

Fig. 3 Estimated varying storey stiffness for damage case-II with different noise levels

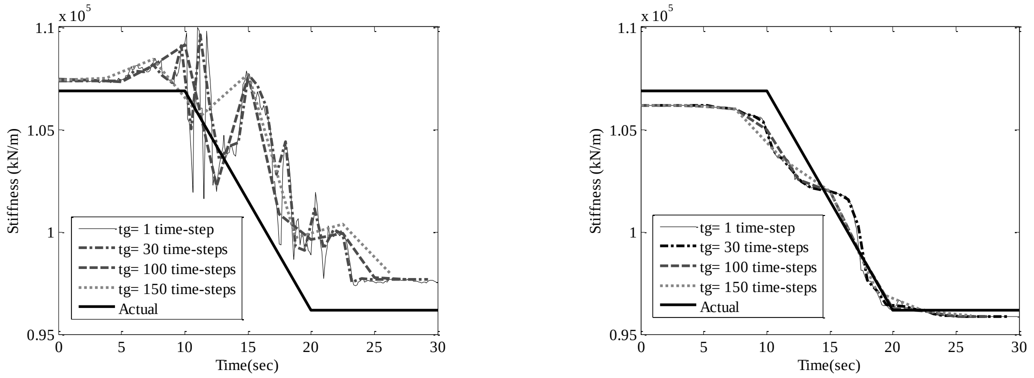


(a) No Noise (b) 100 SNR

(c) 80 SNR (d) 60 SNR

(e) 50 SNR (f) 40 SNR

Fig. 4 Estimated varying storey stiffness for damage case-III with different noise levels



(a) Black-box (b) Gray-box

Fig. 5 Effect of the change in time-gap parameter (t_g) on the estimated storey stiffness (along *x*-direction) (damage case-II) for the black-box and gray-box models at a constant noise level of 100 dB SNR

5.1.4 Possible reasons for the observed results

The underlying mathematical concepts behind both the black-box model and gray-box models are similar to a great extent since both these models are derived from the subspace framework. Yet, the performance of these models are noticeably different as far as stiffness estimation is concerned. This difference becomes more obvious when the data is highly corrupted by sensor noise. One possible reason for this observation can be attributed to the fact that the gray-box model extracts the stiffness parameter directly from the state-space solution. The black-box model, however, requires intermediate mathematical operations which include computation of mode shapes and natural frequencies. The mode shapes obtained are in complex form and need to be converted to real modes. Some error creeps in during this conversion. Moreover, the stiffness matrix is evaluated through a least-squares approach and this also leads to some error. In the presence of high noise, these errors accumulate at each step and this is probably the reason why the gray-box model outperforms the black-box model.

6. Conclusions

This paper compares the traditionally used black-box model and the newly introduced gray-box model of subspace identification. The mathematical concepts underlying these two models are explained in detail. The numerically simulated model of the IASC-ASCE Phase-I benchmark building is used to assess the damage detection potential of the two models. The model of the benchmark building is analysed for scaled El Centro earthquake excitation. Absolute acceleration time-history data is obtained from eight different channels from the structure. To simulate real-world conditions, a zero-mean gaussian white-noise is added to the sensor data of each of the eight channels. Three different damage scenarios are investigated with different levels of sensor noise. Although the two models performed with a relatively similar accuracy in detecting damage at low noise levels, the gray-box model has been found to perform better than the black-box model at high noise levels for LTI systems. The concept of short-time windows has been utilized to extend the gray-box model for the identification of LTV systems. For LTV systems, the gray-box model managed to capture the stiffness variation with more finesse than the black-box model. At very high noise levels of 40dB SNR, the black-box model yielded unusable results. On the other hand, the gray-box model still managed to produce reasonably accurate results. The gray-box model is also found to be far less sensitive to the user-defined time-gap parameter as compared to the black-box model. In all, this study concludes that the gray-box model is a very good candidate for assessing instantaneous damage in structural systems.

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