Structural Monitoring and Maintenance, *Vol. 6, No. 2 (2019) 147-159*
DOI: https:// doi.org/10.12989/smm.2019.6.2.147

Finite element formulation and vibration of nonlocal refined metal foam beams with symmetric and non-symmetric porosities

Ammar F.H. Al-Maliki, Nadhim M. Faleh* and Abbas A. Alasadi

Al-Mustansiriah University, Engineering Collage P.O. Box 46049, Bab-Muadum, Baghdad 10001, Iraq

(Received March 8, 2019, Revised April 11, 2019, Accepted April 25, 2019)

Abstract. In present article, a size-dependent refined thick beam element has been established based upon nonlocal elasticity theory. Next, it is used to explore vibration response of porous metal foam nanobeams on elastic medium. The established beam element introduces ten degrees of freedom. Different porosity distributions called uniform, symmetric and asymmetric will be employed. Herein, introduced thick beam element contains shear deformations without using correction factors. Convergence and verification studies of obtained results from finite element method are also provided. The impacts of nonlocality factor, foundation factors, shear deformation, slenderness ratio, porosity kinds and porosity factor on vibration frequencies of metal foam nano-sized beams have been explored.

Keywords: free vibration; refined beam theory; porous nanobeam; nonlocal elasticity; porosities

1. Introduction

Porosity-dependent materials are a kind of material in which all properties rely on the amount and distribution of pores. Such material introduces novel applications is civil and aerospace structures because of possessing low weight and high stiffness. Pore distribution kind plays a major role is structural performance of beam-type structures (Mirjavadi *et al*. 2018, 2019a,b). In fact, vibration behavior of beam-type structures made of porous material depends on the amount of pores (Bourada *et al.* 2019). Thus, vibrations of the beams made of porous material have been investigated by various researchers.

Small/large amplitude vibrations of a porous beam constructed from metal foam steel have been studied by Chen *et al.* (2016) accounting for pore distribution. Vibrational characteristics of a thick plate made of porous material of non-uniformly distributed pores have been explored by Rezaei and Saidi (2016). Analysis of frequencies of wave propagation in thick porous plates accounting for shear deformations has been performed by Yahia *et al.* (2015). Another investigation on vibrational behavior of a porous beam based on a thick shear deformable theory has been carried out by Atmane *et al*. (2015).

Many theories are available for presentation of the formulation for a nano-size beam such as nonlocal theory Eringen (1983) or strain gradient theory of elasticity. The theories introduced scale factors to describe the size-dependency which is reported for nano-size structures. The scale factor

∗Corresponding author, Professor, E-mail: dr.nadhim@uomustansiriyah.edu.iq

http://www.techno-press.org/?journal=smm&subpage=7 ISSN: 2288-6605 (Print), 2288-6613 (Online)

makes us able to match the mathematical formulation, especially vibration frequencies, with exact experimental behavior of a size-dependent beam. Different beam or plate theories (Bousahla *et al.* 2016, El-Haina *et al.* 2017, Menasria *et al.* 2017, Abdelaziz *et al.* 2017, Chikh *et al.* 2017, Bakhadda *et al*. 2018, Fourn *et al.* 2018, Kaci *et al.* 2018) together with nonlocal elasticity theory have been used to formulate and analysis mechanical behaviors of nanobeams/nanoplates (Belkorissat *et al.* 2015, Bounouara *et al.* 2016, Ahouel *et al*. 2016, Zenkour 2016, Mouffoki *et al.* 2017, Khetir *et al.* 2017, Youcef *et al.* 2018, Mokhtar *et al.* 2018, Yazid *et al*. 2018, Semmah *et al.* 2019, Faleh *et al.* 2018). For analysis of mechanical characteristics of nanobeams, Zemri *et al.* (2015) introduced a novel nonlocal refined beam model based on shear deformation effects. Bellifa *et al.* (2017) presented a novel thick beam model for nonlinear mechanical behaviors of nanobeams based upon nonlocal theory.

This research is devoted to establish a size-dependent refined thick beam element based upon nonlocal elasticity theory. Next, it is used to explore vibration response of porous metal foam nanobeams on elastic medium. The established beam element introduces ten degrees of freedom. Different porosity distributions called uniform, symmetric and asymmetric will be employed. Herein, introduced thick beam element contains shear deformations without using correction factors. Convergence and verification studies of obtained results from finite element method are also provided. The impacts of nonlocality factor, foundation factors, shear deformation, slenderness ratio, porosity kinds and porosity factor on vibration frequencies of metal foam nano-sized beams have been explored.

2. Models of porosities

Pore distribution types have been considered as three models in this article. These models (uniform, symmetric and asymmetric) have been shown in Fig. 1, while the porous beam has been depicted in Fig. 2. Knowing the fact that maximum elastic modulus, shear modulus and mass density are denoted by E_2, G_2 and ρ_2, the materials characteristics based on the type of porosities might be expressed as

• Uniformly

$$E = E_2(1-e_0\chi) \tag{1a}$$

$$G = G_2(1-e_0\chi) \tag{1b}$$

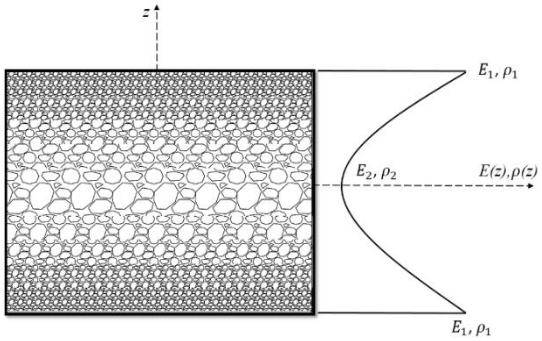
$$\rho = \rho_2\sqrt{(1-e_0\chi)} \tag{1c}$$

• Symmetrically

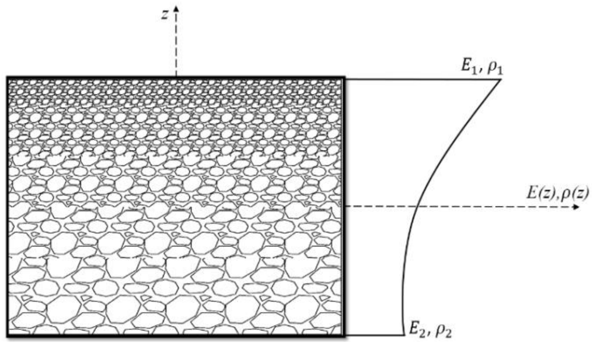
$$E(z) = E_2(1-e_0\cos\left(\frac{\pi z}{h}\right)) \tag{2a}$$

$$G(z) = G_2(1-e_0\cos\left(\frac{\pi z}{h}\right)) \tag{2b}$$

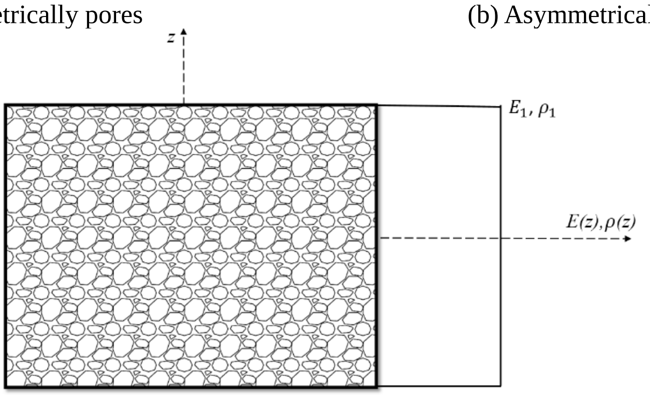
$$\rho(z) = \rho_2 (1 - e_m \cos\left(\frac{\pi z}{h}\right)) \tag{2c}$$



(a) Symmetrically pores



(b) Asymmetrically pores



(c) Uniformly pores

Fig. 1 The kinds of pore dispersions

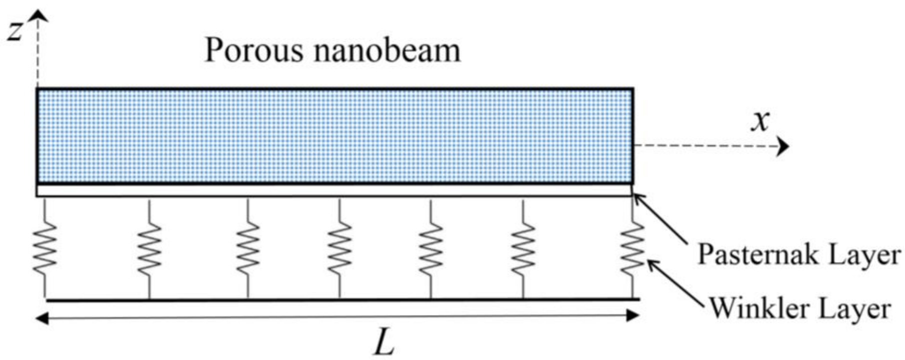


Fig. 2 Configuration and coordinates of nanobeam

- Asymmetrically

$$E(z)=E_2(1-e_0\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)) \tag{3a}$$

$$G(z)=G_2(1-e_0\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)) \tag{3b}$$

$$\rho(z)=\rho_2(1-e_m\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)) \tag{3c}$$

Herein, e_0 and e_m pore and mass factors which are

$$e_0=1-\frac{E_2}{E_1}=1-\frac{G_2}{G_1} \tag{4a}$$

$$e_m=1-\frac{\rho_2}{\rho_1} \tag{4b}$$

and

$$\frac{E_2}{E_1}=\left(\frac{\rho_2}{\rho_1}\right)^2 \tag{5a}$$

$$e_m=1-\sqrt{1-e_0} \tag{5b}$$

Herein, the following function will be used for uniform pore dispersion as

$$\chi=\frac{1}{e_0}-\frac{1}{e_0}\left(\frac{2}{\pi}\sqrt{1-e_0}-\frac{2}{\pi}+1\right)^2 \tag{6}$$

2.1 Formulation based on refined beam theory

Based on the refined beam model accounting for axial (u_0) and transverse (w_b, w_s) displacements, the following 3D field might be defined

$$u_x(x,z,t)=u_0(x,t)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \tag{7a}$$

$$u_y(x,z,t)=0 \tag{7b}$$

$$u_z(x,z,t)=w_b(x,t)+w_s(x,t) \tag{7c}$$

Based upon above definition, the strain field might be defined as

$$\epsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{8a}$$

$$\epsilon_y = \epsilon_z = \gamma_{xy} = \gamma_{yz} = 0 \tag{8b}$$

$$\gamma_{xz} = 2\epsilon_{xz} = g(z)\frac{\partial w_s}{\partial x} \tag{8c}$$

where $g(z) = 1 - f'(z)$.

2.2 A nonlocal refined nanobeam model

In order to formulate a refined beam model at nano-scale, the nonlocal relation in the following form might be used accounting for nonlocal factor (e_0a)

$$(1-(e_0a)^2\nabla^2)\sigma_{kl} = t_{kl} \tag{9}$$

The stress field based on nonlocal theory might be defined as

$$\sigma_{xx} - \mu\frac{\partial^2\sigma_{xx}}{\partial x^2} = E(z)\varepsilon_{xx} \tag{10}$$

$$\sigma_{xz} - \mu\frac{\partial^2\sigma_{xz}}{\partial x^2} = G(z)\gamma_{xz} \tag{11}$$

Next, with the help of the principle of minimum total potential energy, one can get to the following relation

$$\delta\Pi = \delta(U - V - K) = 0 \tag{12}$$

Herein, U is strain energy and T is kinetic energy. The strain energy using Eq. (12) can be stated as

$$U = 0.5\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_{ij}\delta\epsilon_{ij}\right)dzdx$$
$$= \int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}(\sigma_x\delta\epsilon_x + \tau_{xz}\delta\gamma_{xz})dzdx = \int_0^L\Bigg(A_{11}\frac{du_0}{dx}\frac{du_0}{dx} - 2B_{11}\frac{du_0}{dx}\frac{d^2w_b}{dx^2} + D_{11}\frac{d^2w_b}{dx^2}\frac{d^2w_b}{dx^2}$$
$$- 2B_{11}^s\frac{du_0}{dx}\frac{d^2w_s}{dx^2} - (2D_{11}^s + 0.5*(A_{13} + B_{13}))\frac{d^2w_b}{dx^2}\frac{d^2w_s}{dx^2} + H_{11}^s\Bigg)dx \tag{13}$$

The kinetic energy of nanobeam might be defined by

$$K = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)\left[\dot{u}_x\dot{u}_x + \dot{u}_y\dot{u}_y\right] dzdx$$

$$= \int_0^L \left\{ I_0 \left[\dot{u}_0\dot{u}_0 + \mu\left(\frac{d^2\dot{u}_0}{dx^2}\frac{d^2\dot{u}_0}{dx^2}\right) + (\dot{w}_b + \dot{w}_s)(\dot{w}_b + \dot{w}_s) + \mu\left(\frac{d^2(\dot{w}_b + \dot{w}_s)}{dx^2}\frac{d^2(\dot{w}_b + \dot{w}_s)}{dx^2}\right)\right] - 2\left[I_1\left(\dot{u}_0\frac{d\dot{w}_b}{dx}\right) + \mu\left(\frac{d^2\dot{u}_b}{dx^2}\frac{d^3\dot{w}_b}{dx^3}\right)\right] + I_2\left[\left(\frac{d\dot{w}_b}{dx}\frac{d\dot{w}_b}{dx}\right) + \mu\left(\frac{d^3\dot{w}_b}{dx^3}\frac{d^3\dot{w}_b}{dx^3}\right)\right] - 2J_1\left[\left(\dot{u}_0\frac{d\dot{w}_s}{dx}\right) + \mu\left(\frac{d^2\dot{u}_b}{dx^2}\frac{d^3\dot{w}_s}{dx^3}\right)\right] + K_2\left[\left(\frac{d\dot{w}_s}{dx}\frac{d\dot{w}_s}{dx}\right) + \mu\left(\frac{d^3\dot{w}_s}{dx^3}\frac{d^3\dot{w}_s}{dx^3}\right)\right] + 2J_2\left[\left(\frac{d\dot{w}_b}{dx}\frac{d\dot{w}_s}{dx}\right) + \mu\left(\frac{d^3\dot{w}_b}{dx^3}\frac{d^3\dot{w}_s}{dx^3}\right)\right]\right\} dx \quad (14)$$

Also, the mass inertias might be expressed as

$$(I_0, I_1, J_1, I_2, J_2, K_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f, z^2, zf, f^2)\rho(z)dz \quad (15)$$

Next, we define the work of exerted forces in following form

$$V = 0.5\int_0^L [k_p(\frac{\partial(w_b + w_s)}{\partial x}\frac{\partial(w_b + w_s)}{\partial x} + \mu\frac{\partial^2(w_b + w_s)}{\partial x^2}\frac{\partial^2(w_b + w_s)}{\partial x^2} - k_w(w_b + w_s) + \mu k_w\frac{\partial(w_b + w_s)}{\partial x}\frac{\partial(w_b + w_s)}{\partial x})dx \quad (16)$$

where k_w and k_p are Winkler and Pasternak factors and the following edge conditions have for derived for $x = 0$ and $x = L$.

$$\text{Specify} \quad u_o \quad \text{or} \quad N \quad (17)$$

$$\text{Specify} \quad w_b \; or \; V_b = \frac{dM_b}{dx} - I_1\ddot{u}_0 + I_2\frac{d\ddot{w}_b}{dx} + J_2\frac{d\ddot{w}_s}{dx} \quad (18)$$

$$\text{Specify} \quad w_s \; or \; V_s = \frac{dM_s}{dx} + Q - J\ddot{u}_0 + J_2\frac{d\ddot{w}_b}{dx} + K_2\frac{d\ddot{w}_s}{dx} \quad (19)$$

$$\text{Specify} \quad \frac{dw_b}{dx} \; or \; M_b \quad (20)$$

$$\text{Specify} \quad \frac{dw_s}{dx} \; or \; M_s \quad (21)$$

where A_{11} , B_{11}^s , etc., are the beam stiffness, defined by

$$(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)(1, z, z^2, f, zf, f^2)dz \quad (22)$$

$$A_{55}^{s} = \int_{-\frac{h}{2}}^{\frac{h}{2}} G(z) g^2 dz \tag{23}$$

3. Finite element method

Based on present section, finite element approach has been utilized to solve the free vibrational problem of nonlocal porous nanobeams with S-S and C-C boundary conditions. Here, linear shape functions are considered for axial displacement, while Hermitian shape functions are considered for bending and shear displacement components. Accordingly, the displacement field is considered as

$$u_0(x,t) = \sum_{i=1}^{2} U_i(t) N_i(x) = N_1 U_1 + N_2 U_2 \tag{24}$$

$$w_b(x,t) = \sum_{i=1}^{4} W_{bi}(t) \tilde{N}_i(x) = \tilde{N}_1 W_{b1} + \tilde{N}_2 W'_{b1} + \tilde{N}_3 W_{b2} + \tilde{N}_4 W'_{b2} \tag{25}$$

$$w_s(x,t) = \sum_{i=1}^{4} W_{si}(t) \tilde{N}_i(x) = \tilde{N}_1 W_{s1} + \tilde{N}_2 W'_{s1} + \tilde{N}_3 W_{s2} + \tilde{N}_4 W'_{s2} \tag{26}$$

where, U_i , W_{bi} and W_{si} are unknown coefficients and

$$N_1 = 1 - \frac{x}{L_e} \tag{27}$$

$$N_2 = \frac{x}{L_e} \tag{28}$$

$$\tilde{N}_1 = \frac{1}{L_e^3}\left(2x^3 - 3x^2 L_e + L_e^3\right) \tag{29}$$

$$\tilde{N}_2 = \frac{1}{L_e^3}\left(x^3 L_e - 2x^2 L_e^2 + x L_e^3\right) \tag{30}$$

$$\tilde{N}_3 = \frac{1}{L_e^3}\left(-2x^3 + 3x^2 L_e\right) \tag{31}$$

$$\tilde{N}_4 = \frac{1}{L_e^3}\left(x^3 L_e - x^2 L_e^2\right) \tag{32}$$

where L_e is the length of master element.

Substituting Eqs. (24)-(26) in Eq. (12) and minimizing it to the unknown undetermined coefficients U_{i}, W_{bi}, and W_{si}, the following expression yields simultaneous algebraic equations in terms of the unknown coefficients.

$$\frac{\partial \Pi}{\partial U_i}=\frac{\partial \Pi}{\partial W_{bi}}=\frac{\partial \Pi}{\partial W_{si}} \tag{33}$$

Next, the solution trend gives

$$\left\{\begin{pmatrix}[k_{11}] & [k_{12}] & [k_{13}]\\ [k_{21}] & [k_{22}] & [k_{23}]\\ [k_{31}] & [k_{32}] & [k_{33}]\end{pmatrix}_{10*10}+\bar{\omega}_n^{\,2}\begin{pmatrix}[m_{11}] & [m_{12}] & [m_{13}]\\ [m_{21}] & [m_{22}] & [m_{23}]\\ [m_{31}] & [m_{32}] & [m_{33}]\end{pmatrix}_{10*10}\right\}\begin{Bmatrix}U_i\\ W_{bi}\\ W_{si}\end{Bmatrix}=0 \tag{34}$$

in which k_{ij} and m_{ij} are stiffness and mass matrices of master element, respectively. Eq.(34) might be defined as an eigenvalue problem to find vibration frequencies. This is done by defining the determinant of coefficients in this equation and setting it to be zero.

Also, non-dimensional parameters are defined as

$$\hat{\omega}=\omega L^2\sqrt{\frac{\rho_2 A}{E_2 I}},\ K_w=\frac{k_w L^4}{D_2},K_p=\frac{k_p L^2}{D_2},D_2=\frac{E_2 h^3}{12(1-v^2)},\ \ \mu=\frac{e_0 a}{L} \tag{35}$$

4. Numerical results and discussions

Based on established size-dependent refined thick beam element using nonlocal elasticity theory, this section explore vibration response of porous metal foam nanobeams on elastic medium. The established beam element introduces ten degrees of freedom. Different porosity distributions called uniform, symmetric and asymmetric will be employed. Herein, introduced thick beam element contains shear deformations without using correction factors. Convergence and verification studies of obtained results from finite element method are also provided. The impacts of nonlocality factor, foundation factors, shear deformation, slenderness ratio, porosity kinds and porosity factor on vibration frequencies of metal foam nano-sized beams have been explored. Also, the vibration frequency of the present study is validated with those of classical nanobeam model obtained by Ebrahimi and Salari (2015) based on simple supported edge conditions as presented in Table 1. Herein, the material property for the beam might be selected as:

- $E_2\ = 200\ GPa,\ \rho_2=7850\ kg/m^3,\ v\ \ =0.33,$

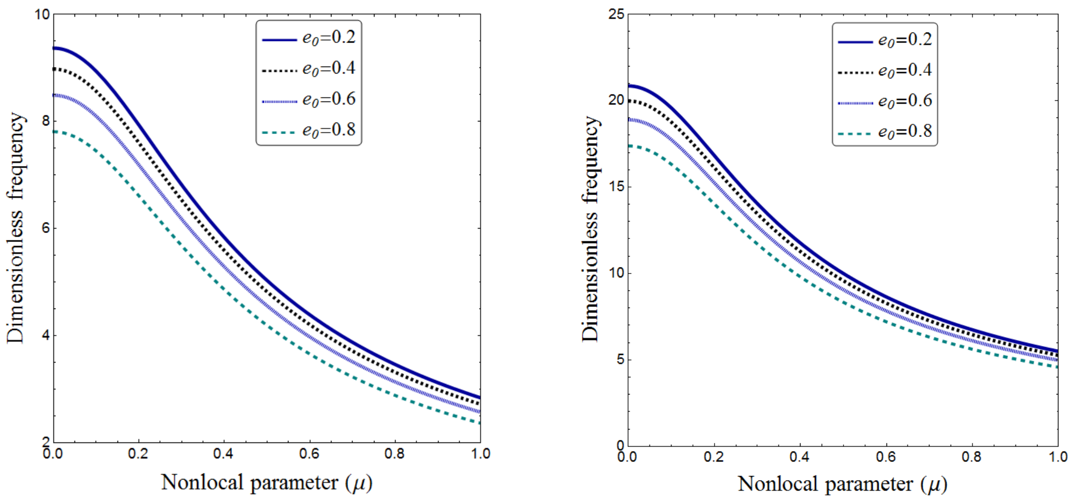
Fig. 3 depicts the influence of nonlocality factor (μ) and porosity factor on the vibrational characteristics of porous nano-sized beams at L/h=10 and K_w=K_p=0 based on uniformly pores. A variety of pore factor have been selected (e_0=0.2, 0.4, 0.6 and 0.8). Vibration frequencies of a macro size beam will be driven by selecting μ =0. It might be seen that nonlocal factor introduces a stiffness decline behavior which yields lower vibration frequencies for all values of porosity coefficient. Thus, non-local models of nanobeams give lower frequencies compared to local models. Also, increase of uniformly porosity factor results in lower frequencies at a fixed nonlocal parameter. This is owning to a remarkable decline in stiffness of nanobeam with presence of pores in material texture.

Pore factor influence on the vibrational frequencies of porous nanobeam has been illustrated in Fig. 4 when μ=0.2, K_w=0 and K_p=0. An increase in porosity coefficient yields larger frequencies for nanobeams with symmetrically pores while lower frequencies for nanobeams with uniformly and asymmetrically pores. Obtained results demonstrate that as the pore factor gets larger, the nanobeams with symmetrically pores have the highest vibration frequency whereas the results of the nanobeam with uniformly and asymmetrically pores become closer. Hence, pore type indicates a notable role on vibrational properties a nano-sized.

In Fig.5, the variation of non-dimensional frequency of nanobeam versus slenderness ratios (L/h) is presented for different types of porosity distribution. One can confirm that a nano-sized beam is less rigid based on a large slenderness ratio. Thus, derived natural frequencies become lower by the growth of slenderness ratio. But, non-dimensional frequency presented increases with the rise of slenderness ratio. However, non-dimensional vibration frequency is more affected by the lower values of slenderness ratio. It can be seen from the figure that shear deformation effect is important at smaller slenderness ratios (larger thickness). While, the effects of shear deformation and slenderness ratio are negligible at high values of slenderness ratio. Once again, one can see that uniform porosity distribution yields smaller vibration frequencies than non-uniform porosity distributions 1 and 2 at a fixed slenderness ratio. While, the results from asymmetrically pores are always between the frequency results of uniformly and symmetrically pores.

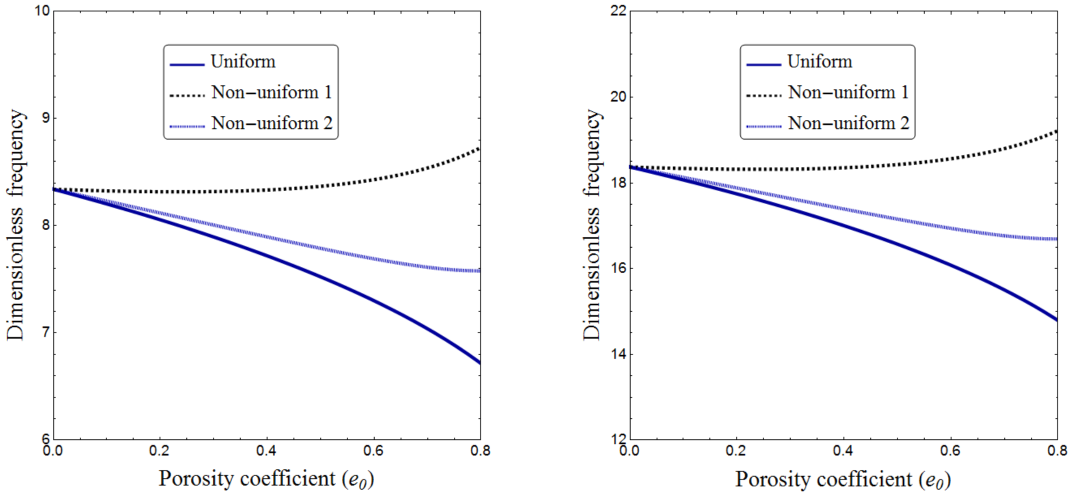
Table 1 Comparison of the dimensionless frequency for nonlocal nanobeams (L/h=20)

μ (nm)	Ebrahimi and Salari (2015)	Present
0	9.8594	9.8567
1	9.4062	9.4036
2	9.0102	9.0077
3	8.6603	8.6579



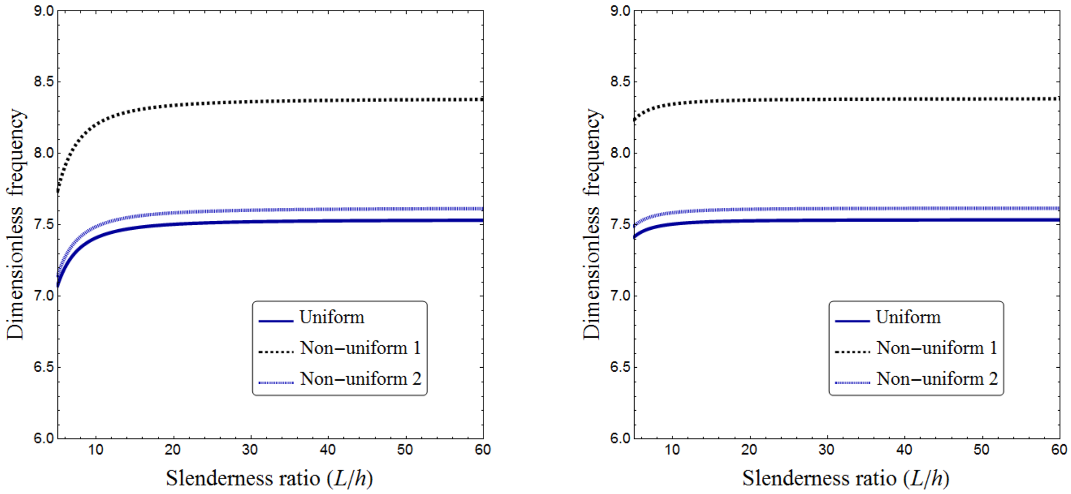
(a) S-S (b) C-C

Fig. 3 Variation of dimensionless frequency versus nonlocal parameter for different uniform porosity coefficient (L/h=10, K_w=0, K_p=0)



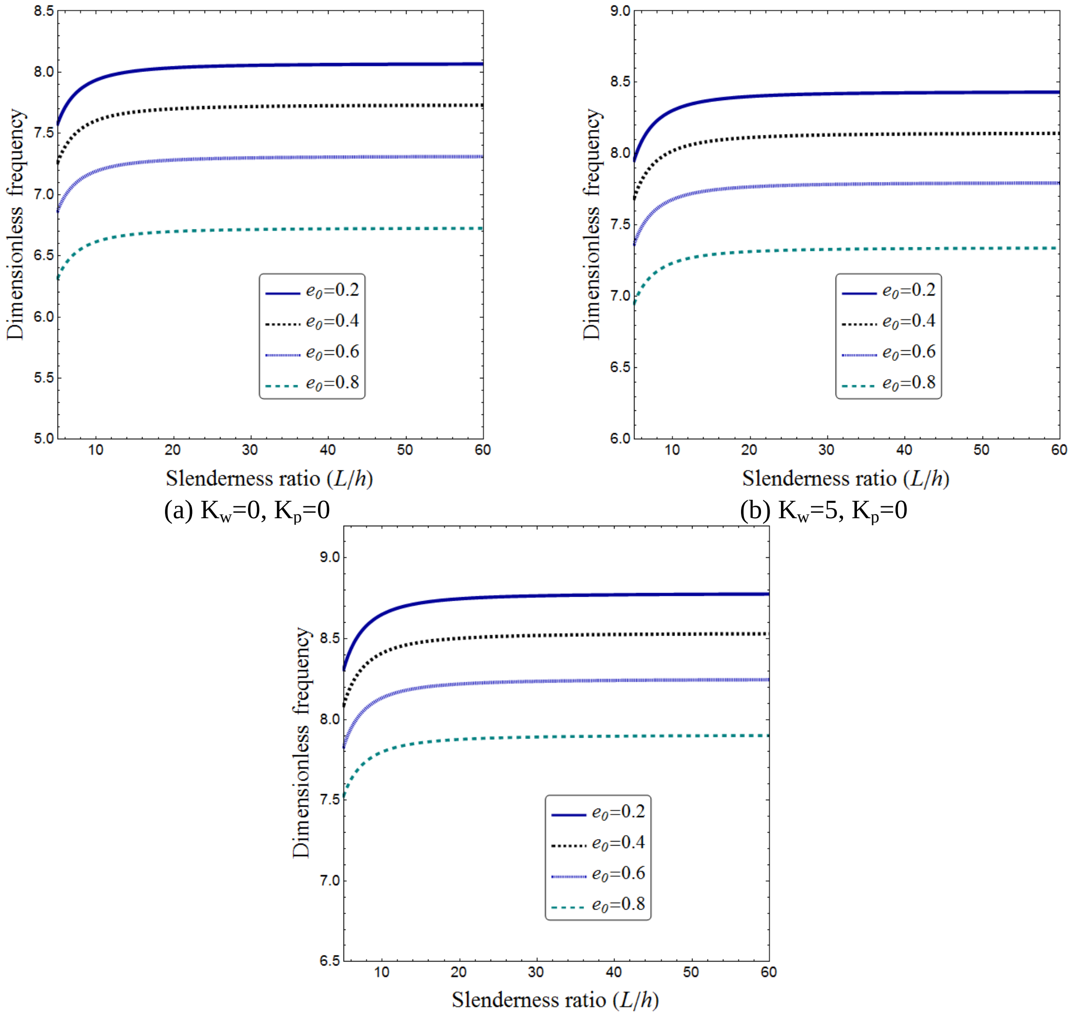
(a) S-S (b) C-C

Fig. 4 Variation of dimensionless frequency versus porosity coefficient for different porosity distributions (L/h=15, K_w=0, K_p=0, μ =*0.2*)



(a) Higher order refined (b) CBT

Fig. 5 Variation of dimensionless frequency versus slenderness ratio for different porosity distributions (K_w=0, K_p=0, μ =*0.2*, e_0=0.5)



(a) K_w=0, K_p=0

(b) K_w=5, K_p=0

(c) K_w=5, K_p=0.5

Fig. 6 Variation of dimensionless frequency versus slenderness ratio for various elastic foundation parameters (μ=0.2)

In Fig. 6, the impacts of Winkler-Pasternak factors on vibration frequencies of porous nanobeams versus slenderness ratio with different pore factors are demonstrated when L/h=10 and μ=0.2. It will be understood that vibration frequencies rely on the amounts of both Winkler and Pasternak factors. Growth of Winkler and Pasternak factors leads to enlargement of the transverse stiffness and vibration frequency of the nano-sized beam. Owning to the fact that Pasternak substrate introduces a continual interaction with the nano-sized beam, its impact on vibration frequencies is more tangible compared to Winkler substrate. At fixed Winkler and Pasternak constant, one can again see that dimensionless vibration frequency increases significantly at lower slenderness ratios, but it remains approximately unchanged at larger slenderness ratios.

5. Conclusions

Based on established size-dependent refined thick beam element using nonlocal elasticity theory, this paper explored vibration response of porous metal foam nanobeams on elastic medium. The established beam element introduced ten degrees of freedom. Different porosity distributions called uniform, symmetric and asymmetric were employed. The impacts of nonlocality factor, foundation factors, shear deformation, slenderness ratio, porosity kinds and porosity factor on vibration frequencies of metal foam nano-sized beams were explored. An increase in porosity coefficient led to larger frequencies for nanobeams with symmetrically pores while lower frequencies for nanobeams with uniformly and asymmetrically pores.

Acknowledgments

The researchers extend their thanks and appreciation to Mustansiriya University and the collage of Engineering.

References

Abdelaziz, H.H., *et al.* (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct.*, **25** (6), 693-704.

Ahouel, M., *et al.* (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct.*, **20**(5), 963-981.

Atmane, H.A., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015b), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct.*, **19**(2), 369-384.

Bakhadda, B., *et al*. (2018), "Dynamic and bending analysis of carbon nanotube-reinforced composite plates with elastic foundation", *Wind Struct.*, **27**(5), 311-324 .

Belkorissat, I., Houari, M.S.A., Tounsi, A., Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., *et al.* (2017), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech*., **62**(6), 695-702.

Bousahla, A.A., *et al*. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech*., **60**(2), 313-335.

Bourada, F., *et al*. (2019), "Dynamic investigation of porous functionally graded beam using a sinusoidal shear deformation theory", *Wind Struct*., **28**(1), 19-30.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation. *Steel Compos. Struct*., **20**(2), 227-249.

Chen, D., Yang, J. and Kitipornchai, S. (2016), "Free and forced vibrations of shear deformable functionally graded porous beams", *Int. J. Mech. Sci*., **108**, 14-22.

Chikh, A., *et al.* (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst*., **19**(3), 289-297.

Ebrahimi, F. and Salari, E. (2015), "Size-dependent free flexural vibrational behavior of functionally graded nanobeams using semi-analytical differential transform method", *Compos. Part B: Eng*., **79**, 156-169.

El-Haina, F., *et al.* (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech*., **63**(5), 585-595.
Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys*., **54**(9), 4703-4710.
Faleh, N.M., Ahmed, R.A. and Fenjan, R.M. (2018), "On vibrations of porous FG nanoshells", *Int. J. Eng. Sci.*, **133**, 1-14.
Faleh, N.M., Fenjan, R.M. and Ahmed, R.A. (2018), "Dynamic analysis of graded small-scale shells with porosity distributions under transverse dynamic loads", *Eur. Phys. J. Plus*, **133**(9), 348.
Fourn, H., *et al.* (2018), "A novel four variable refined plate theory for wave propagation in functionally graded material plates", *Steel Compos. Struct.*, **27**(1), 109-122.
Han, S.C., Park, W.T. and Jung, W.Y. (2015), "A four-variable refined plate theory for dynamic stability analysis of S-FGM plates based on physical neutral surface", *Compos. Struct*., **131**, 1081-1089.
Kaci, A., *et al*. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech*., **65**(5), 621-631.
Khetir, H., *et al.* (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech*., **64**(4), 391-402.
Menasria, A., *et al.* (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct*., **25**(2), 157-175.
Mirjavadi, S.S., Afshari, B.M., Barati, M R. and Hamouda, A.M.S. (2018), "Strain gradient based dynamic response analysis of heterogeneous cylindrical microshells with porosities under a moving load", *Mater. Res. Express*, **6**(3), 035029.
Mirjavadi, S.S., Afshari, B.M., Barati, M.R. and Hamouda, A.M.S. (2019a), "Nonlinear free and forced vibrations of graphene nanoplatelet reinforced microbeams with geometrical imperfection", *Microsyst. Technol*., 1-14.
Mirjavadi, S.S., Afshari, B.M., Barati, M.R. and Hamouda, A.M.S. (2019b), "Transient response of porous FG nanoplates subjected to various pulse loads based on nonlocal stress-strain gradient theory", *Eur. J. Mech. -A/Solids*, **74**, 210-220.
Mokhtar, Y., *et al.* (2018), "A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory", *Smart Struct. Syst*., **21**(4), 397-405.
Mouffoki, A., *et al.* (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst*., **20**(3), 369-383.
Rezaei, A.S. and Saidi, A.R. (2016), "Application of carrera unified formulation to study the effect of porosity on natural frequencies of thick porous–cellular plates", *Compos. Part B: Eng*., **91**, 361-370.
Semmah, A., *et al.* (2019), "Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT", *Adv. Nano Res.*, **7**(2), 89-98.
Yahia, S.A., Atmane, H.A., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech*., **53**(6), 1143-1165.
Yazid, M., *et al.* (2018), "A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium", *Smart Struct. Syst*., **21**(1), 15-25.
Youcef, D.O., *et al*. (2018), "Dynamic analysis of nanoscale beams including surface stress effects", *Smart Struct. Syst.*, **21**(1), 65-74.
Zemri, A., *et al.* (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech*., **54**(4), 693-710.
Zenkour, A.M. (2016), "Nonlocal transient thermal analysis of a single-layered graphene sheet embedded in viscoelastic medium", *Physica E: Low-dimensional Systems and Nanostructures*, **79**, 87-97.

JK