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Dynamic response of size-dependent porous functionally graded beams under thermal and moving load using a numerical approach

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Abstract. Based on differential quadrature method (DQM) and nonlocal strain gradient theory (NSGT), forced vibrations of a porous functionally graded (FG) scale-dependent beam in thermal environments have been investigated in this study. The nanobeam is assumed to be in contact with a moving point load. NSGT contains nonlocal stress field impacts together with the microstructure-dependent strains gradient impacts. The nano-size beam is constructed by functionally graded materials (FGMs) containing even and un-even pore dispersions within the material texture. The gradual material characteristics based upon pore effects have been characterized using refined power-law functions. Dynamical deflections of the nano-size beam have been calculated using DQM and Laplace transform technique. The prominence of temperature rise, nonlocal factor, strain gradient factor, travelling load speed, pore factor/distribution and elastic substrate on forced vibrational behaviors of nano-size beams have been explored.

Keywords: forced vibrations; thermal environment; composites; nonlocal strain gradient theory; travelling load; DQM

1. Introduction

In recent years, diverse kinds of materials have found their applications in nano-scale structures. Vibration behavior of a nano-scale plate is not the same as a macro-scale plate. This is because small-size effects are not present at macro scale. So, mathematical modeling of a nanoplate can be done with the use of nonlocal elasticity (Eringen 1983) incorporating only one scale parameter (Zeighampour and Beni 2014, Akgöz and Civalek 2015). Due to the ignorance of strain gradient effect in nonlocal elasticity theory, a more general theory will be required. Strain gradients at nano-scale are observed by many researchers (Lam *et al.* 2003, Martínez-Criado 2016, Ebrahimi *et al.* 2016, Al-Maliki *et al.* 2019, Nami and Janghorban 2014). Thus, nonlocal-strain gradient theory was introduced as a general theory which contains an additional strain gradient parameter together with nonlocal parameter (Aerfi and Zenkour 2016, Ansari *et al.* 2015, Li *et al.* 2015, Zhang *et al.* 2015, Lou *et al.* 2016, Zeighampour and Shojaeian 2017, Aissani *et al.* 2015, Bouderba *et al.* 2016, Chikh *et al.* 2016, Yahiaoui *et al.* 2018, Achouri *et al.* 2019, Berrabah *et al.* 2013, Barati 2018, She *et al.*

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2018). The scale parameters used in nonlocal strain gradient theory can be obtained by fitting obtained theoretical results with available experimental data and even molecular dynamic simulations.

For functionally graded materials, all material properties may change from one side to another side by means of a prescribed distribution. These two sides may be ceramic or metal. Mechanical characteristics of a FG material can be described based on the percentages of ceramic and metal phases. The material distribution in FG materials may be characterized via a power-law function. FG materials are not always perfect because of porosity production in them (She *et al.* 2018, Ahmed *et al.* 2019, El-Hassar *et al.* 2016). Existence of porosities in the FG materials may significantly change their mechanical characteristics (Atmane *et al.* 2015). For example, the elastic moduli of porous FG material is smaller than that of perfect FG material. Up to now, many authors focused on wave propagation, vibration and buckling analyzes of FG structures having porosities (Mirjavadi *et al.* 2017). Also, there are several investigations concerning with the analysis of FG structures in thermal environments.

With the employment of differential quadrature method (DQM) and nonlocal strain gradient theory (NSGT), forced vibrations of a porous functionally graded (FG) scale-dependent beam in thermal environments have been investigated in this study. The nanobeam is assumed to be in contact with a moving point load. NSGT contains nonlocal stress field impacts together with the microstructure-dependent strains gradient impacts. The nano-size beam is constructed by functionally graded materials (FGMs) containing even and un-even pore dispersions within the material texture. The gradual material characteristics based upon pore effects have been characterized using refined power-law functions. Dynamical deflections of the nano-size beam have been calculated using DQM and Laplace transform technique. The prominence of temperature rise, nonlocal factor, strain gradient factor, travelling load speed, pore factor/distribution and elastic substrate on forced vibrational behaviors of nano-size beams have been examined.

2. Theories and formulations

2.1 Nonlocal strain gradient nano-size beam

In the well-known nonlocal strain gradient elasticity (Lam *et al.* 2003), strain gradient impacts are taken into accounting together with nonlocal stress influences defined in below relation

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla \sigma_{ij}^{(1)} \tag{1}$$

in such a way that stress $\sigma_{ij}^{(0)}$ is corresponding to strain components ε_{kl} and a higher order stress is related to strain gradient components $\nabla\varepsilon_{kl}$ which are

$$\sigma_{ij}^{(0)} = \int_V C_{ijkl}\alpha_0(x,x',e_0a)\varepsilon'_{kl}(x')dx' \tag{2a}$$

$$\sigma_{ij}^{(1)} = l^2\int_V C_{ijkl}\alpha_1(x,x',e_1a)\nabla\varepsilon'_{kl}(x')dx' \tag{2b}$$

in which C_{ijkl} express the elastic properties; Also, $e_0 a$ and $e_1 a$ are corresponding to nonlocality impacts and l is related to strains gradients. Whenever two nonlocality functions $\alpha_0(x,x',e_0a)$ and $\alpha_1(x,x',e_1a)$ verify Eringen's announced conditions, NSGT constitutive relation may be written as follows

$$[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]\sigma_{ij} = C_{ijkl}[1-(e_1a)^2\nabla^2]\varepsilon_{kl} - C_{ijkl}\, l^2[1-(e_0a)^2\nabla^2]\nabla^2\varepsilon_{kl} \quad (3a)$$

so that ∇^2 defines the operator for Laplacian; by selecting $e_1 = e_0 = e$, above relationship decreases to

$$[1-(ea)^2\nabla^2]\sigma_{ij} = C_{ijkl}\,[1-l^2\nabla^2]\varepsilon_{kl} \quad (3b)$$

2.2 Porous FGMs

A porous material, for instance a ceramic-metal composite, might be placed in the category of lightweight materials and can be applied in several structures such as sandwich beams/panels. Often, pore variation along the thickness of beams results in a notable alteration in every kind of material property. When the ceramic-metal distribution inside the material is selected to be non-uniform, the composite might be defined as a functionally graded material since its properties obey some specified functions. Herein, the following definition for elastic modulus based on pore dispersion will be employed (Mirjavadi *et al.* 2017)

$$E(z) = \left(E_c - E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p + E_m - \left(E_c + E_m\right)\frac{\xi}{2} \qquad \text{for even porosities} \quad (4a)$$

$$E(z) = \left(E_c - E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p + E_m - \frac{\xi}{2}\left(E_c + E_m\right)(1-\frac{2|z|}{h}) \qquad \text{for uneven porosities} \quad (4b)$$

where p defines the ceramic-metal gradation exponent; m and c are related with the material coefficients at the top and bottom sides; ξ is the pore factor. Moreover, the material coefficients have been listed in Table 1. Eq. (4) can also be used for definition of mass density.

2.3 Beam modeling via refined theory

So far, a variety of beam theories are introduced for description and analyzes of beam structures (Hussain *et al.* 2019, Karami *et al.* 2019, Balubaid *et al.* 2019, Boutaleb *et al.* 2019, Berghouti *et al.* 2019, Adda Bedia *et al.* 2019, Tlidji *et al.* 2019, Semmah *et al.* 2019, Alimirzaei *et al.* 2019, Addou *et al.* 2019, Medani *et al.* 2019, Batou *et al.* 2019, Abualnour *et al.* 2019, Draiche *et al.* 2019, Belbachir *et al.* 2019, Sahla *et al.* 2019, Chaabane *et al.* 2019, Meksi *et al.* 2019, Khiloun *et al.* 2019, Zarga *et al.* 2019, Zaoui *et al.* 2019, Mahmoudi *et al.* 2019, Draoui *et al.* 2019, Issad *et al.* 2018, Boukhlif *et al.* 2019). The displacement field containing axial displacement (u_1) and transverse displacement (u_3) with respect to the refined beam assumption calculating the precise location of the neutral axis might be defined as

Table 1 Material properties of FGM constituents

Properties	Steel	Alumina (Al_2O_3)
E	210 (GPa)	390 (GPa)
ρ	7800 (kg/m^3)	3960 (kg/m^3)
ν	0.3	0.24

$$u_x(x,z)=u(x)-(z-z^*)\frac{\partial w_b}{\partial x}-[\Gamma(z)-z^{**}]\frac{\partial w_s}{\partial x} \tag{5a}$$

$$u_z(x,z)=w_b(x)+w_s(x) \tag{5b}$$

in which neutral axis location can be described as

$$z^*=\frac{\int_{-h/2}^{h/2}E(z)\,zdz}{\int_{-h/2}^{h/2}E(z)\,dz},\quad z^{**}=\frac{\int_{-h/2}^{h/2}E(z)\Gamma(z)dz}{\int_{-h/2}^{h/2}E(z)\,dz} \tag{6}$$

The shear function may take the below form

$$\Gamma(z)=z-\sin(\chi z)/\chi$$
$$,\chi=\pi/h \tag{7}$$

Above displacement field is calculated form the axial displacement (*u*), together with w_b and w_s as bending and shear displacements. Accordingly, one may calculate the strains of as

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-(z-z^*)\frac{\partial^2 w_b}{\partial x^2}-[\Gamma(z)-z^{**}]\frac{\partial^2 w_s}{\partial x^2} \tag{8a}$$

$$\gamma_{xz}=g(z)\frac{\partial w_s}{\partial x} \tag{8b}$$

where $g(z)=1-d\Gamma(z)/dz$. Based on proposed beam model and using Hamilton’s rule, one can express the governing equations of the porous beam as follows

$$\frac{\partial N}{\partial x}=I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w_b}{\partial x\partial t^2}-J_1\frac{\partial^3 w_s}{\partial x\partial t^2} \tag{9}$$

$$\frac{\partial^2 M_b}{\partial x^2}-q_{dynamic}=+I_0(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2})+I_1\frac{\partial^3 u}{\partial x\partial t^2}-I_2\frac{\partial^4 w_b}{\partial x^2\partial t^2}-J_2\frac{\partial^4 w_s}{\partial x^2\partial t^2}+k_w(w_b+w_s)$$
$$-(k_p-N^T)\frac{\partial^2(w_b+w_s)}{\partial x^2} \tag{10}$$

$$\frac{\partial^2 M_s}{\partial x^2}+\frac{\partial Q}{\partial x}-q_{dynamic}=+I_0(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2})+J_1\frac{\partial^3 u}{\partial x\partial t^2}-J_2\frac{\partial^4 w_b}{\partial x^2\partial t^2}-K_2\frac{\partial^4 w_s}{\partial x^2\partial t^2}$$
$$+k_w(w_b+w_s)-(k_p-N^T)\frac{\partial^2(w_b+w_s)}{\partial x^2} \tag{11}$$

So that forces and moments might be calculated as

$$(N_x,M_x^b,M_x^s)=\int_{-h/2}^{h/2}(1,z-z^*,\Gamma-z^{**})\sigma_x\,dz,$$
$$Q_{xz}=\int_{-h/2}^{h/2}g(z)\sigma_{xz}dz \tag{12}$$

k_w and k_p define Winkler and Pasternak factors of substrate, respectively. $N^T=\int_{-0.5h}^{0.5h}\alpha E\Delta T dz$ is thermal loading and

$$(I_0,I_1,J_1,I_2,J_2,K_2)=\int_{-h/2}^{h/2}(1,z-z^*,(z-z^*)^2,\Gamma-z^{**},(z-z^*)(\Gamma-z^{**}),(\Gamma-z^{**})^2)\rho(z)dz \tag{13}$$

The stress field in the context of NSGT takes the below form

$$\sigma_{xx}-(ea)^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=(1-l^2\frac{\partial^2}{\partial x^2})E(z)\varepsilon_{xx} \tag{14}$$

$$\sigma_{xz}-(ea)^2\frac{\partial^2\sigma_{xz}}{\partial x^2}=(1-l^2\frac{\partial^2}{\partial x^2})G(z)\gamma_{xz} \tag{15}$$

Taking into account nonlocal strain gradient effect and with aid of Eq. (12), the relations for force-strain and the moment-strain might be derived

$$N-(ea)^2\frac{\partial^2 N}{\partial x^2}=(1-l^2\frac{\partial^2}{\partial x^2})[A\frac{\partial u}{\partial x}-B\frac{\partial^2 w_b}{\partial x^2}-B_s\frac{\partial^2 w_s}{\partial x^2}] \tag{16}$$

$$M_b-(ea)^2\frac{\partial^2 M_b}{\partial x^2}=(1-l^2\frac{\partial^2}{\partial x^2})[B\frac{\partial u}{\partial x}-D\frac{\partial^2 w_b}{\partial x^2}-D_s\frac{\partial^2 w_s}{\partial x^2}] \tag{17}$$

$$M_s-(ea)^2\frac{\partial^2 M_s}{\partial x^2}=(1-l^2\frac{\partial^2}{\partial x^2})[B_s\frac{\partial u}{\partial x}-D_s\frac{\partial^2 w_b}{\partial x^2}-H_s\frac{\partial^2 w_s}{\partial x^2}] \tag{18}$$

$$Q-(ea)^2\frac{\partial^2 Q}{\partial x^2}=(1-l^2\frac{\partial^2}{\partial x^2})[A_s\frac{\partial w_s}{\partial x}] \tag{19}$$

in which

$$(A, B, B_s, D, D_s, H_s) = \int_{-h/2}^{h/2} E(z)(1, (z - z^*), (\Gamma - z^{**}), (z - z^*)^2, (z - z^*)(\Gamma - z^{**}), (\Gamma - z^{**})^2)\, dz \tag{20}$$

$$A_s = \int_{-h/2}^{h/2} g^2 G(z) dz \tag{21}$$

There are three nonlinear governing equations for proposed refined beam model which can be written with respect to displacements from inserting Eqs. (16)-(19), into Eqs. (9)-(11) as

$$\begin{aligned}&(1-l^2\frac{\partial^2}{\partial x^2})A\frac{\partial^2 u}{\partial x^2}-(1-l^2\frac{\partial^2}{\partial x^2})B\frac{\partial^3 w_b}{\partial x^3}-(1-l^2\frac{\partial^2}{\partial x^2})B_s\frac{\partial^3 w_s}{\partial x^3}-I_0\frac{\partial^2 u}{\partial t^2}\\&+I_1\frac{\partial^3 w_b}{\partial x\partial t^2}+J_1\frac{\partial^3 w_s}{\partial x\partial t^2}+\mu(I_0\frac{\partial^4 u}{\partial x^2\partial t^2}-I_1\frac{\partial^5 w_b}{\partial x^3\partial t^2}-J_1\frac{\partial^5 w_s}{\partial x^3\partial t^2})=0\end{aligned} \tag{22}$$

$$\begin{aligned}&(1-l^2\frac{\partial^2}{\partial x^2})B(\frac{\partial^3 u}{\partial x^3})-(1-l^2\frac{\partial^2}{\partial x^2})D(\frac{\partial^4 w_b}{\partial x^4})-(1-l^2\frac{\partial^2}{\partial x^2})D_s(\frac{\partial^4 w_s}{\partial x^4})-I_0(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2})\\&-I_1\frac{\partial^3 u}{\partial x\partial t^2}+I_2\frac{\partial^4 w_b}{\partial x^2\partial t^2}+J_2\frac{\partial^4 w_s}{\partial x^2\partial t^2}-k_w(w_b+w_s)+(k_p-N^T)\frac{\partial^2(w_b+w_s)}{\partial x^2}\\&+\mu(+I_0(\frac{\partial^4 w_b}{\partial x^2\partial t^2}+\frac{\partial^4 w_s}{\partial x^2\partial t^2})+I_1\frac{\partial^5 u}{\partial x^3\partial t^2}-I_2\frac{\partial^6 w_b}{\partial x^4\partial t^2}-J_2\frac{\partial^6 w_s}{\partial x^4\partial t^2}\\&+k_w\frac{\partial^2(w_b+w_s)}{\partial x^2}-(k_p-N^T)\frac{\partial^4(w_b+w_s)}{\partial x^4})=q_{dynamic}-(ea)^2\frac{\partial^2 q_{dynamic}}{\partial x^2}\end{aligned} \tag{23}$$

$$\begin{aligned}&(1-l^2\frac{\partial^2}{\partial x^2})B_s(\frac{\partial^3 u}{\partial x^3})-(1-l^2\frac{\partial^2}{\partial x^2})D_s(\frac{\partial^4 w_b}{\partial x^4})-(1-l^2\frac{\partial^2}{\partial x^2})H_s(\frac{\partial^4 w_s}{\partial x^4})+(1-l^2\frac{\partial^2}{\partial x^2})A_s(\frac{\partial^2 w_s}{\partial x^2})\\&-I_0(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2})-J_1\frac{\partial^3 u}{\partial x\partial t^2}+J_2\frac{\partial^4 w_b}{\partial x^2\partial t^2}+K_2\frac{\partial^4 w_s}{\partial x^2\partial t^2}-k_w(w_b+w_s)+(k_p-N^T)\frac{\partial^2(w_b+w_s)}{\partial x^2}\\&+\mu(+I_0(\frac{\partial^4 w_b}{\partial x^2\partial t^2}+\frac{\partial^4 w_s}{\partial x^2\partial t^2})+J_1\frac{\partial^5 u}{\partial x^3\partial t^2}-J_2\frac{\partial^6 w_b}{\partial x^4\partial t^2}-K_2\frac{\partial^6 w_s}{\partial x^4\partial t^2}+k_w\frac{\partial^2(w_b+w_s)}{\partial x^2}\\&-(k_p-N^T)\frac{\partial^4(w_b+w_s)}{\partial x^4})=q_{dynamic}-(ea)^2\frac{\partial^2 q_{dynamic}}{\partial x^2}\end{aligned} \tag{24}$$

The most important issue is that inclusion of neutral axis locations deletes the effect of coupling between bending and axial motions.

3. Solution by differential quadrature method (DQM)

In the present chapter, differential quadrature method (DQM) has been utilized for solving the governing equations for NSGT porous FG nanobeam. According to DQM, at an assumed grid point (x_i, y_j) the derivatives for function F are supposed as weighted linear summation of all functional values within the computation domains as

$$\frac{d^nF}{dx^n}\Big|_{x=x_i} = \sum_{j=1}^{N} c_{ij}^{(n)} F(x_j) \tag{25}$$

where

$$C_{ij}^{(1)} = \frac{\pi(x_i)}{(x_i - x_j)\ \pi(x_j)} \qquad i,j = 1,2,\dots,N, \qquad i \neq j \tag{26}$$

in which $\pi(x_i)$ is defined by

$$\pi(x_i) = \prod_{j=1}^{N} (x_i - x_j), \qquad i \neq j \tag{27}$$

And when $i = j$

$$C_{ij}^{(1)} = c_{ii}^{(1)} = -\sum_{k=1}^{N} C_{ik}^{(1)}, \qquad i = 1,2,\dots,N, \qquad i \neq k,\ i = j \tag{28}$$

Then, weighting coefficients for high orders derivatives may be expressed by

$$C_{ij}^{(2)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(1)}$$

$$C_{ij}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(2)} = \sum_{k=1}^{N} C_{ik}^{(2)} C_{kj}^{(1)}$$

$$C_{ij}^{(4)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(3)} C_{kj}^{(1)} \qquad i,j = 1,2,\dots,N.$$

$$C_{ij}^{(5)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(4)} = \sum_{k=1}^{N} C_{ik}^{(4)} C_{kj}^{(1)}$$

$$C_{ij}^{(6)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(5)} = \sum_{k=1}^{N} C_{ik}^{(5)} C_{kj}^{(1)} \tag{29}$$

According to presented approach, the dispersions of grid points based upon Gauss-Chebyshev-Lobatto assumption are expressed as

$$x_i = \frac{a}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right] \qquad i = 1,2,\dots,N, \tag{30}$$

Next, the displacement components may be determined by

$$w_b(x,t) = W_b(x)e^{i\omega t} \tag{31}$$

$$w_s(x,t) = W_s(x)e^{i\omega t} \tag{32}$$

where W_b and W_n denote vibration amplitudes and ω defines the vibrational frequency. Then, it is possible to express obtained boundary conditions as

$$w_b = w_s = 0, \quad \frac{\partial^2 w_b}{\partial x^2} = \frac{\partial^2 w_s}{\partial x^2} = 0, \frac{\partial^4 w_b}{\partial x^4} = \frac{\partial^4 w_s}{\partial x^4} = 0 \tag{33}$$

Now, one can express the modified weighting coefficients for all edges simply-supported as

$$\bar{C}^{(2)}_{1,j} = \bar{C}^{(2)}_{N,j} = 0, \qquad i = 1, 2, \dots, M,$$

$$\bar{C}^{(2)}_{i,1} = \bar{C}^{(2)}_{1,M} = 0, \qquad i = 1, 2, \dots, N. \tag{34}$$

and

$$\bar{C}^{(3)}_{ij} = \sum_{k=1}^{N} C^{(1)}_{ik} \bar{C}^{(2)}_{kj} \qquad \bar{C}^{(4)}_{ij} = \sum_{k=1}^{N} C^{(1)}_{ik} \bar{C}^{(3)}_{kj} \tag{35}$$

Inserting Eqs. (31) and (32) into Eqs. (22)-(24) respectively, leads to

$$\left\{ [K] + \frac{\partial}{\partial t^2}[M] \right\} \begin{Bmatrix} W_{bn} \\ W_{sn} \end{Bmatrix} = \begin{Bmatrix} q_{dynamic} - (ea)^2 \dfrac{\partial^2 q_{dynamic}}{\partial x^2} \\ q_{dynamic} - (ea)^2 \dfrac{\partial^2 q_{dynamic}}{\partial x^2} \end{Bmatrix} \tag{36}$$

in such a way that [K] and [M] define the stiffness and mass matrices of the structure, respectively. Moreover, the non-dimension foundation factors have been defined by

$$K_w = k_w \frac{L^4}{E_m I}, K_p = k_p \frac{L^2}{E_m I}, \tag{37}$$

It is assumed that the dynamical load is travelling along a straight line causing forced vibrations and is defined in the below form

$$q_{dynamic} = \sum_{n=1}^{\infty} Q_n sin[\frac{n\pi}{L} x] \tag{38}$$

$$Q_n = \frac{2}{L}\int_0^L sin[\frac{n\pi}{L} x] q(x) dx = \frac{2P}{L}\sin[\frac{n\pi}{L} x_p] = \frac{2P}{L}\sin[\frac{n\pi}{L} V_p t] \tag{39}$$

where Q_n defines the Fourier coefficients and $q(x) = P\delta(x - x_p)$ so that P defines the force measure, x_p symbolizes the force position. Moreover, V_p symbolizes the force speed.
Next, based upon zero initial conditions and Laplace transform method, Eq. (36) gives

$$\left\{ [K] + S^2[M] \right\} \begin{Bmatrix} L[W_{bn}] \\ L[W_{sn}] \end{Bmatrix} = \begin{Bmatrix} L[q_{dynamic} - (ea)^2 \dfrac{\partial^2 q_{dynamic}}{\partial x^2}] \\ L[q_{dynamic} - (ea)^2 \dfrac{\partial^2 q_{dynamic}}{\partial x^2}] \end{Bmatrix} \tag{40}$$

Table 2 Comparison of the non-dimension frequency for nonlocal FG nanobeams (L/h=20)

μ	p=0.1		p=0.5		p=1	
	CBT (Eltaher *et al.* 2012)	Present HOBT	CBT (Eltaher et al. 2012	Present HOBT	CBT (Eltaher *et al.* 2012)	Present HOBT
0	9.2129	9.1614	7.8061	7.7151	7.0904	6.9677
1	8.7879	8.7402	7.4458	7.3604	6.7631	6.6473
2	8.4166	8.3722	7.1312	7.0505	6.4774	6.3675
3	8.0887	8.0472	6.8533	6.7768	6.2251	6.1202

Solving Eq. (40) based upon inverse Laplace transform approach leads to the bending (W_{bn}) and shear (W_{sn}) coefficients. The dynamical deflections of higher order refined nanobeams may be calculated as $W= W_{bn}+ W_{sn}$. The non-dimension factors are introduced by

$$\mu = ea/L, \lambda = l/L, \bar{W} = W\frac{100E_mI}{PL^3}, \alpha = \frac{V_p}{V_{cr}}, \quad V_{cr} = \frac{\omega_n L}{\pi}, t^* = \frac{V_p t}{L} \tag{41}$$

4. Discussions with results

According to the section, new findings have been presented for dynamical investigation of porous NSGT and FG beams modeled as a refined thick structure incorporating material imperfectness. Before all, the natural frequencies of FG beams have been verified by using the data of Euler beams reported by Eltaher *et al.* (2012), as presented in Table 2. To do this, a S-S beam having graded distribution of ceramic-metal is selected. With respect to different values of nonlocal factor (μ), an excellent agreement is achieved among obtained natural frequencies with those provided by Eltaher *et al*. (2012).

In Fig. 2, the variations of normalized deflections of a FG nano-dimension beam versus non-dimension time (t^*) of mechanical loading are represented for several elasticity theories (CET, NET and NSGT) when *L/h*=10. Non-dimension speed of the travelling load is selected as V*=0.15 and 0.2. By selecting CET or μ=λ=0, the deflections and vibrational frequencies based upon classic beam assumption will be derived. Actually, selecting μ=0 gives the deflections in the context of classic elasticity theory and discarding nonlocal impacts. Considering NET leads to larger deflections of the beam than NSGT. So, forced vibration behavior of the nanobeam system is dependent on scale effects.

In Fig. 3, the variations of normalized deflections of a FG nano-dimension beam versus non-dimension time (t^*) of mechanical loading are represented for several temperature changes (ΔT). Non-dimension speed of the travelling load is selected as V*=0.15. It can be understand from Fig. 3 that normalized deflection of system will rise with temperature change. Such finding is owning to the lower structural stiffness of the nano-size beam with the rise of temperature variation across the thickness.

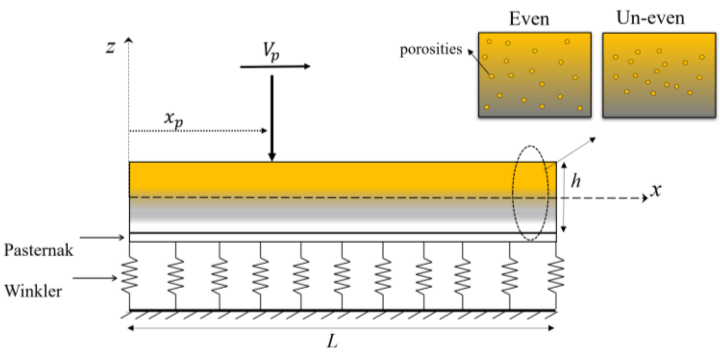
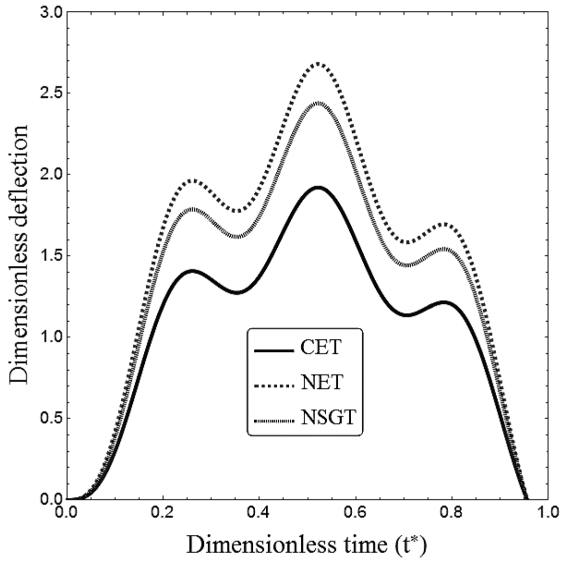
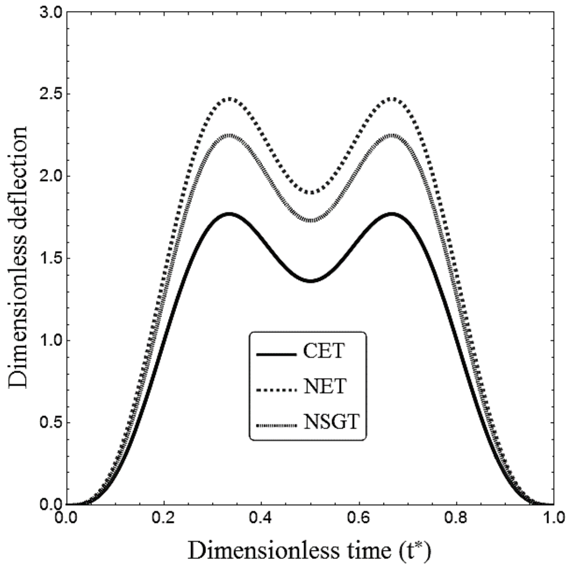


Fig. 1 A nano-size FG beam exposed to moving dynamic force



(a) V*=0.15



(b) V*=0.2

Fig. 2 Effects of different elasticity theories on time responses of the nanobeam ($\Delta T=0$, $L/h=10$, $p=1$, $K_w=0$, $K_p=0$, $\xi=0.1$)

Dynamical deflection variation of the nanobeam according to non-dimension speed based upon even and uneven porosity dispersions at slenderness ratio $L/h=10$, non-dimension time $t^*=0.3$, nonlocal factor $\mu=0.2$, strain gradient factor $\lambda=0.1$, Winkler factor $K_w=25$, Pasternak factor $K_p=5$ and material exponent $p=1$ is shown is Fig. 4. This figure is provided at fixed porosity factor of $\xi=0.1$. The most important finding from the figure is that even porosity dispersion leads to larger dynamical deflections due to representing lower stiffness compared to un-even dispersion.

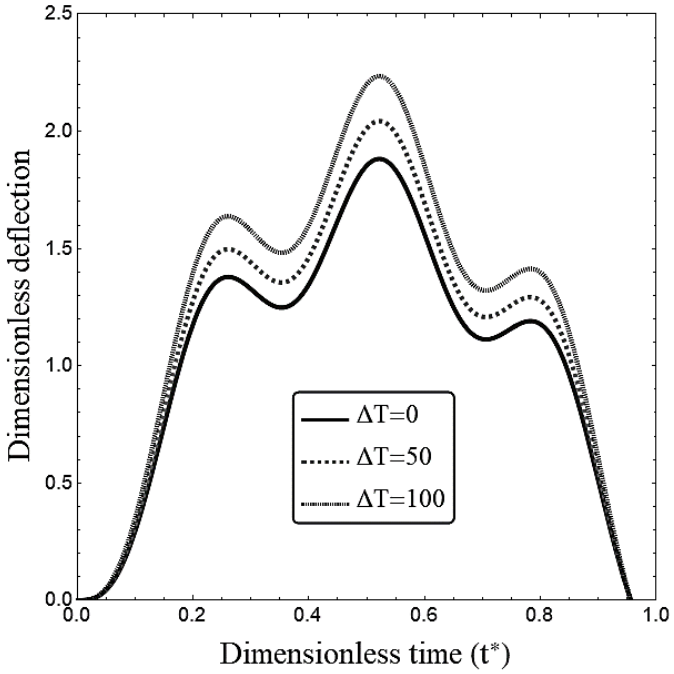


Fig. 3 Temperature effects on time responses of the nanobeam (L/h=10, p=1, K_w=0, K_p=0, ξ=0)

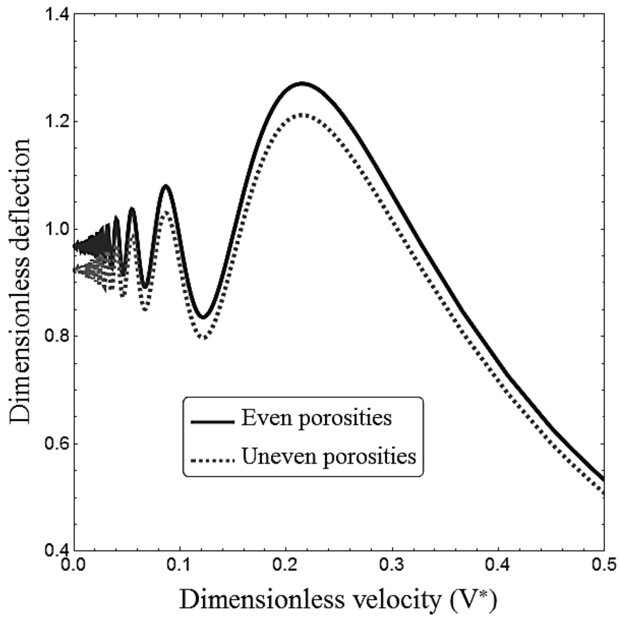
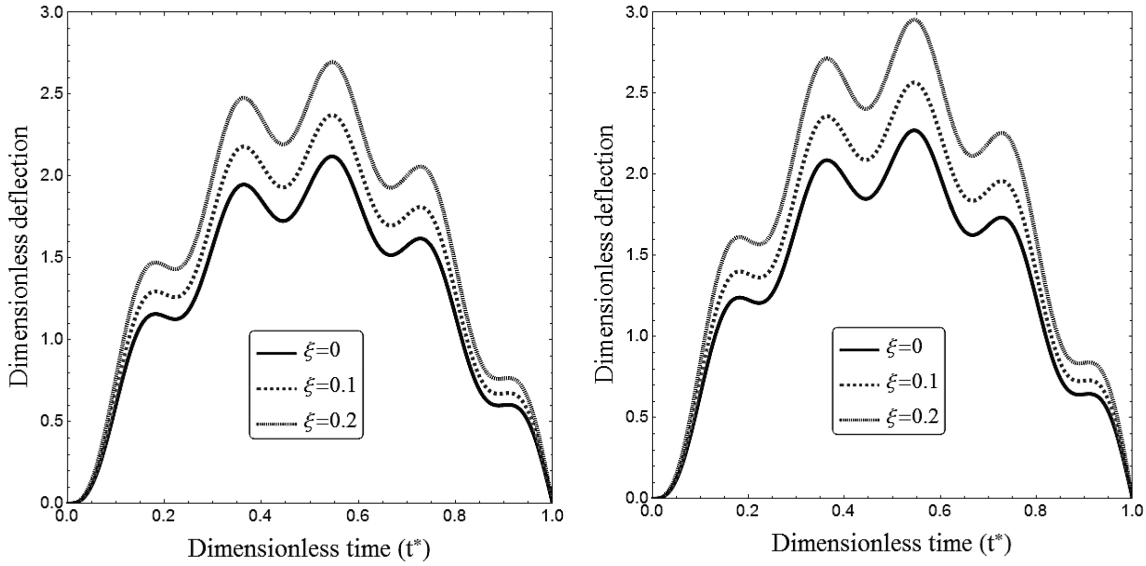
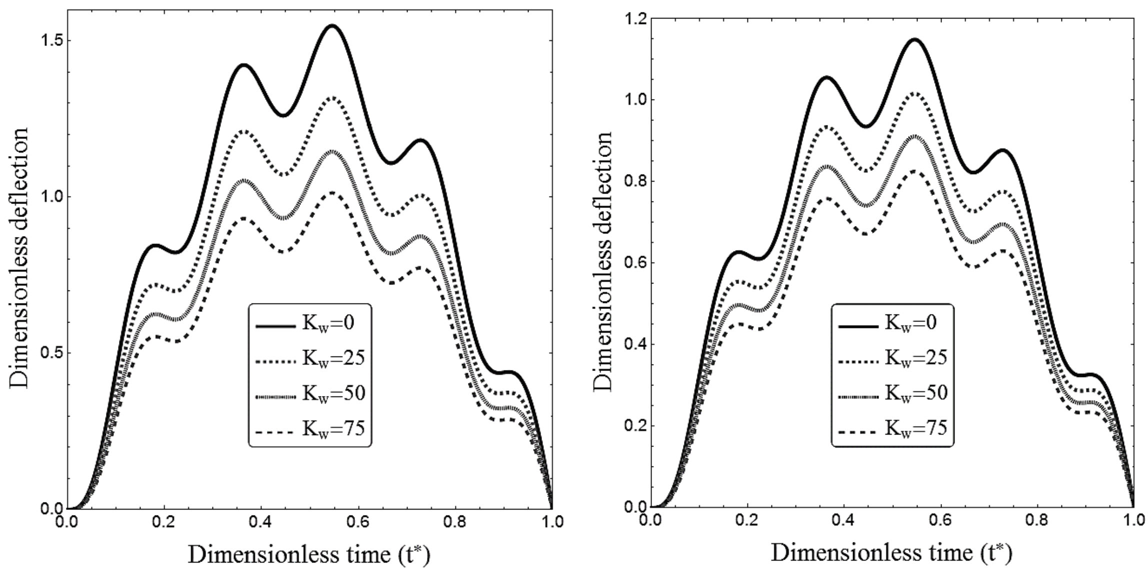


Fig. 4 Dynamical deflection variation of nanobeam according to non-dimension speed based upon even and uneven porosity dispersions (t*=0.3, μ=0.2, L/h=10, λ=0.1, K_w=25, K_p=5, p=1, ξ=0.1)



(a) p=1 (b) p=2

Fig. 5 Porosity factor effects on time responses of the nanobeam (μ=0.2, L/h=10, λ=0.1, K_w=0, K_p=0)



(a) K_p=5 (b) K_p=10

Fig. 6 Foundation factor effects on time responses of the nanobeam (μ=0.2, λ=0.1, L/h=10, p=1, ξ=0.1, V*=0.1)

Impacts of material FG exponent (p=1 and 2) and porosity factor (ξ) on dynamical deflections of FGM nano-size beams exposed to moving dynamical force with respect to the non-dimension time have been illustrated in Fig. 5 assuming L/h=10 and μ=2. One may see that the dynamical bending of FGM nano-size beam is remarkably influenced by FGM gradation. It is found that the magnitudes of dynamical deflections increase via increase of material exponent (*p*). This is owning to higher portions of metallic constituent via increase of material exponent. Moreover, increment of porosity factor results in higher dynamical deflections. Thus, choosing reliable values for material exponent and porosity is crucial for reasonable design of FG nano-size structures when they are exposed to dynamical excitation.

Fig. 6 indicates the influences of Winkler and Pasternak factors on dynamical deflections of FGM nano-size beams versus non-dimension time (t*) assuming L/h=10 and p=1. Different magnitudes of Winkler factor (K_w=0, 25, 50, 75) and Pasternak factor (K_p=5, 10) have been selected. One can observe that a rise in the magnitude of Winkler and Pasternak factors leads to reduction in vibration amplitudes of FGM nano-size beams. Actually, the nano-size beams become more rigid via increasing in foundation factors. An important finding is that Pasternak factor indicates more significant impact on deferment of dynamical deflection. This is because Pasternak factor is corresponding to continuous interactions with the nano-size beam but Winkler factor leads to discontinuous interactions with the nano-size beam. Accordingly, the forced vibrations of FGM nano-size beams have been significantly influenced by elastic substrate.

5. Conclusions

The presented article employed a higher order shear deformation beam formulation having three variables without using of shear correction factor. Based upon differential quadrature (DQ) approach and nonlocal strain gradient elasticity formulation, forced vibrational analysis of shear deformable functionally graded (FG) nanobeam on elastic medium under moving dynamical load was performed. The presented formulation incorporated two scale factors for examining vibrational behaviors of nano-dimension beams. The material properties for FG beam were defined employing a power-law form. The governing equations achieved by Hamilton's principle were solved implementing DQM. Presented results indicated the prominence of material gradient index, nonlocal coefficient, material gradient coefficient, porosity, load velocity and substrate factors on vibrational properties of FG nano-size beam. Especially, it was found that as the porosity factor increases, the dynamical deflections increase. Also, it was observed that nonlocal factor increment results in larger values for dynamical deflection of FGM nano-size beam.

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