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An innovative design method for nonlinear tuned mass damper

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Abstract. The commonly used TMD design method in the project assumes the TMD has pure linearity. However, in real engineering TMD will exhibit nonlinear behaviors. Without considering the nonlinearity of TMD, the control effect of the TMD that is designed by the linear design method, may be worse and even enlarge the structural response. In this paper, based on the previous study results of nonlinear TMD, the improved design method for engineering application is proposed. The linear design method and the improved design method are compared. Taking the best parameter obtained by the improved design method is less than or equal to 90% of that obtained by the original design method as the dividing line. The critical nonlinear coefficient, reaching which value the improved design method needs to be used, is given. Finally, numerical simulations on two engineering examples are conducted to proof the results.

Keywords: TMD; engineering design; nonlinearity; improved design method; critical coefficient

1. Introduction

Tuned Mass Damper (TMD) has been widely used in civil engineering due to its simple structure, no external energy consumption and good stability. Up to now, the working principle of TMD has been basically improved (Den Hartog 1947, Yao 1972, Warburton 1982, Tsai and Lin 1993, Zuo and Nayfeh 2004, Kareem and Kline 1995, Wu and Chen 2000, Kwok and Samali 1995). It has been widely recognized that TMD can control structural vibration induced by wind loads. The John Hancock building in Boston, the Sydney TV Tower in Australia, and the Chiba Port tower in Japan all installed the TMD device to reduce the vibration of the structure (Kwok and Samali 1995).

Linear TMD has the above advantages and is widely used, but there are still limitations, such as small control frequency range. In order to improve the performance of linear TMD, nonlinear TMD has been proposed and studied. Nonlinear Energy Sinks (NES), as one of the nonlinear TMD, has been proved to have wider control frequency bandwidth than linear TMD (Bert *et al*. 1990). Gendelman *et al*. (2011) proposed that NES with multi degree of freedom has wider effective control frequency range. NES can distribute the input energy from lower mode to higher order mode (Quinn *et al*. 2012). However, there is also instability phenomenon and amplification of the structural response amplitude in the nonlinear TMD. Djemal *et al*. (2015) have proved that the nonlinear TMD has a jump phenomenon by experiments. Alexarder and Schilder (2009) pointed

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out that only by eliminating or reducing the periodic solutions with high amplitude generated by nonlinear TMD, the steady-state response of the system can be better than that under linear TMD control. Eason (2015) confirmed the effectiveness of the adjustable length pendulum TMD, and effectively eliminated the periodic solutions with high amplitude. For the experimental research of nonlinear TMD, Wang *et al*. (2015) designed a NES device according to the characteristics of mass block sliding on a curve track. Luo *et al*. (2014) designed a NES with two degrees of freedom by using the constitutive relation of synthetic rubber foam. In the aspect of nonlinear TMD parameter optimization, there are also extensive research results. Sam Fallahpasand *et al.* (2015) analyzed the control performance of nonlinear single pendulum TMD. They used H_∞ and H_2 method to optimize the parameters of TMD. Chen Yong carried out the vibration reduction analysis of high-rise structure based on NES, obtained the analytical solution of nonlinear modal, and gave the empirical formula of optimal parameter calculation (Chen and Xu 2014). Habib *et al.* (2015) used the equal peak method of Den Hartog to study the parameter optimization of nonlinear TMD, and gave an analytic form of parameter optimization.

In practical applications, the TMD design plays a very important role. Tsai and Lin gave the design method of the optimal parameters (frequency ratio, damping ratio) of linear TMD used in damping structures (Tsai and Lin 1993). Based on this theory, Linear TMD design method of multi degree of freedom structure is applied to practical engineering. However, in real engineering, there is no purely linear TMD, and the nonlinear behavior of TMD may have an adverse effect on its control effect.

Aiming at the nonlinear behavior of TMD, Li and Cui (2017) put forward an improved design method. The simulation results show that compared with the original linear TMD design method, the improved design method can effectively eliminate the adverse effects of TMD's nonlinear behavior on control effect. Because most of the nonlinearity in reality is weak nonlinearity, and when the nonlinearity is too weak, there may not be much influence on the vibration control. Thus, this paper focuses on how strong the nonlinearity is that the control effect will be influenced and the improved design method should be used. Simulation of two engineering examples are conducted for verification.

2. Design Method of TMD

In this paper, the TMD with duffing nonlinear stiffness is considered. A single degree of freedom structure with a nonlinear TMD is chosen to be the simplified analytical model, and the equation of motion can be expressed as follow (Li and Cui 2017)

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_{21}(x_1 - x_2) + k_{22}(x_1 - x_2)^3 = F_0cos(\omega t) \quad (1)$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_{21}(x_2 - x_1) + k_{22}(x_2 - x_1)^3 = 0 \quad (2)$$

Where m_1, c_1, k_1 each represents the mass, damping and stiffness of the structure, m_2, c_2, k_{21}, k_{22} each represents the mass, damping, linear stiffness and nonlinear stiffness of the TMD, x_1 and x_2 represent the displacement of the structure and TMD.

2.1 Design method of linear TMD

In the design of optimal parameters of linear TMD, the two factors that should be determined are the optimum frequency ratio γ_{opt} and optimal damping ratio ζ_{opt}. For the structure with multi degree of freedom, taking the ith mode of vibration as the controlled mode, combined with the parameter optimization theory of Tsai and Lin (1993), the parameter optimization formula of multi degree of freedom TMD vibration control is as follows

$$\mu_{eff} = \frac{m_t}{M_{eff}} \tag{3}$$

$$\gamma_{opt} = \frac{\omega_t}{\omega_i} = \frac{1}{1+\mu_{eff}} \tag{4}$$

$$\zeta_{opt} = \sqrt{\frac{3\mu_{eff}}{8(1+\mu_{eff})}} \tag{5}$$

In the formulas, m_t and ω_t are the mass and natural frequency of TMD, ω_i is the ith order frequency of the main structure, M_{eff} is the equivalent mass, defined as the centralized mass of point j, at which point the TMD attach to the main structure, $M_{eff} = M_i/{\emptyset_{ji}}^2$, M_i is the generalized mass, $M_i = \sum {\emptyset_{ji}}^2 m_j$, m_j is the mass of point j, $\emptyset_{ji}$ is the displacement of point j in the ith mode.

2.2 Improved design method of TMD

The improved design method of TMD, which is based on the original linear design method, takes the nonlinearity (duffing form) of the stiffness of TMD into consideration in the process of parameter optimization design. In the simulation and experiment, it is found that the nonlinear TMD has a lower optimal frequency ratio than that designed by the linear method (Li and Cui 2017). Therefore, based on the original design method, by changing the mass of TMD to change the frequency ratio and ensure that the stiffness and damping remain unchanged, which ultimately achieves the purpose of parameter optimization.

Considering the vibration control in the resonance condition is more concerned in the actual project, so the improved optimal frequency ratio is as follows

$$\Upsilon_{opt} = 1\Big/\left(\sqrt[3]{-\frac{q}{2}+\sqrt{(\frac{q}{2})^2+(\frac{p}{3})^3}} + \sqrt[3]{-\frac{q}{2}-\sqrt{(\frac{q}{2})^2+(\frac{p}{3})^3}} - \frac{k}{3}\right)^{\frac{1}{2}} \tag{6}$$

Where, $p = -\frac{k^2}{3} + m$, $q = 2(\frac{k}{3})^3 - \frac{km}{3} + n$, $k = 9{\zeta_2}^2 - 3$, $m = 3 - 9{\zeta_2}^2$, $n = -1 - \frac{81\alpha f^2}{16}$, $\alpha = \frac{k_{22}}{k_{21}}$, $f = \frac{F}{k_{21}}$, $F = m_2\omega^2 A$.

In the formula, k_{21} and k_{22} each represents the linear stiffness and nonlinear stiffness of TMD, ζ_2 represents the damping ratio of TMD, f represents the equivalent static displacement, F represents the amplitude of the equivalent external load, m_2 represents the mass of TMD, ω represents the frequency of external load and A represents the amplitude of the steady state response of the structure and α represents the nonlinear coefficient of TMD, which is the ratio of nonlinear stiffness and linear stiffness of the TMD and shows whether the nonlinearity is strong or weak.

It can be seen from the formula that the improved optimal frequency ratio is directly related to the equivalent static displacement, damping ratio and nonlinear coefficient of TMD, and is also inseparable from the original linear design method, which is also related to the mass ratio. To find out how strong the nonlinearity is that the improved design method should be accepted, the optimal frequency ratio of the two design methods are compared and obtain λ

$$\lambda = \frac{1+\mu}{\sqrt{\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}} - \frac{k}{3}}} \tag{7}$$

When the difference between the two methods is more than 10%, which means $\lambda<0.9$, the improved design method should be adopted. Considering that the optimal frequency ratio in the improved design method is related to more than one factors, and according to the general value range of the relevant factors, the critical α is calculated for the convenience of engineering design, which is shown in Tables 1-6. When the nonlinear coefficient of TMD in the real design is equal to or greater than the critical value of the nonlinear coefficient in the tables, the nonlinearity needs to be considered in the design and the improved design method should be used.

This paper will verify the results in the tables in the next section by two examples.

Table 1 Critical nonlinear coefficient at which value the nonlinearity is needed to be considered (μ=0.005)

Damping ratio of TMD		ζ_2=0.05	ζ_2=0.075	ζ_2=0.1	ζ_2=0.125	ζ_2=0.15	ζ_2=0.175	ζ_2=0.2
Equivalent static displacement	f=0.4	0.028	0.0389	0.0541	0.0736	0.0975	0.1257	0.1583
	f=0.5	0.018	0.0249	0.0346	0.0471	0.0624	0.0805	0.1013
	f=0.75	0.008	0.0111	0.0154	0.021	0.0278	0.0358	0.0451
	f=1	0.0045	0.0063	0.0087	0.0118	0.0156	0.0202	0.0254
	f=1.5	0.002	0.0028	0.0039	0.0053	0.007	0.009	0.0113
	f=2	0.0012	0.0016	0.0022	0.003	0.0039	0.0051	0.0064
	f=2.5	0.0008	0.001	0.0014	0.0019	0.0025	0.0033	0.0041
	f=3	0.0005	0.0007	0.001	0.0014	0.0018	0.0023	0.0029
	f=3.5	0.0004	0.0006	0.0008	0.001	0.0013	0.0017	0.0021
	f=4	0.0003	0.0004	0.0006	0.0008	0.001	0.0013	0.0016
	f=5	0.0002	0.0003	0.0004	0.0005	0.0007	0.0009	0.0011

Table 2 Critical nonlinear coefficient at which value the nonlinearity is needed to be considered (μ=0.01)

Damping ratio of TMD		ζ2=0.05	ζ2=0.075	ζ2=0.1	ζ2=0.125	ζ2=0.15	ζ2=0.175	ζ2=0.2
Equivalent static displacement	f=0.4	0.0326	0.0442	0.0606	0.0816	0.1073	0.1377	0.1727
	f=0.5	0.0209	0.0283	0.0388	0.0522	0.0687	0.0881	0.1105
	f=0.75	0.0093	0.0126	0.0173	0.0232	0.0306	0.0392	0.0492
	f=1	0.0053	0.0071	0.0097	0.0131	0.0172	0.0221	0.0277
	f=1.5	0.0024	0.0032	0.0044	0.0058	0.0077	0.0098	0.0123
	f=2	0.0014	0.0018	0.0025	0.0033	0.0043	0.0056	0.007
	f=2.5	0.0009	0.0012	0.0016	0.0021	0.0028	0.0036	0.0045
	f=3	0.0006	0.0008	0.0011	0.0015	0.002	0.0025	0.0031
	f=3.5	0.0005	0.0006	0.0008	0.0011	0.0015	0.0018	0.0023
	f=4	0.0004	0.0005	0.0007	0.0009	0.0011	0.0014	0.0018
	f=5	0.0003	0.0003	0.0004	0.0006	0.0007	0.0009	0.0012

Table 3 Critical nonlinear coefficient at which value the nonlinearity is needed to be considered (μ=0.02)

Damping ratio of TMD		ζ_2=0.05	ζ_2=0.075	ζ_2=0.1	ζ_2=0.125	ζ_2=0.15	ζ_2=0.175	ζ_2=0.2
Equivalent static displacement	f=0.4	0.0432	0.0566	0.0754	0.0995	0.129	0.1638	0.204
	f=0.5	0.0277	0.0363	0.0483	0.0637	0.0826	0.1049	0.1306
	f=0.75	0.0123	0.0161	0.0215	0.0283	0.0367	0.0466	0.0581
	f=1	0.007	0.0091	0.0121	0.0116	0.0207	0.0263	0.0327
	f=1.5	0.0031	0.0041	0.0054	0.0071	0.0092	0.0117	0.0146
	f=2	0.0018	0.0023	0.0031	0.004	0.0052	0.0066	0.0082
	f=2.5	0.0012	0.0015	0.002	0.0026	0.0034	0.0042	0.0053
	f=3	0.0008	0.0011	0.0014	0.0018	0.0023	0.003	0.0037
	f=3.5	0.0006	0.0008	0.001	0.0013	0.0017	0.0022	0.0027
	f=4	0.0005	0.0006	0.0008	0.001	0.0013	0.0017	0.0021
	f=5	0.0003	0.0004	0.0005	0.0007	0.0009	0.0011	0.0014

Table 4 Critical nonlinear coefficient at which value the nonlinearity is needed to be considered (μ=0.03)

Damping ratio of TMD		ζ_2=0.05	ζ_2=0.075	ζ_2=0.1	ζ_2=0.125	ζ_2=0.15	ζ_2=0.175	ζ_2=0.2
Equivalent static displacement	f=0.4	0.0565	0.0717	0.093	0.1204	0.1539	0.1935	0.2391
	f=0.5	0.0362	0.0459	0.0595	0.0771	0.0985	0.1238	0.1531
	f=0.75	0.0161	0.0204	0.0265	0.0343	0.0438	0.0551	0.0681
	f=1	0.0091	0.0115	0.0149	0.0193	0.0247	0.031	0.0383
	f=1.5	0.0041	0.0051	0.0067	0.0086	0.011	0.0138	0.0171
	f=2	0.0023	0.0029	0.0038	0.0049	0.0062	0.0078	0.0096
	f=2.5	0.0015	0.0019	0.0024	0.0031	0.004	0.005	0.0062
	f=3	0.0011	0.0013	0.0017	0.0022	0.0028	0.0035	0.0043
	f=3.5	0.0008	0.001	0.0013	0.0016	0.0021	0.0026	0.0032
	f=4	0.0006	0.0008	0.001	0.0013	0.0016	0.002	0.0024
	f=5	0.0004	0.0005	0.0006	0.0008	0.001	0.0013	0.0016

Table 5 Critical nonlinear coefficient at which value the nonlinearity is needed to be considered (μ=0.04)

Damping ratio of TMD		ζ_2=0.05	ζ_2=0.075	ζ_2=0.1	ζ_2=0.125	ζ_2=0.15	ζ_2=0.175	ζ_2=0.2
Equivalent static displacement	f=0.4	0.0726	0.0898	0.1138	0.1447	0.1824	0.227	0.2784
	f=0.5	0.0465	0.0575	0.0728	0.0926	0.1167	0.1453	0.1782
	f=0.75	0.0207	0.0256	0.0324	0.0412	0.0519	0.0646	0.0792
	f=1	0.0117	0.0144	0.0182	0.0232	0.0292	0.0364	0.0446
	f=1.5	0.0052	0.0064	0.0081	0.0103	0.013	0.0162	0.0198
	f=2	0.003	0.0036	0.0046	0.0058	0.0073	0.0091	0.0112
	f=2.5	0.0019	0.0023	0.003	0.0038	0.0047	0.0059	0.0072
	f=3	0.0013	0.0016	0.0021	0.0026	0.0033	0.0041	0.005
	f=3.5	0.001	0.0012	0.0015	0.0019	0.0024	0.003	0.0037
	f=4	0.0008	0.0009	0.0012	0.0015	0.0019	0.0023	0.0028
	f=5	0.0005	0.0006	0.0008	0.001	0.0012	0.0015	0.0018

Table 6 Critical nonlinear coefficient at which value the nonlinearity is needed to be considered (μ=0.05)

Damping ratio of TMD		ζ_2=0.05	ζ_2=0.075	ζ_2=0.1	ζ_2=0.125	ζ_2=0.15	ζ_2=0.175	ζ_2=0.2
Equivalent static displacement	f=0.4	0.0921	0.1113	0.1381	0.1727	0.2149	0.2648	0.3224
	f=0.5	0.0589	0.0712	0.0884	0.1105	0.1376	0.1695	0.2064
	f=0.75	0.0262	0.0317	0.0393	0.0492	0.0612	0.0754	0.0917
	f=1	0.0148	0.0178	0.0221	0.0277	0.0344	0.0424	0.0516
	f=1.5	0.0066	0.008	0.0099	0.0123	0.0086	0.0189	0.023
	f=2	0.0037	0.0045	0.0056	0.007	0.0056	0.0106	0.0129
	f=2.5	0.0024	0.0029	0.0036	0.0045	0.0039	0.0068	0.0083
	f=3	0.0017	0.002	0.0025	0.0031	0.0029	0.0048	0.0058
	f=3.5	0.0013	0.0015	0.0019	0.0023	0.0022	0.0035	0.0043
	f=4	0.001	0.0012	0.0014	0.0018	0.0017	0.0027	0.0033
	f=5	0.0006	0.0008	0.0009	0.0012	0.0014	0.0017	0.0021

3. Numerical simulations

3.1 Canton Tower

Canton Tower (Guangzhou TV Tower) is the highest TV tower in China. It is 610 m high, with totally 102 thousand square meters construction area, and weighs 189000t. It is composed of a main tower up to 454 m (core-tube structure) and a 156 m high antenna mast. It is a super high-rise structure with unique, complex, fine soft structure and small damping (Lin *et al.* 2009).

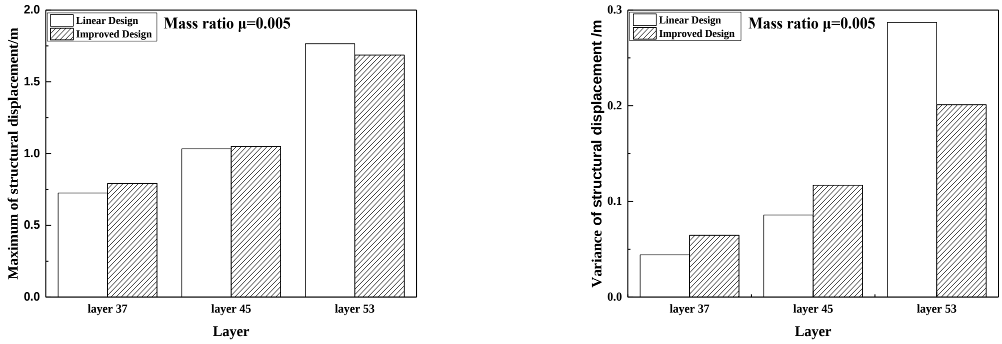
Taking Guangzhou TV Tower as an object of analysis, it is simplified into a frame structure with 53 degrees of freedom. A TMD is placed on thirty-seventh degrees of freedom. Considering the related data in the actual project, the parameters are set up and analyzed by simulation. To prove the data in the tables, according to the different values of the three parameters associated with the improved design method (mass ratio μ, damping ratio ζ_2 of TMD, equivalent static displacement f), we compared the control effect of the two design methods. The equivalent static displacement is calculated to be about 4.1 m. At the same time, because the TMD damping ratio is designed according to the mass ratio, the mass ratio is the only parameter that needs to be selected to confirm the critical value of the nonlinear coefficient α by searching the tables. The displacements of the 37, 45 and 53 layers are mainly considered while the displacements of the lower layers are smaller.

The parameters set according to different mass ratio are shown in Table 7.

After simulation and displacement analysis, the maximum and variance of structural displacement are calculated. The obtained results are shown in Figs. 1-6. We can get the follow conclusions from the figures. When the TMD mass ratio is less than 0.02, for the 37^{th} and 45^{th} layers, the maximum and the variance of the structural displacement of the improved method are both slightly larger than that of the original design method. When the mass ratio is more than 0.02, both the maximum and variance of the improved method become smaller. For the 53^{rd} layer, the maximum and variance of improved method are reduced with every mass ratio. This indicates that the improved design method has an optimized effect on the control effect of nonlinear TMD. When the nonlinearity of TMD reaches the critical value in Tables 1-6, the TMD designed by the improved design method behaves better than that of the original linear design method.

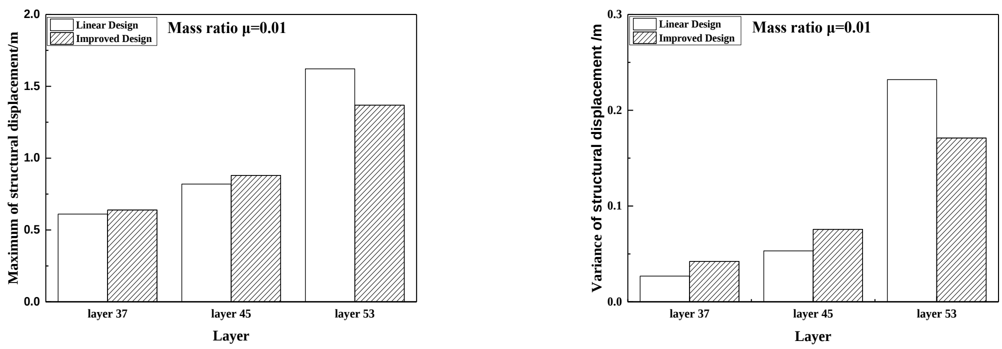
Table 7 Parameters of Guangzhou TV Tower for simulation (equivalent static displacement is 4.1 m)

Group	Mass Ratio	TMD Damping Ratio	Nonlinear Coefficient
Group 1	0.005	0.0432	0.00028
Group 2	0.01	0.0609	0.00044
Group 3	0.02	0.0857	0.00069
Group 4	0.03	0.1045	0.001
Group 5	0.04	0.1201	0.00144
Group 6	0.05	0.1336	0.0019



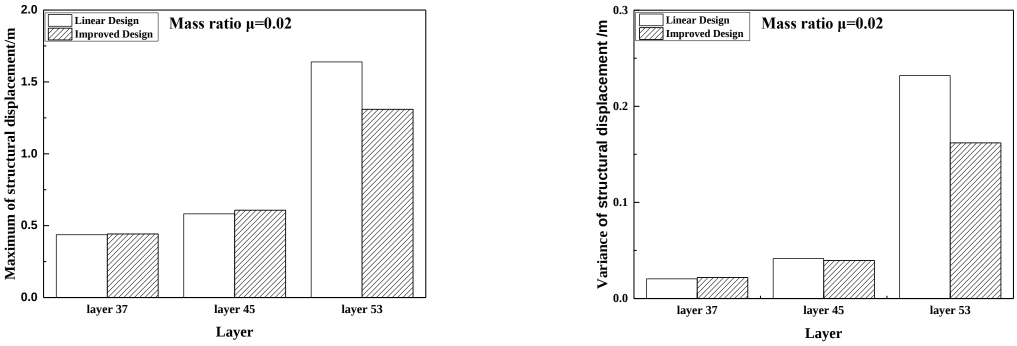
(a) Maximum (b) Variance

Fig.1 The maximum and variance of the structural displacement when μ=0.005



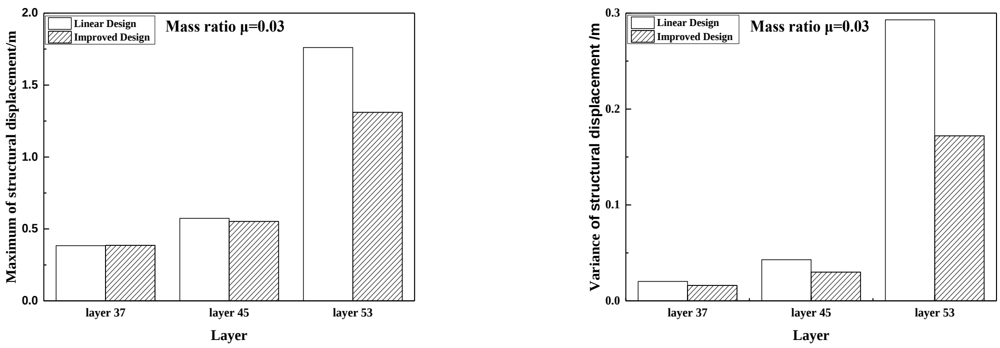
(a) Maximum (b) Variance

Fig. 2 The maximum and variance of the structural displacement when μ=0.01



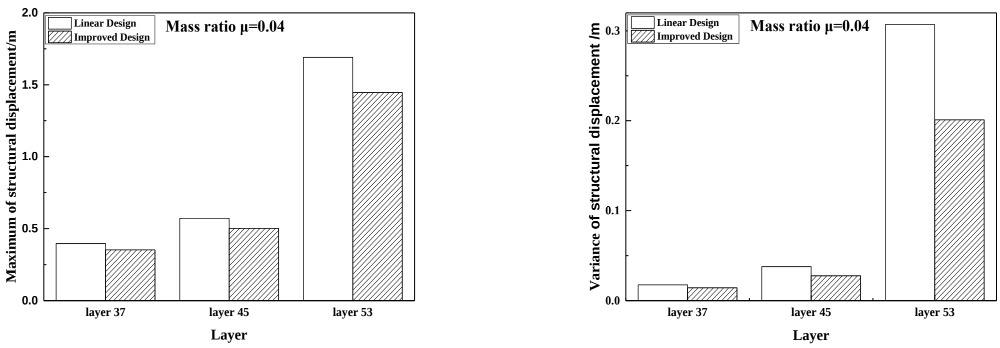
(a) Maximum (b) Variance

Fig. 3 The maximum and variance of the structural displacement when μ=0.02



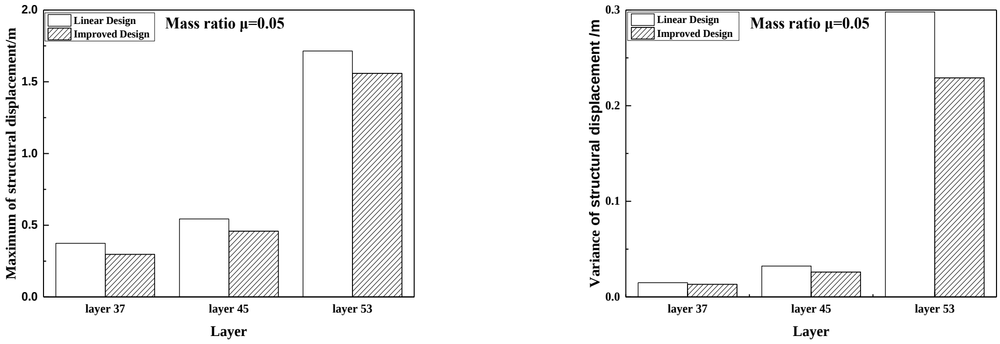
(a) Maximum (b) Variance

Fig. 4 The maximum and variance of the structural displacement when μ=0.03



(a) Maximum (b) Variance

Fig. 5 The maximum and variance of the structural displacement when μ=0.04



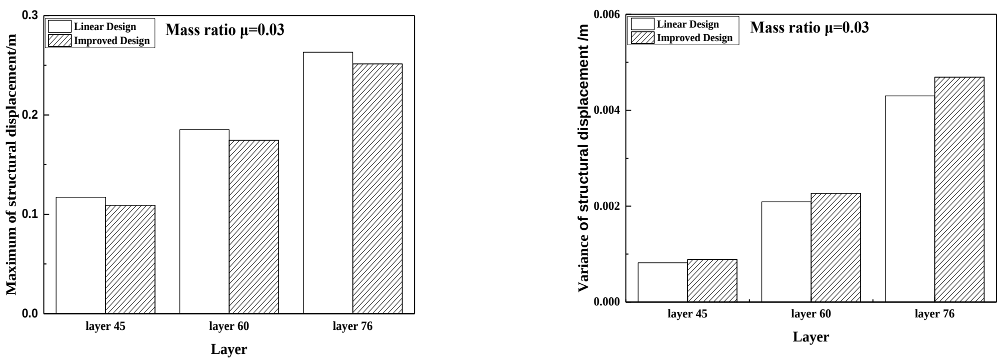
(a) Maximum (b) Variance

Fig. 6 The maximum and variance of the structural displacement when μ=0.05

3.2 76-story benchmark structure model

The benchmark model of 76 levels is a tower structure built in Melbourne, Australia and is 306 meters high. The weight of the structure is 153000 t, total volume is 510000 m^3, and the mass density is about 300 kg/m^3. The ratio of height to width of the structure is 306.1/42=7.3, which belongs to the wind sensitive structure (Wang *et al*. 2009).

In this work, the 45th layer, 60th layer and 76th layer are mainly considered. The mass ratio is set to 0.03, 0.04 and 0.05 and the parameters are shown in Table 8. Figs. 7-9 are the results. It can be seen from the analysis results that when the nonlinearity reaches the critical value in Tables 1-6, the improved design method has a significant reduction in the maximum displacement of the structure, and the variance is only slightly larger than that of the original design method.

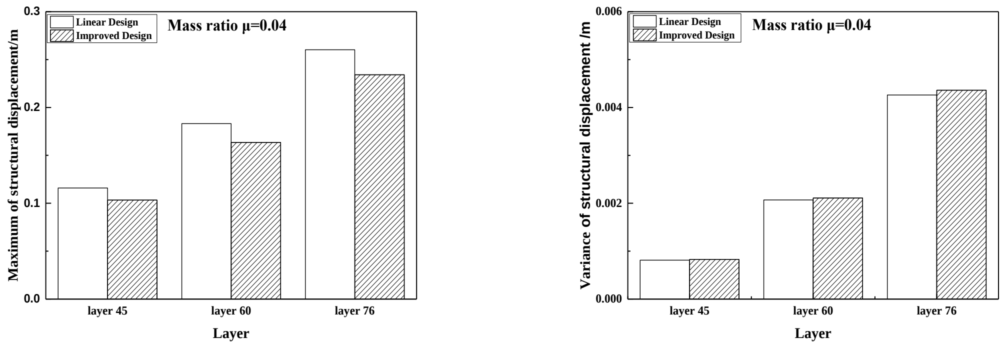


(a) Maximum (b) Variance

Fig. 7 The maximum and variance of the structural displacement when μ=0.03

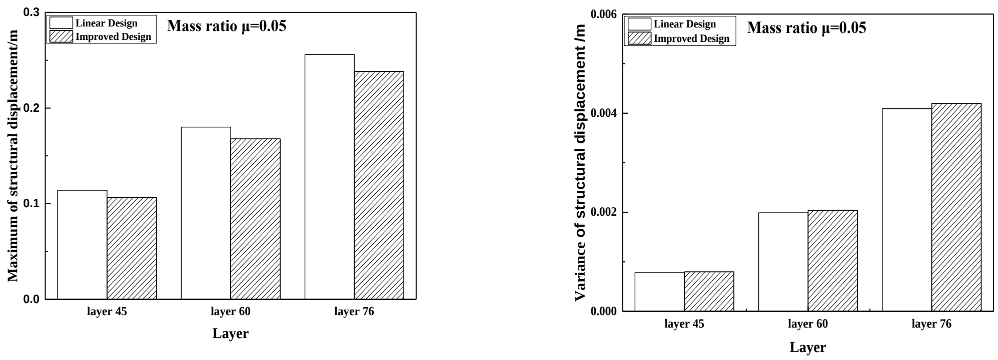
Table 8 Parameters of 76 DOF structure for simulation (equivalent static displacement is 3.5 m)

Group	Mass Ratio	TMD Damping Ratio	Nonlinear Coefficient
Group 1	0.03	0.1045	0.00135
Group 2	0.04	0.1201	0.00182
Group 3	0.05	0.1336	0.00251



(a) Maximum (b) Variance

Fig. 8 The maximum and variance of the structural displacement when μ=0.04



(a) Maximum (b) Variance

Fig. 9 The maximum and variance of the structural displacement when μ=0.05

4. Conclusions

Through the above simulation analysis, the improved design method has optimizing effect on the vibration control of TMD placed on the multi degree of freedom structure. When the nonlinear coefficient of TMD reaches the critical value of Tables 1-6, the improved design method has achieved a more obvious optimizing effect. As the nonlinearity continues to increase, compared with the TMD designed by the linear method, the control effect of TMD designed by the improved design method will become better. Therefore, in practical engineering design, when the nonlinear coefficient of TMD is equal to or greater than the critical value, the improved nonlinear design method should be used to design.

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