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Structural damage detection based on changes of wavelet transform coefficients of correlation functions

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Abstract. In this paper, an innovative finite element updating method is presented based on the variation wavelet transform coefficients of Auto/cross-correlations function (WTCF). The Quasi-linear sensitivity of the wavelet coefficients of the WTCF concerning the structural parameters is evaluated based on incomplete measured structural responses. The proposed algorithm is used to estimate the structural parameters of truss and plate models. By the solution of the sensitivity equation through the least-squares method, the finite element model of the structure is updated for estimation of the location and severity of structural damages simultaneously. Several damage scenarios have been considered for the studied structure. The parameter estimation results prove the high accuracy of the method considering measurement and mass modeling errors.

Keywords: auto correlation; finite element model updating; sensitivity analysis; structures health monitoring; wavelet transform

1. Introduction

Large structures such as offshore platforms, ships, bridges, etc are designed for long-time operation. Inadequate performance of these structures or failure may result in human casualties and economic losses, so identifying the structural threat factors is very important in their maintenance. In order to prevent these hazards, researchers are always looking for methods at the lowest cost to identify the structural damages and estimate the remaining structural lifetime. Therefore, health monitoring of the structures in various engineering fields is of great interest (Sohn *et al*. 2002, Dackermann *et al*. 2013). Vibration-based damage detection approaches are one of the most common methods to monitor the condition of structures. They could be based on the modal data such as natural frequency (Kim *et al*. 2003, Ercolani *et al*. 2018) and mode shape (Lee *et al*. 2018), which are intrinsic characteristics of a structure and provide valuable information about the state of the structure. Furthermore, mode shape curvature (Zhang *et al*. 2022, Ho and Ewins 2000, Zhang *et al*. 2012), strain mode shape (Lee *et al*. 2009, Wei *et al*. 2016), flexibility matrix (Doebling *et al*. 1996), modal flexibility (Seyedpoor and Montazer 2016), static response (Ren *et al*. 2019) and mechanical impedances (Providakis *et al*. 2015, Huynh *et al*. 2017) are structural stiffness, mass

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,and damping parameters. The accuracy of the modal methods decreases when a few modes are available. Hence, innovative and combinatorial methods is essential to increase the accuracy of structural model updating and detecting structural damages based on structural responses (Modak *et al*. 2002). Mousavi *et al*. (2020) evaluated the performance of Hilbert-Huang Transform (HHT) for damage detection of a scaled steel-truss bridge model. They were able to estimate the severity and location of the damage with acceptable accuracy. Kordestani *et al*. (2021) proposed an algorithm for decomposing the structural response signal using the Savitzky-Golay filter and isolating the accompanying signal noise to reconstruct the original signal. They evaluated their proposed algorithm for damage detection of a bridge with free supports and moving load. They showed that the proposed method is able to detect the damage with appropriate accuracy.

The frequency-domain presentation structural responses provides more comprehensive information about the structure than the mode shapes and natural frequency. Low sensitivity to measurement errors and high sensitivity to structural parameter variations are the other advantages of the frequency response function (FRF) and transformations of the correlation function. Based on these features, various studies have been proposed for damage detection based on FRF and transformations of correlation function (Garcia-Palencia *et al*. 2015, Li *et al*. 2016). Dackermann *et al*. (2013) proposed a damage detection method based on FRF and Artifical Neural network (ANN) to identify the location and severity of notch-type damage. Nandakumar and Shankar (2016)used experimental transfer function data to identify defects of beam models. Another feature based on structural response is Power Spectral Density (PSD). PSD is more sensitive to damage as compared to FRF and modal data and, it embraces both auto and cross-spectral terms, hence providing more data for model updating (Kammer and Nimityongskul 2009). Bayissa and Haritos (2007) employed spectral strain energy (SSE) for damage diagnosis. They compared their results with the modal strain energy indicator and discussed the merits of spectral analysis over modal approaches for damage detection. Li *et al*. (2015) used the forward finite difference method for calculation of the sensitivity equation of power spectral density transmissibility (PSDT). Despite the simplicity of the calculation,the quality of the sensitivity equation highly depends on the assumed increments for change in structural parameter.

A group of the SHM methods minimizes the differences of a numerical and experimental feature to update the finite element model. These methods use response sensitivity concerning to unknown variables to update structural parameters at the element level for identification of the location and severity of structural damages. By the sensitivity-based methods, changes of structural responses with respect to structural parameters are computed using the gradient, or formulated based on innovative approaches applied on the governing equation. Zheng *et al*. (2015) proposed a sensitivity-based method based on power spectral density of acceleration response. Acceleration responses and power spectral density data are extracted under stationary and random excitations using the pseudo excitation method. Razavi and Hadidi (2020) presented a sensitivity-based FEM updating technique for damage detection in large space structures. Pedram *et al*. (2016) used sensitivity of power spectral density (PSD) of strain data to identify of structural damages. They extracted the exact sensitivity equation of PSD of strain data. The impact of essential factors such as incompleteness is investigated. The advances show that much of the valuable information is hidden in the vibration signal. Therefore, different types of domain transformation is necessary to obtain essential and non-extractable information (Granlund and Knutsson 2013). One of these tools is the wavelet transform function. A wavelet transform is an efficient tool in the time-frequency analysis of a signal. The wavelet transform enables shorter time properties in areas where more frequency information is desirable. The signifcant advantage of wavelets compared to other signal processing methods, such as Fourier transform, is locally analyzing a particular area of a more signifcant signal. The wavelet transform-based methods were used to detect the structural damages due to the high sensitivity of wavelet transform coefficients to the changes of vibrational signals structural parameter (Rioul and Vetterli 1991, Huang and Nagarajaiah 2021).

In order to model the asymmetric behavior of opening and closing cracks, Valente and Spina (1997) utilized the wavelet transform. Douka *et al*. (2003) used continuous wavelet transform of the first mode shape of the beam to identify the crack location based on the changes of the wavelet transform coefficients. Chang and Chen (2004) have proposed a method for detecting structural damages using the wavelet transform. In this research, the finite element modeling of a plate has been used. The proposed method is highly sensitive to damage and by observing the distribution of wavelet transform coefficients, the location of the damage can also be identified. Law and Li (2007) used the wavelet transform to provide sensitivity functions and indicated that the extracted wavelet coefficients are highly sensitive to local changes of structural parameters and less sensitive to noise and errors. Rucka and Wilde (2006) as well as Fan and Qiao (2009) proposed a damage detection algorithm based on continuous wavelet transform for structures consisting of plates. In these studies, it has been shown that the two-dimensional continuous wavelet transform method is very effective for detecting structural damages including plates and shells damages. Zhong and Oyadiji (2011) identified the damage in reinforced plates based on the finite element model and discrete wavelet transform analysis. Lam and Yin (2012) used the two-dimensional spatial wavelet transform coefficients in order to identify the damage in plate-type structures. They showed that the proposed scheme is able to detect the location, length and depth of crack in the considered model. Asgarian *et al*. (2016) calculated the energy rate of the vibrational response signals using the wavelet transform. Their results indicated that any sensor that detects a more significant rate of changes of energy is closer to the damage location.

In this paper, a new sensitivity equation of the wavelet transform coefficients of the correlation function is developed for the finite element model updating. This WTCF sensitivity is more sensitive to local structural changes than the wavelet transform function response sensitivity. The proposed sensitivity equation presents a quasi-linear relation for the change of the wavelet coefficient concerning the change of structural parameters. The robustness of the method against modeling and mass measurement errors is investigated by adding random errors to the mass parameters of the truss and plate model.

2. Theory

The wavelet transform decomposes a function $f(t)$ into a superposition of the elementary function $\psi_{(a.b)}$ obtained from a mother wavelet $\psi(t)$ by scaling and translation as

$$\psi_{(a.b)}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \tag{1}$$

and the wavelet transform of a function $f(t)$ is defined as

$$W^{f}_{(a.b)} = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\psi^{*}(\frac{t-b}{a})dt \tag{2}$$

Where $\psi^{*}(t)$ is the complex conjugate of the mother wavelet function, a and b are the time scale and the time shift of the function $\psi(t)$, respectively. The defined integral will be the wavelet transform coefficients, which present a correlation between the wavelet function and the $f(t)$ (Rioul and Vetterli 1991). In this study, a binary discrete wavelet transform is used for improvement and more straightforward implementation of wavelet transformation. In this transformation, the scale parameter a and the time shift b are sampled in a binary grid as $a = 2^j$ and $b = k2^j$ $(j\epsilon N$, $k\epsilon Z)$ and j and k are the time scale and time shift in the discrete wavelet transform. The equation of motion of multi degrees of freedom structure, under an external force, is

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \tag{3}$$

where $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$, and $F(t)$ are displacement, velocity, acceleration and external force vectors, respectively, and $[M]$, $[K]$ and $[C]$ are the mass, stiffness, and damping matrices of the system, respectively. The wavelet transform of structures response can be defined as follows

$$x(t) = \int_{-\infty}^{\infty} x_{j.k}\psi_{j.k}(t)\mathrm{dt} \tag{4}$$

By applying the discrete wavelet transforms to the vectors $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$, and $F(t)$ (Law and Li 2007)

$$\{x(t)\} = \sum_{j.k}\{x_{j.k}\}\,\psi_{j.k}(t) \tag{5}$$

$$\{\dot{x}(t)\} = \sum_{j.k}\{x_{j.k}\}\,\dot{\psi}_{j.k}(t) \tag{6}$$

$$\{\ddot{x}(t)\} = \sum_{j.k}\{x_{j.k}\}\,\ddot{\psi}_{j.k}(t) \tag{7}$$

$$\{F(t)\} = \sum_{j.k}\{x^F_{j.k}\}\,\psi_{j.k}(t) \tag{8}$$

Substituting Eqs. (5) to (8) into the equation of motion of the structure and considering the orthogonality of the wavelets, yields (Law and Li 2007)

$$\left[M\int \ddot{\psi}_{j.k}(t)\,\psi_{j.k}(t)dt + C\int \dot{\psi}_{j.k}(t)\,\psi_{j.k}(t)dt + K\right]\begin{Bmatrix} x^1_{j.k} \\ x^2_{j.k} \\ \vdots \\ x^N_{j.k} \end{Bmatrix} = \{x^F_{j.k}\} \tag{9}$$

where N is the number of degrees of freedoms. The superscripts over X present the related DOF numbers. The appeared integrals in Eq. (9) are only related to the mother wavelet functions. Hence these integrals can be written as

$$\begin{aligned} a_{(j.k)} &= \int \ddot{\psi}_{(j.k)}(t)\psi_{(j.k)}\,(t)dt \\ b_{(j.k)} &= \int \dot{\psi}_{(j.k)}(t)\psi_{(j.k)}\,(t)dt \end{aligned} \tag{10}$$

In the above equation, $\psi_{(j.k)}$, $\dot{\psi}_{(j.k)}$ and $\ddot{\psi}_{(j.k)}$ are the desired wavelet, the first-order derivative and the second order derivative of the mother wavelet. Hence, Eq. (9) can be rewritten as

$$\left[Ma_{(j.k)} + Cb_{(j.k)} + K\right]\begin{Bmatrix} x_{j.k}^{1} \\ x_{j.k}^{2} \\ \vdots \\ x_{j.k}^{N} \end{Bmatrix} = \left\{x_{j.k}^{F}\right\} \tag{11}$$

By definition of the wavelet transfer function as (Law and Li 2007)

$$H_{\psi_{(j.k)}} = Z_{\psi_{(j.k)}}{}^{-1} = (Ma_{(j.k)} + \mathrm{C}b_{(j.k)} + K)^{-1} \tag{12}$$

Where $Z_{\psi_{(j.k)}}$ is the impedance matrix and it is the inverse of the wavelet transfer function, $H_{\psi_{(j.k)}}$. The wavelet transform of the structural response can be expressed by

$$\left\{x_{j.k}\right\} = H_{\psi_{(j.k)}}\left\{x_{j.k}^{F}\right\} \tag{13}$$

The relationship between the wavelet transform of the correlation function of the applied force and the wavelet transform of the correlation function of structural response is defined as (Kong, Spanos *et al*. 2014)

$$S_{\psi_{(j.k)}}^{xx} = H_{\psi_{(j.k)}} S_{\psi_{(j.k)}}^{ff} H_{\psi_{(j.k)}}^{*} \tag{14}$$

Where $H_{\psi_{(j.k)}}$ is the wavelet transfer function, $H_{\psi_{(j.k)}}^{*}$ is the complex conjugate transpose (Hermit matrix) of the wavelet transfer function, and $S_{\psi_{(j.k)}}^{xx}$ is the wavelet transform of the correlation functions matrix of structural responses. Its diagonal terms are auto spectral density and the non-diagonal ones are cross spectral density terms. $S_{\psi_{(j.k)}}^{ff}$ is the wavelet transform of the correlation functions of the input in all the active degree of freedom, respectively. Eq. (15) can be rewritten as

$$Z^{*}{}_{\psi_{(j.k)}} S_{\psi_{(j.k)}}^{xx} = H_{\psi_{(j.k)}} S_{\psi_{(j.k)}}^{ff} \tag{15}$$

Now, Eq. (15) can be expressed for a damaged structure as follows

$$\left[Z^{*}{}_{\psi_{(j.k)}} + \Delta Z^{*}{}_{\psi_{(j.k)}}\right]\left[S_{\psi_{(j.k)}}^{xx} + \Delta S_{\psi_{(j.k)}}^{xx}\right] = \left[H_{\psi_{(j.k)}} + \Delta H_{\psi_{(j.k)}}\right] S_{\psi_{(j.k)}}^{ff} \tag{16}$$

Expanding Eq. (16) and subtracting Eq. (15), yields

$$\left[Z^{*}{}_{\psi_{(j.k)}} + \Delta Z^{*}{}_{\psi_{(j.k)}}\right]\Delta S_{\psi_{(j.k)}}^{xx} = S_{\psi_{(j.k)}}^{ff}\Delta H_{\psi_{(j.k)}} - \Delta Z^{*}{}_{\psi_{(j.k)}} S_{\psi_{(j.k)}}^{xx} \tag{17}$$

Expressing the change of $\Delta Z^{*}{}_{\psi_{(j.k)}}$ as (Law and Li 2007)

$$\Delta Z_{\psi_{(j.k)}} = (\Delta Ma_{(j.k)} + \Delta \mathrm{C}b_{(j.k)} + \Delta K) \tag{18}$$

and

$$H^{D}{}_{\psi_{(j.k)}} = \left[Z_{\psi_{(j.k)}} + \Delta Z_{\psi_{(j.k)}}\right]^{-1} \tag{19}$$

The change of wavelet transform of the correlation function of structural response is

$$\Delta S^{xx}_{\psi(j.k)} = H^{*D}{}_{\psi(j.k)} S^{ff}_{\psi(j.k)} \Delta H_{\psi(j.k)} - H^{*D}{}_{\psi(j.k)} \Delta Z^{*}{}_{\psi(j.k)} S^{xx}_{\psi(j.k)}. \tag{20}$$

The estimation of $\Delta H_{\psi(j.k)}$ is expressed as (Pedram *et al.* 2017)

$$\Delta H_{\psi(j.k)} = -H^{D}{}_{\psi(j.k)} (\Delta M a_{(j.k)} + \Delta C b_{(j.k)} + \Delta K) H_{\psi(j.k)} \tag{21}$$

Substituting Eq. (15) and Eq. (21) into Eq. (17), the change of the spectral density is obtained as follows

$$\begin{aligned} \Delta S^{xx}_{\psi(j.k)} &= -H^{*D}{}_{\psi(j.k)} S^{ff}_{\psi(j.k)} H^{D}_{\psi(j.k)} (\Delta Z_{\psi(j.k)}) H_{\psi(j.k)} \\ &- H^{*D}{}_{\psi(j.k)} (\Delta Z^{*}{}_{\psi(j.k)}) H^{*}{}_{\psi(j.k)} S^{ff}_{\psi(j.k)} H_{\psi(j.k)} \end{aligned} \tag{22}$$

The extracted sensitivity equation is an exact formulation to relate the changes of the wavelet transform coefficient of the response correlation function to the changes of structural parameters. Because of the appeared term of $H^{*D}{}_{\psi(j.k)}$, the developed sensitivity equation needs to calculate the wavelet transfer function at all degrees of freedom of the damaged structure. Due to technical limitations such as sensor installation at all degrees of freedom, and unavailability of some degrees of freedom and the high costs of a sensor network, the incomplete measurement problem is inevitable. There are methods to reduce the model or extend the data to solve the problem of incomplete measurement. However, these methods lead to the intensification of nonlinear and non-uniform behavior in the finite element model updating process. The wavelet transfer function can be presented as (Mansourabadi and Esfandiari 2019)

$$H_{\psi(j.k)} = \sum_{r=1}^{n} \frac{\phi_r^T \phi_r}{\Omega_r^2 + a_{(j.k)} + i b_{(j.k)} \Omega_r} \tag{23}$$

In Eq. (23), the complete measurement of the structural responses will not be possible due to practical limitation. Hence, in this study, adverse effects of incompleteness in measurement of $H^{D}_{\psi(j.k)}$, is avoided by using an approximated wavelet transfer function of the damaged structure is evaluated as (Sanayei *et al.* 2012)

$$H^{D(Approx.)}_{\psi(j.k)} \cong \sum_{r=1}^{nm} \frac{\phi_r^T \phi_r}{\Omega_{rD}^2 + a_{(j.k)} + i b_{(j.k)} \Omega_{rD}} + \sum_{r=nm+1}^{n} \frac{\phi_r^T \phi_r}{\Omega_r^2 + a_{(j.k)} + i b_{(j.k)} \Omega_r} \tag{24}$$

Where, index D indicates the damage state for measured DOFs. $H^{D(App)}_{\psi(j.k)}$ is used for approximation of unmeasured DOFs. nm is the number of measured natural frequencies of the damaged structure. Also, Ω_r and ϕ_r are the rth natural frequency and mode shape of the numerical model. The values of $a_{(j.k)}$ are selected to be around to the natural frequencies of the damaged structure. For such values of $a_{(j.k)}$, Eq. (24) is dominated by its denominator and will be more accurate.

In order to reduce other errors in updating model, it should be noted that data very close to natural frequencies should not be considered. At such frequencies, the term $i b_{(j.k)} \Omega_r$ and consequently the damping effects become significant. In such cases, the damping parameters must be modeled very accurately. in order to alleviate the shortcomings of the damping model and related measurements.

It is recommended that values of $a_{(j.k)}$ be selected at the vicinity of resonances. The appropriate differences from the resonances are depended in the damping ratios. For a structural model of very low damping ratios, the denominator of Eq. (24) and consequently the equation of sensitivity is controlled by $\Omega_{rD}^2 + a_{(j.k)}$. If the values of $a_{(j.k)}$ are selected very close to resonances, a small error in the measured resonances, will cause significant changes in the values $\Omega_{rD}^2 + a_{(j.k)}$, cause significant large deviations in the sensitivity equations. Therefore, by appropriate selection of the ranges of $a_{(j.k)}$ several adverse effects can be prevented. The changes in the stiffness and mass matrices in term of change in dimensionless structural parameters are

$$\Delta K = \sum_{n=1}^{ne} K_n \Delta P_n^K$$
$$\Delta M = \sum_{n=1}^{ne} M_n \Delta P_n^M \tag{25}$$

In Eq. (25), K_n and M_n present the element stiffness and mass structural matrices, ΔP_n^K and ΔP_n^M are the normalized coefficients which present the relative changes of the parameters. Using these definitions, the sensitivity matrices for the nth structural elements are obtained from Eq. (26) as

$$S_{\psi(j.k)}^K = -H^{*D}{}_{\psi(j.k)} S_{\psi(j.k)}^{ff} H^D{}_{\psi(j.k)} K_n H_{\psi(j.k)} - H^{*D}{}_{\psi(j.k)} K_n H_{\psi(j.k)} S_{\psi(j.k)}^{ff} H_{\psi(j.k)}$$
$$S_{\psi(j.k)}^M = a_{(j.k)} \left[H^{*D}{}_{\psi(j.k)} S_{\psi(j.k)}^{ff} H^D{}_{\psi(j.k)} M_n H_{\psi(j.k)} + H^{*D}{}_{\psi(j.k)} M_n H_{\psi(j.k)} S_{\psi(j.k)}^{ff} H_{\psi(j.k)} \right] \tag{26}$$

The sensitivity equations for estimation of the change of the structural parameters based on all available data is presented as

$$\Delta S_{\psi(j.k)} = S_{\psi(j.k)}^K \Delta P_K + S_{\psi(j.k)}^M \Delta P_M \tag{27}$$

Where, $S_{\psi(j.k)}^K$ and $S_{\psi(j.k)}^M$ are the sensitivity equations concerning the stiffness and mass parameters. The performance of the sensitivity-based model updating methods is affected by several factors such as the number of available data, measurement and mass modeling errors, selection of measurement and excitation locations, the proper weighting of the equations, algorithm used to solve the optimization problem, and applied constraints on the variation of unknowns. Hence, the optimization problem is defined as follows

$$\min_{\Delta P} \|S\Delta P - \Delta S\|_2 \qquad Subject\ to \quad -1 \le \Delta P \le 1 \tag{28}$$

To calculate the parameter of the damaged structure, and solve the equations by the least square method, the “lsqlin” solver of the MATLAB optimization toolbox is used (Mathworks 2014).

The implementing steps of the introduced damage detection method are summarized in the flowchart given by Fig. 1. This procedure is repeated for each simulated damage cases by Monte Carlo analysis, and the mean and COVs of predicted parameters are reported as damage detection results.

3. Numerical results and discussion

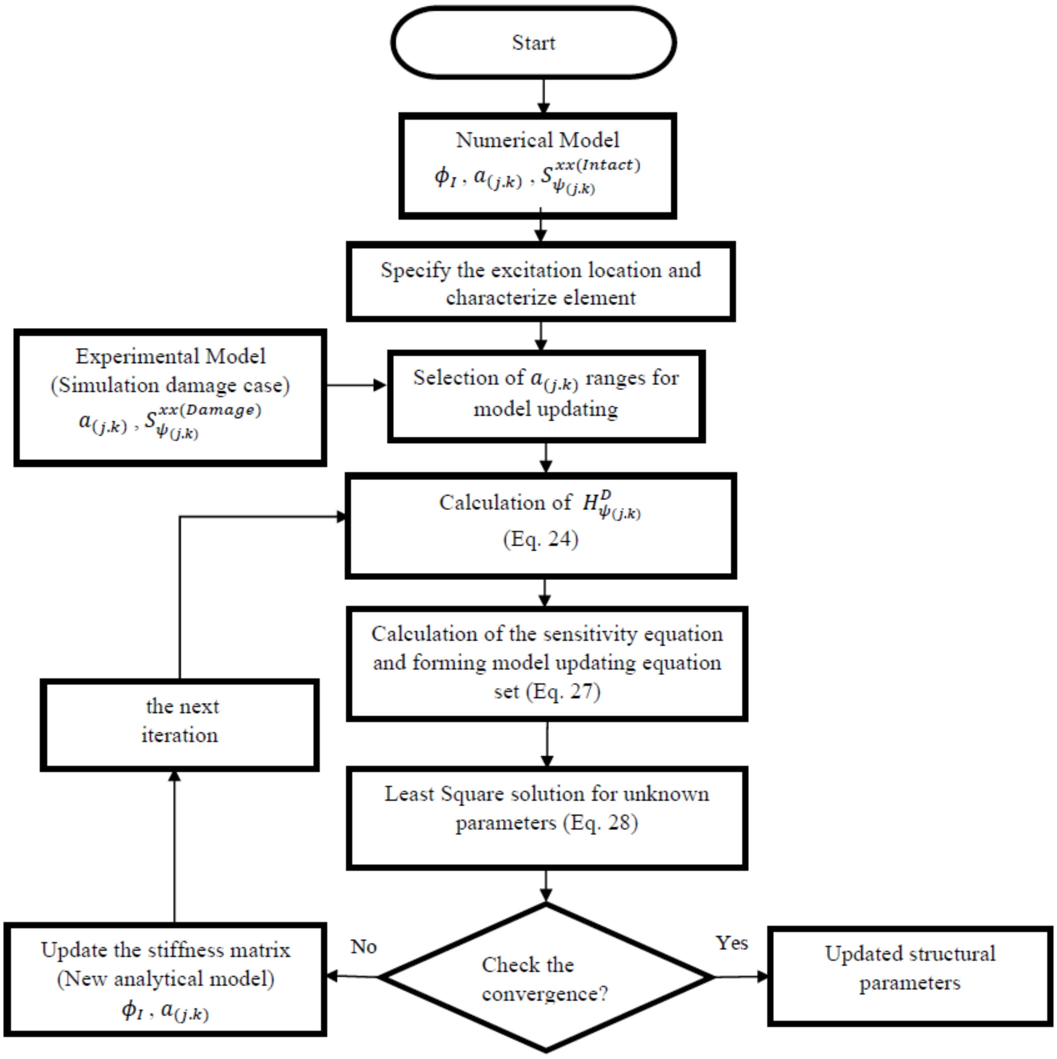


Fig. 1 The flowchart of the proposed damage detection method

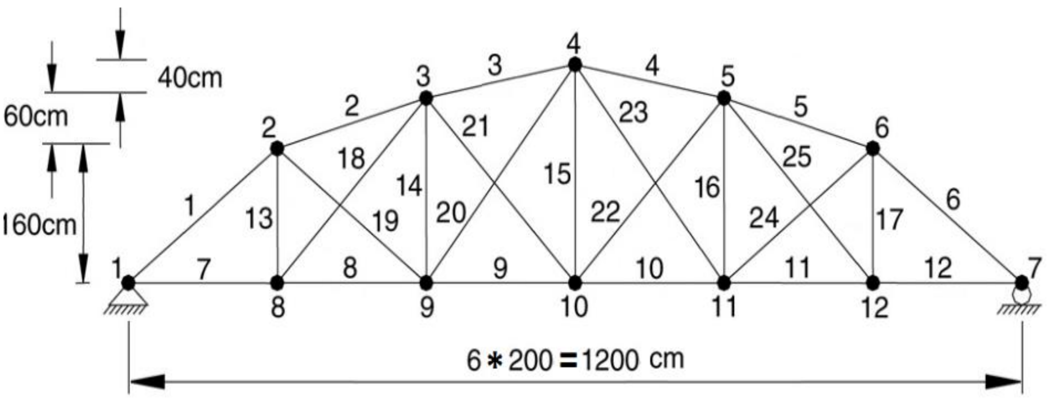


Fig. 2 The geometry of a bowstring truss model

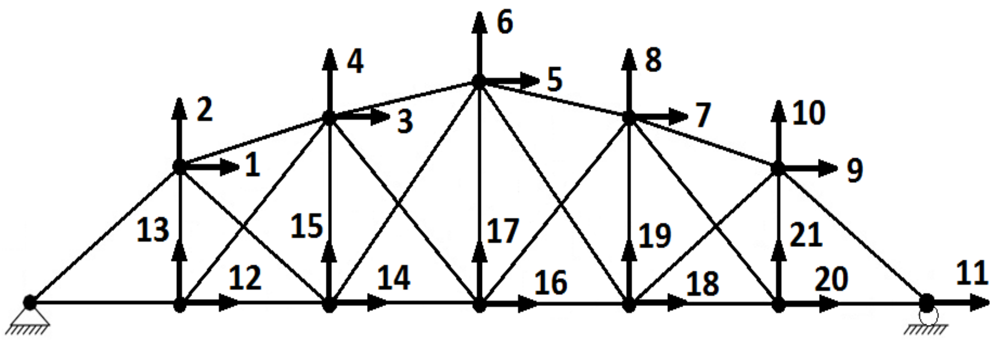


Fig. 3 Degrees of freedom of truss model

Table 1 Cross-sectional area of truss members

Member	Area ($\mathbf{cm^2}$)
1-6	18
7-12	15
13-17	10
18-25	12

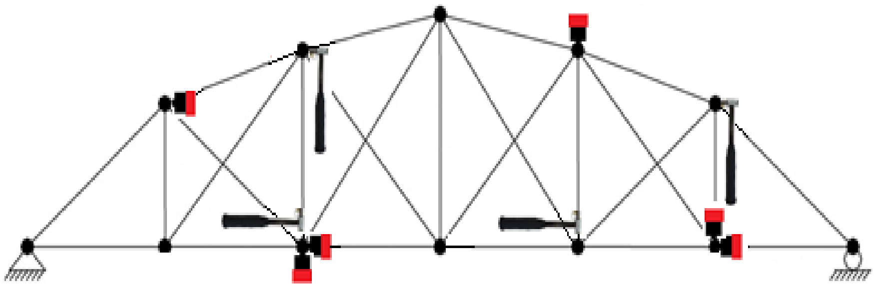


Fig. 4 Schematic excitation and measurement setup of truss model (excited and measurement degrees of freedom)

3.1 2D Truss model

The robustness of the presented algorithm is studied numerically for investigation of the effects of the location, severity, and number of the damaged elements on the model updating results. The presented method is applied on a six-bay truss structure, as shown by Fig. 2, Two-Dimensional axial truss elements are used to model the structure by finite element method.

All truss elements are considered as steel material with Young's modulus of 200 GPa, and mass density of $7800\,\frac{\mathrm{kg}}{\mathrm{m}^3}$. The cross-sectional areas of the truss members are given in Table 1. The truss structure is modeled by 25 elements. The active DOFs of the numerical model are presented in Fig. 3. For a truss element, unknown parameter is axial rigidity EA, where A is cross-sectional area and E is Young's modulus.

The time history analysis of the structural response, and consequently the extraction of wavelet transform coefficients are conducted based on the Newmark-beta method (Naeim 2007). In this study, DOF numbers 1, 8, 14, 15, 20, and 21 are selected as measurement locations, and DOF numbers 3, 9, 15, and 19 are selected as the excitation locations as presented schematically by Fig. 4. It is assumed that excitation loads are applied individually, and structural responses are measured at the selected DOFs. Measurement and excitation locations are considered similar for all considered damage cases.

The excitation force is assumed as $F(t)$=5sin($6\pi t$). The sampling rate is considered as 1000 Hz. The following damage scenarios of different percentage reduction of the flexural rigidity are considered:

Scenario 1 – 30% and 50% reduction in elements 4 and 10.

Scenario 2 – 20%,30%,30% and 30% reduction in elements 3,9,20 and 25.

Scenario 3 – 40%,40%,50%.40% and 30% reduction in elements 5,10,13,20 and 24.

The natural frequencies of intact and damaged structures at the simulated damage cases are given in Table 2.

Displacement responses at the desired DOFs is extracted for two seconds after application of excitation. The structural responses are decomposed into four levels of Daubechies Db4 wavelets. The selection of $a_{(j.k)}$ is a paramount issue for successful structural parameter estimation. At the selected $a_{(j.k)}$ the wavelet coefficient must be sensitive to the changes of structural parameters and less sensitive to the measurement errors. In order to improve the accuracy of the approximated $H^D{}_{\psi(j.k)}$ by Eq. (24) and consequently the sensitivity equation, $a_{(j.k)}$ are selected close to the square of the natural frequencies of the damaged and intact structure. absolute values of $a_{(j.k)}$ is essential. The selected ranges of $a_{(j.k)}$ based on Eq. (10) for a different time and shift scales are given by Fig. 4. The selected ranges values of $a_{(j.k)}$ for the structural model updating are given in Table 1.

Also, in all damage cases, four decomposition steps of response signal were used for structural parameter estimation. The total number of elements in $a_{(j.k)}$ is 3850 that 275, 194, and 288 data has been used at different time scale for the first damage case Table 3. These data have been selected

Table 2 Natural frequencies (Hz) for the intact and damaged cases

Mode number.	Intact model	Damage case		
		1	2	3
1	30.34	28.53	29.34	28.81
2	68.95	67.89	67.31	66.89
3	96.34	94.39	95.10	92.95
4	181.76	170.41	166.17	159.98
5	223.23	219.64	209.06	211.59
6	275.59	271.81	265.56	263.85

Table 3 The selected range for aj,k Value for Model Updating

Damage Case	1	2	3
$\boldsymbol{a}_{(j.k)}$range	(-3.45e+4)~(-4.22e+4)	(-3.4e+4)~(-4.22e+4)	(-3.42e+4)~(-4.3e+4)
	(-5.14e+4)~(-5.41e+4)	(-5.58e+4)~(-6.12e+4)	(-5.64e+4)~(-6.16e+4)
	(-7.58e+4)~(-8.27e+4)	(-7.6e+4)~(-8.36e+4)	(-7.76e+4)~(-8.48e+4)
	-	(-10.47e+4)~(-11.61e+4)	(-10.17e+4)~(-11.45e+4)

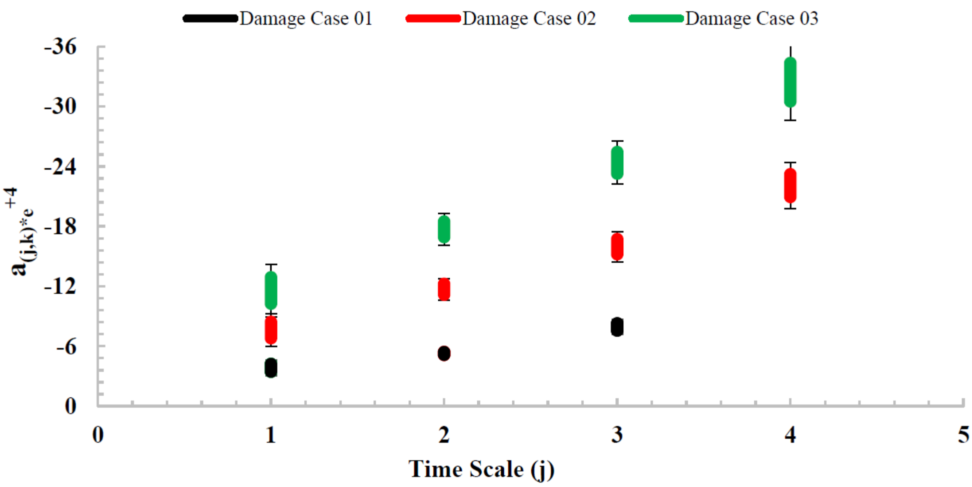


Fig. 4 The range of collected $a_{(j.k)}$ in different damage scenarios of the truss model

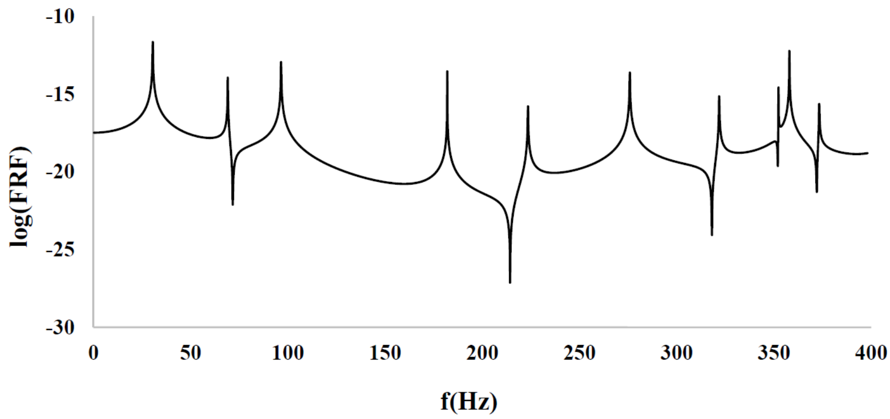


Fig. 5 The frequency response function (FRF) of the intact Truss model (excited at DOF 15 and measured at DOF 19)

close to the natural frequencies of the damaged structure. The selected ranges of $a_{(j.k)}$ at second and third damage scenarios, which are also shown in Fig. 5, the number of collected data is different for achieving a better estimation of the location and severity the damage. To allow comparison in a single diagram, the data of second and third scenarios are vertically shifted by the factors 2 and 3, respectively.

Due to practical limitations, it is assumed that only a few numbers of the natural frequencies are measurable, because the amplitude of oscillation at higher mode shapes decreases and accurate measurements will be impractical. By a numerical simulation, the structural response function at domain of frequency and wavelet transform are plotted as shown in Figs. 6 to 7 in order to show the ranges of coefficients $a_{(j.k)}$ around natural frequencies of the intact truss structure subjected to the applied load at DOF 15 and measurement at DOF 19. Due to the negligible changes of the structural response in the lower frequencies, the selection of the coefficient aij around resonances 1, 2, and 3 are omitted.

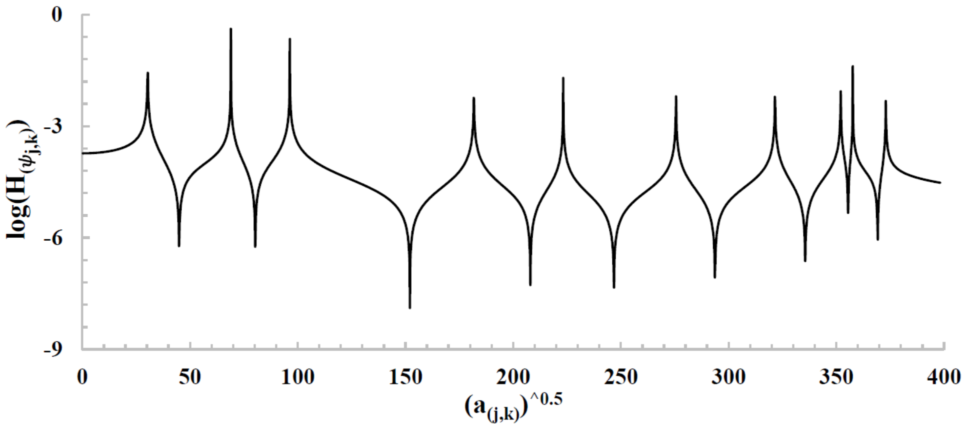
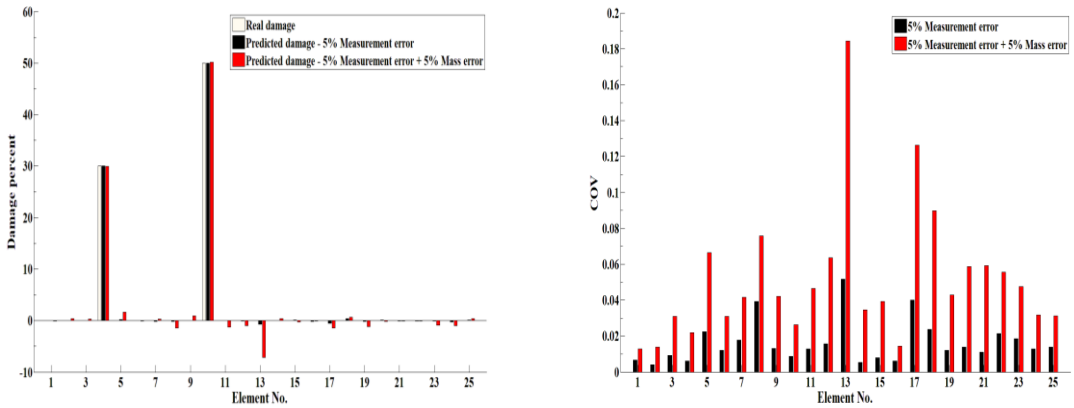


Fig. 6 The wavelet transform function (WTF) of the intact Truss model (excited at DOF 15 and measured at DOF 19)

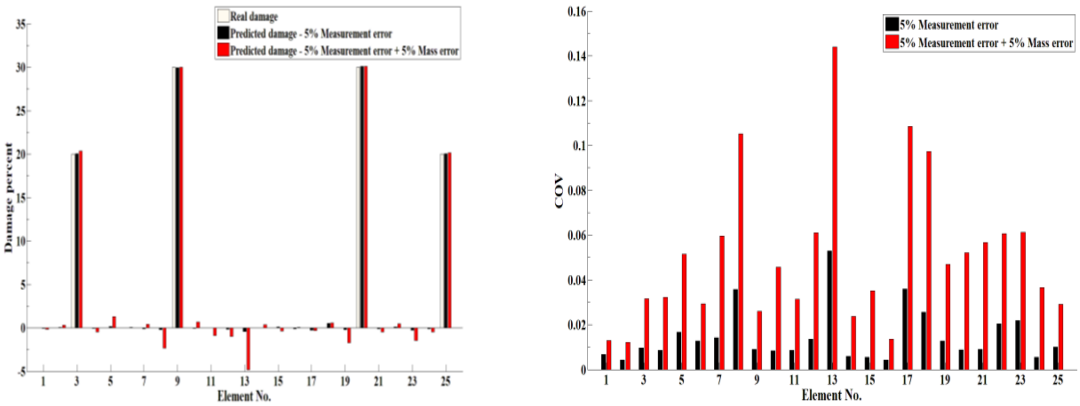


(a) (b)

Fig. 7 (a) First case damage predicted parameter at considering 5% measurement error and 5% modeling error, (b) COV of the predicted parameters

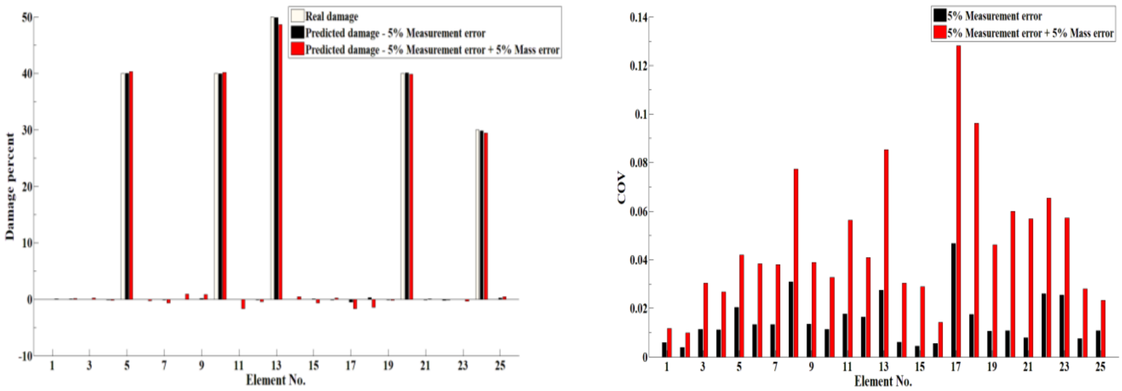
The results of parameters estimation are shown in Fig. 8(a) to 10(a). The results demonstrate the acceptable performance of the proposed method to identify structural damages.

There are unavoidable errors in the finite element updating process, such as environmental, modeling, and measurement errors. In order to simulate the impacts of measurement errors, random values are added to the numerically evaluated responses. Thus, 5% of uniformly distributed random errors are added proportionally to the simulated data. The proposed method uses the mass parameters of the structure for evaluation of the structural responses, mode shapes, and consequently, the proposed approximation of ${H^D}_{\psi(j.k)}$ For most real cases, structural damage influences the stiffness parameters and the mass parameters are identical for the intact and damaged structures. However, the assumed mass parameters may not be accurate. Hence, 5% of the random errors are added to the mass matrix as the modeling error. The average of estimated parameters do not reflect the robustness and confidence of the parameters estimation process. Scattering of predicted stiffness parameters around mean values is studied through by evaluating of standard deviations of predicted stiffness



(a) (b)

Fig. 8 (a) Second case damage predicted parameter at considering 5% measurement error and 5% modeling error, (b) COV of the predicted parameters



(a) (b)

Fig. 9 (a) Third case damage predicted parameter at considering 5% measurement error and 5% modeling error, (b) COV of the predicted parameters

parameters. The coefficient of variation (COV) for each of the predicted unknown parameters is evaluated by normalization of standard deviation concerning its mean value. Low (COV) indicates the resistance of the method to measurement error. Fifty sets of random errors are considered to examine the robustness of the proposed method against measurement and mass modeling errors.

The model updating results presented in Fig. 8(a) to 10(a) prove that the proposed sensitivity equation is able to accurately identify the location of the defects as well as the severity of the structural damages. Furthermore, the stability and robustness of the achieved results by the proposed sensitivity equation are assessed by evaluating the coefficient of variation (COV) of the predicted parameters. The coefficient of variation for each of the predicted parameters is calculated by normalization of the standard deviation concerning the mean value. The coefficients of variation of the estimated parameters are plotted in Fig. 8(b) to 10(b). Low COVs of the predicted parameters indicate the robustness of the proposed method against measurement errors and less scattering of the obtained results.

Table 4 Closeness index of truss model considering measurement and modeling error

Damage Cases	CI	
	With measurement error	with measurement error and modelling error
1	0.96	0.93
2	0.95	0.92
3	0.98	0.95

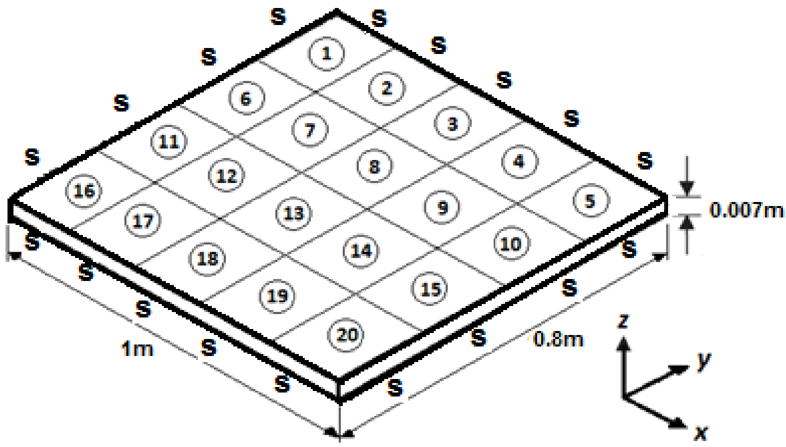


Fig. 10 The geometry of plate model

In order to have a quantitative evaluation of the accuracy of results, some indices can be used. Accuracy of the results can be assessed by closeness index (CI) based on the distance between the actual and estimated damage vectors as (Bakhtiari-Nejad *et al*. 2005)

$$CI = 1 - \frac{\|\delta P_t - \delta P_p\|}{\|\delta P_t\|} \tag{29}$$

Where, δp_t and δp_p are the actual and predicted damage ratios, for an accurate evaluation of the damaged parameters, the CI value is one. CI values of the considered damage cases are given in Table 4.

CI values confirm the validity of the updated model results. The presented results show that the proposed sensitivity equation can update the model in the presence of measurement errors and mass modeling with acceptable accuracy.

3.2 Plate model

The proposed method is applied numerically on a flat plate structure. The plate is simply supported at all edges (SSSS), as depicted in Fig. 11.

The plate is 1 m×0.8 m, and its finite-element model consists of 20 elements (5 divisions along x and four divisions along y-direction). The plate is considered steel material with a Young modulus of 210 GPa, Poisson ratio of 0.3, and mass density of $7800\,\frac{\mathrm{kg}}{\mathrm{m}^2}$. The thickness of the plate uniform is 7 mm.

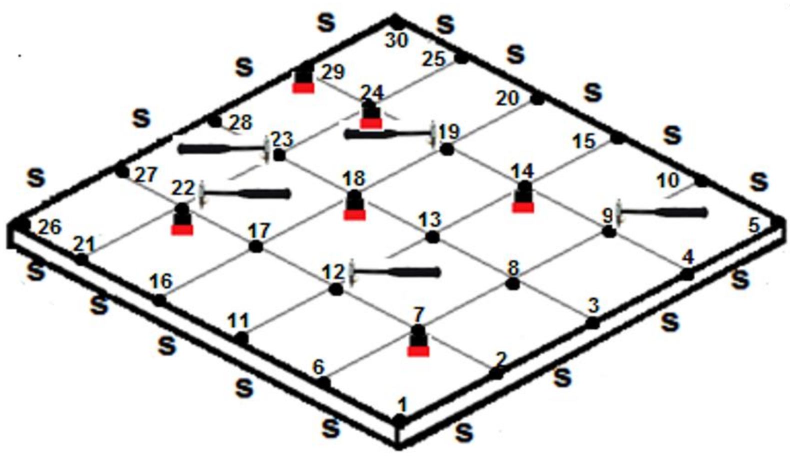


Fig. 11 Schematic of excitation and measurement setup of the plate model

Table 5 Natural frequencies (Hz) for the intact and damaged cases

Mode Number.	Intact model	Damage Case		
		1	2	3
1	23.4	21.9	17.21	18.06
2	55.5	53.74	48.84	45.12
3	139.7	137.45	127.28	131.87
4	298.1	295.1	284.74	278.23
5	307.5	303.52	296.89	291.46
6	336.2	332.86	318.1	316.84

The responses measurement at the rotational DOFs is a challenging issue. Therefore, in this study, the transitional DOFs perpendicular to the plate, i.e., the Z-axis, are considered excitation and measurement locations. The DOF numbers 7, 14, 18, 22, 24, and 29 are selected for response measurement and DOF numbers 9, 12, 19, 22, and 23 are selected as the excitation points. The locations of excitation and measurements are shown in Fig. 12.

The natural frequencies of intact and damaged structures at the simulated damage cases are given in Table 5. The considered damage scenarios are of a different percentage reduction in the flexural rigidity in an element as:

Scenario 1 –40% reduction in element 4.

Scenario 2 –30% reduction in elements 6 and 13.

Scenario 3 – 45%,reduction in elements 1, 5, 16 and 20.

In this simulation, excitation frequency points are selected around the resonance frequencies at low modes. The response signal analysis is decomposed up to 4 levels for parameter estimation.

Fig. 13 shows the obtained values $a_{(j.k)}$ based on Eq. (10) for different time and shift scales to identify the damage to the structural vibration response. The selected ranges of $a_{(j.k)}$ for the structural model updating are given in Table 6. Also, in all damage cases, four decomposition steps of response signal were used for damage detection. The total number of elements in $a_{(j.k)}$ is 3850, that 288, 275, 192, and 212 data have been used at different time scales for the first damage case Table 6.

Table 6 The selected range for *aj,k* Value for Model Updating

Damage Case	1	2	3
$a_{(j.k)}$range	(-2.27e+4)~(-2.68e+4) (-4.12e+4)~(-8.37e+4) (-8.6e+4)~(-9.11e+4) (-11.42e+4)~(-11.61e+4)	(-2.34e+4)~(-2.76e+4) (-4.23e+4)~(-4.53e+4) (-8.45e+4)~(-9.18e+4) (-11.25e+4)~(-11.57e+4)	(-2.24e+4)~(-2.64e+4) (-4.28e+4)~(-4.76e+4) (-8.38e+4)~(-8.97e+4) (-11.28e+4)~(-11.54e+4)

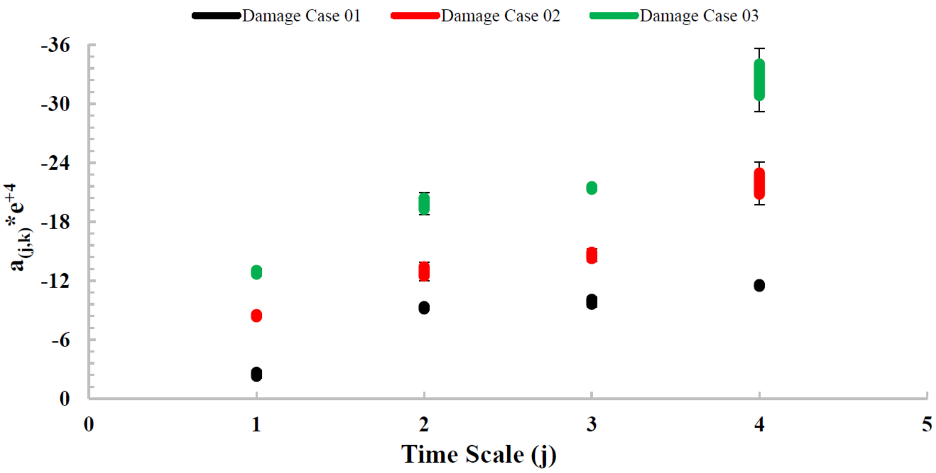


Fig. 12 The range of collected sensitivity coefficients in damage scenarios with different time scales of the plate model

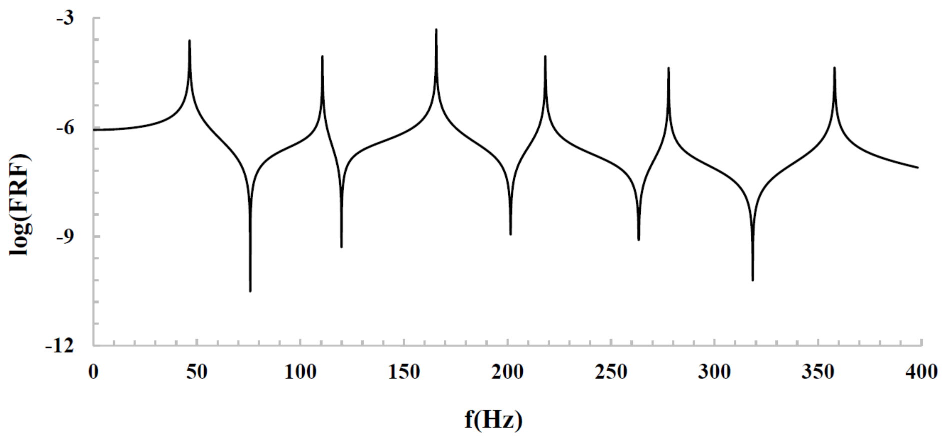


Fig. 13 The frequency response function (FRF) of the intact plate model (excited at DOF 22 and measured at DOF 18)

In the second and third damage scenarios, shown in Fig. 13, the number of collected data is different for achieving a better estimation of the location and severity of the damage. By a numerical simulation, the structural response function at domain of frequency and wavelet transform are plotted as shown in Figs. 14 to 15 in order to show the ranges of coefficients $a_{(j.k)}$ around natural frequencies of the intact truss structure subjected to the applied load at DOF 22 and

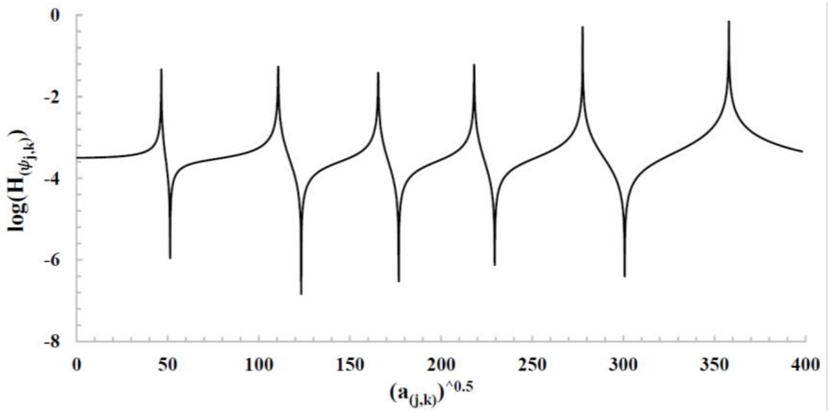
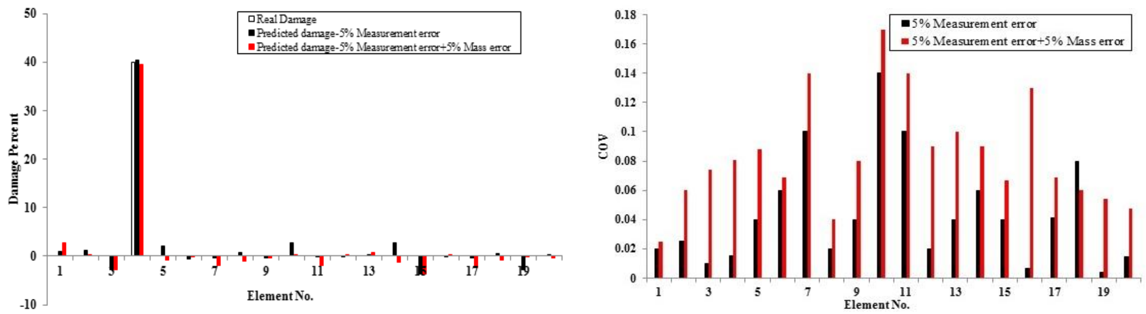
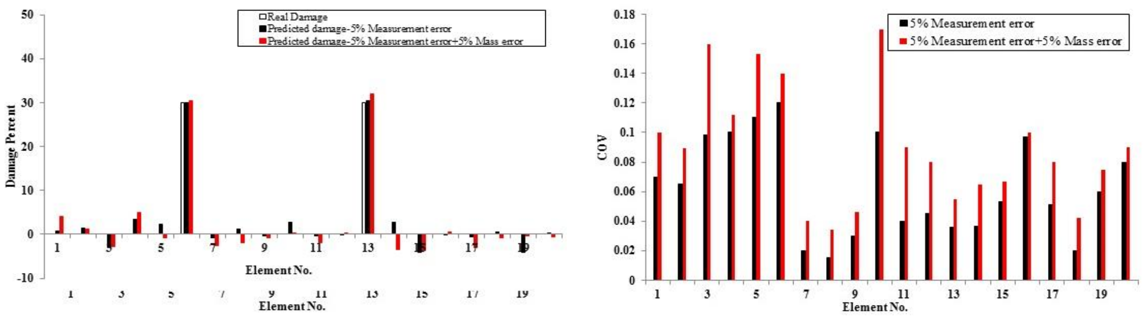


Fig. 14 The wavelet transform function (WTF) of the intact Truss model (excited at DOF 22 and measured at DOF 18)



(a) (b)

Fig. 15 (a) First case damage predicted parameter at considering 5% measurement error and 5% modeling error, (b) COV of the predicted parameters

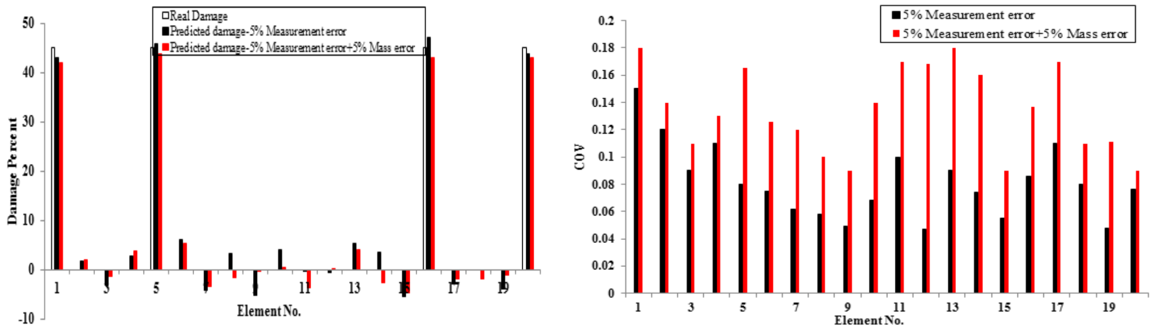


(a) (b)

Fig. 16 (a) Second case damage predicted parameter at considering 5% measurement error and 5% modeling error, (b) COV of the predicted parameters

measurement at DOF 18.

In this study, 5% uniformly distributed random values are added to the simulated data. Also, 5% of the random error is added to the mass matrix as the modeling error. in addition, 50 sets of random



(a) (b)

Fig. 17 (a) Third case damage predicted parameter at considering 5% measurement error and 5% modeling error, (b) COV of the predicted parameter

Table 7 Closeness index of the predicted parameters of the plate model considering measurement modelling error

Damage Cases	CI	
	With measurement error	with measurement error and modelling error
1	0.93	0.90
2	0.91	0.87
3	0.87	0.85

data are considered for structural parameter estimation. The obtained results are presented by Figs. 16 to 18. The results are presented in Fig. 16(a) to 18(a) show that the proposed sensitivity equation can accurately identify the location and severity of the simulated damage cases using error-contaminated data. The COVs of the estimated unknown parameters are plotted in Fig. 16(b) to 18(b).

The CI values confirm the accuracy of the model updating results. The presented results prove that the proposed sensitivity relation is capable of the structural model updating with acceptable accuracy in the presence of measurement and mass modeling errors.

4. Conclusions

In this study, the sensitivity equation of the wavelet transform coefficients of the correlation function was used for damage detection of the structures. An approximated equation is applied to estimate the unmeasured wavelet-based transform function of the damaged structure. The model is updated by the wavelet transform coefficients achieved in the frequency range in the vicinity of the resonances, in which damping and incomplete measurements have no significant effect on the results of the parameter estimation. Sensitivity equations are solved to obtain the change of structural parameters by the least-squares approach. The proposed method was successfully applied to a 2D truss and a plate model using simulated data contaminated by measurement and modeling errors. Results indicate the ability of the method to identify the location and severity of the structural damages. The robustness of the proposed method was confirmed by low values of the COVs of the parameter estimation results.

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