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The remarkable story of Portogruaro Civic Tower's probabilistic health monitoring

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Abstract. This is the story of a bell-tower and its monitoring. The Civic Tower in Portogruaro is a 59 m high masonry bell-tower, originally built in the XIII century, today leaning more than a meter out of plumb. Since 2003, the building inclination has been continuously monitored with an optical inclinometer in an effort to see whether the tilt is still in progress. When the monitoring started, it was thought highly unlikely that the Tower would tilt further. After three years of monitoring and historical investigation, this idea was completely overturned. We show here how the initial view developed to a final awareness via a probabilistic analysis of the information acquired, based on Bayesian logic. We illustrate how the joint use of instrumental monitoring and historical documentation allowed timely recognition of signs of ongoing tilting and accurate calculation not only of the mean inclination trend, but also the credibility of this information.

Keywords: Bayesian analysis; real-time monitoring; historic building

1. Introduction

In September 2002 the Municipality of Portogruaro, a town located in North-Eastern Italy some 70 km away from the city of Venice, contacted the first author of this paper at the University of Trento to install and operate instrumentation to monitor the tilt of its Civic Tower. This building is an ancient masonry bell-tower, 59 m tall, leaning towards its north-east corner with an out of plumb measured at the time as 1.197 m. Although the inclination was -and still is- striking, at the time there was no real specific concern about stability because the Tower had always been known to be leaning, as documented in local chronicles. Also, there was no evidence of inclination still in progress, and such an occurrence was judged at the time to be very unlikely.

The monitoring system started recording the Tower inclination in October 2003. In September 2004, we reported to the Municipality that the changes in inclination recorded during this period were too small to raise any concern over the short-term safety of the Tower.

In September 2005, after almost two years of recording, we communicated to the Municipality of Portogruaro that analysis of the data acquired to date allowed calculation of a possible linear trend of 1.7 mm per year. Particularly, the direction of the possible motion was very close to the

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maximum lean direction. Nonetheless, the leaning progression was too small and the monitoring period too short to state with certainty whether the trend calculated was a sign of an ongoing process.

During the third year of monitoring, we recovered in the Municipality archives some unpublished historical documents about the Tower. Among these, an original design project revealing an elevation built in 1879, and documents reporting old measurements of the inclination dating 1962 and 1997. Based on this new information and on the fresh instrumental data, we concluded, in the annual monitoring report issued in September 2006, that an increasing inclination was very likely.

This final judgment, in sharp contrast with the initial view, is the result of a rigorous and quantitative logical analysis of the information gathered over the three preceding years. This paper follows this logical route again, showing how the initial view evolved to final awareness via a probabilistic model, based on Bayesian logic. In the next Section we introduce the Tower at issue, its history and the monitoring system installed; Section 3 formulates the algorithm used to update the posterior judgment based on the information acquired; the application of this procedure to the Civic Tower case is reported in Section 4; the results of Bayesian identification are presented and discussed in Section 5; finally, some concluding remarks are reported at the end of the paper.

2. The tower and its monitorıng

2.1 Tower description and characterization

As seen today, the bell-tower of the Cathedral of S. Andrea in Portogruaro, also known as Civic Tower, is a 59 m tall leaning free-standing bell-tower. The building was probably started in the XIII century. The tower was surveyed for the last time by Busetto & Romanin (2001) using theodolite and diastimeter, and its main geometrical features are reported in Fig.1. From the architectural point of view the Tower has a masonry body, a belfry and a spire. The body has a roughly square cross section, of size varying with height, from 7.30 m on a side at ground level to 6.45 m at the top. The walls are masonry infilled, with thickness varying from 1.3 m at the base to 0.9 m at the top. There are four wooden floors at levels 5.58 m, 12.07 m, 18.45 m and 22.74 m, and an old masonry cross vault, now reinforced with a concrete slab, at level 26.20 m. The main column ends with a second cross vault, similar to that below, which supports the floor of the belfry, at level 31.43 m. The belfry balcony carries an octagonal tambour 5.45 m in diameter and 4.9 m height, in turn topped by a pyramidal spire 15.84 m in height overall.

The Tower leans visibly to the North-East. However, the angle of slant of the Tower is not constant over the height, because the higher parts were apparently rebuilt with lower inclination in an effort to compensate for a pre-existing tilt. Based on the 2001 survey, the out of plumb with respect to the northeast edge of the balcony, at level 36.62 m, is 1.197 m, corresponding to an inclination of $35.47\times10^{-3\ \mathrm{rd}}$; while the out of plumb of the spire is 0.310 m, corresponding to an angle of $19.57\times10^{-3\ \mathrm{rd}}$.

Examining the outer masonry of the Tower we can observe at least four different masonry textures, a clear sign of successive building phases carried out in the past. Nevertheless, at the time monitoring started, little documentation was available about these works. Local chronicles report a reconstruction of the spire in 1879, but little was known about the extent of this intervention at the beginning of this story. The Municipality conserved incomplete documentation of work done by the architect Mario De Goetzen between 1962 and 1963; this included reinforcement of the Tower with concrete ring beams and steel ties, and consolidation of the lower level of masonry at the North and East sides. As visible today, this consolidation involved replacing the original low quality yellow brickwork of the outer layer with red brick of better mechanical characteristics.

Between 2002 and 2003, the Tower underwent an extensive experimental campaign including material sampling, core drillings, endoscopies, flat jacks and chemical analysis of the outer masonry foil. The results of the investigation are reported in detail by Molteni (2003). The core drillings (labelled C1 to C3 on Fig. 1) confirmed that the masonry is in-filled: the external leafs consist of a single layer of bricks, while the quality of the infill is extremely poor and inhomogeneous. The flat jack tests showed the better qualities of the new brickwork with respect to the original: a compression strength of 8.5 MPa was measured at the North side (M1 on Fig. 1), against a strength of 3MPa measured at the South side (M2 on Fig. 1).

2.2 Monitoring system

Since October 2003, the tilt of the Tower has been observed continuously by a monitoring system. The main instrument installed is an inclinometer, based on a pendulum hung from the ceiling of the upper vault at level H=29.90 m. The pendulum consists of a 1.5 mm diameter steel wire with a brass mass attached: the mass hangs in a water tank located at ground level, to dampen pendulum motion, as shown in Fig. 2(a). The position of the pendulum is permanently recorded by two digital network cameras, carried on a steel frame at level 1.1 m which in turn is fastened to the floor.

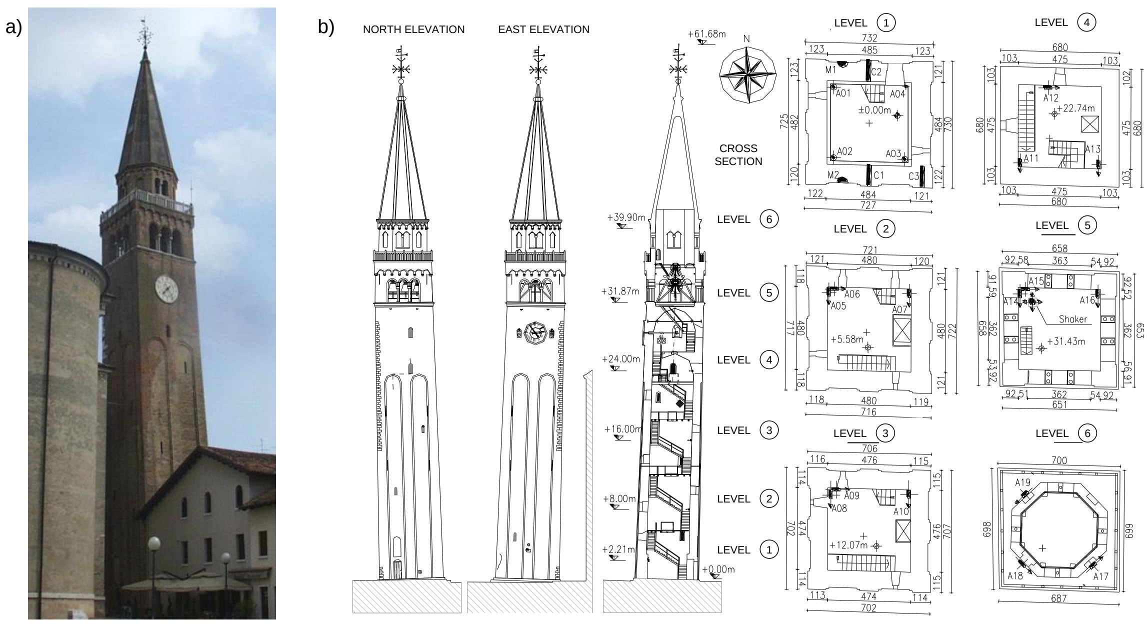


Fig. 1 Overview of the Tower (a); North and East elevation, cross-section and plan views at different levels (b)

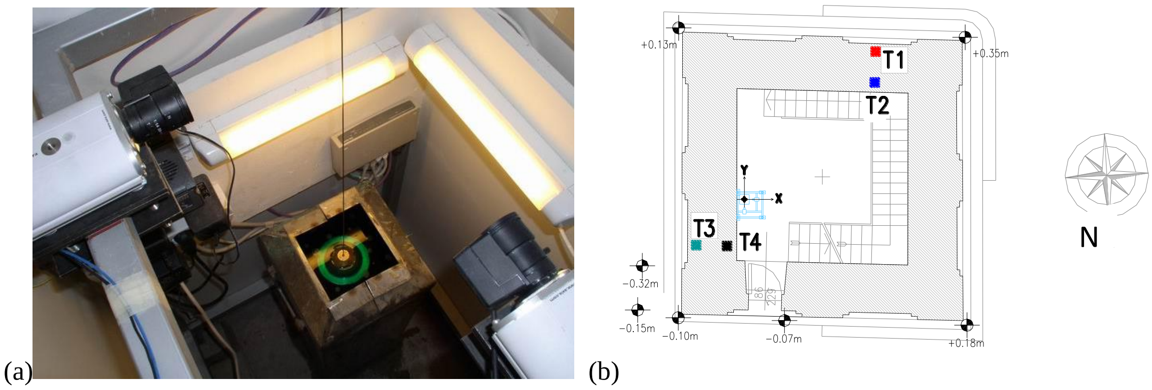


Fig. 2 View of the pendulum inclinometer wire, including the network cameras (a); plan-view of the Tower highlighting the thermocouples and plumb wire position (b)

The cameras permanently acquire pictures of the wire and transmit them though the Internet every 10 minutes to the monitoring station, physically located at the University of Trento. Using image recognition software, these images are processed in real-time to calculate the position of the wire with respect to a reference background. The position of the pendulum is returned in the form of two coordinates, x and y, representing the shifts in direction West-East and North-South, respectively, with respect to the intersection of the axes of the two cameras. The instrumental resolution of these measurements is 0.15 mm. In addition to the pendulum inclinometer, the system records the temperature with an accuracy of about 0.1C at four thermocouples, two (T1 and T2) installed on the outer surface of the masonry, the other two (T3 and T4) on the inside, as depicted in Fig. 2(b). The temperature data is acquired through a National Instrument Field Point device, also remotely controlled by the monitoring station at the University of Trento.

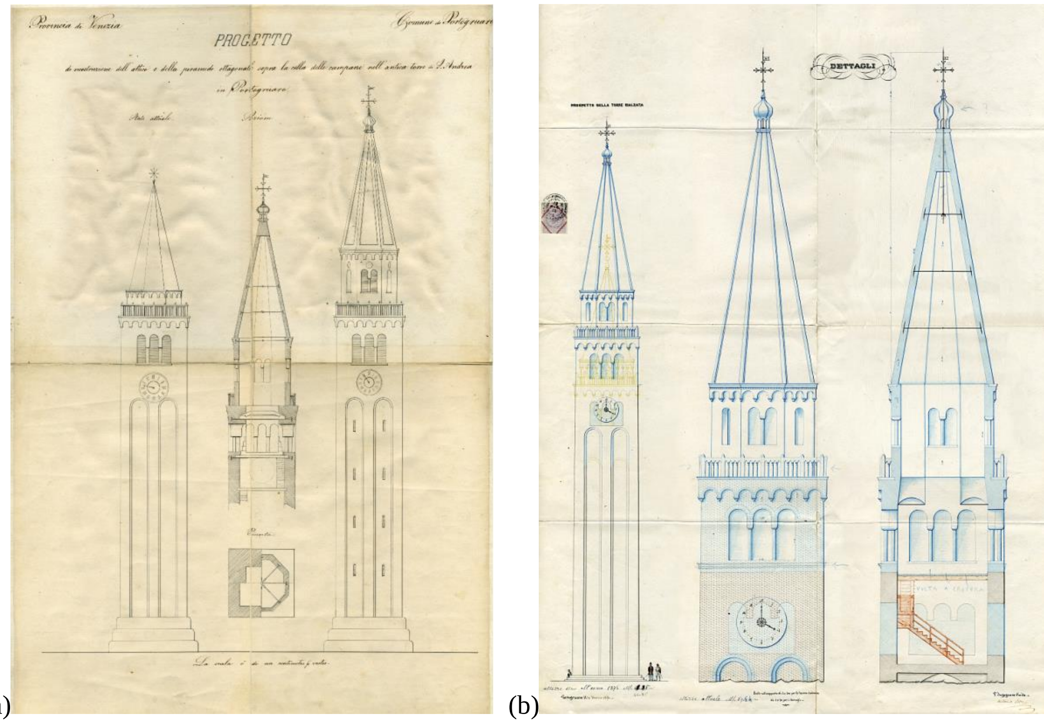
The server processes all the data, and publishes the state of the Tower in real time. As an example, Fig. 5(a) shows the time history of the temperature T_1 recorded at thermocouple T1 from October 2003 to September 2007, while Fig. 5(b) plots the out of plumb in x and y directions recorded in the same period.

2.3 Discovery of historical documentation

In late 2005, the discovery of unpublished documents threw new light on the recent history of the Tower. A first step was the retrieval of a geometrical survey of the Tower carried out by De Goetzen in October 1962, immediately before the restoration work. One of the designs reports an out of plumb of 740 mm to the East and 760 mm to the North, with respect to the North-East edge of the balcony, at level 36.74 m. The corresponding angles are 20.14×10^{-3} rd East and 20.68×10^{-3} rd North. The same survey also reports an out of plumb of the spire of 150 mm North and 150 mm East, apparently taken over a difference of level of 22.12 m: i.e., inclinations of 7.90×10^{-3} rd East and North. During an interview with the first author of this paper, in September 2005, De Goetzen specified that these measurements were taken using a plumb wire, thus with a precision that can be estimated in the order of two centimeters. In the same interview, De Goetzen recollected that many documents about the Tower were found a few years ago by a student of the University of Padova, as part of the research work in preparation for his Master Thesis.

The first author of this paper met Massimo Zanet, the former student and now professional engineer, in November 2005, learning from him some key facts which turned out to be critical for understanding the Tower's behavior. First: Zanet surveyed the Tower in July 1997, apparently using the same method and reference system as De Goetzen in 1962. As reported in his Thesis (Zanet 1997), he measured an out of plumb of 770 mm to the East and 760 mm North, values corresponding to inclinations of 20.96×10^{-3} rd East and 20.68×10^{-3} rd North. Second: during his search, Zanet was able to access and examine the original designs of the restoration work carried out between 1877 and 1879 by civil engineer Antonio Bon.

Based on Zanet's hints, we were eventually able to locate these designs in the archives of the Municipality. From analysis of the design documentation, it is clear that the XIX century work was not just simple repair, but rather a radical modification of the existing campanile. Fig. 3(a) reproduces a sketch from Bon's preliminary project, dating 1877, where the original campanile is compared with one of the new proposals. In another design drawing, reproduced in Fig. 3(b), the final refurbishment solution is drawn in blue, overlapped on the 'as was' state in sepia. We note that the original Tower was only 46.85 m tall, considerably lower than 59 m, the height we see today. In the technical report that accompanies the design, Bon mentions an existing tilt of the Tower, without specifying its extent.



(a) (b)

Fig. 3 Original restoration projects of the Civic Tower by Antonio Bon, dating 1877: (a) preliminary design showing on the left the Tower 'as was' and on the right one of the refurbishment proposals, not actually realized and (b) final refurbishment design (in blue) overlying the Tower 'as was' (in sepia)

Table 1 Historical measurement of the Tower inclination

Authors	Date	Body inclination*		Spire inclination	
		φ_x [rd]	φ_y [rd]	φ_x [rd]	φ_y [rd]
De Goetzen	October 1962	20.14×10^{-3}	20.68×10^{-3}	7.90×10^{-3}	7.90×10^{-3}
Zanet	July 1997	20.96×10^{-3}	20.68×10^{-3}	-	-
Busetto & Romanin	January 2001	21.50×10^{-3}	24.49×10^{-3}	6.01×10^{-3}	12.65×10^{-3}

* Measured at the North-East corner

Table 2 List of events in chronological order

Date	Event
1877	Restoration projects of the Civic Tower by Bon.
1879	The restauration works are completed.
1962, October	De Goetzen surveys the Tower and measure its out of plumb.
1997, July	Zanet measures the Tower out of plumb as part of his Master Thesis.
2001, January	Busetto & Romanin survey the Tower and measure its out of plumb.
2002, September	The Municipality of Portogruaro contacts the University of Trento.
2003, October	The monitoring system starts recording.
2004, September	The University of Trento reports to the Municipality that the changes in inclination recorded during this period are too small to raise concern
2005, September	The University of Trento reports to the Municipality a possible linear trend of 1.7 mm per year; De Goetzen's 1962 documentation is found in the Municipality's archives; the University of Trento interview De Goetzen.
2005, November	The University of Trento meets Zanet and learns of his 1997 measurement; the original 1877 Bon's design documentation is found in the Municipality's archives.
2006, September	The University of Trento reports to the Municipality that an increasing inclination is very likely.

Also, there is no mention of the fact that the new spire would have to be built with a different inclination to that of the body below.

Table 1 compares the inclinations surveyed in 1962 by De Goetzen, in 1997 by Zanet and in 2001 by Busetto. Even though the precision of the first two measurements is presumably quite low, it is still remarkable to observe that only one of the measurements is smaller than the preceding values. A summary of the events cited in this story is reported in Table 2 in chronological order.

3. Bayesian identification concept

At this point, our goal is to exploit appropriately our knowledge about the Tower in order to make inference about its tilting and be able to warn the owner of a possible hazardous situation as it arises. A critical issue that we encounter when we attempt to formulate this problem is that the nature of the information that we are handling is twofold: on one side, we have the instrumental data recorded by the monitoring system; on the other we have unverified historical measurements and educated guesses stemming from the analysis of the historical documentation found. Here we present first the general paradigm of the method; then, in the next Section, we clarify its practical application to the specific case of the Civic Tower.

The general goal of the method is to try to recognize in real-time symptoms of a specific hazardous scenario (in this case: leaning in progress), from a set of instrumental measurements, using the principle of Bayesian statistical analysis. Bayesian theory of probability originates from Bayes' well known essay (Bayes 1763); reference works on the subject are those by Jaynes (2003) and Skilling (1998) while many modern specialized textbooks provide the reader with a critical review and applications of this theory to data analysis (see for instance those of Gregory 2005, Sivia 2006, Murphy 2012).

Among the many applications to structural health monitoring, we wish to mention Papadimitriou *et al.* (1997), Beck and Katafygiotis (1998), and Beck *et al.* (2002), which defined a consistent framework for probabilistic data processing. An advantage of Bayesian methods over deterministic ones is that they allow for the modelling of all uncertainties involved in the analysis, and for consistently combining information of completely different natures. In addition, they allow not only estimation of the most likely values of the unknown parameters, but also their distribution, which is of paramount importance when monitoring is addressing a critical decision process as in this case.

Bayesian analysis has been applied to system identification and dynamic modelling (Chatzi and Smyth 2009), updating of Finite Element Models (Capecchi and Vestroni 1993, Mthembu *et al.* 2011), prediction of extreme response (Tien *et al.* 2013), inference on deterioration (Straub 2009) and damage detection (Sohn and Law 1997, 2000), and it has been integrated in system operation and maintenance (Memarzadeh *et al.* 2014), decision about sensor deployment (Zonta *et al.* 2014), and sensor placement (Flynn and Todd 2010, Malings and Pozzi 2014).

Here, we will follow a logical route and the formal notation as in Zonta *et al.* (2008, 2010). A typical structural health monitoring system is equipped with both sensors measuring structural response features (e.g., strain gauges, accelerometers, inclinometers…) and sensors recording environmental actions (such as temperature or external loads) which are needed for performing compensation. For convenience, we will refer to the former as *response sensors*, and to the latter as *environmental sensors*. During monitoring operation, these sensors, both response and environmental, record measurements at time instants $(t_1,t_2,\ldots,t_N)$. Label $\mathbf{z}_k$ the observations recorded by the response sensor set at time t_k and $\mathbf{z}_{1:k}=[\mathbf{z}_1\ \mathbf{z}_2\ \ \ldots\ \mathbf{z}_k]$ the whole dataset collected to time t_k. Similarly, indicate with $\mathbf{T}_k$ and $\mathbf{T}_{1:k}$, the corresponding observations from the environment sensors set (T for 'temperature').

Generally speaking, the objective of structural health monitoring is to understand the state of the structure based on observation of its response $\mathbf{z}_{1:k}$. We can divide the domain of the possible structural response into a mutually exclusive and exhaustive set of n scenarios $(S_1,S_2,\ldots,S_n)$, each defining a specific structural condition state. If we assume our structure to be in a specific state j, then we can attempt to predict its response to environmental actions $\mathbf{T}(t)$ at time t using a response model. We generally indicate with $\hat{\mathbf{z}}_j\left(\mathbf{T}(t),t\right)$ the prediction of the model at time t, where the hat over the $\mathbf{z}$ vector indicates that this is a prediction rather than an observation. In general, the response model changes with the state and the j index is there exactly to remind us that the prediction refers to the specific scenario j.

Often, the response $\hat{\mathbf{z}}_j$ to the action **T** is controlled by a number of parameters whose values are unknown, or known with some uncertainty. We indicate with vector $\boldsymbol{\theta}_j$ the set of parameters which control scenario *j*, a vector whose dimension and value generally changes with the scenario. Thus, the structural prediction at time t can be generally indicated as a function of the scenario parameter $\boldsymbol{\theta}_j$, the action **T** and the time: $\hat{\mathbf{z}}_j\left(\boldsymbol{\theta}_j;\mathbf{T}(t),t\right)$. For those familiar with Bayesian model selection theory (Bretthorst 1996, Mackay 2003, Gregory 2005), the discrete scenario here introduced can be seen as a meta-parameter which qualitatively identifies the type of response function (e.g., constant, linear, exponential) which in turn is controlled by a parameter set.

Assuming scenario S_j, our problem is to learn about parameter $\boldsymbol{\theta}_j$, for example identifying the value that best fits the model prediction $\hat{\mathbf{z}}_j$ to the observations $\mathbf{z}_{1:k}$. Even after appropriate selection of the scenario's parameter $\boldsymbol{\theta}_j$, we don't expect that observation $\mathbf{z}_j$ exactly matches the prediction $\hat{\mathbf{z}}_j$ at any time t_j. In general we can write

$$\mathbf{z}_i = \hat{\mathbf{z}}_j\left(\boldsymbol{\theta}_j;\mathbf{T}(t_i),t_i\right)+\mathbf{e}_{i,j} \tag{1}$$

where $\mathbf{e}_{j,i}$ is a residual at time t_i and for scenario *j*, that accounts for both instrumental noise and the incompleteness of the model assumed.

Once measurements $\mathbf{z}_{1:k}$ become available from the monitoring system, Bayes' theorem allows calculation of the *posterior* (posterior meaning in simple words: after having acquired the data) probability $\mathrm{P}\left(S_j\middle|\mathbf{z}_{1:k}\right)$ of each possible scenario S_j, using the expression

$$\mathrm{P}\left(S_j\middle|\mathbf{z}_{1:k}\right) \propto \mathrm{p}\left(\mathbf{z}_{1:k}\middle|S_j\right)\cdot \mathrm{P}\left(S_j\right) \tag{2}$$

where: p generally stands for *probability density function* of a random variable; P for *probability mass function*, $\mathrm{P}(S_j)$ is the *prior* ('before acquiring data') probability of the scenario S_j; $\mathrm{p}\left(\mathbf{z}_{1:k}\middle|S_j\right)$ is known as the likelihood of that scenario. Prior probabilities assigned to each scenario reflect the initial knowledge, or judgment, of the evaluator, independent of monitoring observations. On the contrary, the likelihood is connected with the dataset $\mathbf{z}_{1:k}$ acquired: it basically tells us how likely it is, on the basis of our interpretation model, to get that specific set of measurements having assumed the structure is in a specific scenario. It is therefore clear that the likelihood also depends on the assumed interpretation model and on the prior assumption of its parameters.

To start, assume that we fix the parameters $\boldsymbol{\theta}_j$, and therefore that we can calculate the prediction $\hat{\mathbf{z}}_j(t)$, and we want to calculate likelihood of a single sample $\mathbf{z}_i$. We have already noted that the observation does not necessarily match the prediction, and we have indicated the residual with **e**. We expect this residual to be a random quantity with zero mean and covariance $\boldsymbol{\Sigma}_e$ independent of time; specifically, we adopt a Gaussian model (Beck 2010)

$$\mathrm{p}\left(\mathbf{z}_i\middle|\boldsymbol{\theta}_j,S_j\right)=\mathscr{N}\left\{\hat{\mathbf{z}}_j\left(\boldsymbol{\theta}_j;\mathbf{T}(t_i),t_i\right),\boldsymbol{\Sigma}_e;\mathbf{z}_i\right\} \tag{3}$$

where the notation $\mathscr{N}\{\boldsymbol{\mu},\boldsymbol{\Sigma};\mathbf{z}\}$ indicates a normal distribution with mean value $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$, evaluated at **z**. As long as errors are assumed to be independent for each time, the likelihood for the whole measure set $\mathbf{z}_{1:k}$ is obtained by combining the likelihoods of the samples for all time intervals recorded

$$\mathrm{p}\left(\mathbf{z}_{1:k}\left|\boldsymbol{\theta}_j,S_j\right.\right)=\prod_{i=1}^{k}\mathrm{p}\left(\mathbf{z}_i\left|\boldsymbol{\theta}_j,S_j\right.\right) \tag{4}$$

This equation provides the likelihood of the scenario once we have assumed its model parameters. In reality, we don't know the exact value of the parameters, but we have an initial idea of their probability distribution, that we indicate with $\mathrm{p}\left(\boldsymbol{\theta}_j\left|S_j\right.\right)$. Therefore, the likelihood of scenario S_j is calculated by marginalization of parameters $\boldsymbol{\theta}_j$

$$\mathrm{p}\left(\mathbf{z}_{1:k}\left|S_j\right.\right)=\int_{D_j}\mathrm{p}\left(\mathbf{z}_{1:k}\left|\boldsymbol{\theta}_j,S_j\right.\right)\cdot\mathrm{p}\left(\boldsymbol{\theta}_j\left|S_j\right.\right)\cdot d\boldsymbol{\theta}_j \tag{5}$$

which basically means integrating the likelihood over the parameters' domain D_j, using their prior distribution $\mathrm{p}\left(\boldsymbol{\theta}_j\left|S_j\right.\right)$ as the weighting function. Once the probability of a scenario has been calculated, Bayes' theorem also allows us to estimate the posterior distribution of the corresponding parameter $\boldsymbol{\theta}_j$, using

$$\mathrm{p}\left(\boldsymbol{\theta}_j\left|\mathbf{z}_{1:k},S_j\right.\right)=\frac{\mathrm{p}\left(\mathbf{z}_{1:k}\left|\boldsymbol{\theta}_j,S_j\right.\right)\cdot\mathrm{p}\left(\boldsymbol{\theta}_j\left|S_j\right.\right)}{\mathrm{p}\left(\mathbf{z}_{1:k}\left|S_j\right.\right)} \tag{6}$$

In summary, we initially have a prior idea of the probability of each scenario and we have models for interpreting the response of the structure in each scenario. When we a acquire fresh set of measurements $\mathbf{z}_{1:k}$, we can calculate the likelihood using Eq. (3) to (5) and then the posterior probability of the scenarios and the parameters using Bayes' theorem in the form of Eqs. (2) and (6).

4. Application to the case study

4.1 Scenarios and models

The procedure presented in the previous Section is stated in a general form, and applies to any type of monitoring problem involving structural and environmental measurements. Below, we explain in more detail how this procedure applies to the Portogruaro Civic Tower. In this case, the problem is to understand as soon as possible if the Tower is still tilting. Using the formal approach introduced in the previous Section, two scenarios are possible: according to the first scenario, S_1, the Tower inclination basically does not change with time, with any shift from the mean position being due to daily and annual temperature changes; according to the second, S_2, the Tower tilt is increasing, with a trend we can assume is linear.

In scenario S_1 the compensated inclination $\varphi_x(t)$ of the Tower in the x direction (i.e., east-west) is modelled as constant and equal to parameter $\varphi_{0,x}^{(1)}$. Conversely, in scenario S_2 the compensated inclination is a linear function with trend w_x and an offset $\varphi_{0,x}^{(2)}$. Response measurements $\mathbf{z}=[x \quad y]^{\mathrm{T}}$

are in this case the out of line components, *x* and *y*, measured at the base of the pendulum while vector $\mathbf{T}$ collects the 4 thermocouple measurements. The predictions used in Eq. (1) can be formulated as

$$\begin{cases} \hat{\mathbf{z}}_1\left(\boldsymbol{\theta}_1;\mathbf{T}(t_i),t_i\right)=\left[\varphi_{0,x}^{(1)}\cdot H + \mathbf{a}_x^{(1)}\cdot\mathbf{T}_i \quad \varphi_{0,y}^{(1)}\cdot H + \mathbf{a}_y^{(1)}\cdot\mathbf{T}_i\right]^{\mathrm{T}} \\ \hat{\mathbf{z}}_2\left(\boldsymbol{\theta}_2;\mathbf{T}(t_i),t_i\right)=\left[\left(\varphi_{0,x}^{(2)}+w_x\cdot t_i\right)\cdot H + \mathbf{a}_x^{(2)}\cdot\mathbf{T}_i \quad \left(\varphi_{0,y}^{(2)}+w_y\cdot t_i\right)\cdot H + \mathbf{a}_y^{(2)}\cdot\mathbf{T}_i\right]^{\mathrm{T}} \end{cases} \tag{7}$$

where H=29.90 m is the reference level for the out of line measurement, $\mathbf{a}_x$ is the linear transformation that correlates the temperatures to the out of plumb, while indices 1 or 2 indicate that this vector generally can assume different values in different scenarios. Following the notation outlined in Section 3, we group the parameters to be updated into a single vector: in scenario S_1, this vector is $\boldsymbol{\theta}_1=\left[\varphi_{0,x}^{(1)} \quad \mathbf{a}_x^{(1)} \quad \varphi_{0,y}^{(1)} \quad \mathbf{a}_y^{(1)}\right]$, while in scenario S_2 it is $\boldsymbol{\theta}_2=\left[\varphi_{0,x}^{(2)} \quad w_x \quad \mathbf{a}_x^{(2)} \quad \varphi_{0,y}^{(2)} \quad w_y \quad \mathbf{a}_y^{(2)}\right]$. Errors $\mathbf{e}_{i,j}=\left[e_{i,1}^{(x)} \quad e_{i,2}^{(y)}\right]^{\mathrm{T}}$ are such that independent components are zero-mean normally distributed, with standard deviation equal to σ_e =10 mm for each sensor and scenario: this value, much larger than that strictly related to the instrumental noise, also takes into account that the actual noise is correlated, while we assume an equivalent white noise model.

4.2 Prior knowledge

To make inference using Bayes' rule, we must define prior knowledge quantitatively. This means assigning a prior probability to the two scenarios and prior distributions to the scenarios' parameters. As mentioned, in October 2003, the probability of the Tower tilting further was reputed to be very low: we can formalize this initial perception by assuming for the tilting scenario a prior probability equal to P(S_2)=1/1000, and therefore P(S_1)=99.9% for the no-trend scenario.

As evident in Eq. (9), scenarios S_1 and S_2 share parameters with the same physical meaning: the offsets in each direction ($\varphi_{0,x}^{(1)}$ and $\varphi_{0,x}^{(2)}$, $\varphi_{0,y}^{(1)}$ and $\varphi_{0,y}^{(2)}$) and the linear transformations in each direction ($\mathbf{a}_x^{(1)}$ and $\mathbf{a}_x^{(2)}$, $\mathbf{a}_y^{(1)}$ and $\mathbf{a}_y^{(2)}$). They are referred to with different indices because they follow a different updating path, and therefore in general their posterior distribution is different. However a priori their value is independent of the scenario, as there is no logical reason to differentiate them based on the fact that the Tower is actually titling or not.

In details, the two offsets, $\varphi_{0,x}$ and $\varphi_{0,y}$, are a purely auxiliary parameters that serve to establish a conventional offset of the Tower inclination at time 0 and uniform temperature equal to 0°C. There is no reason to prefer a priori one value to another, so we could assume for them a uniform initial distribution. Alternatively, we could assume a normal distribution with large variance, which is practically equivalent in terms of effect. In our case, we chose for both a zero-mean normal distribution, with standard deviation of 500 $\times 10^{-6\,\mathrm{rd}}$, corresponding to a horizontal displacement of the wire ($\varphi_0 \cdot H$) of 15 mm. The two parameters are assumed uncorrelated.

Parameters $\mathbf{a}_x$ and $\mathbf{a}_y$ are the sensitivity of tilt to temperature and thus have a very clear physical interpretation. In principle, their distribution could be derived a priori with a decent degree of confidence: take a finite element model of the Tower, apply a number of independent plausible temperature fields to the model and predict for each the resulting tilt of the Tower; then estimate backward the linear relationship between tilt and temperature observed at the four thermocouple points. In practice, there is no need to spend effort in the search of a sophisticated prior distribution: it suffices to take a look at the data to note the strict correlation between temperature and tilt. In other words, the dataset carries very strong information on tilt-temperature correlation and the Bayesian updating converges very fast to the posterior values of $\mathbf{a}_x$ and $\mathbf{a}_y$ regardless of the prior distribution assigned. So, even in this case, their prior distribution can be taken to be uniform or, as we did, zero-mean Gaussian with large standard deviation, 2 mm °C^{-1} in our case.

We must also define the prior knowledge of trends w_x and w_y: because these variables only appear in S_2, their distribution has to be carefully selected. Based on the limited documentation available in October 2003, the only hint is that at the time of construction, say XIV century, the Tower was presumably straight. Compared to Busetto's measurements, carried out in 2001, and assuming a linear trend, we can roughly estimate average shifts of -1.5 mm·year^{-1} in both directions, corresponding to inclination trends of -40.8×10^{-6} rd·year^{-1}. In summary, given the scarce information available at that time, this was assumed as the most likely value of current inclination trend, if any. Of course, this information is very imprecise: to account for this uncertainty, we can assume a prior scatter of, say, 4mm·year^{-1}, corresponding to an angular trend of 108.9×10^{-6} rd.year^{-1}. Note that, although the specific value of selected scatter is not critical, it cannot be arbitrarily chosen, but must reflect our prior ignorance of the trends.

In summary, in each scenario S_j the parameter vector $\boldsymbol{\theta}_j$ is modelled a priori with a multivariate Gaussian distribution $\mathrm{p}\left(\boldsymbol{\theta}_j \middle| S_j\right) = \mathcal{N}\left\{\boldsymbol{\mu}_{\boldsymbol{\theta}}^{(j)}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{(j)}; \boldsymbol{\theta}_j\right\}$ and the components are treated as uncorrelated, which is to say that covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{(j)}$ is diagonal.

4.3 Implementation of the updating procedure

In this Section, we provide the details for applying the procedure outlined in Section 3 to our case. Integration of likelihood as in Eq. (5) generally requires application of numerical methods, which are computationally demanding. However, we can drastically simplify the computation when the problem is linear and all distribution are Gaussian. Going back to our case, first note that the relation between the parameters and prediction stated in Eq. (7), and the corresponding relation for the *y* direction, is linear and Gaussian. We can easily demonstrate that in this case the likelihood is also proportional to a Gaussian distribution on $\boldsymbol{\theta}_j$: $\mathrm{p}\left(\mathbf{z}_{1:k} \middle| \boldsymbol{\theta}_j, S_j\right) \propto \mathcal{N}\left\{\boldsymbol{\mu}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)}, \boldsymbol{\Sigma}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)}; \boldsymbol{\theta}_j\right\}$.

Second, in Section 4.2 we have assumed that the prior parameters are normally distributed: since both prior and likelihood are Gaussian, the posterior is normally distributed as well $\mathrm{p}\left(\boldsymbol{\theta}_j \middle| \mathbf{z}_{1:k}, S_j\right) = \mathcal{N}\left\{\boldsymbol{\mu}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)}; \boldsymbol{\theta}_j\right\}$, and the corresponding mean value $\boldsymbol{\mu}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)}$ and covariance $\boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)}$ have simple closed form expressions (Murphy 2012)

$$\begin{aligned} \boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)} &= \left(\boldsymbol{\Sigma}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)\,-1} + \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{(j)\,-1}\right)^{-1} \\ \boldsymbol{\mu}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)} &= \boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)} \cdot \left(\boldsymbol{\Sigma}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)\,-1} \cdot \boldsymbol{\mu}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)} + \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{(j)\,-1} \cdot \boldsymbol{\mu}_{\boldsymbol{\theta}}^{(j)}\right) \end{aligned} \qquad (8a,b)$$

These equations encode a rule sometimes referred to as *inverse covariance weighting*. Under the same conditions (linear Gaussian model and Gaussian prior), it is also demonstrated (Gregory

2005) that the likelihood of the scenario $\mathrm{p}\left(\mathbf{z}_{1:k}\left|S_j\right.\right)$, required to calculate the posterior probability of the scenario, has a simple closed form solution

$$\mathrm{p}\left(\mathbf{z}_{1:k}\left|S_j\right.\right) = L_{\max}^{(j)} \cdot \Omega^{(j)} \tag{9}$$

where the first term, $L_{\max}^{(j)}$, indicates the maximum value of the likelihood function $\mathrm{p}\left(\mathbf{z}_{1:k}\left|\boldsymbol{\theta}_j, S_j\right.\right)$, corresponding to the most likely value $\boldsymbol{\mu}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)}$ of the parameters

$$L_{\max}^{(j)} = \max_{\boldsymbol{\theta}_j}\left\{\mathrm{p}\left(\mathbf{z}_{1:k}\left|\boldsymbol{\theta}_j, S_j\right.\right)\right\} = \mathrm{p}\left(\mathbf{z}_{1:k}\left|\boldsymbol{\mu}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)}, S_j\right.\right) \tag{10}$$

while the second term, $\Omega^{(j)}$, is the so-called *Ockham factor* which, in the present case, has the following form

$$\Omega^{(j)} = \sqrt{\frac{\left|\boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)}\right|}{\left|\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{(j)}\right|}} \cdot \exp\left[-\frac{1}{2}\cdot\left(\boldsymbol{\mu}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)\mathrm{T}} \cdot \boldsymbol{\Sigma}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)\ -1} \cdot \boldsymbol{\mu}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)} + \boldsymbol{\mu}_{\boldsymbol{\theta}}^{(j)\mathrm{T}} \cdot \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{(j)\ -1} \cdot \boldsymbol{\mu}_{\boldsymbol{\theta}}^{(j)} - \boldsymbol{\mu}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)\mathrm{T}} \cdot \boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)\ -1} \cdot \boldsymbol{\mu}_{\boldsymbol{\theta}|\mathbf{z}}^{(j)}\right)\right] \tag{11}$$

While mean value and covariance of the prior, $\boldsymbol{\mu}_{\boldsymbol{\theta}}^{(j)}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{(j)}$, are assumed as explained in Section 4.2, our problem at this point is to calculate mean value and covariance of the likelihood, $\boldsymbol{\mu}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)}$ and $\boldsymbol{\Sigma}_{\mathbf{z}|\boldsymbol{\theta}}^{(j)}$. To do so, it is convenient to rewrite the relationship between observations and parameters, stated in Eq. (7), in the canonical form

$$\mathbf{z} = \mathbf{A}_j \cdot \boldsymbol{\theta}_j + \mathbf{e}_j \tag{12}$$

where $\mathbf{z} = \left[x_{1:k} \quad y_{1:k}\right]^{\mathrm{T}}$ is the whole set of observations rearranged in a single vector, $\mathbf{A}_j$ is a linear transformation relating prediction to scenario parameter, according to Eq. (7), and $\mathbf{e}_j$ is the vector collecting all the residuals between observation and prediction.

As residuals are assumed to be a zero mean Gaussian noise with variance σ_e^2, the likelihood function can be expressed as

$$\mathrm{p}\left(\mathbf{z}_{1:k}\left|\boldsymbol{\theta}_j, S_j\right.\right) = \mathscr{N}\left\{\mathbf{0}, \sigma_e^2\mathbf{I}; \mathbf{z} - \mathbf{A}_j\boldsymbol{\theta}_j\right\} \propto \mathscr{N}\left\{\boldsymbol{\mu}_{\mathbf{y}|\boldsymbol{\theta}}^{(j)}, \boldsymbol{\Sigma}_{\mathbf{y}|\boldsymbol{\theta}}^{(j)}; \boldsymbol{\theta}_j\right\} \tag{13}$$

where likelihood parameters can be expressed in terms of the pseudo inverse matrix $\mathbf{A}^{+}$ are (Gregory 2005)

$$\begin{aligned} \boldsymbol{\mu}_{\mathbf{y}|\boldsymbol{\theta}}^{(j)} &= \mathbf{A}_j^{+} \cdot \mathbf{z} \\ \boldsymbol{\Sigma}_{\mathbf{y}|\boldsymbol{\theta}}^{(j)} &= \left(\mathbf{A}_j^{\mathrm{T}} \cdot \mathbf{A}_j\right)^{-1} \sigma_e^2 \end{aligned} \tag{14a,b}$$

At this point we can calculate the posterior distribution of the parameters using Eq. (6), and finally calculate probability of the Tower tilting or not using Eqs. (9) and (2). In addition, for scenario S_2, we are also interested in learning the distribution of the trends of inclination w_x and w_y, and this can be done simply through marginalization of their posterior distributions with respect to the other parameters. As the updated distribution is Gaussian, this step turns out to be trivial, as it can be easily proved that the m-th component of the $\boldsymbol{\theta}_j$ vector, $\theta_{j,m}$, is distributed with a uni-variate

Gaussian distribution: $\mathscr{N}\left\{\mu_m^{(j)}, \sigma_m^{2(j)}; \theta_{j,m}\right\}$ (Sivia 2006). The mean value and variance can be directly extracted from the corresponding mean vector and covariance matrix of the multivariate distribution.

We wish in the end to comment on the practical meaning of the *Ockham factor* introduced in Eq. (9). It is worth noting that the two scenarios have differing degrees of complexity: scenario S_2 involves free parameters w_x and w_y, while scenario S_1 can be regarded as a special sub-case of scenario S_2, when w_x and w_y are forced to be null. So, by an appropriate tuning of these trend parameters, S_2 allows the model to follow the measurements more closely, obtaining a better fit. The reader might argue that, because of this, the probability of scenario S_2 will always be greater than that of S_1. Actually, this is not necessarily the case. In fact, it is true, for the above argument, that $L_{\max}^{(2)}$ will be necessary higher than $L_{\max}^{(1)}$, but this is not generally true for the corresponding likelihood of scenarios, because of the Ockham factors. The latter values act as penalty against complexity and flexibility: the Ockham factor is low for models that require fine tuning, while it is high for robust models providing good fitting for a large range of parameters. The first term in Eq. (11), in fact, quantifies the ratio between volumes of posterior and prior distributions: a high ratio indicates model robustness. Overall, it should be noted from Eq. (5) that the likelihood of a scenario is related to the *mean* fitting, taking prior knowledge into account, not to the best fitting only, and the role of $\Omega^{(j)}$ is exactly that of relating the best fitting to the average one (MacKay 2003). In the case-study application, assuming a possible ongoing tilt of the Tower, without precise knowledge about the actual velocity and direction of the motion, could lead to poor average prediction. Bayesian model selection for civil applications is presented in Beck and Yuen (2004), and Yuen (2010).

5. Results and discussion

Based on the prior information (i.e., without considering all the historical information learned in 2005) we can apply the above procedure to update our knowledge using the data acquired real-time by the monitoring system. The thinner line of Fig. 5(e) shows how the monitoring data modifies the perception of having a trend. We can see that during the first two years of monitoring the probability of the trend scenario is always close to zero. Only starting in the third year does the monitoring information begins overturning the initial perception, to the point that in April 2006 the data is sufficient to convince us that the Tower is tilting. Similarly, the thin lines of Figs. 5(c) and 5(d) show the evolution of the distributions of trends w_x and w_y: we see that the trend estimates, which are very uncertain during the first two years, rapidly converge to more reliable values.

The documentation acquired in 2005 radically changed the initial judgment on the stability of the Tower. Common sense suggests that this new information supports the idea that the Tower is tilting. However, our goal here is to quantify the impact of this information on the probability of there being further tilt. The approach we followed is to reduce the historical information to additional samples of tilt measurement that a hypothetical monitoring system would have acquired in the past, and to use them recursively in Eq. (2) to update the prior probability of Scenario 2. To do so, it is convenient to cluster the historical information into three separate datasets, each characterized with its own uncertainty.

The first dataset identified (labelled A) consists of the two out of plumb measurements taken in1962 and 1997, as compared with the 2001 measurements. We have already observed that these measurements, taken with a plumb wire, are very imprecise: keeping in mind the measurement procedure, 20 mm seems a reasonable estimate of the noise scatter.

Another dataset (B) results from comparison of the two spire inclination measurements taken in 1962 and 2001. The 1962 measurement is particularly imprecise: first because it was taken with a plumb wire, second because we don't know exactly the reference of the measurement; under these conditions, a standard deviation of 50 mm is assumed.

The third dataset (C) stems from the guess that the spire was rebuilt vertical in 1879; of course, this is an unproven conjecture, that we could assume true with a likelihood of, say, 50%. If true, the mean value of the presumed body inclination in 1879 can be calculated as the difference between the body inclination and the spire inclination as observed in 2001. To reproduce the very high uncertainty of this estimation, we assumed a standard deviation of 8×10^{-3} rd for each direction.

To give the reader a qualitative idea of the information carried by the historical documentation, Fig. 4 reports in a graph the alleged inclination of the Tower along the two directions in the past years, including its uncertainty. Although the data appear to grow with time, their high uncertainty doesn't allow drawing a sharp conclusion as of the Tower leaning. Moreover, in judging these graphs, we should keep in mind that the 1879 datum is an unverified conjecture.

Table 3 summarizes the outcomes of the prior information update, recursively adding the three historical data sets. With respect to the initial judgment, P(S_2) grows up to 3.7% when all the datasets are considered, while the standard deviations of the angular trend decrease to 17.3×10^{-6} rd year^{-1}.

Merging this new prior knowledge with the instrumental monitoring data, we obtain, day by day, the posterior trend distributions and the probability of scenario S_2, plotted in bold in Figs. 5(c)-5(e).

In detail, Fig. 5(e) illustrates that, assuming all historical data (i.e., sets A, B and C) is known from the beginning, the probability of tilting exceeds 10% after one year and 50% as early as September 2005.

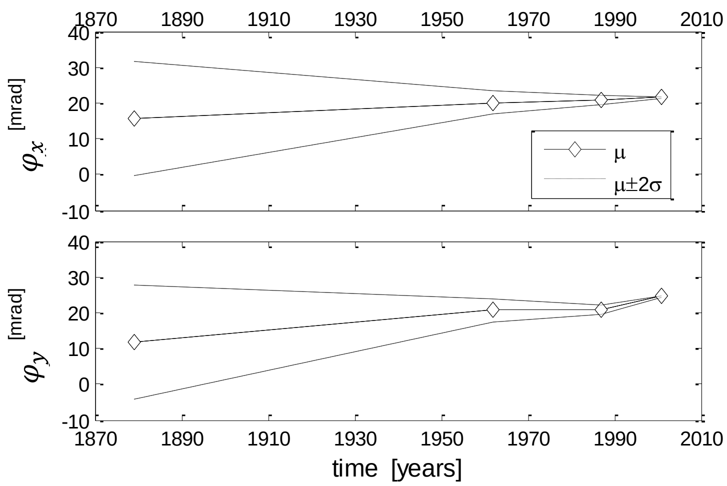


Fig. 4 Alleged Tower inclination in x (a) and y (b) directions at years 1879,1962, 1997 and 2001 based on the historical documentation

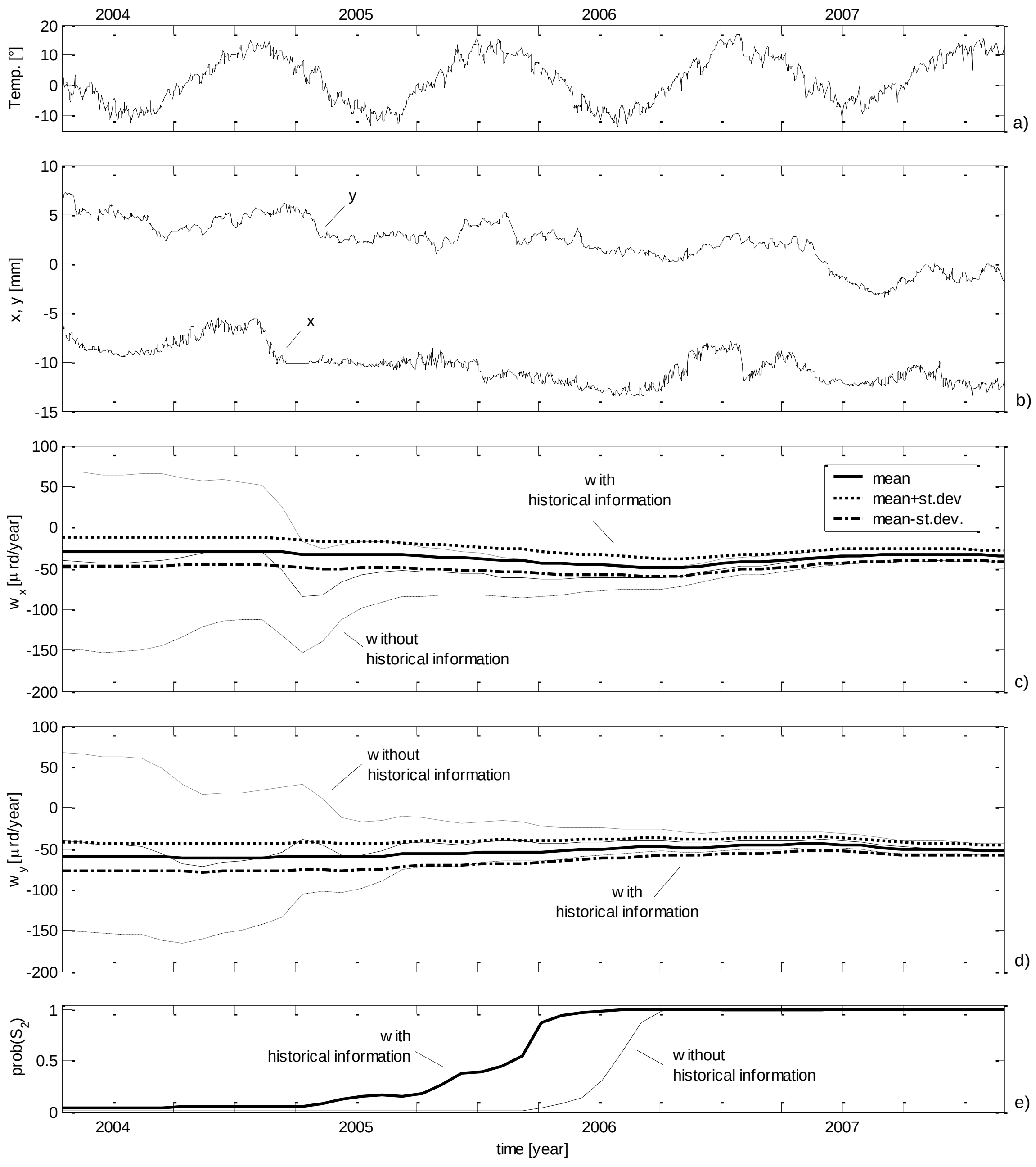


Fig. 5 Temperature measurements at thermocouple T4 (a) out of plumb measurements, (b) posterior distribution of angular trend w_x, (c) posterior distribution of angular trend w_y, (d) posterior probability of scenario S_2 and (e) Historical information refers to all sets: A, B and C

Table 3 Update of prior information based on the historical information

Prior knowledge	P(S_2)	w_x [rd year^{-1}] mean value	scatter	w_x [rd year^{-1}] mean value	scatter
No dataset considered	0.1%	-40.8 × 10^{-6}	108.9 × 10^{-6}	-40.8 × 10^{-6}	108.9 × 10^{-6}
Considering dataset A	2.0%	-30.8 × 10^{-6}	18.0 × 10^{-6}	-58.5 × 10^{-6}	18.0 × 10^{-6}
Considering dataset B	1.3%	-27.0 × 10^{-6}	17.8 × 10^{-6}	-57.3 × 10^{-6}	17.8 × 10^{-6}
Considering dataset C	3.7%	-29.2 × 10^{-6}	17.3 × 10^{-6}	-60.1 × 10^{-6}	17.3 × 10^{-6}

In rough terms, prior knowledge of the historical information makes the system suspicious after one year and aware of the trend after less than two years. Comparison of the two curves (with and w/o the historical data) reveals that historical information has roughly the same effect as 6 to 11 months of instrumental monitoring, thus allowing for a much earlier recognition of the ongoing tilting trend. Observe that, because of the consistency of the Bayesian logic, the outcome of the updating process is invariant with respect to the order of information processing. Thus, the effect of finding the historical documentation is to shift from the thin plot to the bold plot at the time this information is available, which is November 2005.

Worthy of note is that after three years of monitoring, the chance of tilting of the Tower is close to certainty, regardless of the historical data assumed. In the same way the angular trends converge to similar values after the first years of monitoring. This means that, after a certain time, the information acquired by the monitoring system becomes dominant over any type of prior knowledge. In detail, the trends identified at the end of the monitoring are w_x=-34.6×$10^{-6\ rd}$ ·year^{-1} and w_y=-52.1×$10^{-6\ rd}$ year^{-1} with a standard deviation of 6.7×$10^{-6\ rd}$ year^{-1}: in terms of the out of plumb with respect to level H=29.90 m, these values correspond to 1.03 mm year^{-1} West-East and 1.56 mm year^{-1} South-North, with a scatter of only 0.2 mm year^{-1}.

6. Conclusions

We have told the story of a leaning Tower, its monitoring and the attempt at early recognition of possible evidence for ongoing tilting. As often happens when monitoring real-world things, the correct interpretation of the structural behavior is not just a matter of mathematical handling of the rough data, but also of educated interpretation. The general Bayesian methodology introduced deals flexibly with all the uncertainties involved in the recognition problem: measurement noise, uncertainties in the model and inaccurate prior information. Moreover, it lets us quantitatively combine information of completely different natures, including incomplete datasets, subjective experience or even unproven conjectures. We have shown that the Bayesian fusion of instrumental monitoring and historical information allowed timely recognition of signs of an ongoing tilt and estimation of the tilting trend. Further, Bayesian logic allowed not only estimation of the most plausible value of this trend, but the level of reliability of this parameter, which is critical when we are at making decisions which, as in this case, have a huge impact in terms of cost and safety.

So, what happened next to the Tower? Based on the logical route described above, we concluded, in September 2006, that a progressive tilt was very likely. Following these conclusions, in 2007 the Department of Public Works of the Municipality of Portogruaro initiated an investigation campaign to analyze the state of the Tower foundations: it was found that the original timber foundation piles lay in an extremely poor preservation state. At the time these results were officially disclosed, in September 2009, it was measured that the Tower out of plumb had increased 14.3 mm since the beginning of monitoring. In late 2009 the Municipality appointed two local consultants, Busetto Consultants and Colleselli & Partnes, to design a reinforcement intervention of the Tower foundation in order stop the tilt progression and to prevent any possible future risk of collapse. The preliminary design was delivered and approved by the Municipality in January 2011: it included enlargement of the original masonry footing, driving of 19 prefabricated concrete piles and erection of a 15 m tall steel frame to prevent overturning of the Tower during underpinning works. The detailed intervention design has been completed in December 2014. Meanwhile, starting September 2013, the existing monitoring system was integrated, in view of the forthcoming intervention, with an automatic topographic station which tracks the real-time deformation of the Tower. The new system has once again confirmed the ongoing tilting trend, reporting, as of November 2014, a total out of plumb of 1.227 m: this is 30 mm more than the slant measured at the beginning of the monitoring. Funding permitting, the reinforcement works are expected to begin June 2015. But that's another story and shall be told another time.

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References

Bayes, T. (1763), "An Essay toward solving a Problem in the doctrine of Chances", *Philos. T. R. Soc. London*, **53**, 370-418.

Beck, J.L. (2010), "Bayesian system identification based on probability logic", *Struct. Control Health Monit.*, **17**, 825-847

Beck, J.L. and Au, S.K. (2002), "Bayesian updating of structural models and reliability using markov chain Monte Carlo simulation", *J. Eng. Mech. – ASCE*, **128**(4), 380-391.

Beck, J.L. and Katafygiotis, L.S. (1998), "Updating models and their uncertainties, I: Bayesian statistical framework", *J. Eng. Mech. – ASCE*, **124**(2), 455-461.

Beck, J.L. and Yuen, K. (2004), "Model selection using response measurements: Bayesian probabilistic approach", *J. Eng. Mech.- ASCE*, **130**(2), 192-203.

Bretthorst, G.L. (1996), *An introduction to model selection using probability theory as logic*, (Ed., G.R. Heidbreder), Maximum Entropy and Bayesian Methods, Springer New York, NY.

Busetto, A. and Romanin, L. (2001), *Studio preliminare per l'individuazione del percorso atto a definire lo stato patologico della Torre Civica*, Tecnical report for the Municipality of Portogruaro.

Capecchi D. and Vestroni F. (1993), "Identification of finite element models in structural dynamics", *Eng.*

Struct., **15**(1), 21-30.

Chatzi, E. and Smyth A.W. (2009), "The unscented Kalman filter and particle filter methods for nonlinear structural system identification with non-collocated heterogeneous sensing", *Struct. Control Health Monit.*, **16**(1), 99-123.

Flynn E.B. and Todd M.D. (2010), "Bayesian probabilistic structural modeling for optimal sensor placement in ultrasonic guided wave-based structural health monitoring", *Proc. SPIE 7648, Smart Sensor Phenomena, Technology, Networks, and Systems* 2010, 76480Z (April 08, 2010); doi:10.1117/12.847744

Gregory, P. (2005), *Bayesian Logical Data Analysis for the Physical Sciences*, Cambridge: Cambridge University Press.

Jaynes, E.T. (2003), *Probability Theory: The Logic of Science. Cambridge*, UK: Cambridge University Press.

MacKay, D.J.C. (2003), *Information Theory, Inference and Learning Algorithms*, Cambridge, UK: Cambridge University Press.

Malings, C. and Pozzi, M. (2014), "Conditional entropy and value of information based metrics for sensor placement in infrastructure systems", *Proceedings of the 6th World Conference on Structural Control and Monitoring*, Barcelona, 15-17 July 2014.

Memarzadeh M., Pozzi M. and Kolter J.Z. (2014), "Optimal planning and learning in uncertain environments for the management of wind farms", *J. Comput. Civ. Eng.*, 10.1061/(ASCE)CP.1943-5487.0000390 , 04014076.

Molteni, M. (2003), *Torre Civica Duomo di S. Andrea: Campagna di studi, analisi, indagini e prove sperimentali per la valutazione dello stato patologico della torre campanaria: Relazione tecnica conclusiva*, Technical report for the Municipality of Portogruaro.

Mthembu, L., Marwala, T., Friswell, M.I. and Adhikarid, S. (2011), "Model selection in finite element model updating using the Bayesian evidence statistic", *Mech. Syst. Signal Pr.*, **25**(7), 2399-2412.

Murphy, K. (2012), *Machine Learning, a probabilistic perspective*, MIT press.

Papadimitriou, C., Beck, J.L. and Katafyogiotis, L.S. (1997), "Asymptotic expansion for reliability and moments of uncertain systems", *J. Eng. Mech. – ASCE*, **123**(12), 380-391.

Sivia, D.S. (2006), *Data Analysis: a Bayesian Tutorial*, Oxford: Oxford University Press.

Skilling, J. (1998), "Probabilistic data analysis; an introductory guide", *J. Microscopy*, **190**(1-2), 28-36.

Sohn, H. and Law, K.H. (1997), "A bayesian probabilistic approach for structure damage detection", *Earthq. Eng. Struct. D.*, **26**(12), 1259-1281.

Sohn, H and Law, K.H. (2000), "Bayesian probabilistic damage detection of a reinforced concrete bridge column", *Earthq. Eng. Struct. D.*, **29**(8), 1131-1152.

Straub D. (2009), "Stochastic modeling of deterioration processes through dynamic Bayesian networks", *J. Eng. Mech. - ASCE*, **135**(10).

Tien, I., Pozzi, M. and Der Kiureghian, A. (2013), "Assessing maximum structural response under seismic load using dynamic Bayesian network", (Eds., G. Deodatis, B.R. Ellingwood and D.M. Frangopol), *Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures, Proceedings of the 11th International Conference on Structural Safety And Reliability (ICOSSAR2013)*, New York, NY. 16-20 June 2013.

Yuen, K.V. (2010), "Recent developments of Bayesian model class selection and applications in civil engineering", *Struct. Saf.*, **32**(5), 338-346.

Zanet, M. (1997), *Studio storico strutturale della Torre Civica della città di Portogruaro*. Laurea Thesis, Facoltà di Ingegneria, University of Padova.

Zonta, D., Glisic, B. and Adriaenssens, S., (2014) "Value of Information: impact of monitoring on decision making", *Struct. Control Health Monit.*, **21**(7), 1043-1056.

Zonta, D., Pozzi, M. and Zanon, P. (2008) "Managing the historical heritage using distributed technologies". *Int. J. Architect. Heritage*, **2**(3), 200-225.

Zonta, D., Wu, H., Pozzi, M., Zanon, P., Ceriotti, M., Mottola, L., Picco, G.P., Murphy, A.L., Guna, S. and Corrà, M., (2010) "Wireless sensor networks for permanent health monitoring of historic constructions", *Smart Struct. Syst.*, **6**(5-6), 595-618.