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Damage identification for high-speed railway truss arch bridge using fuzzy clustering analysis

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Abstract. This study aims to perform damage identification for Da-Sheng-Guan (DSG) high-speed railway truss arch bridge using fuzzy clustering analysis. Firstly, structural health monitoring (SHM) system is established for the DSG Bridge. Long-term field monitoring strain data in 8 different cases caused by high-speed trains are taken as classification reference for other unknown cases. And finite element model (FEM) of DSG Bridge is established to simulate damage cases of the bridge. Then, effectiveness of one fuzzy clustering analysis method named transitive closure method and FEM results are verified using the monitoring strain data. Three standardization methods at the first step of fuzzy clustering transitive closure method are compared: extreme difference method, maximum method and non-standard method. At last, the fuzzy clustering method is taken to identify damage with different degrees and different locations. The results show that: non-standard method is the best for the data with the same dimension at the first step of fuzzy clustering analysis. Clustering result is the best when 8 carriage and 16 carriage train in the same line are in a category. For DSG Bridge, the damage is identified when the strain mode change caused by damage is more significant than it caused by different carriages. The corresponding critical damage degree called damage threshold varies with damage location and reduces with the increase of damage locations.

Keywords: railway bridge; steel truss arch; structural health monitoring; damage identification; fuzzy clustering; finite element analysis

1. Introduction

In the past few decades, structural health monitoring (SHM) has been one of the most popular research areas in the bridge engineering field (Garden and Fanning 2004, Farrar and Worden 2007, Ou and Li 2010 and Yu and Xu 2011). SHM process is to collect data from the monitored structure using periodically sampled measurements by an array of sensors, then extract features from these measurements and conduct statistical analysis of these features to assess the structural degradation (Fan and Qiao 2011, Sabatto, Mikhail *et al.* 2011 and Kovvali, Das *et al.* 2007).

The detection of damage is the most fundamental issue in SHM. Damage may be defined as a state of change that affects the present or future performance of a system. Implicit in the above definition is the fact that damage detection involves comparison with some initial undamaged state

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(Meyyappan, Jose *et al.* 2003). In this project the sensors were connected to the bridge, which was monitored. SHM system with a great quantity of various types of sensors is usually employed by large infrastructure engineering for long-term health monitoring. As an alternative to field monitoring method, test and numerical simulation methods are also adopted as a supplement in research bridge damage (Yu, Zhu *et al.* 2011 and Erdogan, Catbas *et al.* 2014). The numerical analysis model is calibrated using SHM data and better represents the existing structure behavior under different loading conditions.

Recently, the fuzzy approaches have been applied to solve damage detection problems. Fuzzy logic is utilized to handle uncertainties and imprecision involved. Fuzzy clustering is an unsupervised learning operation that aims at decomposing a given set of objects into subgroups or clusters based on similarity. The goal is to divide the dataset in such a way that objects or cases belonging to the same cluster are as similar as possible, whereas objects belonging to different clusters are dissimilar (Kruse, Doring *et al.* 2007). Fuzzy cluster analysis methods mainly include: transitive closure method based on fuzzy equivalence relation, the method based on similarity relation and fuzzy relationship, the maximum tree method based on fuzzy graph theory and the convex decomposition based on data sets and the dynamic rules (Zhou, Zhang *et al.* 2015).

Fuzzy clustering method has been used in many areas by researchers. Tarighat and Miyamoto (2009) introduced a new fuzzy method to deal with uncertainties from inspection data, which was practically based on both subjective and objective results of existing inspection methods and tools. Wang and Elhag (2007) proposed a fuzzy group decision making (FGDM) approach for bridge risk assessment. Silva, Dias *et al.* (2008) compared two fuzzy clustering algorithms: fuzzy c-means (FCM) and Gustafson–Kessel (GK) algorithms by applying them to data from a benchmark frame structure in the Los Alamos National Laboratory. Palomino, Steffen *et al.* (2014) and Salah, Sabatto *et al.* (2013) use fuzzy cluster analysis methods for aircraft's damage classification. Zhou, Zhang *et al.* (2015) evaluate health state of shield tunnel SHM using fuzzy cluster method. Zhao and Chen (2002) use fuzzy inference system to do concrete bridge deterioration diagnosis. Jiao, Liu *et al.* (2013) assess durability of the bridge based on fuzzy clustering and field data. Meyyappaq, Jose *et al.* (2003) has done damage accumulation analysis based on bridge health monitoring vibration data using fuzzy-neuro system.

Even though many researches have done damage analysis of different kinds of structures using fuzzy logic, there are few studies on high-speed railway truss arch bridges according to previous studies, especially based on field monitoring data. Nanjing DSG Bridge is a steel truss arch bridge with the longest span throughout the world. Its 336 m main span and 6-track railways rank itself the largest bridge with heaviest design loading among the high-speed railway bridges by far. And the design speed 300 km/h is also one of the most advanced level in the world. Thus damage identification of DSG Bridge is important. In this study, long-term field monitoring sensors are installed on the Nanjing DSG Bridge to collect strain extreme value caused by high-speed trains. The finite element model of DSG Bridge is also established to study damage as a supplement. Then, effectiveness of fuzzy clustering method and FEM results are verified using SHM data. Three standard methods are compared in the fuzzy clustering analysis. Finally, the fuzzy clustering method is taken to identify damage with different degrees and locations.

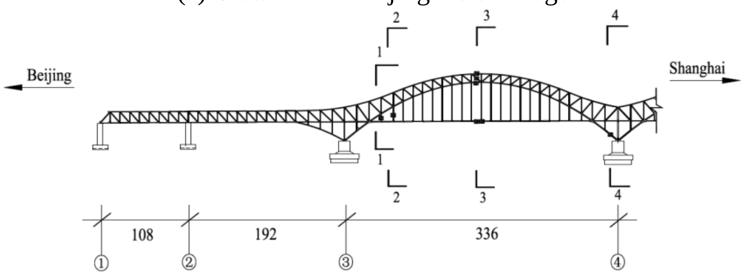
2. SHM system of DSG Bridge

The panoramic view of Nanjing DSG Bridge is shown in Fig. 1(a), which is a steel truss arch bridge with the span arrangement (108+192+336+336+192+108) m. The elevation drawing of the bridge is shown in Fig. 1(b). Due to the remarkable characteristics of DSG Bridge including long span of the main girder, heavy design loading and high speed of trains, a long-term SHM system was installed on the DSG Bridge shortly after it was opened to railway traffic. As shown in Fig. 1(b), dynamic strain monitoring of steel truss arch is performed at the 1-1, 2-2, 3-3 and 4-4 cross-section in the first main span of the bridge. Location of twenty strain sensors on the bridge is shown in Fig. 2 and instructions of these sensors are given in Table 1. Sampling frequency of dynamic strain data collection is set to 50 Hz.

Table 1 Location instructions of twenty strain sensors

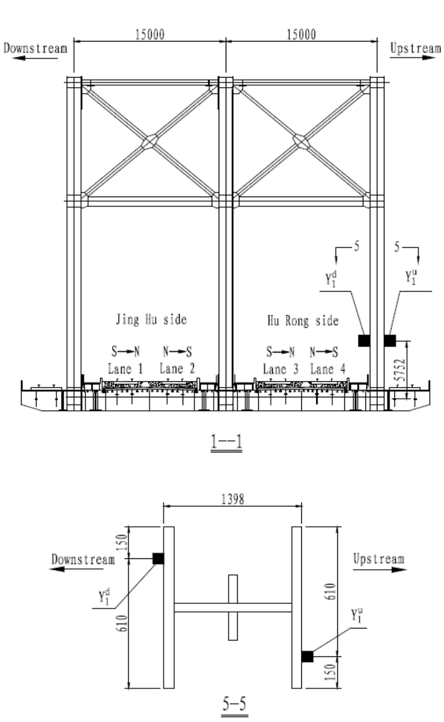
Cross-section number of bridge	Strain sensors number	Location instructions
1-1 cross section	Y_1^u,Y_1^d	5-5 section of hanger
2-2 cross section	Y_2^u,Y_2^d	6-6 section of hanger
3-3 cross section	Y_3^u,Y_3^d	8-8 section of top chord member
	Y_4^u,Y_4^d	9-9 section of diagonal web member
	Y_5^u,Y_5^d	10-10 section of bottom chord member
	Y_6^u,Y_6^d	11-11 section of deck chord member
	Y_7,Y_8	on the steel deck plate member
	Y_9,Y_{10}	on the horizontal beam member
4-4 cross section	Y_{11}^u,Y_{11}^d	14-14 section of arch foot chord member
	Y_{12}^u,Y_{12}^d	14-15 section of arch foot chord member

(a) View of the Nanjing DSG Bridge

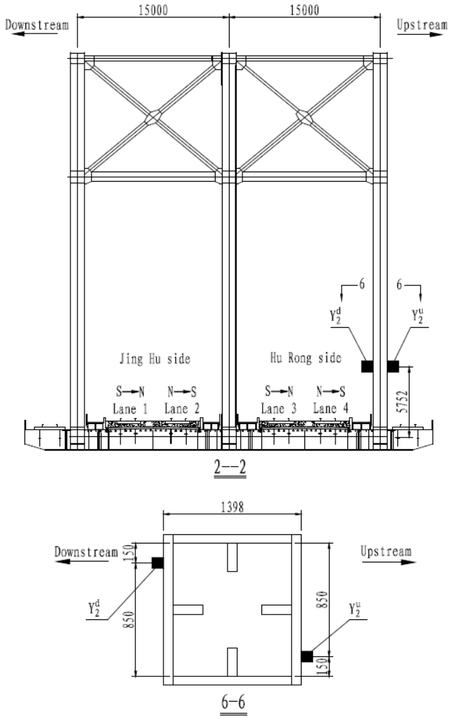


(b) Elevation drawing of half part bridge (Unit: m)

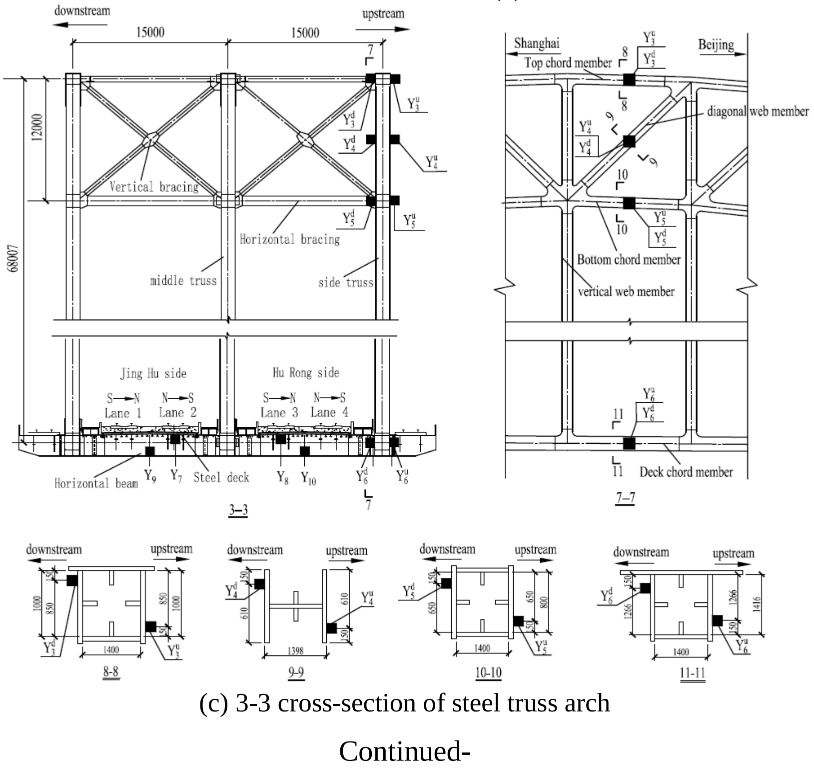
Fig. 1 Nanjing DSG Bridge



(a) 1-1 cross-section of steel truss arch



(b) 2-2 cross-section of steel truss arch



(c) 3-3 cross-section of steel truss arch

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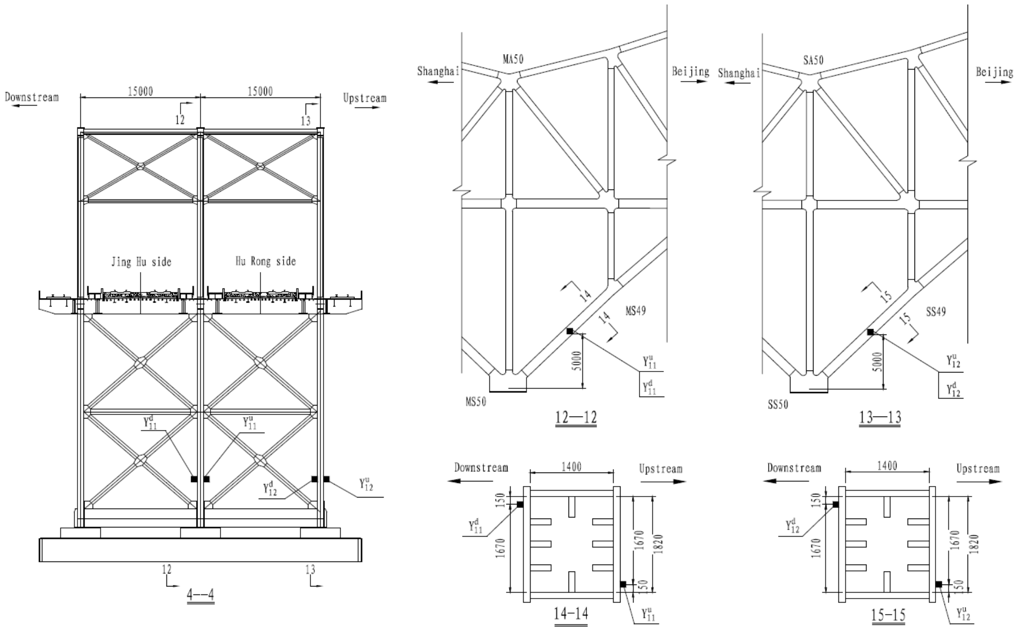


(d) 4-4 cross-section of steel truss arch

Fig. 2 Location of strain sensors on the steel truss arch bridge (unit: mm)

3. Finite element model of DSG Bridge

As an alternative to the field monitoring method, we can also obtain strain value of DSG Bridge by finite element modeling (FEM) method. DSG Bridge operates well and no damage has appeared till now in practical. The strain state of DSG Bridge in damage can be obtained through finite element simulation. Then damage identification method and damage indicators are introduced. Finally, damage can be identified based on SHM data using a certain method when the bridge really suffer damage during the future service.

Fig. 3 shows the three-dimensional finite element model of the DSG Bridge using ANSYS software. A total of 59760 nodes and 112706 elements are built in the model, 58370 of which are beam elements and 54336 of which are shell elements. The top chords, bottom chords, deck chords, diagonal web members, vertical web members, horizontal and vertical bracings of the steel truss arch are simulated by BEAM188 element; the diaphragm members and top plates of the steel bridge deck are simulated by SHELL181 element. Moreover, the finite element model has 7 bearings. The restraints of 7 bearings are set as follows: the middle bearing is constrained with three degrees of translational freedom in directions of longitudinal X, transverse Y, and vertical Z; the other bearings are constrained with two degrees of translational freedom in directions of transverse Y and vertical Z. The elastic modulus and Poisson ratio of the steel is assigned as 210GPa and 0.30. The acceleration of gravity is set to 9.8 m/s^2 and the damping ratio is set to 0.02.

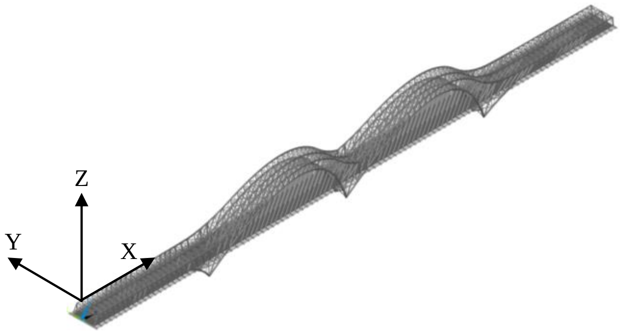


Fig. 3 Three-dimensional FEM of Nanjing DSG Bridge

4. Theory of fuzzy clustering

Traditional sample classification method belongs to supervised learning style which realizes the classification through specific standards. However, fuzzy clustering method can conduct the process based on properties of the sample characteristics, and it is unsupervised. The criterion for classification is not consistent and possesses apparent dynamic characteristics. It can establish the uncertainty description of samples and more precisely reveals the actual situation (Sebzalli and Wang 2001, Podofillini, Steffen *et al.* 2010 and Li 2004). Steps of one fuzzy clustering analysis method named transitive closure method used in this paper are given as follows.

(1) Standardization for clustering data

$X=\{x_1, x_2, \cdots, x_n\}$ is the vector of data for classification, and each data possesses *m* properties. x_i can be represented by Eq. (1).

$$x_i=[x_{i1}, x_{i2}, \cdots, x_{im}] \tag{1}$$

An original data matrix can be constructed as (2).

$$X=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \tag{2}$$

where x_{ij} is the *j*th property of the *i*th classification object.

The first step for fuzzy clustering analysis is standardization. That is transforming original data to the interval [0, 1] in order to eliminate dimensional effect and making each property do same contribution to the analysis. There are many standardization methods such as standard deviation method, extreme difference method, mean value method, center method, and logarithm method and so on. Extreme difference method shown in Eq. (3) is the most widely used in many papers.

① Standard1-extreme difference method

$$x_{ij}^{'} = \frac{x_{ij} - x_{j\min}}{x_{j\max} - x_{j\min}} \quad i = 1, 2, \cdots, n; \; j = 1, 2, \cdots, m \tag{3}$$

$$x_{j\max} = \max\left\{x_{1j}, x_{2j}, \cdots, x_{nj}\right\}, \; x_{j\min} = \min\left\{x_{1j}, x_{2j}, \cdots, x_{nj}\right\}$$

Step1: $$\overline{x_{ij}} = x_{ij} - x_{j\min} \quad i = 1, 2, \cdots n; \; j = 1, 2, \cdots m \tag{4a}$$

Step2: $$x_{ij}^{'} = \frac{\overline{x_{ij}}}{\overline{x_{j\max}}} \quad i = 1, 2, \cdots, n; \; j = 1, 2, \cdots, m \tag{4b}$$

Standard1 method can be divided into two steps just shown as Eqs. (4(a)) and (4(b)). The first step shown in Eq. (4(a)) is each member x_{ij} in the original matrix subtracts the minimum member $x_{j\min}$ of each column. Then we get a new matrix. The second step shown in Eq. (4(b)) is each element $\overline{x_{ij}}$ in the new matrix divided by the maximum $\overline{x_{j\max}}$ of each column to transform data to the interval [0, 1]. We can see the first step in this place is not necessary to eliminate dimensional effect. So we can try to skip the first step and only do the second step. This is standard2 method shown in Eq. (5).

② Standard2-the maximum method

$$x_{ij}^{'} = \frac{x_{ij}}{x_{j\max}} \quad i = 1, 2, \cdots, n; \; j = 1, 2, \cdots, m \tag{5}$$

Take each row of the original data matrix for classification as a *m* dimension vector $x_i = \{x_{i1}, x_{i2}, \cdots, x_{im}\}, \; i = 1, 2, \cdots n$. Fuzzy clustering analysis is to compare the relationship between these different rows according to the m different properties. Then do classification for the *n* row vectors. Both standard methods above have transformed the original data and brought changes to some extent about the relationship between the row vectors. And in the problem which will be analyzed in this paper, the dimensional for each property is the same. So we could also not standardize the original data and not disturb the original characteristic as much as possible. This idea brings the third method that is the non-standard method.

(2) Construction of fuzzy similarity matrix

Fuzzy similarity matrix is constructed mainly according to distance or ratio of data. Similarity coefficient r_{ij} describes the similarity degree between x_i and x_j . r_{ij} calculation methods mainly include dot product method, angle cosine method, correlation coefficient method, exponent similarity coefficient, the maximum minimum method and so on. In this paper, Similarity coefficient r_{ij} will be obtained by calculating the angle cosine value between x_i and x_j . It is defined as Eq. (6)

$$r_{ij} = \frac{\sum_{k=1}^{m} x_{ik}^{'} \cdot x_{jk}^{'}}{\sqrt{\sum_{k=1}^{m} x_{ik}^{'2}} \cdot \sqrt{\sum_{k=1}^{m} x_{jk}^{'2}}} \quad (i, j = 1, 2, \cdots, n) \tag{6}$$

(3) Calculate fuzzy equivalent matrix

The fuzzy similarity matrix calculated by Eq. (6) satisfies the reflexivity and symmetry but does not satisfy transitivity. The corresponding fuzzy equivalent matrix which satisfies reflexivity, symmetry and transitivity must be obtained in order to do clustering analysis. In this paper, the successive square method is used to calculate the equivalent matrix as shown in Eq. (7).

$$R^* = t(R) = R^{2k}, \quad R^{2k} = R^{2k-1} \tag{7}$$

R^* is the fuzzy equivalent matrix. By selecting appropriate thresholds $\lambda \in [0,1]$, truncated matrix $R^*_\lambda = t_\lambda(R)$ is obtained.

(4) Determination of best classification

$X = \{x_1, x_2, \cdots, x_n\}$ is the object for classification. $x_j = [x_{j1}, x_{j2}, \cdots, x_{jm}]$ is the *j*th member of $X(j = 1, 2, \cdots n)$. And x_{jk} is the *k*th feature of $x_j (k = 1, 2, \cdots, m)$. r is the classification number corresponding to λ, and n_i is the number for the *i*th category. The average value for *k*th eigenvalue of *i*th category can be calculated as shown in Eq. (8).

$$\overline{x_{ik}} = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{jk}, \quad k = 1, 2, \cdots, m \tag{8}$$

The average value for *k*th eigenvalue of all data can be calculated by Eq. (9).

$$\overline{x_k} = \frac{1}{n}\sum_{j=1}^{n} x_{jk}, \quad k = 1, 2, \cdots, m \tag{9}$$

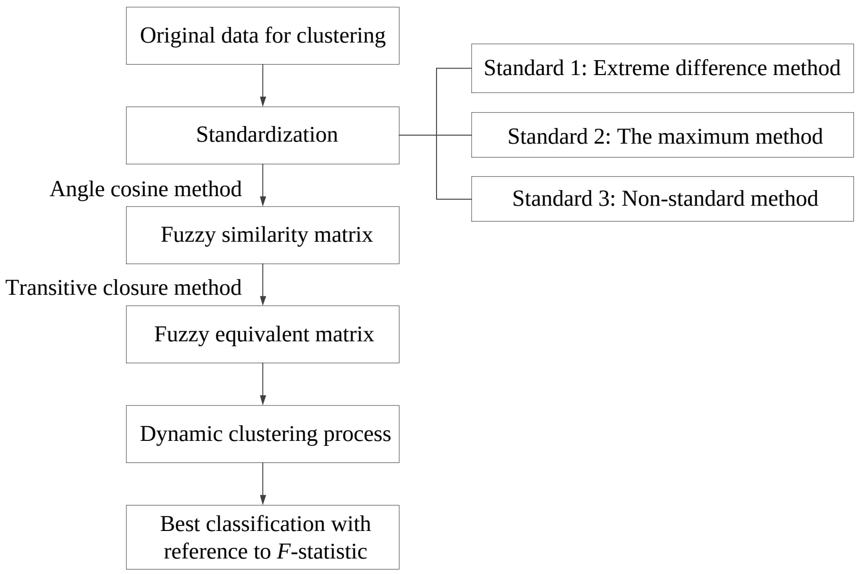


Fig. 4 Flow chart of transitive closure fuzzy clustering analysis method

F-statistics analysis is used for determining the best classification threshold; it can be calculated by Eq. (10). *F*-statistics obeys distribution $F(r-1, n-r)$. Its numerator stands for the distance between different categories while its denominator stands for the distance of samples in one category. So, the bigger the *F* is, the further distance between different categories is. If $F > F_{0.05}(r-1, n-r)$, the classification results are reasonable. And the bigger (F-$F_{0.05}$) value is, the better the classification results is.

$$F = \frac{\sum_{i=1}^{r} n_i \sum_{k=1}^{m} (\overline{x_{ik}} - \overline{x_k})^2 / (r-1)}{\sum_{i=1}^{r} \sum_{j=1}^{n_i} \sum_{k=1}^{m} (x_{jk} - \overline{x_{ik}})^2 / (n-r)} \tag{10}$$

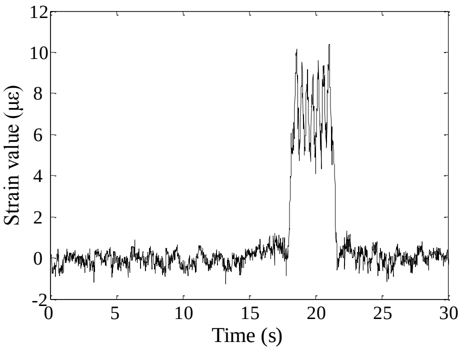
The flow chart of transitive closure fuzzy clustering analysis method is summarized in Fig. 4.

5. Effectiveness verification for fuzzy clustering method and FEM

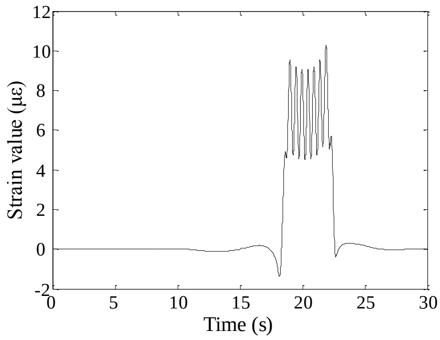
8 different load cases of DSG Bridge are shown in Table 2 with reference to Fig. 2.

Strain value of deck plate members (Y_7,Y_8) and horizontal beam members(Y_9,Y_{10}) is equal to stain sensor field monitoring value. But for truss members including hanger(Y_1^u, Y_1^d, Y_2^u, Y_2^d), web member(Y_4^u, Y_4^d) and chord member(Y_3^u, Y_3^d , Y_5^u, Y_5^d , Y_6^u, Y_6^d , Y_{11}^u, Y_{11}^d , Y_{12}^u, Y_{12}^d), the strain value is the mean of strain sensor monitoring value in two sides of each truss member because truss members mainly subject axial stress. For example, strain value Y_1 is the mean value of Y_1^u and Y_1^d. Y_1 is the time history curve of strain value when the train goes through the bridge, shown in Fig. 5. MaxY_1 and MinY_1 is the maximum and minimum value of Y_1, respectively.

Figs. 5(a) and 5(b) show Y_1 strain value of signal drive in case 6 by field SHM method and FEM simulation method, respectively. From Fig. 5 we can see that: the results by SHM and FEM are similar. The SHM data is subject to random disturbance outside, so the strain value appears slight fluctuations. But the strain value acquired by the random disturbance is much lower than by trains. The slight fluctuations caused by random disturbance can be ignored in this place. The curve pattern and strain value in Figs. 5(a) and 5(b) is close. It indicates the FEM results are reliable.



(a) SHM



(b) FEM

Fig. 5 Time history curve of Y_1 strain value of single drive in case 6

Table 2 Load case of DSG Bridge

Load case	Case instruction
Case1	8 carriage train from north to south on Jing Hu side
Case2	8 carriage train from south to north on Jing Hu side
Case 3	16 carriage train from north to south on Jing Hu side
Case 4	16 carriage train from south to north on Jing Hu side
Case 5	8 carriage train from north to south on Hu Rong side
Case 6	8 carriage train from south to north on Hu Rong side
Case 7	16 carriage train from north to south on Hu Rong side
Case 8	16 carriage train from south to north on Hu Rong side

Strain maximum and minimum in 12 field monitoring locations are shown in Table 3. Column 1 to column 8 is the year mean value of strain maximum and minimum in 2014. Column X1 and X3 is strain maximum and minimum by field SHM under case 1 and case 6 of single drive, respectively. Column X2 and X4 is strain maximum and minimum by FEM under case 2 and case 6 of single drive, respectively. Each column in Table 3 is a kind of strain mode, which is a group of 24 strain maxima and minima at 12 monitoring locations. If the bridge suffers damage, the strain mode will change.

Standard1 and Standard2 methods are shown in Eqs. (3) and (5), respectively. In the problem we considered, the dimension of each property is the same, just dimensionless. So non-standard method can be applied. In this part, we take three different standardization methods (standard1, standard2 and non-standard method) to conduct fuzzy clustering analysis for the 12 group data in Table 3. Fuzzy similarity matrix *R*, fuzzy equivalent matrix R^* and truncated matrix R_λ^* when $\lambda = 0.9959, r = 4$ using non-standard method are shown in Eqs. (11)-(13), respectively. Figs. 6(a)-6(c) show dynamic fuzzy clustering process for the three different standard methods, respectively. Fig. 6(d) shows the comparison of (F-$F_{0.05}$) value of the classification results of the three standard methods. The value of $F_{0.05}$ and F in non-standard method are listed in Table 4.

$$R = \begin{bmatrix} 1.0000 & 0.9836 & 0.9968 & 0.9721 & 0.3824 & 0.4017 & 0.3874 & 0.3949 & 0.9993 & 0.9834 & 0.3747 & 0.3973 \\ 0.9836 & 1.0000 & 0.9863 & 0.9959 & 0.2982 & 0.3224 & 0.3070 & 0.3183 & 0.9788 & 0.9996 & 0.2891 & 0.3166 \\ 0.9968 & 0.9863 & 1.0000 & 0.9813 & 0.3770 & 0.4011 & 0.3848 & 0.3978 & 0.9961 & 0.9860 & 0.3689 & 0.3959 \\ 0.9721 & 0.9959 & 0.9813 & 1.0000 & 0.2855 & 0.3151 & 0.2970 & 0.3139 & 0.9676 & 0.9953 & 0.2759 & 0.3088 \\ 0.3824 & 0.2982 & 0.3770 & 0.2855 & 1.0000 & 0.9719 & 0.9971 & 0.9695 & 0.3787 & 0.3015 & 0.9996 & 0.9682 \\ 0.4017 & 0.3224 & 0.4011 & 0.3151 & 0.9719 & 1.0000 & 0.9733 & 0.9980 & 0.3999 & 0.3283 & 0.9679 & 0.9988 \\ 0.3874 & 0.3070 & 0.3848 & 0.2970 & 0.9971 & 0.9733 & 1.0000 & 0.9739 & 0.3835 & 0.3113 & 0.9960 & 0.9705 \\ 0.3949 & 0.3183 & 0.3978 & 0.3139 & 0.9695 & 0.9980 & 0.9739 & 1.0000 & 0.3930 & 0.3243 & 0.9656 & 0.9962 \\ 0.9993 & 0.9788 & 0.9961 & 0.9676 & 0.3787 & 0.3999 & 0.3835 & 0.3930 & 1.0000 & 0.9785 & 0.3710 & 0.3958 \\ 0.9834 & 0.9996 & 0.9860 & 0.9953 & 0.3015 & 0.3283 & 0.3113 & 0.3243 & 0.9785 & 1.0000 & 0.2922 & 0.3235 \\ 0.3747 & 0.2891 & 0.3689 & 0.2759 & 0.9996 & 0.9679 & 0.9960 & 0.9656 & 0.3710 & 0.2922 & 1.0000 & 0.9645 \\ 0.3973 & 0.3166 & 0.3959 & 0.3088 & 0.9682 & 0.9988 & 0.9705 & 0.9962 & 0.3958 & 0.3235 & 0.9645 & 1.0000 \end{bmatrix} \quad (11)$$

Table 3 Strain maximum and minimum in 12 field monitoring locations

Strain max and min	Case1	Case2	Case3	Case4	Case5	Case6	Case7	Case8	X1	X2	X3	X4
MaxY_1	2.38	2.43	2.76	2.65	20.15	10.43	20.82	11.68	2.09	2.37	22.11	10.26
MaxY_2	3.24	3.02	4.25	2.94	24.55	13.75	25.30	15.84	2.66	2.09	26.41	11.13
MaxY_3	2.01	1.95	2.03	2.20	1.73	2.26	1.87	1.87	1.94	1.78	1.24	1.73
MaxY_4	11.31	7.77	9.48	6.36	21.93	17.05	20.58	16.15	11.47	7.33	24.32	16.61
MaxY_5	5.82	6.22	4.56	5.85	4.86	5.57	5.24	4.08	5.40	6.26	5.06	5.64
MaxY_6	5.08	4.52	6.38	5.72	8.76	7.80	10.41	8.90	5.36	5.11	9.66	7.68
MaxY_7	9.00	3.31	8.81	3.68	3.17	3.22	3.20	3.61	10.36	3.11	3.92	3.16
MaxY_8	2.43	2.44	2.63	2.61	2.41	4.61	2.26	5.30	2.18	2.17	2.37	2.58
MaxY_9	67.73	66.41	67.27	65.85	4.93	7.65	6.22	7.80	66.06	68.10	4.24	7.37
MaxY_{10}	7.33	3.20	8.03	3.41	58.79	63.37	63.43	67.11	7.31	4.30	62.05	62.52
MaxY_{11}	2.00	2.03	1.90	2.05	2.10	1.71	2.23	1.91	1.65	2.34	2.31	1.49
MaxY_{12}	1.94	1.65	1.82	1.51	2.42	1.95	3.01	2.66	1.47	1.79	2.51	2.57
MinY_1	-2.66	-2.20	-2.17	-2.18	-1.69	-1.65	-2.19	-1.85	-2.41	-2.22	-2.00	-1.36
MinY_2	-2.98	-3.09	-2.28	-3.11	-2.54	-2.98	-3.20	-2.35	-3.15	-3.60	-3.05	-4.22
MinY_3	-14.42	-9.73	-14.24	-9.77	-27.64	-21.28	-28.88	-23.17	-14.09	-10.03	-30.15	-21.46
MinY_4	-12.32	-8.17	-9.07	-5.01	-22.42	-16.59	-19.74	-14.17	-11.47	-8.03	-24.22	-15.76
MinY_5	-11.27	-8.61	-10.18	-7.47	-21.63	-16.81	-23.93	-18.28	-10.54	-8.93	-24.52	-16.86
MinY_6	-2.98	-2.73	-2.32	-2.26	-3.61	-2.79	-3.38	-2.10	-2.94	-2.30	-3.55	-2.76
MinY_7	-9.07	-3.04	-8.79	-3.50	-3.31	-3.31	-2.97	-3.24	-10.75	-2.62	-2.93	-2.79
MinY_8	-2.37	-2.36	-2.71	-2.26	-3.34	-3.25	-4.04	-4.10	-2.21	-2.39	-3.35	-3.33
MinY_9	-3.91	-3.23	-3.87	-3.88	-13.72	-7.06	-17.44	-6.85	-3.54	-3.98	-14.36	-7.33
MinY_{10}	-6.34	-12.24	-6.25	-13.63	-4.81	-4.07	-6.43	-5.22	-5.46	-12.15	-4.36	-3.10
MinY_{11}	-13.50	-12.76	-15.52	-14.59	-11.45	-11.94	-14.61	-13.98	-13.06	-12.34	-12.37	-11.16
MinY_{12}	-15.56	-19.58	-19.11	-24.69	-2.91	-5.78	-2.12	-6.06	-15.02	-19.58	-3.04	-5.23

$$R^* = \begin{bmatrix}
1.0000 & 0.9863 & 0.9968 & 0.9863 & 0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9993 & 0.9863 & 0.4017 & 0.4017 \\
0.9863 & 1.0000 & 0.9863 & 0.9959 & 0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9863 & 0.9996 & 0.4017 & 0.4017 \\
0.9968 & 0.9863 & 1.0000 & 0.9863 & 0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9968 & 0.9863 & 0.4017 & 0.4017 \\
0.9863 & 0.9959 & 0.9863 & 1.0000 & 0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9863 & 0.9959 & 0.4017 & 0.4017 \\
0.4017 & 0.4017 & 0.4017 & 0.4017 & 1.0000 & 0.9739 & 0.9971 & 0.9739 & 0.4017 & 0.4017 & 0.9996 & 0.9739 \\
0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9739 & 1.0000 & 0.9739 & 0.9980 & 0.4017 & 0.4017 & 0.9739 & 0.9988 \\
0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9971 & 0.9739 & 1.0000 & 0.9739 & 0.4017 & 0.4017 & 0.9971 & 0.9739 \\
0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9739 & 0.9980 & 0.9739 & 1.0000 & 0.4017 & 0.4017 & 0.9739 & 0.9980 \\
0.9993 & 0.9863 & 0.9968 & 0.9863 & 0.4017 & 0.4017 & 0.4017 & 0.4017 & 1.0000 & 0.9863 & 0.4017 & 0.4017 \\
0.9863 & 0.9996 & 0.9863 & 0.9959 & 0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9863 & 1.0000 & 0.4017 & 0.4017 \\
0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9996 & 0.9739 & 0.9971 & 0.9739 & 0.4017 & 0.4017 & 1.0000 & 0.9739 \\
0.4017 & 0.4017 & 0.4017 & 0.4017 & 0.9739 & 0.9988 & 0.9739 & 0.9980 & 0.4017 & 0.4017 & 0.9739 & 1.0000
\end{bmatrix} \tag{12}$$

$$R_\lambda^* = \begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{13}$$

From Fig. 6 and Table 4, the following observations can be made:

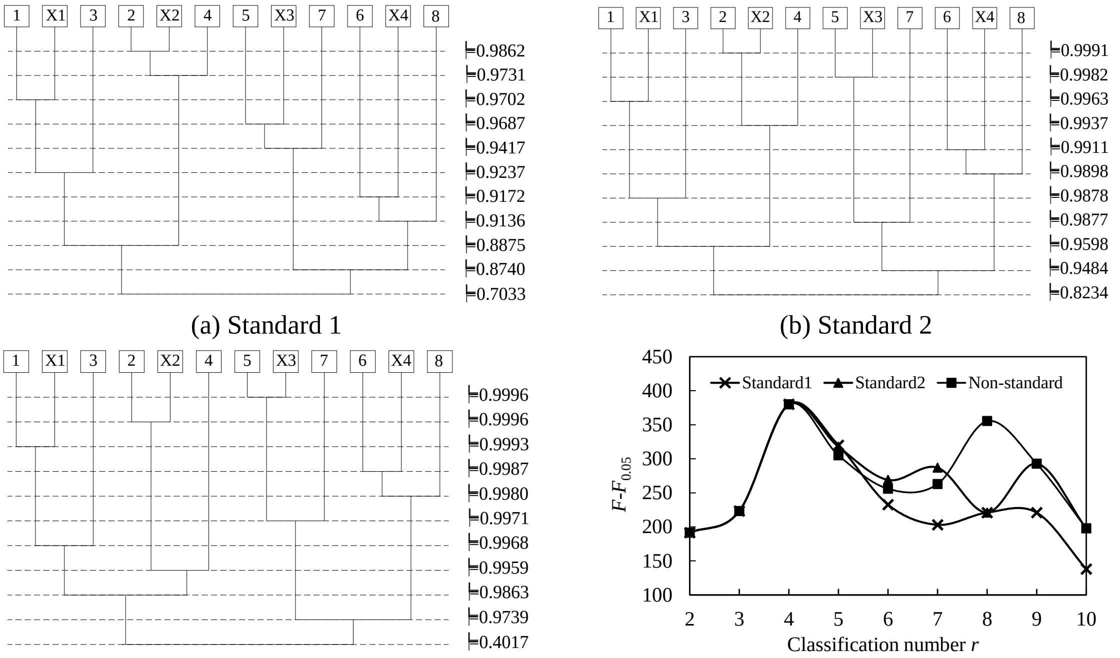
(1) From the same characteristics observed for the three standard methods

① X1 gets into case 1 before others, and X3 gets into case 5 before others. Therefore, X1 belongs to case 1 and X3 belongs to case 5. Clustering results are consistent with field SHM results. So, this clustering method is credible.

② X2 gets into case 2 before others, and X4 gets into case 6 before others. Therefore, X2 belongs to case 2 and X4 belongs to case 6. Clustering results are consistent with FEM results. So, FEM results are credible.

③ As it is mentioned above, the bigger (F-$F_{0.05}$) value is, the better the clustering result is. For these three methods, (F-$F_{0.05}$) gets the maximum value 380 when classification member r equals to 4. The corresponding truncated matrix R_λ^* is shown in Eq. (13). At this time the clustering result is: {case 1, X1, case3}, {case 2, X2, case4}, {case 5, X3, case7}, {case 6, X4, case8}. It means that the clustering result is the best when 8 carriage and 16 carriage train in the same line are in a category.

④ At last, {case 1, X1, case3, case 2, X2, case4} and {case 5, X3, case7, case 6, X4, case8} become a big category, respectively. It means Jing Hu side and Hu Rong side become a category, respectively. At this time, classification member r equals to 2, (F-$F_{0.05}$) value is 191.48. This category result is not good.



(c) Non-standard (d) Compare of different standard method

Fig. 6 Dynamic clustering process

Table 4 The value of $F_{0.05}$ and F in non-standard method

r	2	3	4	5	6	7	8	9	10	11
F	196.44	227.87	384.07	309.27	260.37	267.83	361.47	301.70	217.21	113.46
$F_{0.05}$	4.96	4.26	4.07	4.12	4.39	4.95	6.09	8.85	19.40	242
F-$F_{0.05}$	191.48	223.61	380.00	305.15	255.98	262.88	355.38	292.85	197.81	-128.54

(2) From the different characteristics observed for the three standard methods

① The bigger (F-$F_{0.05}$) value is, the better the clustering result is. It means that the higher the curve in the Fig. 6(d) is, the better the standard method is. So from Fig. 6(d) we can get the conclusion that: standard2 method is better than standard1 and non-standard method is the best.

② In the Fig. 6(d), for non-standard method the curve gets another extreme value 355.38 when classification member r equals to 8. This extreme value is just a little less than the maximum value 380 and more than others. It indicates that the clustering result is also good when classification number r equals to 8. At this time the clustering result is: {case 1, X1}, {case3}, {case 2, X2}, {case4}, {case 5, X3}, {case7}, {case 6, X4}, {case8}. This clustering result means each case in Table 1 is in a category. The result is reasonable according to practical situation. However, standard1 and standard2 methods can't recognize this extreme point when r equals to 8. It also indicates that non-standard method is better than the other two methods in this problem.

(3) The reason for the difference of the three standard methods

What brings the different results of the three standard methods? As we have mentioned in section 4, we need to maintain the characteristic of original data and not to disturb the relationship between the original row vectors as much as possible. The less the original data is disturbed, the more the result is close to real situation. From standard1 to sandand2 to non-standard method, the results become more and more adequate because the impact of the transform operation is minimized. So we will use non-standard method to do fuzzy clustering analysis below.

6. Damage identification using fuzzy clustering analysis

Bridge may suffer various degrees of damage after used for a period of time. Damage identification is a fundamental issue in bridge health monitoring. FEM method is taken to simulate bridge damage as the FEM results are credible as illustrated in section 5. Then we try to identify damage by non-standard fuzzy clustering analysis method.

6.1 Damage with different degrees

As a typical representative, all the damage simulation by FEM given below is in the case 6. Table 5 gives strain maximum and minimum values of the 12 monitoring locations at different degrees of damage at bottom chord member Y_5 which is at side truss of the mid-span. The variable name Dam0, Dam10~ Dam50 in Table 5 mean the area of chord member decreases 0, 10%~50%. Fig. 7 shows the fuzzy clustering process of Y_5 when the damage degree of Y_5 varies from 0 to 50%. Table 6 gives threshold value λ in different damage degrees when the damage case and case 6 become the same category. F statistical value in Table 6 is used to evaluate the difference between damage case and other 8 cases. So, value of F and $F_{0.05}$ is corresponding to classification number r equals to 9 with damage degree changing.

As we have illustrated in section 5, an undamaged case must get into one of the 8 cases in Table 1 before the other 7 cases using this fuzzy clustering analysis method. If an unknown case can't be classified into one of the 8 cases firstly. It means that this unknown case does not belong to the 8 cases. That is to say, this unknown case is abnormal and the bridge strain mode changes. Bridge may be damage. In this paper, simulation by FEM is in the case 6. The threshold value λ is 0.9980 for case 6 and case 8 getting into the same category. So if threshold value λ of an unknown case with case 6 is less than 0.9980. It means that the change of bridge strain mode caused by the unknown case is more than it caused by the different carriages in the same lane. So this unknown case is abnormal and bridge may be damage. At this time, the unknown case is identified as damage. Or just in brief, the damage case is identified.

From Fig. 7 and Table 6, the following observations can be made:

(1) When damage degree is no more than 10%, damage case gets into case 6 before the other 7 cases. Threshold value λ is greater than 0.9980. The damage case can't be identified in the degree of 10%.

(2) When damage degree reaches 20%, damage case gets into case 6 after case 8. Threshold value λ is less than 0.9980. It means that strain mode change caused by damage in this degree is more than it caused by different carriages. So the damage case can be identified when the damage degree is more than 20%.

(3) When damage degree reaches 50%, damage case is getting into case 6 just after case 8 and before others. It illustrates that strain mode change caused by damage is no more than it caused by

different lanes although the damage degree reaches 50%.

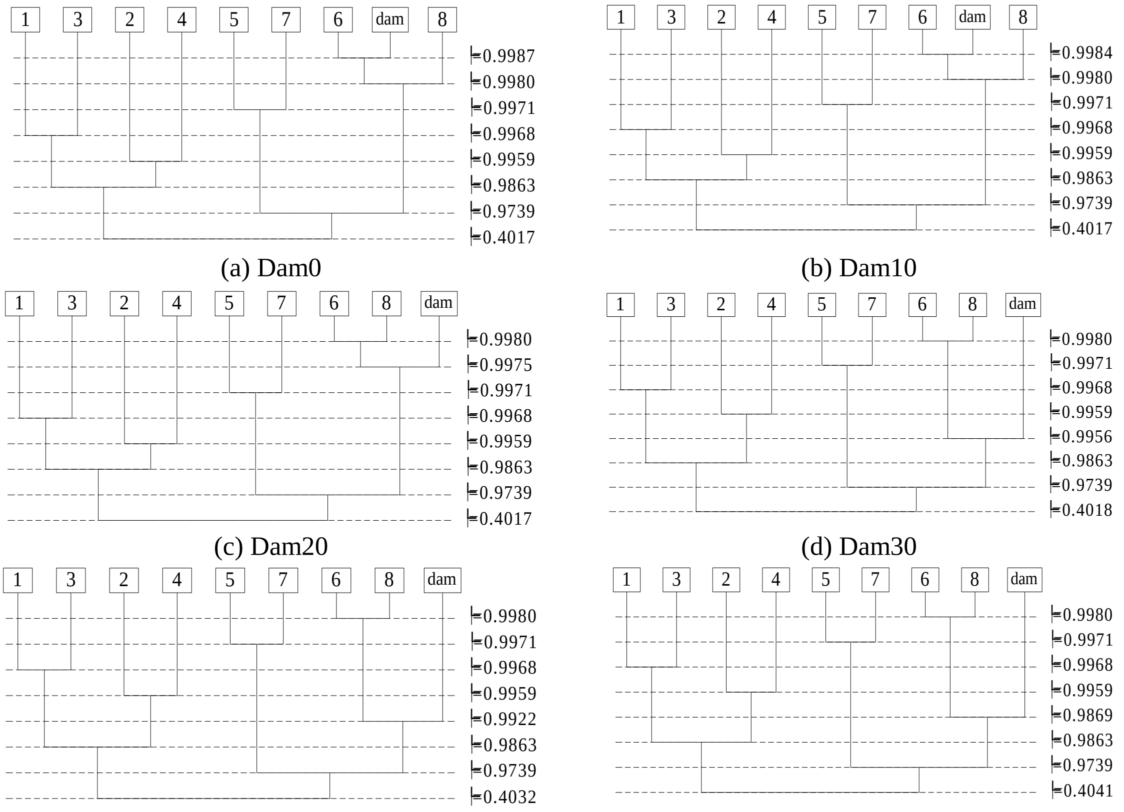
(4) The higher the damage degree is, the lower threshold value λ and (F-$F_{0.05}$) value is. It means that the difference between damage case and case 6 increases with the growth of damage.

Table 5 Strain maximum and minimum at different degree damage of bottom chord Y_5

Damage degree	Dam0	Dam10	Dam20	Dam30	Dam40	Dam50
MaxY_1	10.26	10.26	10.26	10.26	10.26	10.26
MaxY_2	11.13	11.13	11.13	11.13	11.13	11.13
MaxY_3	1.73	1.83	1.96	2.10	2.28	2.48
MaxY_4	16.61	16.57	16.51	16.44	16.34	16.21
MaxY_5	5.64	6.16	6.79	7.52	8.44	9.52
MaxY_6	7.68	7.68	7.68	7.67	7.67	7.66
MaxY_7	3.16	3.14	3.11	3.08	3.04	2.99
MaxY_8	2.58	2.60	2.63	2.65	2.69	2.74
MaxY_9	7.37	7.37	7.38	7.38	7.39	7.40
MaxY_{10}	62.52	62.52	62.52	62.52	62.52	62.52
MaxY_{11}	1.49	1.49	1.49	1.49	1.49	1.49
MaxY_{12}	2.57	2.57	2.57	2.57	2.57	2.56
MinY_1	-1.36	-1.36	-1.37	-1.37	-1.38	-1.38
MinY_2	-4.22	-4.23	-4.25	-4.27	-4.29	-4.32
MinY_3	-21.46	-21.51	-21.58	-21.65	-21.75	-21.86
MinY_4	-15.76	-15.82	-15.87	-15.93	-16.01	-16.09
MinY_5	-16.86	-18.42	-20.30	-22.50	-25.26	-28.47
MinY_6	-2.76	-2.75	-2.75	-2.74	-2.73	-2.72
MinY_7	-2.79	-2.78	-2.78	-2.78	-2.78	-2.77
MinY_8	-3.33	-3.33	-3.34	-3.34	-3.34	-3.34
MinY_9	-7.33	-7.33	-7.33	-7.33	-7.33	-7.34
MinY_{10}	-3.10	-3.10	-3.10	-3.10	-3.10	-3.09
MinY_{11}	-11.16	-11.16	-11.17	-11.17	-11.17	-11.17
MinY_{12}	-5.23	-5.23	-5.23	-5.23	-5.23	-5.23

Table 6 threshold value λ and F statistical value in different damage degree

Dam(%)	0	10	20	30	40	50
λ	0.9987	0.9984	0.9975	0.9956	0.9922	0.9869
F	306.54	264.63	174.01	100.64	55.56	31.91
$F_{0.05}$	237	237	237	237	237	237
F-$F_{0.05}$	69.54	27.63	-62.99	-136.36	-181.44	-205.09



(a) Dam0 (b) Dam10

(c) Dam20 (d) Dam30

(e) Dam40 (f) Dam50

Fig. 7 Dynamic clustering process of Y_5 when damage degree varied from 10% to 50%

6.2 Damage at different locations

The same with bottom chord member Y_5, we have also simulated damage of top chord member Y_3 at mid-span side truss and damage of Y_3 and Y_5 meanwhile by FEM. Simulation damage degree is 0, 10%, 20%, 30%, 40% and 50%, respectively. And then fuzzy clustering analysis is taken to do damage identification. Threshold value λ of different locations(Y_3, Y_5, Y_3andY_5) varied with damage degree is shown in Fig. 8.

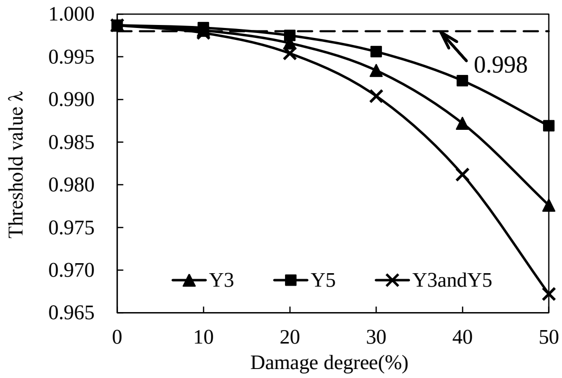


Fig. 8 Threshold value λ of different locations varied with damage degree

Table 7 Fitting coefficient

Coefficient	*a*	*b*	*c*	*d*
Y_3	-1.519×10^{-7}	-3.968×10^{-9}	-4.239×10^{-5}	0.9987
Y_5	-7.407×10^{-8}	-6.944×10^{-7}	-1.612×10^{-5}	0.9987
Y_3 and Y_5	-1.972×10^{-7}	-1.762×10^{-6}	-1.762×10^{-5}	0.9987

$$\lambda = a\cdot x^3 + b\cdot x^2 + c\cdot x + d \tag{14}$$

Fitting error formula

$$fitting\ \ error\ \ \gamma = \frac{fitting\ \ value - real\ \ value}{real\ \ value}\times 100\% \tag{15}$$

From Fig. 8 we can see:

(1) Each of the three curves presents parabolic shape. So, polynomial fitting is taken for the three curves in this paper. Fitting formula is shown in Eq. (14) and fitting coefficient is in Table 7. Fitting error formula is shown in Eq. (15) and fitting error γ is no more than 0.01% .

(2) As it is referred in section 6.1, when threshold value λ of an unknown case getting into case 6 is less than 0.998, this unknown case is identified as damage. Intersecting *x*-coordinate of the curve Y_3, Y_5 and (Y_3 and Y_5) with λ=0.998 is 11.34, 15.64 and 8.80, respectively. It indicates that for Y_3, Y_5, and Y_3 and Y_5 damage simultaneously, when the damage degree reaches to 11.34%, 15.64% and 8.8%, respectively, the damage case is identified, which is the strain mode change caused by damage is more significant than it caused by different carriages. The corresponding critical damage degree is called as damage threshold in this paper. It shows that: The damage threshold varies with damage location and reduces with the increase of damage locations. Bridge integrity is good, local small degree damage (less than 10%) of one chord member will not bring obvious changes of stress distribution mode. But when damage of one member reaches certain degree or small damage occurs in two or more places, stress distribution mode will produce obvious changes and action should be taken now.

(3) For the same degree, threshold value λ is different at different location. Top chord member is more sensitive to damage than bottom chord member.

(4) The threshold value λ of Hu Rong side in the same category is 0.9739. When Y_3 and Y_5 damage at the same time, λ is 0.9672 at the damage of 50%, less than 0.9739. That is to say: when two locations are damaged at the same time and its damage degree reaches 50%, strain mode change caused by damage is more significant than it caused by different lanes. In this case, damage is serious.

7. Conclusions

● In fuzzy clustering analysis, for the problem which dimension of different properties is the same, the first step of standardization can be omitted as standardization is not necessary at this time. The results may be better because any standardization method disturbs the characteristic of original data while non-standard method keeps the characteristic as much as possible.

● Clustering result is the best when 8 carriage and 16 carriage train in the same line are in a category. If an unknown case gets into one of the 8 cases in Table 1 before the other 7 cases by fuzzy clustering analysis, this unknown case belongs to this one.

● For DSG Bridge, the damage is identified when the strain mode change caused by damage is more significant than it caused by different carriages. The corresponding critical damage degree is called damage threshold. The damage thresholds are 11.34% for top chord member, 15.64% for bottom chord member and 8.8% for these two chord members at side truss of the mid-span damage simultaneously respectively. The damage threshold varies with damage location and reduces with the increase of damage locations.

● When damage of two or more locations reaches a certain degree (50% for the top and bottom chord members damage simultaneously), the strain mode change caused by damage is more significant than it caused by different lanes. In this case, damage is serious.

● The curve of threshold value λ which is damage case and its corresponding case being the same category varied with damage degree presents parabolic shape and can be fitted with a cubic polynomial well.

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