Structural Engineering and Mechanics, Vol. 18, No. 5 (2004) 565-589
DOI: http://dx.doi.org/10.12989/sem.2004.18.5.565

Non-linear rheology of tension structural element under single and variable loading history Part I: Theoretical derivations

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(Received July 16, 2003, Accepted June 24, 2004)

Abstract. The present paper concerns the macroscopic overall description of rheologic properties for steel wire and synthetic fibre cables under variable loading actions according to non-linear creep and/or relaxation theory. The general constitutive equations of non-linear creep and/or relaxation of tension elements - cables under one-step and the variable stress or strain inputs using the product and two types of additive approximations of the kernel functions are presented in the paper. The derived non-linear constitutive equations describe a non-linear rheologic behaviour of the cables for a variable stress or strain history using the kernel functions determined only by one-step – constant creep or relaxation tests. The developed constitutive equations enable to simulate and to predict in a general way non-linear rheologic behaviour of the cables under an arbitrary loading or straining history. The derived constitutive equations can be used for the various tension structural elements with the non-linear rheologic properties under uniaxial variable stressing or straining.

Key words: non-linear creep of cable; non-linear relaxation of cable; variable stress or strain history; non-linear constitutive equation; kernel functions; approximations of the kernel functions.

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1. Introduction

The engineering community is facing growing demands for state of the art modelling and realistic predictions in many fields. Such demands lead to models of very complex characteristics. In many cases, a non-linear time-dependent behaviour has to be taken into account, including complex non-linear constitutive models. Design of the modern high performance tension structures often have to take into consideration the time-dependent effect of complex states of stress, strain and environment on the mechanical behaviour of high strength cables.

Tension structural elements like spiral strands, locked coil strands and structural wire ropes are widely used in the engineering practice. Spiral strands are mainly used as carrying, stabilizing or edge cables for light-weight cable structures, fabric membranes, cable nets, hybrid wood and steel structures etc. Locked coil strands are used for cable suspended and cable stayed bridges.

The remarkable progress in the polymer industry over the last decades has produced new materials with revolutionary mechanical properties. One example is the new generation of the high-strength

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synthetic fibre ropes, which has already found wide engineering application in tension structures.

In contrast to the classical tension steel rods and bars, which operate in the linear elastic range, steel cables and mainly fibre ropes have time-dependent non-linear viscoelastic properties. Modelling and prediction of the time-dependent behaviour of wire and fibre cables for complex loading history is therefore a necessary step in the design of these tension elements. In most applications of wire and fibre cables, load and deformation vary with time. In general, due to non-linear viscoelasticity of the material stiffness will depend on the magnitude of permanent and variable load and on the length of their time affect, cyclic load range, previous loading history, temperature etc. The actual value of the stiffness is needed to know in order to compute the response of a total structural system in the required time.

Several different approaches have been considered in the development of ropes time-dependent properties prediction concepts applicable in designers work in the past. Some researchers prefer micromechanical analytical approach, other experts are advocating applicability of macrorheological methods.

Microrheology derives the rheological behaviour of complex cables from their structural configuration and from the known rheological behaviour of their constituents – wires or fibres (creep or relaxation tests on a single wire or fibre are carried out). The analytical method for the calculation of creep strain and relaxation values of rope has been established by Nakai *et al.* (1975). This method involves the creep strain and stress relaxation values of a single wire in calculating the mechanical properties of spiral ropes, if the tensile load acting on each wire of the rope can be known. Conway and Costello (1993) present a method in which the axial viscoelastic response of a simple strand may be predicted given the stress relaxation of the filament's construction material. This approach utilizes the Schapery collocation method to determine the coefficients for the elements of a Wiechert model. The geometric effects of the strand are then combined with the analytical solution for the Wiechert model to develop a system of convolution integrals which satisfy the equilibrium and boundary conditions for the strand construction. The solutions of these integrals are approximated numerically using a modified Newton's iterative method combined with technique developed by Lee and Rogers. Leech *et al.* (1993) outline a quasi-static analysis which has been developed for structured ropes and embodied in computer codes for prediction of the quasi-static and long-term properties of ropes.

Many analytical and numerical models have been developed to describe the mechanical behaviour of cables. Many suggested methods do not involve the rheological properties. Huang and Vinogradov (1996) investigate the effect of friction forces and the interwire slip on the mechanical properties of tension cables. A mathematical model of a cable as a system of interacting wires is presented. Costello (1997) formulates the equations of equilibrium for a thin curved wire in space, solves these equations and applies the solution to determine the stresses in a simple strand. These results are then extended to ropes with more complex cross sections. Raoof and Kraincanic (1998) propose a theoretical model for predicting the patterns of interwire contact forces, wire kinematics, and the coupled axial/torsional stiffness coefficients in fully bedded-in locked-coil ropes experiencing either static monotonic axial/torsional loading or a mean axial load superimposed on which are external load perturbations. Roshan Fekr *et al.* (1999) constructed a three-dimensional finite element model of a cable to predict the stress distribution in each component when the cable is subjected to a given elongation. The model was used for analysis of an optical ground wire which is composed of an envelope of one or two layers of metallic helical strands wound around a core containing optical fibres. Nawrocki and Labrosse (2000) present a finite element model for simple straight wire rope strands, which allows for the study of all the possible interwire motions. The role of the contact conditions in pure axial loading is investigated. It appears that the interwire pivoting and interwire sliding govern the cable response, respectively, for axial and bending loads. Jiang *et al.* (2000) present a finite element model of three-layered straight helical wire rope strand under axial loads. Three-dimensional solid elements were used for structural discretization and contact, friction and plastic yielding were taken into account. Labrosse *et al.* (2000) investigate the quantity of energy dissipated through friction due the motions between wires when a cable is loaded. Evans *et al.* (2001) investigate wire strain variations in tension-tension fatigue for two six-strand rope constructions under normal and overloaded conditions. It has been found that for rope tension there is a considerable variation in wire strains both on different wires at the same cross-section and to a marginally lesser extent along the length of the same wire. Leech (2002) surveys the various rope constructions and describes the analysis for polymer fibre rope deformations and loads considering the internal components. The analysis for the various friction modes and the mechanisms of component slip are presented. Lefik and Schrefler (2002) present an example of the use of an artificial neural network for parameter identifications of a theoretical elasto-plastic model of the behaviour of a superconducting cable under cyclic loading.

Macrorheology regards the deformation of a cable as macroscopic changes of the whole. The cables are investigated as if they were homogeneous or devoid of structure or are what may be called quasi-homogeneous. In such cases a cable is tested for creep or relaxation behaviour as the whole and the description of time-dependent behaviour of cables by the constitutive equations is based on the experimental evaluation of the material time functions and constants. A series of dead load tests on a parallel-lay aramid rope has been performed by Guimaraes and Burgoyne (1992) with the purpose of studying its creep behaviour. It was found that creep and recovery are adequately described by a logharitmic time law and that the creep coefficient for the material can be considered stress independent. An empirical expression for prediction of long-term creep at ambient temperature was presented.

Banfield *et al.* (2003) discussed, that it is too difficult a problem to determine the vast amount of data needed to fully characterise the response of fibre rope to any increment or cycle of loading history by the rheological model, which includes elastic, viscous and frictional elements in series and in parallel. As reported by Banfield *et al.* (2003), there is no adequate theory of non-linear viscoelasticity to apply. Current work of the authors is directed to finding ways round this problem.

One of the possible alternatives of macrorheology-based description and prediction of the ropes non-linear time-dependent behaviour, the constitutive equations of nonlinear creep and stress relaxation of ropes under variable stress and strain, respectively, with involving the theoretical determination of the material time-dependent constants based on experimental evaluation, is presented in this paper.

2. Problem formulation

The various empirical expressions for creep of engineering materials have been used to represent constant stress creep. If the situation involves a non-steady stress history, the creep rate even in the secondary stage cannot be described as a function of stress alone. One approach to this problem has been to describe the creep rate as a function of strain or time in addition to stress. The strain rate equation including stress and time as variables is called a time-hardening law, while it is called a strain-hardening law if it is a function of stress and strain. Both laws have been widely used to represent creep in stress analysis of time-dependent materials. It has been observed by most investigators that the strain-hardening law usually yields a better representation and predictability of creep than the time-hardening law (for both metals and plastics).

Most of the stress-strain-time relations (they are primarily empirical) and analytical rheological models were developed to fit experimental creep curves obtained under constant stress and constant temperature. The non-linear behaviour of viscoelastic materials has also been represented by means of rheological models. One of the simplest viscoelastic materials is represented by the Kelvin-Voigt or Maxwell rheological models. The generalized Kelvin-Voigt model (several Kelvin-Voigt models in series) is more convenient than the generalized Maxwell model for viscoelastic analysis in case where the stress history is prescribed, whereas the generalized Maxwell model is the more convenient in cases where the strain history is prescribed. Because of the range of different retardation or relaxation times, which can be brought into play, both of these models permit a close description of real behaviour over a wider time span than is possible with simpler models. However, for the non-linear viscoelastic cables more accurate rheological models must be available. The time-dependent response of cable to any increment of loading could, in principle be represented by the rheological model which includes in series Hookean elastic spring, Newtonian viscous dashpot, Burgers model where a Maxwell and a Kelvin-Voigt model are connected in series and Saint Venant frictional elements in series and in parallel with Hookean elastic spring, provided the values of the parameters are allowed to change with the loading history. It is too difficult to be precise about the determination of the vast amount of data needed to fully characterise the rheological model. Non-linear rheological models can not be described mathematically as conveniently and generally as linear models can. Consequently, it has been common practice to approximate non-linear behaviour by simpler and linear rheological models. The development of adequate non-linear rheological model for the solution of non-linear viscoelastic cables is still an active area of research. Because of the mathematical difficulties that can arise, functional types of non-linear constitutive equations are the more popular and can be in this case of the ropes more practical and precise. It is also easier to incorporate to them temperature effects. The actual behaviour of cables has shown that the strain at a given time depends on all of the values of the stress in the past, not just on its final value. Thus the creep phenomenon is affected by the magnitude and sequence of stresses or strains in all of the past loading history of the cable. Based on this fact, the mathematical method has been used to represent the time dependence or viscoelastic behavior of cables.

In stress analysis problems involving linear viscoelastic materials, linear differential equations have been widely used. Some such problems have been treated by removing the time variable in the system of equations and in the boundary conditions by employing the Laplace transformation with respect to time. The viscoelastic problem thus becomes an elastic problem.

Besides the differential operator type of constitutive equations of linear viscoelastic behaviour and associated mechanical rheological models, there is another means of describing behaviour of viscoelastic materials, the integral operator representation. Any stress-time curve may be approximated by the sum of a series of step functions which correspond to a series of step-like increments in load. This integral representation contains a hereditary integral, which was first suggested by Volterra. The kernel function of the integral is a memory function which describes the stress history dependence of strain. The stress relaxation of a linear viscoelastic material can be expressed in a similar way. Guimaraes and Burgoyne (1992) for prediction of creep of a parallel-lay aramid rope under varying load applied Boltzmann superposition principle. The linearity of the viscoelastic behaviour of material is assumed. The principle states that the creep strain is a function of the entire loading history, each loading step makes an independent contribution to the final strain, and the final creep strain can be obtained by the simple addition of each contribution. Both the differential operator method and the integral representation have been used in stress analysis of cable approximated as a linear viscoelastic material (Husiar and Switka 1986, Sobotka 1984). By performing a proper set of experiments the material constants and the kernel functions for a given cable can be determined and used to predict behavior under other stress histories.

In recent years, there has been considerable effort to develop a general constitutive equation for non-linear material with rheological properties. Such constitutive equations are much more complex than the linear theory. For generality, they require a large number of functions with higher order stress terms to describe creep behaviour satisfactorily. By making use of the functional analysis, the more general constitutive equations for stress relaxation were expressed by Green *et al.* (1959), Green and Rivlin (1960) and by Pipkin (1964) as a series of multiple hereditary integrals. They considered that the current stress at a point of materials is not only a function of the current deformation gradient but also the deformation gradients at all previous times. In a similar manner, Findley *et al.* (1976) and Sobotka (1984) described creep as a series of multiple integrals.

Many constitutive models and equations as well as rheological methods have been developed to describe the linear and non-linear viscoelastic behaviour of materials. Schapery (2000) gives an overview of rheological constitutive equations and models for fracture and strength of non-linear viscoelastic solids. Recent work is emphasized. Drozdov (1998) propose a model for the non-linear viscoelastic response in polymers at finite strains. Reese and Govindjee (1998) present a model for finite deformation viscoelasticity that utilizes a non-linear evolution law for the viscous material behaviour. Non-linear constitutive models for the special purposes were developed. Wineman *et al.* (1998) present a non-linear viscoelastic model suitable for one dimensional response analysis of elastomeric bushing. Jung and Youn (1999) propose a non-linear viscoelastic constitutive model for solid propellant.

Park and Schapery (1999) present an efficient and accurate numerical method of interconversion between linear viscoelastic material functions based on a Prony series representation. The method is tested using experimental data from selected polymeric materials. The tensile relaxation data from polymethyl methacrylate are used in illustrating the method. Schapery and Park (1999) propose a new, simple approximate interconversion method and verify by examples. Poon and Fouad Ahmad (1999) present a new scheme for performing integration point constitutive updates for anisotropic, small strain, non-linear viscoelasticity, within the context of implicit, non-linear finite element structural analysis. The work illustrates the generality of the derived fundamentals by extending to Schapery's non-linear model. Numerical examples involving homogeneous stress state such as uniaxial extension are carried out by incorporating the present scheme into a general purpose FEM package. Fafard *et al.* (2001) present a fully 3D viscoelastic model to predict the creep and relaxation behaviour of anisotropic materials. This model is based on phenomenological approach using internal variables and is applicable to nonconstant coefficients. From the general 3D model, the analytical solution in 1D is derived and a connection with the classical rheological model is made. Frank and Brockman (2001) present a constitutive model which combines non-linear viscoelasticity and viscoplasticity into a unified set of equations. Kaliske *et al.* (2001) consider different physical non-linear constitutive formulations in order to model elastic and viscoelastic materials in a format ready for a finite element implementation. A novel approach of rubber elasticity is introduced.

In quasi-linear viscoelasticity, the time-dependence is separated from the strain-dependence. This form has been used widely in the modelling of some materials, which are nonlinearly viscoelastic. Kwon and Soo Cho (2001) present an analysis that verifies time-strain separability in viscoelastic constitutive equations. They investigate unstability of differential and integral constitutive equations based on time-strain separability. It has been shown by Kwon and Soo Cho (2001), in the case of fluids, that both the differential and integral constitutive equations based on time-strain separability have either Hadamard or dissipative type instabilities. Hadamard instabilities are associated with rapid elastic response. Dissipative instabilities are related to the dissipative viscous nature of the constitutive equations. Both these conditions relate the quality of rheological equations to the laws of thermodynamics. Therefore, the separability hypothesis is invalid for short time periods and not free of mathematical instabilities.

Bonet (2001) propose a new model based on the multiplicative decomposition of the isochronic component of deformation gradient into elastic and viscous contribution and the generalized Maxwell rheological model. Patlashenko *et al.* (2001) propose several new families of time-stepping schemes for solution of linear Volterra-type systems of integro-differential equations with kernels in the forms of sums of exponentials. Anderssen and Loy (2002) analyze how fading memory in the constitutive equations of a viscoelastic material should be defined. Beijer and Spoormaker (2002) implement with a user subroutine the Schapery model for non-linear viscoelastic behaviour of polymers into the FEM package MARC. Hartmann (2002) propose the interpretation of current non-linear finite element calculations applied to constitutive equations of evolutionary type as a system of differential – algebraic equations. This procedure is applied to a model of finite-strain viscoelasticity which incorporates a particular model of non-linear rate dependence. Meo *et al.* (2002) deal with a thermoviscoelastic model and its analysis. The mechanical formulation is based on the generalization to large strain of the Poynting-Thomson rheological model. The heat transfers are governed by the classical heat equation and the Fourier law. Ponter and Boulbibane (2002) derive a model to provide an intermediary description between perfect plasticity, for which general minimum theorems are already known, and more complex and realistic creep constitutive relationships involving internal state variable of a body subjected to cyclic load and temperature.

Efficient numerical simulations of rheological problems generally require the use of quasi-static formulations with conjugate-gradient techniques for solving the large number of algebraic equations. Implicit in the approach is the need to solve the constitutive equation several times for large time steps and for loading actions. Therefore, an unconditionally stable and efficient algorithm for solving the constitutive equation is essential for the overall efficiency of the solution procedure. Schreyer (2002) propose an algorithm, which is a combination of the use of a trapezoidal rule and an iterative Newton-Raphson method for solving implicitly the non-linear constitutive equations.

Zheng and Weng (2002) derive a new constitutive equation to describe the long-term creep behaviour of polymers. During long-term creep the viscosity of polymers will continue to increase due to physical aging. Haitian and Yan (2003) combine element free Galerkin method with a precise algorithm in the time domain for solving viscoelasticity problems. Meskuita and Coda (2003) describe an efficient procedure to analyse two dimensional viscoelastic coupling problems. The coupling between the finite element method and the boundary element method is based on the sub-regions technique. The viscous behaviour is taken into account by rheological constitutive relations properly imposed in the development of the integral representations. The Kelvin-Voigt viscoelastic model is implemented in order to validate the idea. Quintanilla (2004) propose an alternative method to study the decay estimates for the transient problem in non-linear viscoelasticity.

Functional types of the constitutive equations contain kernel functions that represent the time dependence of materials. In order to use these constitutive equations for stress or strain analysis, under characteristic short- or long-term loading history, the kernal functions must be determined experimentally. A more complete characterization of the cable kernel functions requires experimental data from multiple steps loading or straining and the number of creep or relaxation tests needed is in a higher order. This creates a great problem for both experimentation and analysis. Besides, the constitutive equations thus obtained are quite complicated. Therefore, several kinds of the numerical approximations of kernel functions were applied, to provide a less – time consuming test and less involved means of dealing with non-linear rheological behaviour of cables under variable input programs (variable stress for non-linear creep and/or variable strain for non-linear relaxation analyses). In these methods, the kernel functions determined from constant input programs (one-step creep test under constant stress and/or one-step relaxation test under constant strain) are used to describe the non-linear rheological behaviour under a variable input program. Method enables to describe a creep (or a stress relaxation) during a variable history of stress (and/or loading) or strain which is divided into the constant levels in studied – required time intervals. It enables to simulate and predict in a design stage the influence of loading history on a behaviour history of the cable structural elements and the whole cable or cable hybrid structure.

Since the theory of non-linear memory-dependent behaviour of materials was developed, there are only few studies involving the experimental evaluation of the cable constants. Because of the varying inputs, this determination is rather complicated. The characterization of the non-linear constitutive equations of cables for one-step creep and stress relaxation were treated by the authors, Kmet (1989, 1994), Kmet and Holickova (2000).

The present paper concerns the macroscopic overall description of rheologic properties for steel wire and synthetic fibre cables – ropes under variable loading history according to non-linear creep and/or relaxation theory.

In the first part of the present paper, the mathematical derivations of the general constitutive equations for non-linear creep and relaxation of the cables under one-step and the variable stress or strain inputs using the product and two types of additive approximations of the kernel functions are presented. The time-dependent material constants – kernel functions determined from the results of one-step creep or relaxation tests are used for the description of constitutive equations for variable stress or strain history. The derived constitutive equations enable to describe non-linear rheologic behaviour of the cables for variable stress or strain history by using the kernel functions determined only from one-step constant creep or relaxation tests.

The numerical applications of the obtained constitutive equations with the input data from the tests will be discussed in the second part of the paper.

3. Assumptions for the analysis

The derivation of the constitutive equations for non-linear creep and relaxation of the structural tension elements under one-step and the variable stress or strain inputs is built up upon the following assumptions:

a) The tension structural element is assumed steel wire or synthetic fibre cable, which is treated as line element exposed by uniaxial tension stress for creep and uniaxial strain for relaxation analysis.

b) The material of cable is non-linearly viscoelastic. The stress is not proportional to strain at a given time and the linear Boltzmann superposition principle does not hold.
c) The macroscopic approach is adopted. Therefore, a cable is tested for creep or relaxation behaviour as the whole and the mathematical-physical description of time-dependent behaviour of cables by the constitutive equations is based on the non-linear creep or relaxation theory and on the experimental evaluation of the material time-dependent functions and constants.
d) The approximation methods using the product and two types of additive approximations which utilize the time-dependent kernel functions determined from one-step creep or relaxation tests to describe nonlinear creep behaviour under arbitrary stress history (relaxation under arbitrary strain history) are applied.
e) The various types of stress or strain history can be accounted. The following step of stress or strain can be greater or smaller than the previous one.
f) The various magnitudes and time sequences of stresses or strains can occur. The assuming loading history of cable can be transform to the stress or strain history at the required time interval, which enables to simulate, predict and assess the rheological behaviour characteristics of cable at the design stage. By gradually changing of stress or strain levels the deformation history of cable can be modelled.

Remark 1: The non-linear time-dependent behaviour of viscoelastic cables can be expressed by a constitutive equation, which includes time as a variable in addition to the stress and strain variables. The constitutive equations characterize the individual cable and its reaction to external excitations. Each of these equations is a mathematical formulation designed to approximately describe the observed response of a real cable material over a certain restricted range of the variables involved. These equations must satisfy certain physical requirements if they are to faithfully represent cable time-dependent stress - creep strain behaviour. In order to satisfy these physical requirements of constitutive equations, their kernel time functions must be determined experimentally. Just these time-dependent functions and/or time-dependent material characteristics of cables create the physical substance of this approach.

Remark 2: The representation of the uniaxial non-linear constitutive relations derived in the papers can be extended to a multi-axial stress state. It is necessary to state that the complete formulation as well as the determination of the material time-dependent parameters is too complicated for practical use in many stress analysis situations. Therefore, some simplification of the constitutive equations is desirable in order to obtain a practically useful theory. There are two possible approaches. One is to simplify the constitutive equation by restricting the types of stress or strain states or restricted loading conditions. The other approach is simplification of the time-dependent material functions.

4. Non-linear viscoelastic behaviour of cable under uniaxial generally variable stress history

The strain of a cable of hereditary type can be directly expressed by the functional (Onaran and Findley 1965, Sobotka 1984)

$$\varepsilon(t) = \tilde{G}\left[\underset{\tau = t_0}{\overset{t}{\sigma(\tau)}}\right] \tag{1}$$

According to the Fréchet theorem which states that, subject to certain continuity requirements, any functional may be expressed as a series of integrals, we can write strain of a cable $\varepsilon(t)$ at the studied time $t(t_0 \le \tau < t)$ under arbitrarily varying stress history in the form

$$\begin{aligned}
\varepsilon(t) \; = \; & \int_{t_0}^{t} F_1(t-\tau_1)\frac{\partial\sigma(\tau_1)}{\partial\tau_1}d\tau_1 + \\
& + \int_{t_0}^{t}\int_{t_0}^{t} F_2(t-\tau_1, t-\tau_2)\frac{\partial\sigma(\tau_1)}{\partial\tau_1}\frac{\partial\sigma(\tau_2)}{\partial\tau_2}d\tau_1 d\tau_2 + \\
& + \int_{t_0}^{t}\int_{t_0}^{t}\int_{t_0}^{t} F_3(t-\tau_1, t-\tau_2, t-\tau_3)\frac{\partial\sigma(\tau_1)}{\partial\tau_1}\frac{\partial\sigma(\tau_2)}{\partial\tau_2}\frac{\partial\sigma(\tau_3)}{\partial\tau_3}d\tau_1 d\tau_2 d\tau_3 + \ldots + \\
& + \int_{t_0}^{t}\ldots\int_{t_0}^{t} F_n(t-\tau_1, t-\tau_2, \ldots, t-\tau_n)\frac{\partial\sigma(\tau_1)}{\partial\tau_1}\frac{\partial\sigma(\tau_2)}{\partial\tau_2}\ldots\frac{\partial\sigma(\tau_n)}{\partial\tau_n}d\tau_1 d\tau_2 \ldots d\tau_n
\end{aligned} \tag{2}$$

In case, that a general stress history is divided into the subintervals with N constant levels of stresses, which have increasing tendency, Eq. (2) can be modified as follows

$$\begin{aligned}
\varepsilon(t) \; = \; & \sum_{i=0}^{N} F_1(t-t_i)\Delta\sigma_i + \sum_{i=0}^{N}\sum_{j=0}^{N} F_2(t-t_i, t-t_j)\Delta\sigma_i\Delta\sigma_j + \\
& + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N} F_3(t-t_i, t-t_j, t-t_k)\Delta\sigma_i\Delta\sigma_j\Delta\sigma_k + \\
& + \ldots + \sum_{i=0}^{N}\ldots\sum_{n=0}^{N} F_n(t-t_i, t-t_j, \ldots, t-t_n)\Delta\sigma_i\Delta\sigma_j\ldots\Delta\sigma_n
\end{aligned} \tag{3}$$

The kernel functions $F_1, F_2, F_3, \ldots, F_n$ in Eqs. (2-3) are decreasing functions of their arguments $t-t_i, t-t_j, \ldots, t-t_n$ which tend to zero as the arguments tend to infinity. The influence of effects decreases with time and this fact represents the fading memory of the cable material.

Properties of the steel cables depend on many factors, especially on the wires material and structure – geometry of the cable (on the number of layers of wires in a strand, on the number of strands in a cable, on number and shape of wires in a layer, on the height of wires stranding in a strand and that of strands in a cable), and further on the magnitude of loading and the number of loading cycles, treatment and lubrication etc. Further the properties of fibre ropes partly resulting from the organic polymeric materials used to make them and partly from friction between fibres and the compacting of the structure.

The creep strain of the steel cable consists of two kinds of the creep strains. One is caused by the tension load acting on a wire. The other is an extension due to the construction of the cable when a clearance exists between wire layers of the cable, because all wire layers of the cable do not contact each other perfectly. The greater the number of layers a cable is, the greater the creep strain and the stress relaxation values become, because the total clearances between wire layers grow larger. If a tensile load acts on the cable, the cable is hardened – tightened and the creep strain increments gradually decrease with a time (fading memory). It is evident that the creep strain of a spiral cable is several times larger than that of a single wire, because the creep strain due to the construction of a cable increases with an increase in the number of layers. The decrease of the creep rate with time depends on the decrease in the creep strain due to the construction of the cable. By the use of initial pretension the creep strain and relaxation values can be significantly reduced due to a reduction of clearance.

5. Non-linear creep under three constant stress levels

If a cable structural element is exposed to the three constant uniaxial stress steps as shown in Fig. 1, Eq. (3) takes the modified form for $i = j = k = 0$, 1 and 2. Thus the constitutive equations of non-linear creep for the individual time subintervals have the forms as follows

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 \tag{4}$$

for $t_0 < t \le t_1$, (usually for t_0 valid as follows $t_0 = 0$),

$$\begin{aligned} \varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_2(t, t)\Delta\sigma_0^2 + F_3(t, t, t)\Delta\sigma_0^3 + \\ & + F_1(t - t_1)\Delta\sigma_1 + F_2(t - t_1, t - t_1)\Delta\sigma_1^2 + F_3(t - t_1, t - t_1, t - t_1)\Delta\sigma_1^3 + \\ & + 2F_2(t, t - t_1)\Delta\sigma_0\Delta\sigma_1 + 3F_3(t, t, t - t_1)\Delta\sigma_0^2\Delta\sigma_1 + 3F_3(t, t - t_1, t - t_1)\Delta\sigma_0\Delta\sigma_1^2 \end{aligned} \tag{5}$$

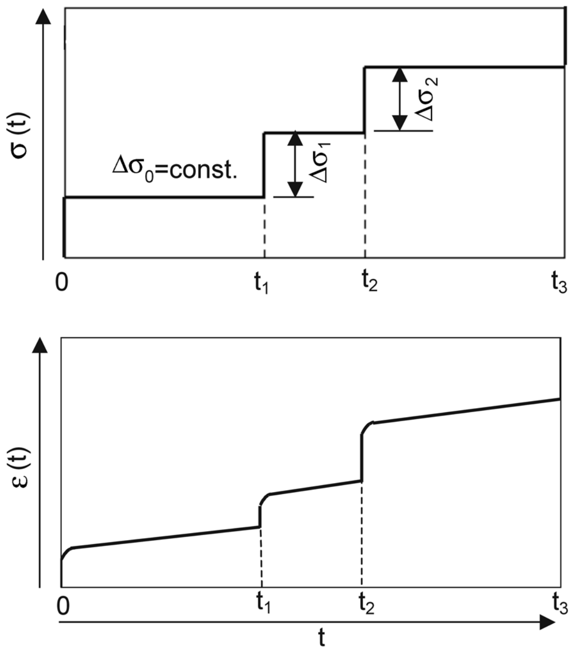

Fig. 1 Creep under variable stress history

for $t_1 < t \le t_2$,

$$\begin{aligned}\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_2(t,t)\Delta\sigma_0^2 + F_3(t,t,t)\Delta\sigma_0^3 + \\ & + F_1(t-t_1)\Delta\sigma_1 + F_2(t-t_1,t-t_1)\Delta\sigma_1^2 + F_3(t-t_1,t-t_1,t-t_1)\Delta\sigma_1^3 + \\ & + F_1(t-t_2)\Delta\sigma_2 + F_2(t-t_2,t-t_2)\Delta\sigma_2^2 + F_3(t-t_2,t-t_2,t-t_2)\Delta\sigma_2^3 + \\ & + 2F_2(t,t-t_1)\Delta\sigma_0\Delta\sigma_1 + 2F_2(t,t-t_2)\Delta\sigma_0\Delta\sigma_2 + 2F_2(t-t_1,t-t_2)\Delta\sigma_1\Delta\sigma_2 + \\ & + 3F_3(t,t,t-t_1)\Delta\sigma_0^2\Delta\sigma_1 + 3F_3(t,t-t_1,t-t_2)\Delta\sigma_0\Delta\sigma_1^2 + 3F_3(t,t,t-t_2)\Delta\sigma_0^2\Delta\sigma_2 + \\ & + 3F_3(t,t-t_2,t-t_2)\Delta\sigma_0\Delta\sigma_2^2 + 3F_3(t-t_1,t-t_1,t-t_2)\Delta\sigma_1^2\Delta\sigma_2 + \\ & + 3F_3(t-t_1,t-t_2,t-t_2)\Delta\sigma_1\Delta\sigma_2^2 + 6F_3(t,t-t_1,t-t_2)\Delta\sigma_0\Delta\sigma_1\Delta\sigma_2\end{aligned} \tag{6}$$

for $t_2 < t \le t_3$,
where $\Delta\sigma_0 = \sigma_0, \Delta\sigma_1 = \sigma_1 - \sigma_0, \Delta\sigma_2 = \sigma_2 - \sigma_1$ and $\sigma_0 < \sigma_1 < \sigma_2$.
The members $F_1(t), F_2(t), F_3(t), F_2(t,t), F_3(t,t,t), F_1(t-t_1), F_2(t-t_1,t-t_1)$,

$$\begin{gathered}F_3(t-t_1,t-t_1,t-t_1),\ F_2(t,t-t_1),\ F_3(t,t,t-t_1),\ F_3(t,t-t_1,t-t_1),\ F_1(t-t_2), \\ F_2(t-t_2,t-t_2),\ F_3(t-t_2,t-t_2,t-t_2),\ F_2(t,t-t_2),\ F_3(t,t,t-t_2),\ F_3(t,t-t_2,t-t_2), \\ F_2(t-t_1,t-t_2),\ F_3(t-t_1,t-t_1,t-t_2),\ F_3(t-t_1,t-t_2,t-t_2)\ \text{and}\ F_3(t,t-t_1,t-t_2)\end{gathered} \tag{7}$$

are the kernel functions which represent the time dependence of the cables.

The constitutive Eq. (4) in the first time interval is the creep constitutive equation in the form of polynomial of third order for an arbitrary one-step constant stress.

A more complete characterization of the kernel functions requires experimental data from multiple steps loading and the number of tests needed is in a high order. This creates a problem for both experimentation and analysis. Besides, the constitutive equations thus obtained (by means of a direct determination of the kernel functions) are quite complicated. This creates mathematical difficulty in solving boundary value problems. Therefore two types of approximation methods are applied to simplify the constitutive equations of non-linear creep under variable stress history - the product and additive forms of an approximation of the kernel functions.

Methods to be discussed in the following sections enable to derive the non-linear creep constitutive equations of cables for variable three stress levels by using the kernel functions determined only from one-step creep tests.

6. Non-linear creep constitutive equations with the product form of approximation of the kernel functions

The product form method of approximation of the kernel functions is based on an assumption that all the kernel functions which depend on the time arguments have the explicit form of the product of their time argument was originally suggested by Nakada (1960) and was further explored in the investigation of non-linear viscoelasticity of PVC by Findley and Onaran (1968). According to this assumption, the kernel functions of the higher order term can be written as follows $F_2(t,t) = [F_2(t)]^{1/2} \cdot [F_2(t)]^{1/2} = F_2(t)$,

$$F_3(t,t,t-t_2) = [F_3(t)]^{2/3} \cdot [F_3(t-t_2)]^{1/3}$$
$$F_3(t,t-t_1,t-t_2) = [F_3(t)]^{1/3} \cdot [F_3(t-t_1)]^{1/3} \cdot [F_3(t-t_2)]^{1/3} \text{ etc.}$$

where $F_2(t), F_3(t), F_3(t-t_1), F_3(t-t_2)$, …. can be determined from the one-step constant stress creep tests (Kmet and Holickova 2000).

According to an application of the product form of approximation for the kernel functions (7) new simplified forms are obtained, as follows

$$\begin{gathered}
F_1(t) = F_1(t), F_2(t) = F_2(t), F_3(t) = F_3(t), F_2(t,t) = F_2(t), F_3(t,t,t) = F_3(t), \\
F_2(t-t_1, t-t_1) = F_2(t-t_1), F_3(t-t_1, t-t_1, t-t_1) = F_3(t-t_1), F_2(t-t_2, t-t_2) = F_2(t-t_2), \\
F_3(t-t_2, t-t_2, t-t_2) = F_3(t-t_2), F_1(t-t_1) = F_1(t-t_1), F_1(t-t_2) = F_1(t-t_2), \\
F_2(t, t-t_1) = [F_2(t)]^{1/2} \cdot [F_2(t-t_1)]^{1/2}, F_2(t, t-t_2) = [F_2(t)]^{1/2} \cdot [F_2(t-t_2)]^{1/2}, \\
F_2(t-t_1, t-t_1) = [F_2(t-t_1)]^{1/2} \cdot [F_2(t-t_2)]^{1/2}, \\
F_3(t,t,t-t_2) = [F_3(t)]^{2/3} \cdot [F_3(t-t_2)]^{1/3}, F_3(t, t-t_2, t-t_2) = [F_3(t)]^{1/3} \cdot [F_3(t-t_2)]^{2/3}, \\
F_3(t-t_1, t-t_1, t-t_2) = [F_3(t-t_1)]^{2/3} \cdot [F_3(t-t_2)]^{1/3}, \\
F_3(t, t-t_1, t-t_2) = [F_3(t)]^{1/3} \cdot [F_3(t-t_1)]^{1/3} \cdot [F_3(t-t_2)]^{1/3}, \\
F_3(t,t,t-t_1) = [F_3(t)]^{2/3} \cdot [F_3(t-t_1)]^{1/3}, F_3(t, t-t_1, t-t_1) = [F_3(t)]^{1/3} \cdot [F_3(t-t_1)]^{2/3}, \\
F_3(t-t_1, t-t_2, t-t_2) = [F_3(t-t_1)]^{1/3} \cdot [F_3(t-t_2)]^{2/3}
\end{gathered} \tag{8}$$

After substituting (8) into Eqs. (4-6), the following constitutive equations are obtained

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 \tag{9}$$

for $t_0 < t \le t_1$,

$$\begin{aligned}
\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 + F_1(t-t_1)\Delta\sigma_1 + F_2(t-t_1)\Delta\sigma_1^2 + F_3(t-t_1)\Delta\sigma_1^3 + \\
& + 2[F_2(t)]^{1/2} \cdot [F_2(t-t_1)]^{1/2}\Delta\sigma_0\Delta\sigma_1 + 3[F_3(t)]^{2/3} \cdot [F_3(t-t_1)]^{1/3}\Delta\sigma_0^2\Delta\sigma_1 + \\
& + 3[F_3(t)]^{1/3} \cdot [F_3(t-t_1)]^{2/3}\Delta\sigma_0\Delta\sigma_1^2
\end{aligned} \tag{10}$$

for $t_1 < t \le t_2$,

$$\begin{aligned}
\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 + F_1(t-t_1)\Delta\sigma_1 + F_2(t-t_1)\Delta\sigma_1^2 + F_3(t-t_1)\Delta\sigma_1^3 + \\
& + F_1(t-t_2)\Delta\sigma_2 + F_2(t-t_2)\Delta\sigma_2^2 + F_3(t-t_2)\Delta\sigma_2^3 + \\
& + 2[F_2(t)]^{1/2}[F_2(t-t_1)]^{1/2}\Delta\sigma_0\Delta\sigma_1 + 2[F_2(t)]^{1/2}[F_2(t-t_2)]^{1/2}\Delta\sigma_0\Delta\sigma_2 + \\
& + 2[F_2(t-t_1)]^{1/2}[F_2(t-t_2)]^{1/2}\Delta\sigma_1\Delta\sigma_2 + 3[F_3(t)]^{2/3}[F_3(t-t_1)]^{1/3}\Delta\sigma_0^3\Delta\sigma_1 +
\end{aligned}$$

$$+ 3[F_3(t)]^{1/3}[F_3(t-t_1)]^{2/3}\Delta\sigma_0\Delta\sigma_1^2 + 3[F_3(t)]^{2/3}[F_3(t-t_2)]^{1/3}\Delta\sigma_0^2\Delta\sigma_2 +$$
$$+ 3[F_3(t)]^{1/3}[F_3(t-t_2)]^{2/3}\Delta\sigma_0\Delta\sigma_2^2 + 3[F_3(t-t_1)]^{2/3}[F_3(t-t_2)]^{1/3}\Delta\sigma_1^2\Delta\sigma_2 +$$
$$+ 3[F_3(t-t_1)]^{1/3}[F_3(t-t_2)]^{2/3}\Delta\sigma_1\Delta\sigma_2^2 +$$
$$+ 6[F_3(t)]^{1/3}[F_3(t-t_1)]^{1/3}[F_3(t-t_2)]^{1/3}\Delta\sigma_0\Delta\sigma_1\Delta\sigma_2 \tag{11}$$

for $t_2 < t \le t_3$.

After a simple algebraic arrangement and by means of an application of the binomial theorem, we obtain the final forms of the transform non-linear creep constitutive equations under three stress levels for the product form of an approximation of the kernel functions as follows

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 \tag{12}$$

for $t_0 < t \le t_1$,

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + \{[F_2(t)]^{1/2}\Delta\sigma_0 + [F_2(t-t_1)]^{1/2}\Delta\sigma_1\}^2 +$$
$$+ \{[F_3(t)]^{1/3}\Delta\sigma_0 + [F_3(t-t_1)]^{1/3}\Delta\sigma_1\}^3 \tag{13}$$

for $t_1 < t \le t_2$,

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_1(t-t_2)\Delta\sigma_2 +$$
$$+ \{[F_2(t)]^{1/2}\Delta\sigma_0 + [F_2(t-t_1)]^{1/2}\Delta\sigma_1 + [F_2(t-t_2)]^{1/2}\Delta\sigma_2\}^2$$
$$+ \{[F_3(t)]^{1/3}\Delta\sigma_0 + [F_3(t-t_1)]^{1/3}\Delta\sigma_1 + [F_3(t-t_2)]^{1/3}\Delta\sigma_2\}^3 \tag{14}$$

for $t_2 < t \le t_3$.

7. Non-linear creep constitutive equations with the additive forms of approximation of the kernel functions

The additive form approximation of kernel functions has two modifications. First modification can be obtained by means of sum of times inside of brackets of kernel functions. Gottenberg *et al.* (1969) first used this approach to describe the non-linear viscoelastic behaviour of plasticized cellulose acetate magnetic recording tape under constant strain rate inputs from single-step relaxation tests. The second modification can be obtained by means of sum of the functional values of the times arguments. Cheung (1970) first utilized this approach to describe the non-linear viscoelastic behaviour of blood vessels.

Taking into account, that in the case of the first modification of the additive forms of approximation valid, for instance $F_3(t-t_2, t-t_2, t-t_2) = F_3(t-t_2+t-t_2+t-t_2) = F_3(3t-3t_2)$ etc., the values of the kernel functions (7) obtained the forms as follows

$$F_1(t) = F_1(t),\ F_2(t) = F_2(t),\ F_3(t) = F_3(t),\ F_2(t,t) = F_2(2t),\ F_3(t,t,t) = F_3(3t),$$
$$F_2(t-t_1, t-t_1) = F_2(2t-2t_1), F_3(t-t_1, t-t_1, t-t_1) = F_3(3t-3t_1),$$
$$F_2(t-t_2, t-t_2) = F_2(2t-2t_2), F_3(t-t_2, t-t_2, t-t_2) = F_3(3t-3t_2), F_1(t-t_1) = F_1(t-t_1),$$
$$F_1(t-t_2) = F_1(t-t_2), F_2(t, t-t_1) = F_2(2t-t_1), F_2(t, t-t_2) = F_2(2t-t_2),$$
$$F_2(t-t_1, t-t_2) = F_2(2t-t_1-t_2), F_3(t,t,t-t_2) = F_3(3t-t_2), F_3(t, t-t_2, t-t_2) = F_3(3t-2t_2),$$
$$F_3(t-t_1, t-t_1, t-t_2) = F_3(3t-2t_1-t_2), F_3(t, t-t_1, t-t_2) = F_3(3t-t_1-t_2),$$
$$F_3(t,t,t-t_1) = F_3(3t-t_1), F_3(t, t-t_1, t-t_1) = F_3(3t-2t_1) \text{ and}$$
$$F_3(t-t_1, t-t_2, t-t_2) = F_3(3t-t_1-2t_2) \tag{15}$$

Taking into account, that in the case of the second modification valid, for instance $F_3(t-t_2, t-t_2, t-t_2)$ $= \frac{1}{3}F_3(t-t_2) + \frac{1}{3}F_3(t-t_2) + \frac{1}{3}F_3(t-t_2) = F_3(t-t_2)$, etc., the values of the kernel functions (7) have the following forms

$$F_1(t) = F_1(t),\ F_2(t) = F_2(t),\ F_3(t) = F_3(t),\ F_2(t,t) = F_2(t),\ F_3(t,t,t) = F_3(t),$$
$$F_2(t-t_1, t-t_1) = F_2(t-t_1), F_3(t-t_1, t-t_1, t-t_1) = F_3(t-t_1), F_2(t-t_2, t-t_2) = F_2(t-t_2)$$
$$F_3(t-t_2, t-t_2, t-t_2) = F_3(t-t_2), F_1(t-t_1) = F_1(t-t_1), F_1(t-t_2) = F_1(t-t_2),$$
$$F_2(t, t-t_1) = \frac{1}{2}F_2(t) + \frac{1}{2}F_2(t-t_1), F_2(t, t-t_2) = \frac{1}{2}F_2(t) + \frac{1}{2}F_2(t-t_2),$$
$$F_2(t-t_1, t-t_2) = \frac{1}{2}F_2(t-t_1) + \frac{1}{2}F_2(t-t_2), F_3(t,t,t-t_2) = \frac{2}{3}F_3(t) + \frac{1}{3}F_3(t-t_2),$$
$$F_3(t, t-t_2, t-t_2) = \frac{1}{3}F_3(t) + \frac{2}{3}F_3(t-t_2), F_3(t-t_1, t-t_1, t-t_2) = \frac{2}{3}F_3(t-t_1) + \frac{1}{3}F_3(t-t_2),$$
$$F_3(t, t-t_1, t-t_2) = \frac{1}{3}F_3(t) + \frac{1}{3}F_3(t-t_1) + \frac{1}{3}F_3(t-t_2), F_3(t,t,t-t_1) = \frac{2}{3}F_3(t) + \frac{1}{3}F_3(t-t_1),$$
$$F_3(t, t-t_1, t-t_1) = \frac{1}{3}F_3(t) + \frac{2}{3}F_3(t-t_1) \text{ and}$$
$$F_3(t-t_1, t-t_2, t-t_2) = \frac{1}{3}F_3(t-t_1) + \frac{2}{3}F_3(t-t_2) \tag{16}$$

After substituting (15) into Eqs. (4-6), the final forms of transform constitutive equations of non-linear creep for 1st modification of the additive forms approximation of the kernel functions are obtained as follows

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 \tag{17}$$

for $t_0 < t \le t_1$,

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_2(2t)\Delta\sigma_0^2 + F_2(2t-2t_1)\Delta\sigma_1^2 + 2F_2(2t-t_1)\Delta\sigma_0\Delta\sigma_1 + \\ + F_3(3t)\Delta\sigma_0^3 + F_3(3t-3t_1)\Delta\sigma_1^3 + 3F_3(3t-t_1)\Delta\sigma_0^2\Delta\sigma_1 + 3F_3(3t-2t_1)\Delta\sigma_0\Delta\sigma_1^2 \quad (18)$$

for $t_1 < t \le t_2$,

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_1(t-t_2)\Delta\sigma_2 + F_2(2t)\Delta\sigma_0^2 + F_2(2t-2t_1)\Delta\sigma_1^2 + \\ + F_2(2t-2t_2)\Delta\sigma_2^2 + 2F_2(2t-t_1)\Delta\sigma_0\Delta\sigma_1 + 2F_2(2t-t_2)\Delta\sigma_0\Delta\sigma_2 + \\ + 2F_2(2t-t_1-t_2)\Delta\sigma_1\Delta\sigma_2 + F_3(3t)\Delta\sigma_0^3 + F_3(3t-3t_1)\Delta\sigma_1^3 + F_3(3t-3t_2)\Delta\sigma_2^3 + \\ + 3F_3(3t-t_1)\Delta\sigma_0^2\Delta\sigma_1 + 3F_3(3t-2t_1)\Delta\sigma_0\Delta\sigma_1^2 + 3F_3(3t-t_2)\Delta\sigma_0^2\Delta\sigma_2 + \\ + 3F_3(3t-3t_2)\Delta\sigma_0\Delta\sigma_2^2 + 3F_3(3t-2t_1-t_2)\Delta\sigma_1^2\Delta\sigma_2 + 3F_3(3t-2t_2-t_1)\Delta\sigma_1\Delta\sigma_2^2 + \\ + 6F_3(3t-t_1-t_2)\Delta\sigma_0\Delta\sigma_1\Delta\sigma_2 \quad (19)$$

for $t_2 < t \le t_3$.

After substituting (16) into Eqs. (4-6) and after algebraic arrangements the transform constitutive equations of non-linear creep for 2nd modification of additive forms approximation of kernel functions are obtained in the following final forms

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 \quad (20)$$

for $t_0 < t \le t_1$,

$$\varepsilon_c(t) = F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + [F_2(t)\Delta\sigma_0 + F_2(t-t_1)\Delta\sigma_1](\Delta\sigma_0 + \Delta\sigma_1) + \\ + [F_3(t)\Delta\sigma_0 + F_3(t-t_1)\Delta\sigma_1](\Delta\sigma_0 + \Delta\sigma_1)^2 \quad (21)$$

for $t_1 < t \le t_2$,

$$\varepsilon_c(t) = F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_1(t-t_2)\Delta\sigma_2 + \\ + [F_2(t)\Delta\sigma_0 + F_2(t-t_1)\Delta\sigma_1 + F_2(t-t_2)\Delta\sigma_2](\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2) + \\ + [F_3(t)\Delta\sigma_0 + F_3(t-t_1)\Delta\sigma_1 + F_3(t-t_2)\Delta\sigma_2](\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2)^2 \quad (22)$$

for $t_2 < t \le t_3$.

8. Analysis of the constitutive equations

For a practical application of the derived constitutive equations of non-linear creep under variable stress history is necessary to determine values of the kernel function F_1, F_2 and F_3 for the required times arguments.

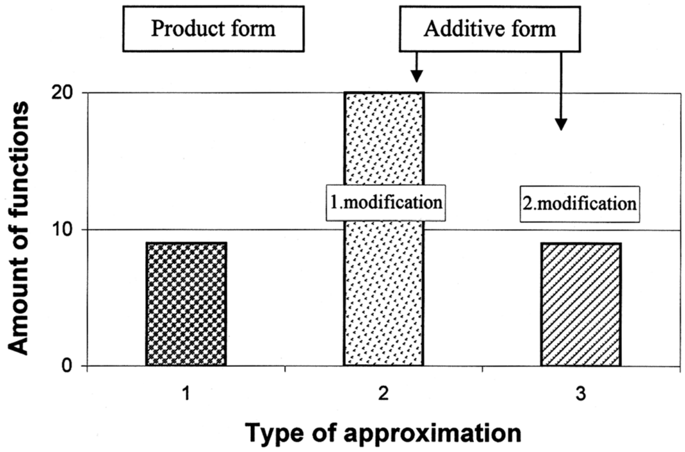

Fig. 2 Number of the necessary kernel functions according to the used method of an approximation

In case of the product form (see Eqs. (12-14)) and 2nd modification of the additive forms (see Eqs. (20-22)) of approximation 9 kernel functions are necessary against the 20 values of the kernel functions in the case of 1st modification (see Eqs. (17-19)), which is not so suitable. The first two methods are from a practical use equivalent. Comparison of the number of the necessary kernel functions according to the used method of an approximation is shown in Fig. 2.

9. Determination of the kernel functions values

Required stress interval can be covered by three various stress levels σ_A, σ_B and σ_C, for which there were obtained creep curves $\varepsilon_A(t)$, $\varepsilon_B(t)$ and $\varepsilon_C(t)$ gradually from one-step creep tests during time interval $<t_0, t_3>$, as shown in Fig. 3(a). Therefore, for known values of stresses and creep strains (taking into account the constitutive equations of creep in the form of third order polynomials) for calculation of kernel function F_1, F_2 and F_3 in the times t and $t - t_i$ for i = 1, 2 we can write gradually systems of three linear equations (Fig. 3b), as follows

$$\begin{Bmatrix} \varepsilon_A(t) \\ \varepsilon_B(t) \\ \varepsilon_C(t) \end{Bmatrix} = \begin{bmatrix} \sigma_A & \sigma_A^2 & \sigma_A^3 \\ \sigma_B & \sigma_B^2 & \sigma_B^3 \\ \sigma_C & \sigma_C^2 & \sigma_C^3 \end{bmatrix} \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix} \tag{23}$$

$$\begin{Bmatrix} \varepsilon_A(t-t_i) \\ \varepsilon_B(t-t_i) \\ \varepsilon_C(t-t_i) \end{Bmatrix} = \begin{bmatrix} \sigma_A & \sigma_A^2 & \sigma_A^3 \\ \sigma_B & \sigma_B^2 & \sigma_B^3 \\ \sigma_C & \sigma_C^2 & \sigma_C^3 \end{bmatrix} \begin{Bmatrix} F_1(t-t_i) \\ F_2(t-t_i) \\ F_3(t-t_i) \end{Bmatrix} \tag{24}$$

for i = 1, 2.

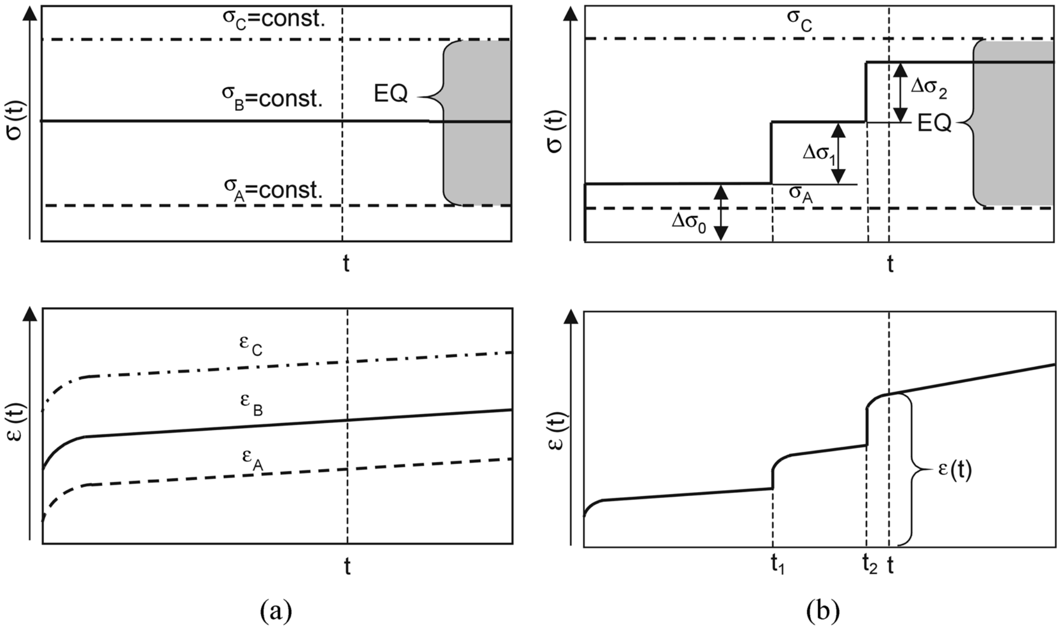

Fig. 3 Creep under single, i.e., one-step stress (a) and under varying stress history (b)

By solving the linear systems of Eqs. (23) and (24), which can be written in the following forms

$$\mathbf{F}(t) = \boldsymbol{\sigma}^{-1}\boldsymbol{\varepsilon}(t) \tag{25}$$

$$\mathbf{F}(t-t_i) = \boldsymbol{\sigma}^{-1}\boldsymbol{\varepsilon}(t-t_i), \quad \text{for} \quad i = 1, 2 \tag{26}$$

one can obtain the required values of the kernel functions.

10. Computer simulation of non-linear creep behaviour under variable stress

Computer simulation of non-linear creep behaviour history of the cables, based on the numerical modelling, becomes the most powerful tool. The creep behaviour of the cables can be represented by an arbitrarily varying stress history consisting of a k number of stepwise stress inputs.

The non-linear creep constitutive equations under the generally stepwise k stress levels and time domains for the individual types of an approximation of the kernel functions have the following forms

- product form of the approximation

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 \tag{27}$$

for $t_0 < t \le t_1$,

$$\begin{aligned}\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + \{[F_2(t)]^{1/2}\Delta\sigma_0 + [F_2(t-t_1)]^{1/2}\Delta\sigma_1\}^2 + \\ & + \{[F_3(t)]^{1/3}\Delta\sigma_0 + [F_3(t-t_1)]^{1/3}\Delta\sigma_1\}^3\end{aligned} \tag{28}$$

for $t_1 < t \le t_2$,

$$\begin{aligned}\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_1(t-t_2)\Delta\sigma_2 + \\ & + \{[F_2(t)]^{1/2}\Delta\sigma_0 + [F_2(t-t_1)]^{1/2}\Delta\sigma_1 + [F_2(t-t_2)]^{1/2}\Delta\sigma_2\}^2 \\ & + \{[F_3(t)]^{1/3}\Delta\sigma_0 + [F_3(t-t_1)]^{1/3}\Delta\sigma_1 + [F_3(t-t_2)]^{1/3}\Delta\sigma_2\}^3\end{aligned} \tag{29}$$

for $t_2 < t \le t_3$,

$$\begin{aligned}\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_1(t-t_2)\Delta\sigma_2 + \ldots + F_1(t-t_{k-1})\Delta\sigma_{k-1} + \\ & + \{[F_2(t)]^{1/2}\Delta\sigma_0 + [F_2(t-t_1)]^{1/2}\Delta\sigma_1 + [F_2(t-t_2)]^{1/2}\Delta\sigma_2 + \ldots + [F_2(t-t_{k-1})]^{1/2}\Delta\sigma_{k-1}\}^2 + \\ & + \{[F_3(t)]^{1/3}\Delta\sigma_0 + [F_3(t-t_1)]^{1/3}\Delta\sigma_1 + [F_3(t-t_2)]^{1/3}\Delta\sigma_2 + \ldots + [F_k(t-t_{k-1})]^{1/3}\Delta\sigma_{k-1}\}^3 + \\ & + \ldots + \{[F_k(t)]^{1/k}\Delta\sigma_0 + [F_k(t-t_1)]^{1/k}\Delta\sigma_1 + [F_k(t-t_2)]^{1/k}\Delta\sigma_2 + \\ & + \ldots + [F_k(t-t_{k-1})]^{1/k}\Delta\sigma_{k-1}\}^k\end{aligned} \tag{30}$$

for $t_{k-1} < t < t_k$, $\Delta\sigma_{k-1} = \sigma_{k-1} - \sigma_{k-2}$ and $k = 4, 5, 6, \ldots, n$.

- additive form of the approximation

$$\varepsilon(t) = F_1(t)\Delta\sigma_0 + F_2(t)\Delta\sigma_0^2 + F_3(t)\Delta\sigma_0^3 \tag{31}$$

for $t_0 < t \le t_1$,

$$\begin{aligned}\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + [F_2(t)\Delta\sigma_0 + F_2(t-t_1)\Delta\sigma_1](\Delta\sigma_0 + \Delta\sigma_1) + \\ & + [F_3(t)\Delta\sigma_0 + F_3(t-t_1)\Delta\sigma_1](\Delta\sigma_0 + \Delta\sigma_1)^2\end{aligned} \tag{32}$$

for $t_1 < t \le t_2$,

$$\begin{aligned}\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_1(t-t_2)\Delta\sigma_2 + \\ & + [F_2(t)\Delta\sigma_0 + F_2(t-t_1)\Delta\sigma_1 + F_2(t-t_2)\Delta\sigma_2](\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2) + \\ & + [F_3(t)\Delta\sigma_0 + F_3(t-t_1)\Delta\sigma_1 + F_3(t-t_2)\Delta\sigma_2](\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2)^2\end{aligned} \tag{33}$$

for $t_2 < t \le t_3$,

$$\begin{aligned}\varepsilon(t) = {} & F_1(t)\Delta\sigma_0 + F_1(t-t_1)\Delta\sigma_1 + F_1(t-t_2)\Delta\sigma_2 + \ldots + F_1(t-t_{k-1})\Delta\sigma_{k-1} + \\ & + [F_2(t)\Delta\sigma_0 + F_2(t-t_1)\Delta\sigma_1 + F_2(t-t_2)\Delta\sigma_2 + \ldots + F_2(t-t_{k-1})\Delta\sigma_{k-1}]. \\ & \cdot (\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2 + \ldots + \Delta\sigma_{k-1}) + \\ & + [F_3(t)\Delta\sigma_0 + F_3(t-t_1)\Delta\sigma_1 + F_3(t-t_2)\Delta\sigma_2 + \ldots + F_3(t-t_{k-1})\Delta\sigma_{k-1}]. \\ & \cdot (\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2 + \ldots + \Delta\sigma_{k-1})^2 + \\ & + \ldots + [F_k(t)\Delta\sigma_0 + F_k(t-t_1)\Delta\sigma_1 + F_k(t-t_2)\Delta\sigma_2 + \ldots + F_k(t-t_{k-1})\Delta\sigma_{k-1}]. \\ & \cdot (\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2 + \ldots + \Delta\sigma_{k-1})^{k-1}\end{aligned} \tag{34}$$

for $t_{k-1} < t < t_k$, $\Delta\sigma_{k-1} = \sigma_{k-1} - \sigma_{k-2}$ and $k = 4, 5, 6, \ldots, n$.

For the known values of the k stresses and k creep strain increments (determined only from one-step creep tests) the kernel functions $F_1, F_2, F_3, \ldots, F_k$ in the times $t, t-t_1, t-t_2, \ldots, t-t_{k-1}$ can be obtained from the systems of linear equations as follows

$$\boldsymbol{\varepsilon}(t) = \boldsymbol{\sigma}\mathbf{F}(t) \tag{35}$$

$$\boldsymbol{\varepsilon}(t-t_1) = \boldsymbol{\sigma}\mathbf{F}(t-t_1) \tag{36}$$

$$\boldsymbol{\varepsilon}(t-t_2) = \boldsymbol{\sigma}\mathbf{F}(t-t_2) \tag{37}$$

$$\boldsymbol{\varepsilon}(t-t_{k-1}) = \boldsymbol{\sigma}\mathbf{F}(t-t_{k-1}) \tag{38}$$

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where

$$\boldsymbol{\varepsilon}(t) = \{\varepsilon_1(t), \varepsilon_2(t), \ldots\}^T, \boldsymbol{\varepsilon}(t-t_1) = \{\varepsilon_1(t-t_1), \varepsilon_2(t-t_1), \ldots\}^T,$$

$$\boldsymbol{\varepsilon}(t-t_2) = \{\varepsilon_1(t-t_2), \varepsilon_2(t-t_2), \ldots\}^T, \boldsymbol{\varepsilon}(t-t_{k-1}) = \{\varepsilon_1(t-t_{k-1}), \varepsilon_2(t-t_{k-1}), \ldots\}^T,$$

$$\mathbf{F}(t) = \{F_1(t), F_2(t), \ldots\}^T, \mathbf{F}(t-t_1) = \{F_1(t-t_1), F_2(t-t_1), \ldots\}^T,$$

$$\mathbf{F}(t-t_2) = \{F_1(t-t_2), F_2(t-t_2), \ldots\}^T \text{ and } \mathbf{F}(t-t_{k-1}) = \{F_1(t-t_{k-1}), F_2(t-t_{k-1}), \ldots\}^T$$

and

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & \sigma_1^2 & \sigma_1^3 & \sigma_1^k & . \\ \sigma_2 & \sigma_2^2 & \sigma_2^3 & \sigma_2^k & . \\ \sigma_3 & \sigma_3^2 & \sigma_3^3 & \sigma_3^k & . \\ \sigma_k & \sigma_k^2 & \sigma_k^3 & \sigma_k^k & . \\ . & . & . & . & . \end{bmatrix},$$

for $k = 4, 5, 6, \ldots, n$.

These Eqs. (35-38) are solved for the determination of kernel functions as follows

$$\mathbf{F}(t) = \boldsymbol{\sigma}^{-1}\boldsymbol{\varepsilon}(t) \tag{39}$$

$$\mathbf{F}(t-t_1) = \boldsymbol{\sigma}^{-1}\boldsymbol{\varepsilon}(t-t_1) \tag{40}$$

$$\mathbf{F}(t-t_2) = \boldsymbol{\sigma}^{-1}\boldsymbol{\varepsilon}(t-t_2) \tag{41}$$

$$\mathbf{F}(t-t_{k-1}) = \boldsymbol{\sigma}^{-1}\boldsymbol{\varepsilon}(t-t_{k-1}) \tag{42}$$

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for $k = 4, 5, 6, \ldots, n$.

The created rheologic model has been implemented into the computer software that can be used as a subroutine of the program for time-dependent structural analysis.

11. Conclusions

The macroscopic approach is adopted for the mathematical-physical representation of the non-linear creep and relaxation of cables. Moreover, the overall rheological characteristics of creep or relaxation of cables under varying loading history (variable stress or strain history) used here (macroscopic approach) are much simpler when solving analytical model with detailed investigation of the influence of internal cable structure (microscopic approach).

The general constitutive equations of non-linear creep and/or relaxation of cables under one-step and the variable stress or strain inputs using the product and two types of additive approximations of the kernel functions have been presented. The derived non-linear constitutive equations describe the non-linear rheologic behaviour of the cables for variable stress or strain history by using the kernel functions determined only from one-step – constant creep or relaxation tests. The constitutive equations developed have been designed to simulate and predict in a general way non-linear rheologic behaviour of the prestressed cables under an arbitrary loading or straining history. Any stress-time or strain-time history may be approximated by the sum of a series of the step functions that correspond to a series of step – like increments in load. By using the approximation forms for a characterization of the kernel functions in the constitutive equations a significant simplification in view of an amount of the required experimental data has been achieved.

Successful application of the derived non-linear constitutive equations for steel wire or synthetic fibre cables depends on performing a proper set of one-step creep or relaxation tests, which can be used as inputs to constitutive equations to reflect the material properties. The time-dependent values of the kernel functions and the concrete forms of the constitutive equations can be obtained and used to predict rheologic behaviour under other variable stress or strain histories. The derived constitutive equations can be used for the various non-linear rheologic materials under uniaxial variable stressing or straining.

Acknowledgements

This work is a part of the research project No. 1/0357/03, partially founded by the Scientific Grant Agency of the Ministry of Education of Slovak Republic and the Slovak Academy of Sciences.

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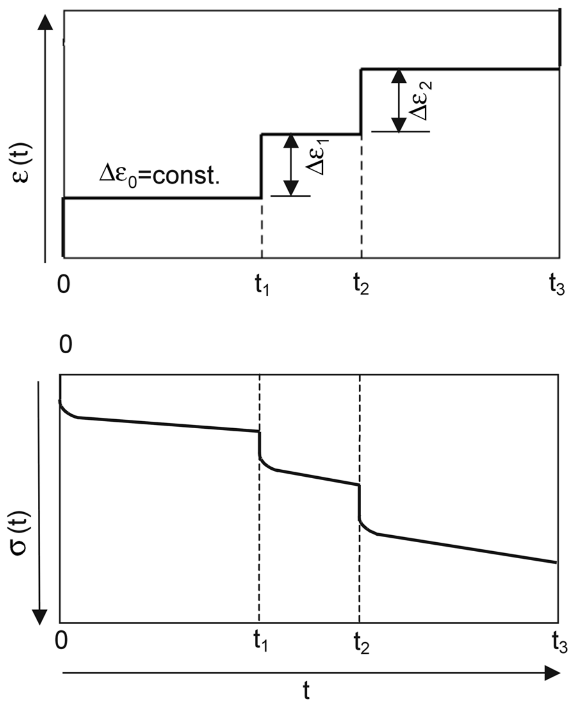

Fig. 4 Stress relaxation under variable strain history

Appendix A: Non-linear stress relaxation of tension element under variable strain

By the analogous manner the constitutive equations of non-linear relaxation of cable under constant and variable strain can be derived (Fig. 4). Since creep and stress relaxation phenomena are two aspects of the same rheologic behaviour of cables, one should be predictable if the other is known. There are presented here only the final forms of the non-linear stress relaxation constitutive equations under the generally variable of *k* train inputs for the product and additive type of the approximations of the kernel functions, as follows:

- a product form of the approximation

$$\sigma(t) = G_1(t)\Delta\varepsilon_0 + G_2(t)\Delta\varepsilon_0^2 + G_3(t)\Delta\varepsilon_0^3 \tag{A.1}$$

for $t_0 < t \leq t_1$,

$$\begin{aligned}\sigma(t) = {} & G_1(t)\Delta\varepsilon_0 + G_1(t-t_1)\Delta\varepsilon_1 + \{[G_2(t)]^{1/2}\Delta\varepsilon_0 + [G_2(t-t_1)]^{1/2}\Delta\varepsilon_1\}^2 + \\ & + \{[G_3(t)]^{1/3}\Delta\varepsilon_0 + [G_3(t-t_1)]^{1/3}\Delta\varepsilon_1\}^3\end{aligned} \tag{A.2}$$

for $t_1 < t \leq t_2$,

$$\begin{aligned}\sigma(t) = {} & G_1(t)\Delta\varepsilon_0 + G_1(t-t_1)\Delta\varepsilon_1 + G_1(t-t_2)\Delta\varepsilon_2 + \\ & + \{[G_2(t)]^{1/2}\Delta\varepsilon_0 + [G_2(t-t_1)]^{1/2}\Delta\varepsilon_1 + [G_2(t-t_2)]^{1/2}\Delta\varepsilon_2\}^2 \\ & + \{[G_3(t)]^{1/3}\Delta\varepsilon_0 + [G_3(t-t_1)]^{1/3}\Delta\varepsilon_1 + [G_3(t-t_2)]^{1/3}\Delta\varepsilon_2\}^3\end{aligned} \tag{A.3}$$

for $t_2 < t \le t_3$.

$$\begin{aligned}\sigma(t) \;=\; & G_1(t)\Delta\varepsilon_0 + G_1(t-t_1)\Delta\varepsilon_1 + G_1(t-t_2)\Delta\varepsilon_2 + \ldots + G_1(t-t_{k-1})\Delta\varepsilon_{k-1} + \\ & + \{[G_2(t)]^{1/2}\Delta\varepsilon_0 + [G_2(t-t_1)]^{1/2}\Delta\varepsilon_1 + [G_2(t-t_2)]^{1/2}\Delta\varepsilon_2 + \ldots + [G_2(t-t_{k-1})]^{1/2}\Delta\varepsilon_{k-1}\}^2 + \\ & + \{[G_3(t)]^{1/3}\Delta\varepsilon_0 + [G_3(t-t_1)]^{1/3}\Delta\varepsilon_1 + [G_3(t-t_2)]^{1/3}\Delta\varepsilon_2 + \ldots + [G_k(t-t_{k-1})]^{1/3}\Delta\varepsilon_{k-1}\}^3 + \\ & + \ldots + \{[G_k(t)]^{1/k}\Delta\varepsilon_0 + [G_k(t-t_1)]^{1/k}\Delta\varepsilon_1 + [G_k(t-t_2)]^{1/k}\Delta\varepsilon_2 + \\ & + \ldots + [G_k(t-t_{k-1})]^{1/k}\Delta\varepsilon_{k-1}\}^k \end{aligned} \tag{A.4}$$

for $t_{k-1} < t \le t_k$ and $k = 4, 5, 6, \ldots, n$. For the deformations, valid as follows $\Delta\varepsilon_0 = \varepsilon_0$, and $\Delta\varepsilon_1 = \varepsilon_1 - \varepsilon_0$, $\Delta\varepsilon_2 = \varepsilon_2 - \varepsilon_1$ and $\Delta\varepsilon_{k-1} = \varepsilon_{k-1} - \varepsilon_{k-2}$.

- additive form of the approximation

$$\sigma(t) = G_1(t)\Delta\varepsilon_0 + G_2(t)\Delta\varepsilon_0^2 + G_3(t)\Delta\varepsilon_0^3 \tag{A.5}$$

for $0 < t \le t_1$,

$$\begin{aligned}\sigma(t) \;=\; & G_1(t)\Delta\varepsilon_0 + G_1(t-t_1)\Delta\varepsilon_1 + [G_2(t)\Delta\varepsilon_0 + G_2(t-t_1)\Delta\varepsilon_1](\Delta\varepsilon_0 + \Delta\varepsilon_1) + \\ & + [G_3(t)\Delta\varepsilon_0 + G_3(t-t_1)\Delta\varepsilon_1](\Delta\varepsilon_0 + \Delta\varepsilon_1)^2 \end{aligned} \tag{A.6}$$

for $t_1 < t \le t_2$,

$$\begin{aligned}\sigma(t) \;=\; & G_1(t)\Delta\varepsilon_0 + G_1(t-t_1)\Delta\varepsilon_1 + G_1(t-t_2)\Delta\varepsilon_2 + [G_2(t)\Delta\varepsilon_0 + G_2(t-t_1)\Delta\varepsilon_1 + \\ & + G_2(t-t_2)\Delta\varepsilon_2](\Delta\varepsilon_0 + \Delta\varepsilon_1 + \Delta\varepsilon_2) + [G_3(t)\Delta\varepsilon_0 + G_3(t-t_1)\Delta\varepsilon_1 + \\ & + G_3(t-t_2)\Delta\varepsilon_2](\Delta\varepsilon_0 + \Delta\varepsilon_1 + \Delta\varepsilon_2)^2 \end{aligned} \tag{A.7}$$

for $t_2 < t \le t_3$,

$$\begin{aligned}\sigma(t) \;=\; & G_1(t)\varepsilon\sigma_0 + G_1(t-t_1)\Delta\varepsilon_1 + G_1(t-t_2)\Delta\varepsilon_2 + \ldots + G_1(t-t_{k-1})\Delta\varepsilon_{k-1} + \\ & + [G_2(t)\Delta\varepsilon_0 + G_2(t-t_1)\Delta\varepsilon_1 + G_2(t-t_2)\Delta\varepsilon_2 + \ldots + G_2(t-t_{k-1})\Delta\varepsilon_{k-1}] \cdot \\ & \cdot (\Delta\varepsilon_0 + \Delta\varepsilon_1 + \Delta\varepsilon_2 + \ldots + \Delta\varepsilon_{k-1}) + \\ & + [G_3(t)\Delta\varepsilon_0 + G_3(t-t_1)\Delta\varepsilon_1 + G_3(t-t_2)\Delta\varepsilon_2 + \ldots + G_3(t-t_{k-1})\Delta\varepsilon_{k-1}] \cdot \\ & \cdot (\Delta\varepsilon_0 + \Delta\varepsilon_1 + \Delta\varepsilon_2 + \ldots + \Delta\varepsilon_{k-1})^2 + \\ & + \ldots + [G_k(t)\Delta\varepsilon_0 + G_k(t-t_1)\Delta\varepsilon_1 + G_k(t-t_2)\Delta\varepsilon_2 + \ldots + G_k(t-t_{k-1})\Delta\varepsilon_{k-1}] \cdot \\ & \cdot (\Delta\varepsilon_0 + \Delta\varepsilon_1 + \Delta\varepsilon_2 + \ldots + \Delta\varepsilon_{k-1})^{k-1} \end{aligned} \tag{A.8}$$

for $t_{k-1} < t < t_k$, $\Delta\varepsilon_{k-1} = \varepsilon_{k-1} - \varepsilon_{k-2}$ and $k = 4, 5, 6, \ldots, n$.

The values of the kernel functions can be determined from the following equations (analogously with Eqs. (40-43))

$$\boldsymbol{\sigma}(t) = \boldsymbol{\varepsilon}\, \mathbf{G}(t) \tag{A.9}$$

$$\boldsymbol{\sigma}(t-t_1) = \boldsymbol{\varepsilon}\, \mathbf{G}(t-t_1) \tag{A.10}$$

$$\boldsymbol{\sigma}(t-t_2) = \boldsymbol{\varepsilon}\, \mathbf{G}(t-t_2) \tag{A.11}$$

$$\boldsymbol{\sigma}(t - t_{k-1}) = \boldsymbol{\varepsilon}\, \mathbf{G}(t - t_{k-1}) \tag{A.12}$$

.
.
.

where

$$\boldsymbol{\sigma}(t) = \{\sigma_1(t), \sigma_2(t), \ldots\}^T, \boldsymbol{\sigma}(t-t_1) = \{\sigma_1(t-t_1), \sigma_2(t-t_1), \ldots\}^T,$$

$$\boldsymbol{\sigma}(t-t_2) = \{\sigma_1(t-t_2), \sigma_2(t-t_2), \ldots\}^T, \boldsymbol{\sigma}(t-t_{k-1}) = \{\sigma_1(t-t_{k-1}), \sigma_2(t-t_{k-1}), \ldots\}^T,$$

$$\mathbf{G}(t) = \{G_1(t), G_2(t), \ldots\}^T, \mathbf{G}(t-t_1) = \{G_1(t-t_1), G_2(t-t_1), \ldots\}^T,$$

$$\mathbf{G}(t-t_2) = \{G_1(t-t_2), G_2(t-t_2), \ldots\}^T \text{ and } \mathbf{G}(t-t_{k-1}) = \{G_1(t-t_{k-1}), G_2(t-t_{k-1}), \ldots\}^T$$

and

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & \varepsilon_1^2 & \varepsilon_1^3 & \varepsilon_1^k & . \\ \varepsilon_2 & \varepsilon_2^2 & \varepsilon_2^3 & \varepsilon_2^k & . \\ \varepsilon_3 & \varepsilon_3^2 & \varepsilon_3^3 & \varepsilon_3^k & . \\ \varepsilon_k & \varepsilon_k^2 & \varepsilon_k^3 & \varepsilon_k^k & . \\ . & . & . & . & . \end{bmatrix},$$

for $k = 4, 5, 6, \ldots, n$.
These Eqs. (A.9-A.12) may be solved for determination of the time functions as follows

$$\mathbf{G}(t) = \boldsymbol{\varepsilon}^{-1}\, \boldsymbol{\sigma}(t) \tag{A.13}$$

$$\mathbf{G}(t - t_1) = \boldsymbol{\varepsilon}^{-1}\, \boldsymbol{\sigma}(t - t_1) \tag{A.14}$$

$$\mathbf{G}(t - t_2) = \boldsymbol{\varepsilon}^{-1}\, \boldsymbol{\sigma}(t - t_2) \tag{A.15}$$

$$\mathbf{G}(t - t_{k-1}) = \boldsymbol{\varepsilon}^{-1}\, \boldsymbol{\sigma}(t - t_{k-1}) \tag{A.16}$$

.
.

for $k = 4, 5, 6, \ldots, n$.