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Scour around vertical piles due to random waves alone and random waves plus currents on mild slopes

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Abstract. This paper provides a practical stochastic method by which the maximum equilibrium scour depth around a vertical pile exposed to random waves plus a current on mild slopes can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes including the effect of breaking waves, and using the empirical formulas for the scour depth on the horizontal seabed by Sumer and Fredsøe (2002). The present approach is valid for wave-dominant flow conditions. Results for random waves alone and random wave plus currents have been presented and discussed by varying the seabed slope and water depth. An approximate method is also proposed, and comparisons are made with the present stochastic method. For random waves alone it appears that the approximate method can replace the stochastic method, whereas the stochastic method is required for random waves plus currents. Tentative approaches to related random wave-induced scour cases on mild slopes are also suggested.

Keywords: scour depth; vertical pile; mild slope; random waves; current; stochastic method

1. Introduction

The present work addresses the scour around a cylinder with a circular cross-section mounted vertically on the mild-sloped seabed due to random waves alone and random waves plus a current. Such a vertical pile may represent a part of a coastal and marine structure, e.g., piers, offshore oil platforms and foundations for offshore wind turbines. Especially for the foundation design of offshore wind turbines, it is a challenge for the engineering community. Different foundation options have been used, such as gravity base structures, monopiles, suction piles, jacket structures and multi-leg foundations (tripod/quadruped structures). It is essential to make reliable assessments of the wave and current loads on the structure, as well as the effect of scour around the foundations. For example, the base of a monopile may be affected by scour, which is due to erosion of sediments in the vicinity of the pile caused by the local increase of the wave and current-induced flow velocities due to the flow contraction around the pile. When a scour hole

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develops, this may have considerable effect on the dynamic behaviour and the stability of the structure. After installation, for example, on a plane or sloped seabed consisting of fine sand, it may experience different seabed conditions, e.g., the seabed may be flat or rippled. This is mainly due to the complicated flow generated by the interaction between the incoming flow, the pile, and the seabed. The result will depend on the incoming flow velocity, the geometry of the bed and the bed material, as well as the ratio between the near-bed oscillatory fluid particle excursion amplitude and the pile diameter. Additional details on the background and complexity as well as reviews of the problem are given in, e.g., Whitehouse (1998) and Sumer and Fredsøe (2002). Myrhaug and Ong (2011a) gave a review of the authors' studies on 2D random wave-induced equilibrium scour characteristics around marine structures including comparison with data from random wave-induced scour experiments. Recently the authors have also provided practical stochastic methods for calculating the maximum equilibrium scour depth around vertical piles (Myrhaug and Ong 2013a, Myrhaug and Ong 2013b, Ong *et al.* 2013) and below pipelines (Myrhaug and Ong 2011b) due to 2D and 3D nonlinear random waves. To our knowledge, no studies are available in the open literature dealing with random wave induced scour around a vertical pile on mild slopes.

The purpose of this study is to provide an engineering approach by which the maximum equilibrium scour depth around a vertical pile exposed to random waves alone and random waves plus a current, respectively, on mild slopes can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes including the effect of breaking waves, and using the empirical formulas for the scour depth by Sumer and Fredsøe (2002). Wave-dominant flow conditions are considered in this study. Results are presented and discussed by varying the seabed slope and water depth. An approximate method is proposed and compared with the present stochastic method. Tentative approaches to related random wave-induced scour cases on mild slopes are also suggested, such as scour around single vertical piles with square cross-sections, scour around group of vertical circular piles, and effects of sand-clay mixtures on scour around vertical piles.

2. Scour in regular waves alone and regular waves plus current

The scour around a single, slender vertical pile with a circular cross-section in regular waves was investigated in laboratory tests by Sumer *et al.* (1992b). They obtained the following empirical formula for the equilibrium scour depth S around the pile with diameter, D (see Fig. 1)

$$\frac{S}{D} = C(1 - \exp[-q(KC - r)]) \quad \text{for} \quad KC \geq r \tag{1}$$

where C, q and r are coefficients given by the following values

$$(C, q, r) = (1.3, 0.03, 6) \tag{2}$$

Here the Keulegan-Carpenter number KC is defined as

$$KC = \frac{UT}{D} \tag{3}$$

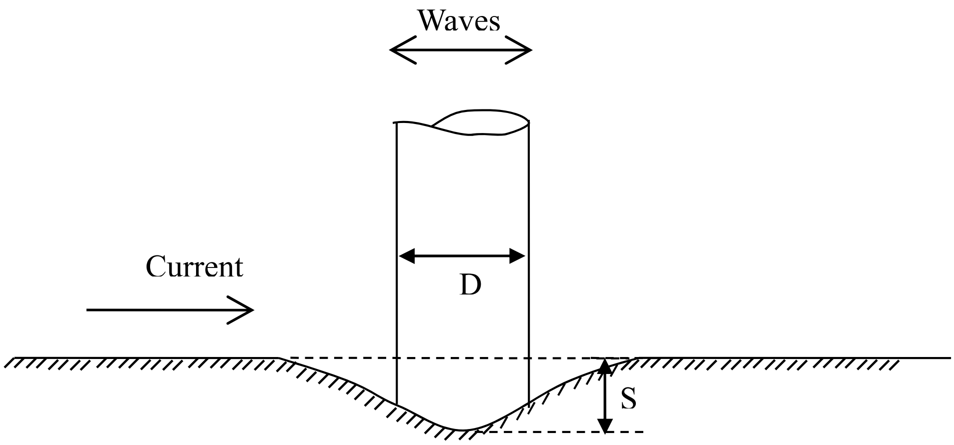

Fig. 1 Definition sketch of the scour depth (S) around a circular vertical pile with diameter (D) on a horizontal bed

where U is the undisturbed linear near-bed orbital velocity amplitude, and T is the wave period. The scour for $KC > 6$ is caused by the vortex structure around the pile, while the scour for KC well below 1 is caused by the steady streaming and the separation vortices around the pile (see Sumer and Fredsøe (2002), Fig. 6.21). The threshold of sediment motion should be exceeded for scouring to occur, which may not be the case for small values of KC.

It should be noted that C =1.3 corresponds to the current-alone case (i.e., for $T \to \infty$ and $KC \to \infty$). Eqs. (1) and (2) are valid for live-bed scour, for which $\theta > \theta_{cr}$ where θ is the undisturbed Shields parameter defined by

$$\theta = \frac{\tau_w}{\rho g(s-1)d_{50}} \tag{4}$$

where τ_w is the maximum bottom shear stress under the waves, ρ is the density of the fluid, g is the acceleration due to gravity, s is the sediment density to fluid density ratio, d_{50} is the median grain size diameter, and θ_{cr} is the critical value of the Shields parameter corresponding to the initiation of motion at the bed, i.e., $\theta_{cr} \approx 0.05$. It should be noted that this is only correct for high grain Reynolds numbers (see Soulsby (1997, Ch. 6.4) for details). One should also note that the scour process attains its equilibrium stage through a transition period. Thus, in the present study, it is assumed that the storm generating random waves has lasted sufficiently longer than the time-scale of the scour. Further details on the time-scale of the scour are given in Sumer *et al.* (1992a).

Table 1 Parameters for the four locations

	Location 1	**Location 2**	**Location 3**	**Location4**
x **(m)**	0	200	400	600
$k_p h$	0.77	0.70	0.64	0.57
KC_{rms}	12.73	13.33	13.91	14.57

The maximum bottom shear stress within a wave cycle is taken as

$$\frac{\tau_w}{\rho} = \frac{1}{2} f_w U^2 \tag{5}$$

where f_w is the friction factor, which here is taken from Myrhaug *et al*. (2001), and is valid for waves plus current for wave-dominated situations (see Myrhaug *et al*. (2001), Table 3)

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} \tag{6}$$

$$(c,d) = (18, 1) \quad \text{for} \quad 20 \le A/z_0 < 200 \tag{7}$$

$$(c,d) = (1.39, 0.52) \quad \text{for} \quad 200 \le A/z_0 < 11000 \tag{8}$$

$$(c,d) = (0.112, 0.25) \quad \text{for} \quad 11000 \le A/z_0 \tag{9}$$

where $A = U/\omega$ is the near-bed orbital displacement amplitude, $\omega = 2\pi/T$ is the angular wave frequency, and $z_0 = d_{50}/12$ is the bed roughness (see e.g., Soulsby (1997)). The advantage of using this friction factor for rough turbulent flow is that it is possible to derive the stochastic approach analytically.

It should be noted that the *KC* number can alternatively be expressed as

$$KC = \frac{2\pi A}{D} \tag{10}$$

Moreover, *A* is related to the linear wave amplitude *a* by

$$A = \frac{a}{\sinh kh} \tag{11}$$

where *h* is the water depth, and *k* is the wave number determined from the dispersion relationship $\omega^2 = gk \tanh kh$.

One should notice that Eqs. (1) and (2) are also valid for nonlinear regular waves with $q = 0.06$, but with *KC* defined according to Eq. (10) where A is the stroke of the wave motion close to the bottom (Carreiras *et al*. 2000). These results were obtained as a best fit to data for *KC* in the range 11 to 23.

Sumer and Fredsøe (2001) presented results of an experimental study on scour around a vertical circular pile subject to combined irregular waves and current with *KC* ranging from 5 to about 30, and sand with $d_{50} = 0.16$ mm. They found that their empirical formula for the equilibrium scour depth for regular waves given in Eqs. (1) and (2) can be used for irregular waves provided that the *KC* number is calculated by $KC_{rms} = U_{rms} T_p / D$. Here $U_{rms} = \sqrt{2}\sigma_u$ is the root-mean-square (*rms*) value of the near-bed wave induced velocity amplitude, and T_p is the spectral peak period. Moreover, $\sigma_u^2 = \int_0^\infty S_u(\omega) d\omega$ where $S_u(\omega)$ is the spectrum of the instantaneous near-bed wave induced velocity $u(t)$. It should be noted that U_{rms} corresponds to U_m defined by Sumer and Fredsøe (2002).

Based on the Sumer and Fredsøe (2001) data for random waves plus current, Sumer and Fredsøe (2002) found that Eq. (1) can be used for irregular waves plus current provided that *KC* is calculated by KC_{rms} and with the coefficients

$$q = 0.03 + 0.75 U_{cwrms}^{2.6} \tag{12}$$

$$r = 6 \exp(-4.7 U_{cwrms}) \tag{13}$$

where $U_{cwrms} = U_c / (U_c + U_{rms})$ and U_c is the current velocity. Now $C = 1.3$ with a standard deviation $\sigma = 0.7$ corresponds to the current alone case (i.e., for $KC_{rms} \to \infty$) according to Sumer and Fredsøe (2002). Moreover, they also stated that one can take $C = 1.3 + \sigma = 2$ or $C = 1.3 + 2\sigma = 2.7$, which gives the maximum non-dimensional scour depths of $S/D = 2$ or $S/D = 2.7$ for design purposes. In the present study, $C = 1.3$ is chosen. It is also noticed that Eqs. (12) and (13) reduce to the values of q and r, respectively, in Eq. (2) for waves alone, i.e., for $U_c = 0$.

The stochastic method proposed here for random waves plus current is valid for wave-dominated flow. Moreover, it is based on assuming that Eqs. (1), (12) and (13) are also valid for regular waves if KC_{rms} and U_{rms} are replaced by KC and U, respectively, i.e., if the coefficients q and r are given by

$$q = 0.03 + 0.75 U_{cw}^{2.6} \tag{14}$$

$$r = 6 \exp(-4.7 U_{cw}) \tag{15}$$

where U_{cwrms} in Eqs. (12) and (13) has been replaced by U_{cw}

$$U_{cw} = \frac{U_c}{U_c + U} \tag{16}$$

The dispersion relationship for regular waves plus current at an angle φ to the direction of the wave propagation is $\omega = kU_c \cos\varphi + (gk \tanh kh)^{1/2}$ (see e.g., Soulsby 1997), which determines the wave number k for given values of ω, U_c and h. However, for wave-dominated situations the effect of U_c on k is small, i.e., k is determined from $\omega^2 = gk \tanh kh$, as previously given for waves alone.

3. Scour in random waves alone and random waves plus currents on mild slopes

Here a stochastic approach will be outlined following the approach presented in Ong *et al.* (2013) and Myrhaug and Ong (2013), except for the modification performed by adopting the Battjes and Groenendijk (2000) wave height distribution. As a first approximation, it is assumed that the scour formulas for the case of a horizontal bed described in Section 2 can be applied for the case of mild slopes as well. Fig. 2 shows the definition sketch of the scour around a vertical pile on a mild slope.

As mentioned the Battjes and Groenendijk (2000) distribution includes the effect of breaking waves. However, as will be discussed in Section 4.2, the effects of wave breaking on mild slopes are of minor importance. It should be noted, however, that some studies of scour around vertical piles in breaking waves are available; Bijker and de Bruyn (1988), Carreiras *et al.* (2000) and Nielsen *et al.* (2012), where a brief summary of the other two references is given in Sumer and Fredsøe (2002). Bijker and de Bruyn (1988) performed tests on scour around a vertical pile in combined breaking waves and current. They found that combined breaking waves and current increased the scour compared with that both for current alone and for waves alone, but they did not present the results for breaking waves alone. Carreiras *et al.* (2000) did tests on scour around a vertical pile in breaking waves on a sloping bed with slope 1:20.

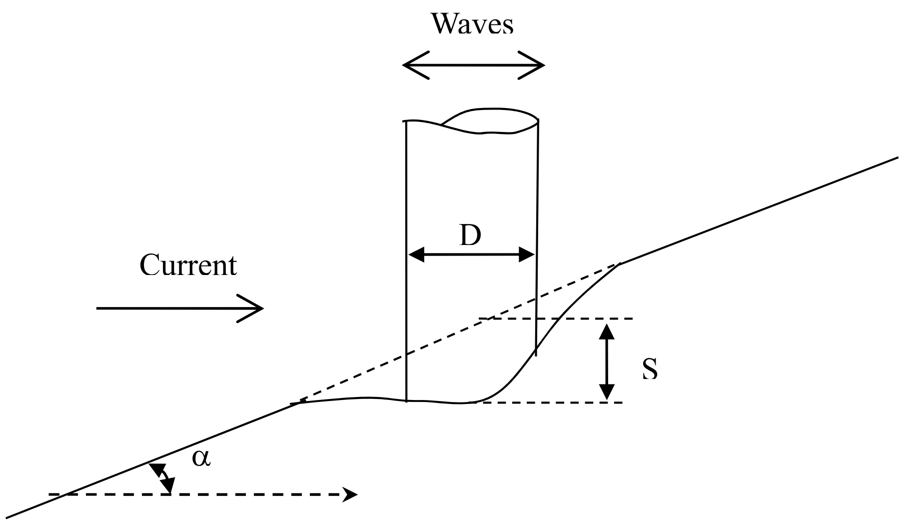

Fig. 2 Definition sketch of the scour depth (S) around a circular vertical pile with diameter (D) on a mild slope (α)

They found that when the pile in offshore relative to the breaking point, the scour depth was well represented by Eqs. (1) and (2) with q = 0.06 and KC defined as in Eq. (10). Moreover, when the pile is located at the breaking point or onshore relative to it, they found that the global large scale bed changes, i.e., the formation of the bar, are superposed to the local scour processes. However, Carreiras *et al*. (2000) did not give any procedure on how to determine the scour around piles in breaking waves. Nielsen *et al.* (2012) performed a systematic experimental study on the scour around a monopile exposed to breaking waves. The waves were breaking as plunging breakers on a flat sand section after shoaling on a mildly sloping ramp. They found that the scour was caused by turbulence generated by the breaking and was diverted towards the bed by the pile. The maximum scour depth was found to be about 0.6 times the pile diameter.

3.1 Theoretical background

At a fixed point in a sea state with stationary narrow-band random waves consistent with regular linear waves in finite water depth h and wave height H = 2a, the near-bed orbital displacement amplitude, A, and the near-bed horizontal orbital velocity amplitude, U, can be taken as, respectively

$$A = \frac{H}{2\sinh k_p h} \tag{17}$$

$$U = \omega_p A = \frac{\omega_p H}{2\sinh k_p h} \tag{18}$$

where $\omega_p = 2\pi / T_p$ is the spectral peak frequency, T_p is the spectral peak period, and k_p is the wave number corresponding to ω_p determined from the dispersion relationship

$$\omega_p^2 = g k_p \tanh k_p h \tag{19}$$

Moreover, A and U are made dimensionless by taking $\hat{A} = A / A_{rms}$ and $\hat{U} = U / U_{rms}$, respectively, where

$$A_{rms} = \frac{H_{rms}}{2\sinh k_p h} \tag{20}$$

$$U_{rms} = \omega_p A_{rms} = \frac{\omega_p H_{rms}}{2\sinh k_p h} \tag{21}$$

and H_{rms} is the rms value of H. By combining Eqs. (17), (18), (20) and (21) it follows that

$$\omega_p = \frac{U}{A} = \frac{U_{rms}}{A_{rms}} \tag{22}$$

and consequently

$$\frac{U}{U_{rms}} = \frac{A}{A_{rms}} = \frac{H}{H_{rms}} \tag{23}$$

Here the Battjes and Groenendijk (2000) parametric wave height distribution based on laboratory experiments on shallow foreshores is adopted. This cumulative distribution function (*cdf*) is composed of two two-parameter Weibull distributions of the non-dimensional wave height $\hat{H} = H / H_{rms}$

$$P(\hat{H}) = \begin{cases} P_1(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_1})^{k_1}] \; ; \; \hat{H} < \hat{H}_{tr} \\ P_2(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_2})^{k_2}] \; ; \; \hat{H} \geq \hat{H}_{tr} \end{cases} \tag{24}$$

where $k_1 = 2$, $k_2 = 3.6$, $\hat{H}_1 = H_1 / H_{rms}$, $\hat{H}_2 = H_2 / H_{rms}$, $\hat{H}_{tr} = H_{tr} / H_{rms}$. Here H_{tr} is the transitional wave height corresponding to the change of wave height where there is a change of the distribution associated with depth-induced wave breaking, given by

$$H_{tr} = (0.35 + 5.8 \tan\alpha) h \tag{25}$$

where α is the slope angle, and H_{rms} is related to the zeroth spectral moment m_0 by

$$H_{rms} = (2.69 + 3.24\sqrt{m_0} / h)\sqrt{m_0} \tag{26}$$

It should be noted that the deep water value of Eq. (26) is $H_{rms} = 2.69\sqrt{m_0}$, corresponding to a 5% reduction of the factor 2.83 obtained from the Rayleigh distribution, accounting for finite bandwidth effects obtained by Goda (1979).

The values of H_1 and H_2 can either be read from Table 2 in Battjes and Groenendijk (2000), or they can be solved by an iteration procedure solving two equations (see Eqs. (32) and (33)). The model is a so-called point model, i.e., depending on local parameters regardless of the history of the waves in deeper water. It should be noted that the effect of the bottom slope is of a secondary nature compared to the effect of water depth (see Battjes and Groenendijk (2000) for more details). Although the *cdf* in Eq. (24) is a continuous function of *H*, but with an abrupt change of its shape at $H = H_{tr}$ (i.e., the derivative and thus the *pdf* is discontinuous at this point), which is physically unrealistic, this feature is acceptable since all the integral statistical properties of the wave height are well defined. This change in the *cdf* (and *pdf*) is related to depth-induced breaking, and thus H_{tr} is expressed as the limiting wave height for non-breaking waves (i.e. defined as for purely depth-limited breaking, by excluding the steepness effect on wave breaking). Thus the effect of wave breaking is inherent in the *cdf* for *H* larger than H_{tr} (see Battjes and Groenendijk (2000) for more details).

If $\hat{H}$ is defined beyond a limited value, i.e., $\hat{H} \geq \hat{H}_t$, then $\hat{H}$ follows the truncated distribution as shown below

$$P(\hat{H}) = \begin{cases} P_1(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_1})^{k_1} + (\frac{\hat{H}_t}{\hat{H}_1})^{k_1}] \ ; \ \hat{H}_t \le \hat{H} < \hat{H}_{tr} \\ P_2(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_2})^{k_2} + (\frac{\hat{H}_t}{\hat{H}_2})^{k_2}] \ ; \ \hat{H} \ge \hat{H}_{tr} \end{cases} \tag{27}$$

The zeroth spectral moment, m_0, is obtained as

$$m_0 = \int_0^\infty S_\eta(\omega, h) d\omega \tag{28}$$

where $S_\eta(\omega, h)$ is the wave spectrum in finite water depth, which can be obtained by multiplying the deep water wave spectrum $S_\eta(\omega)$ with a depth correction factor $\psi(\omega, h)$ as

$$S_\eta(\omega, h) = \psi(\omega, h) S_\eta(\omega) \tag{29}$$

where , according to Young (1999)

$$\psi(\omega, h) = \frac{[k(\omega, h)]^{-3} \partial k(\omega, h) / \partial \omega}{\{[k(\omega, h)]^{-3} \partial k(\omega, h) / \partial \omega\}_{kh \to \infty}} \tag{30}$$

ensuring that the frequency part of the wave spectrum becomes proportional to k^{-3} irrespectively of the water depth (see Young (1999) for more details). From Eq. (30) it follows that (see Jensen, 2002)

$$\psi(\omega, h) = \frac{\omega^6}{(gk)^3 [\tanh kh + kh(1 - \tanh^2 kh)]} \tag{31}$$

For given h, α, and m_0, the values of $\hat{H}_1$ and $\hat{H}_2$ can be either read from Table 2 in Battjes and Groenendijk (2000), or they can be determined by solving the following two equations:

1) The distribution function has to be continuous, i.e.

$$P_1(\hat{H}) = P_2(\hat{H}) \tag{32}$$

2) The mean square normalized wave height, or the second moment of the probability density function (*pdf*) of $\hat{H}$, has to equal unity, i.e.

$$\hat{H}_{rms}^2 = \int_{\hat{H}_t}^{\hat{H}_{tr}} \hat{H}^2 p_1(\hat{H}) d\hat{H} + \int_{\hat{H}_{tr}}^{\infty} \hat{H}^2 p_2(\hat{H}) d\hat{H} = 1 \tag{33}$$

where p_1 and p_2 are the truncated *pdf*s of $\hat{H}$ and defined as $p_1 = dP_1 / d\hat{H}$ and $p_2 = dP_2 / d\hat{H}$, as given in Eq. (42) for random waves alone and in Eq. (43) for random waves plus currents.

3.2 Outline of stochastic method

The highest among random waves in a stationary narrow-band sea-state is considered, as it is reasonable to assume that it is mainly the highest waves which are responsible for the scour response. This is based on earlier comparisons between the stochastic method (Myrhaug and Rue 2003 and Myrhaug *et al.* 2009) and the corresponding Sumer and Fredsøe (2001) experimental data. It is also assumed that the sea-state has lasted long enough to develop the equilibrium scour depth. The highest waves considered here are those exceeding the probability $1/n$, $\hat{H}_{1/n}$ (i.e., $1-P(\hat{H}_{1/n})=1/n$). The parameter of interest is the expected (mean) value of the maximum equilibrium scour characteristics caused by the *(1/n)*th highest waves, which is given as

$$E[S(\hat{H})\,|\,\hat{H}>\hat{H}_{1/n}]=n\int_{\hat{H}_{1/n}}^{\infty}S(\hat{H})p(\hat{H})d\hat{H} \tag{34}$$

where $S(\hat{H})$ represents the scour characteristics, and $p(\hat{H})$ is the *pdf* of $\hat{H}$. More specifically, the present approach is based on the following assumptions: (1) the free surface elevation is a stationary narrow-band process with zero expectation, and (2) the scour response formulas for regular waves plus a current given in the previous section (see Eqs. (1), (3), (14) to (16)), are valid for irregular waves as well. These assumptions are essentially the same as those given in e.g., Myrhaug *et al*. (2009), where further details are found.

For a narrow-band process $T = T_p$ where $T_p = 2\pi/\omega_p = 2\pi A_{rms}/U_{rms}$ and $k = k_p$. Then by referring to Eq. (23) it follows that

$$\hat{U}=\frac{U}{U_{rms}}=\frac{A}{A_{rms}}=\frac{H}{H_{rms}}=\hat{H} \tag{35}$$

By substituting Eq. (35) in Eq. (1), and using Eqs. (3), (14), (15) and (16), Eq. (1) can be re-arranged to (i.e., taken to be valid for $0 \le U_{cwrms} \le 0.4$)

$$\hat{S}\equiv\frac{S}{D}=C\left\{1-\exp\left[-q\left(\hat{H}\right)(KC_{rms}\hat{H}-r\left(\hat{H}\right))\right]\right\}\ ;\ \hat{H}\ge\hat{H}_t=\frac{r\left(\hat{H}\right)}{KC_{rms}} \tag{36}$$

where

$$KC_{rms}=\frac{U_{rms}T_p}{D}=\frac{2\pi A_{rms}}{D} \tag{37}$$

$$U_{cw}(\hat{H})=\left(\frac{U_c}{U_{rms}}\right)/\left(\frac{U_c}{U_{rms}}+\hat{U}\right)=\left(\frac{U_c}{U_{rms}}\right)/\left(\frac{U_c}{U_{rms}}+\hat{H}\right) \tag{38}$$

$$q(\hat{H})=0.03+0.75U_{cw}^{2.6} \tag{39}$$

$$r(\hat{H})=6\exp\left(-4.7U_{cw}\right) \tag{40}$$

Then the mean of the maximum equilibrium scour depth caused by the $(1/n)^{th}$ highest waves follows from Eq. (34) as

$$E\left[\hat{S}\left(\hat{H}\right)\middle|\hat{H}>\hat{H}_{1/n}\right]=n\int_{0}^{\infty}S\left(\hat{H}\right)p\left(\hat{H}\right)H\left(\hat{H}-\hat{H}_{1/n}\right)d\hat{H} \tag{41}$$

where $H\left(\hat{H}-\hat{H}_{1/n}\right)$ is the heaviside-function defined as 1 for $\hat{H}>\hat{H}_{1/n}$ and zero elsewhere (to ensure that the integral only gets contributions for $\hat{H}>\hat{H}_{1/n}$); the *cdf* of $\hat{H}$ is given in Eq. (27). $p\left(\hat{H}\right)$ is the truncated *pdf* and determined from Eq. (24), i.e., $p_1=dP_1/d\hat{H}$ and $p_2=dP_2/d\hat{H}$. The expressions of p_1 and p_2 are given as follows:

a) Random waves alone

$$p(\hat{H})=\begin{cases}p_1(\hat{H})=\dfrac{k_1}{\hat{H}}(\dfrac{\hat{H}}{\hat{H}_1})^{k_1}\exp[-(\dfrac{\hat{H}}{\hat{H}_1})^{k_1}+(\dfrac{\hat{H}_t}{\hat{H}_1})^{k_1}]\ ;\ \hat{H}_t\le\hat{H}<\hat{H}_{tr}\\ p_2(\hat{H})=\dfrac{k_2}{\hat{H}}(\dfrac{\hat{H}}{\hat{H}_2})^{k_2}\exp[-(\dfrac{\hat{H}}{\hat{H}_2})^{k_2}+(\dfrac{\hat{H}_t}{\hat{H}_2})^{k_2}]\ ;\ \hat{H}\ge\hat{H}_{tr}\end{cases} \tag{42}$$

b) Random waves plus current

$$p(\hat{H})=\begin{cases}p_1(\hat{H})=\left(\dfrac{k_1}{\hat{H}}(\dfrac{\hat{H}}{\hat{H}_1})^{k_1}-4.7k_1(\dfrac{\hat{H}_t}{\hat{H}_1})^{k_1}\dfrac{\hat{U}_c}{\left(\hat{U}_c+\hat{H}\right)^2}\right)\exp[-(\dfrac{\hat{H}}{\hat{H}_1})^{k_1}+(\dfrac{\hat{H}_t}{\hat{H}_1})^{k_1}]\ ;\ \hat{H}_t\le\hat{H}<\hat{H}_{tr}\\ p_2(\hat{H})=\left(\dfrac{k_2}{\hat{H}}(\dfrac{\hat{H}}{\hat{H}_2})^{k_2}-4.7k_2(\dfrac{\hat{H}_t}{\hat{H}_2})^{k_2}\dfrac{\hat{U}_c}{\left(\hat{U}_c+\hat{H}\right)^2}\right)\exp[-(\dfrac{\hat{H}}{\hat{H}_2})^{k_2}+(\dfrac{\hat{H}_t}{\hat{H}_2})^{k_2}]\ ;\ \hat{H}\ge\hat{H}_{tr}\end{cases} \tag{43}$$

It should be noted that the formulation in Section 3 is general, i.e., valid for a finite water depth.

3.3 Brief summary of the authors' previous studies on random wave-induced scour characteristics around marine structures

The same stochastic method has been applied in previous studies on random wave-induced scour characteristics around other marine structures. Some of these studies also include comparison with random wave-induced scour data from laboratory experiments (Myrhaug and Rue 2003) on scour below pipelines and vertical piles in random waves; Myrhaug *et al.* (2004, 2009) on scour around breakwaters in random waves and on scour below pipelines and around vertical piles due to second-order random waves plus a current; Myrhaug and Ong (2009a) on random wave-induced scour at the trunk section of a breakwater). Overall, these results give some confidence in the present stochastic method. The method has also been applied in other cases where no data were available to compare with. These are: Myrhaug and Rue (2005) on scour around group of slender vertical piles in random waves, Myrhaug *et al.* (2007) on scour around spherical bodies in random waves, Myrhaug *et al.* (2008) on scour below pipelines in shoaling conditions for random waves, Myrhaug and Ong (2009b) on burial and scour of short cylinders under combined random waves and currents including effects of second-order wave asymmetry, and Myrhaug and Ong (2010) on random wave-induced onshore scour characteristics around submerged breakwaters.

It should also be noticed that validation with data from field observations are required. A few examples are given below. Myrhaug *et al*. (2004) included one data point for the scour depth at the round head of a rubble-mound emerged breakwater from Lillycrop and Hughes (1993). Sumer *et al.* (2005) compared their laboratory experimental results for the roundhead scour around submerged breakwaters with prototype observations. It should also be mentioned that Testik *et al*. (2007) studied scour and burial of mines (i.e., short cylinders) in the shoaling zone and compared their laboratory-based empirical regular wave-induced scour formulas with field observations.

4. Results and discussion

To the authors' knowledge no data exist in the open literature for random wave-induced scour around vertical piles on mild slopes. Hence examples of calculating the scour depth are provided.

4.1 Prediction of parameters

In the present study, the effect of mild slopes on scour around vertical piles in random waves alone and random waves plus currents is investigated. Four bed slopes 1/50, 1/100, 1/150 and 1/250, are considered for this purpose.

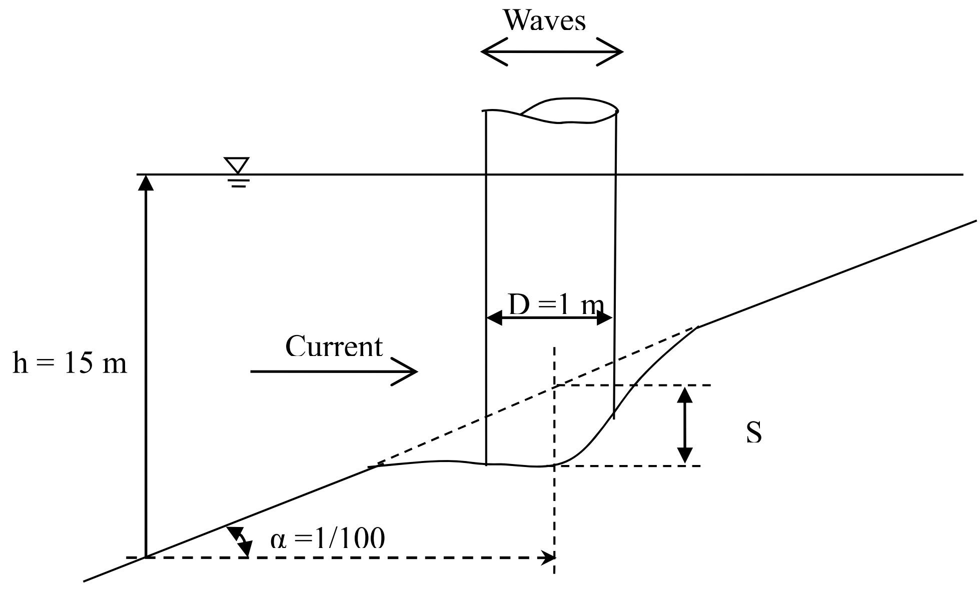

Fig. 3 Definition sketch of the seabed conditions with α =1/100

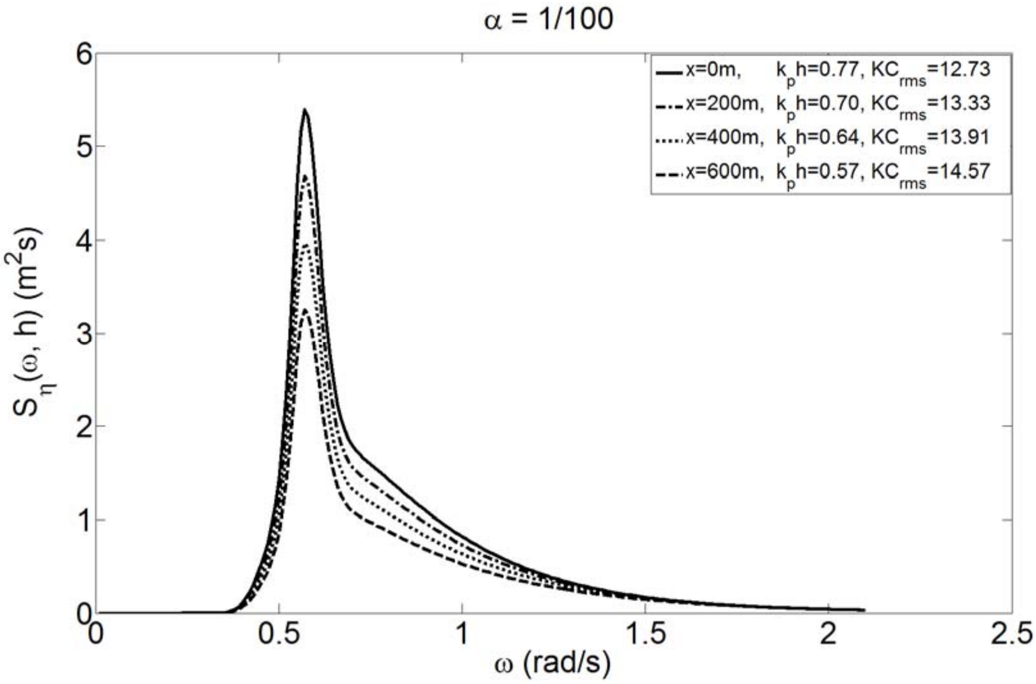

Fig. 4 The transitional wave spectra in finite water depth $S_\eta(\omega, h)$ versus ω at four locations for slope α =1/100

The case with the bed slope α =1/100 is exemplified to show the procedure of calculating all the required parameters. Fig. 3 shows the seabed conditions with α =1/100. The water depth at the seaward location (x = 0 m) is 15 m; the horizontal length of the sloping seabed is 600 m; the diameter of the circular vertical pile D is set to be 1m for all the cases.

The wave spectrum in finite water depth $S_\eta(\omega, h)$ can be obtained from the spectrum in deep water $S_\eta(\omega)$, see Eq. (29). Hence, the random waves with a standard JONSWAP spectrum (γ = 3.3) and significant wave height H_{m0} = 8m and spectral peak period T_p = 11.1s are assumed to describe the sea state in deep water. Fig. 4 shows some results of the wave spectra at four locations transformed from the deep water according to Eqs. (28)-(31). The water depth at each location, as well as the corresponding values of KC_{rms} and k_ph at each location are presented in Table 1. It is clearly seen in Fig. 4 that the wave energy decreases as the water depth decreases. Consequently, Fig. 5 shows that m_0 (Eq. (28)) decreases as the water depth decreases.

With the values of m_0 along x, H_{rms} can be determined by Eq. (26) and therefore KC_{rms} by Eqs. (20) and (37). Fig. 6 shows KC_{rms} versus x for the slope α=1/100. It appears that KC_{rms} increases from 12.73 to 14.57 as the water depth decreases (for x: 0 m→ 600 m). It needs to be noted that although H_{rms} decreases as x approaches to 600 m, k_ph decreases because of the finite water depth. These effects increase KC_{rms} along x.

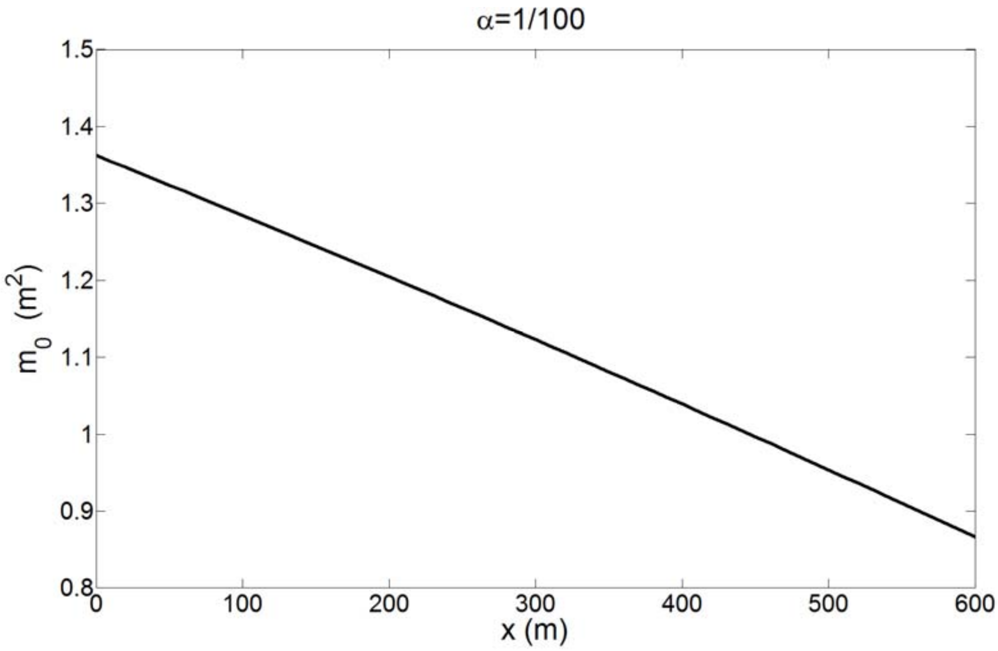

Fig. 5 Zeroth wave spectral moment m_0 versus x in finite water depth for slope α =1/100

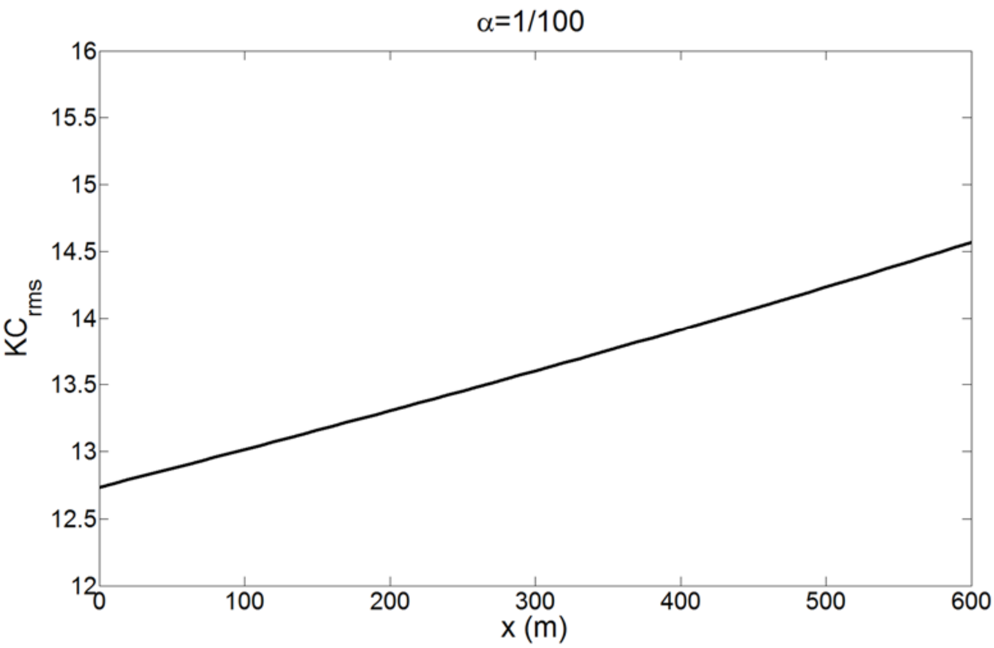

Fig. 6 KC_{rms} versus x in finite water depth for slope α =1/100

4.2 Random waves alone

The random wave-induced scour formulas, Eqs. (36)-(40), are only valid for $\hat{H} \geq \hat{H}_t$, since there is no horseshoe vortex formed for $\hat{H} < \hat{H}_t = r / KC_{rms}$ (see Sumer and Fredsøe (2002, Ch 3.1) for the detailed description of the horseshoe vortex formation). Hence, the truncated Battjes and Groenendijk (2000) wave height distribution in Eq. (27) is employed to ensure that all the waves considered here have physical meaning.

The *pdf* and *cdf* of the truncated wave height distribution of $\hat{H}$ at the four locations referred to earlier (see Table 1) are shown in Figs. 7 and 8, respectively. The truncation point here, $\hat{H}_t$, indicates the smallest wave height at which a horseshoe vortex could be formed, taking sediments away from the pile. Fig. 7 shows that $\hat{H}_t$ decreases slightly from location 1 (x = 0 m) to location 4 (x = 600 m), indicating that the threshold of scour decreases as the water depth decreases. The discontinuous points in Fig. 7 are due to the transitional wave height $\hat{H}_{tr}$, representing the limiting wave height for non-breaking waves. The figure shows that from location 1 to location 4 (as x increasing from 0m to 600m), $\hat{H}_{tr}$ decreases from 1.78 to 1.30, reflecting that the influence of breaking waves on the truncated distributions becomes more significant as the water depth decreases. It should be noted that the area under each truncated *pdf* curve must be equal to one, and this is validated in Fig. 8 where the *cdf* curves are shown.

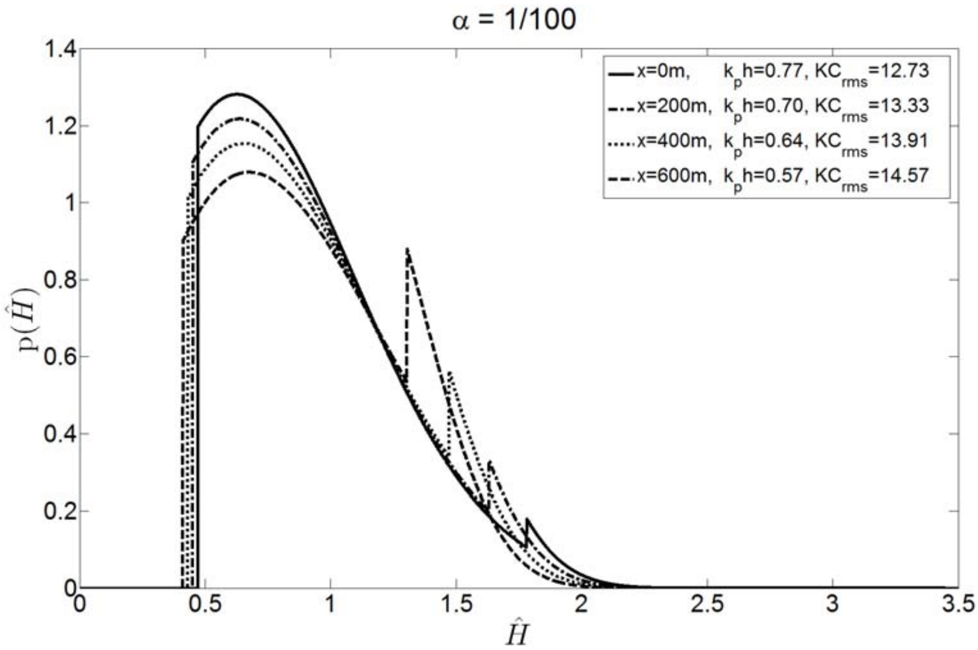

Fig. 7 Truncated *pdf* of $\hat{H}$ at four locations for slope α =1/100

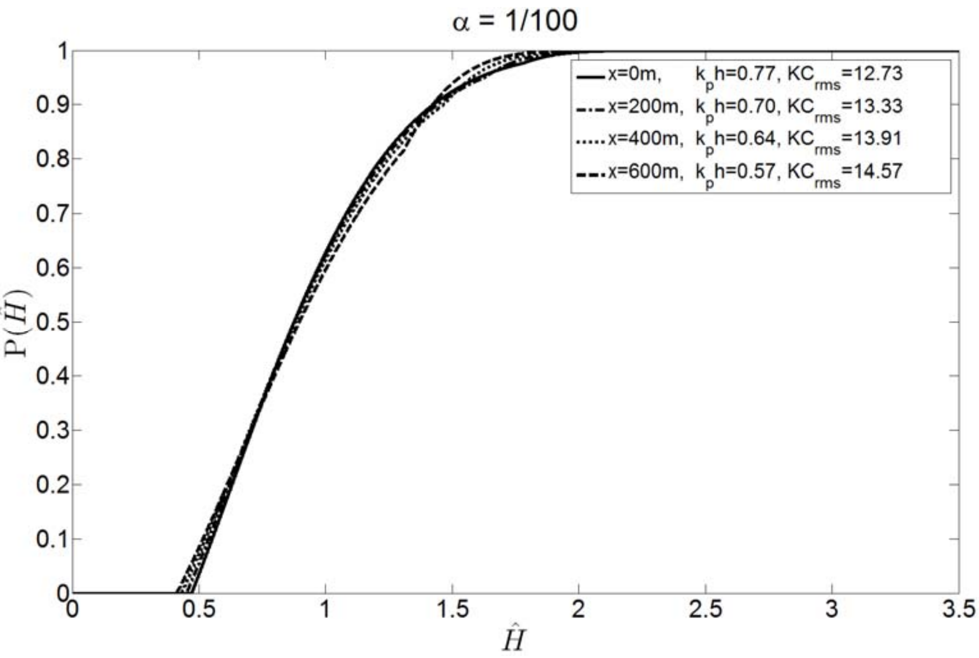

Fig. 8 Truncated *cdf* of $\hat{H}$ at four locations for slope α =1/100

Fig. 9 shows the predictions of S/D for the (1/10)th highest waves ($S/D_{1/10}$) along x for the slope α = 1/100. It appears that $S/D_{1/10}$ increases as the water depth decreases (i.e., x increases from 0m to 600 m). This effect may be attributed to the increase of KC_{rms} (Fig. 6) along the sloping bed since large KC_{rms} induces more scour.

Four different bed slopes (α = 1/50, 1/100, 1/150, 1/250) are considered in the present study. The seabed configuration is illustrated in Fig. 10. Fig. 11 shows KC_{rms} versus x for the four slopes; KC_{rms} increases as the water depth decreases for all the slopes. Futhermore, it appears that KC_{rms} increases as the slope increases at a given location x. Fig. 12 shows $S/D_{1/10}$ for the different slopes. Deeper scour hole is encountered when the slope becomes steeper. For all slopes, $S/D_{1/10}$ increases as x approaches to 600 m. At a given location x, it appears that $S/D_{1/10}$ increases as the slope increases. These results are physically sound and consistent with those observed in Fig. 11.

An assessment of breaking waves is feasible by using the surf parameter $\xi_p = (H_s/((g/2\pi)\ T_p^2))^{-1/2} \tan\alpha$ defined in terms of the significant wave height in deep water H_s = 8 m and the spectral peak period T_p = 11.1s, giving ξ_p = (0.020, 0.033, 0.049, 0.098) for the slopes (1:250, 1:150, 1:100, 1:50). For individual waves the surf parameter is defined as $\xi = (H/((g/2\pi)T^2))^{-1/2} \tan\alpha$, where H is the deep water wave height. Types of breaking waves are defined in terms of this surf parameter (see e.g., Battjes (1974)); spilling if $0 < \xi < 0.5$ and plunging if $0.5 < \xi < 3$. Thus, if breaking occurs in this example there will most likely be spilling breakers; and therefore the effect of breaking waves on scour is of minor importance.

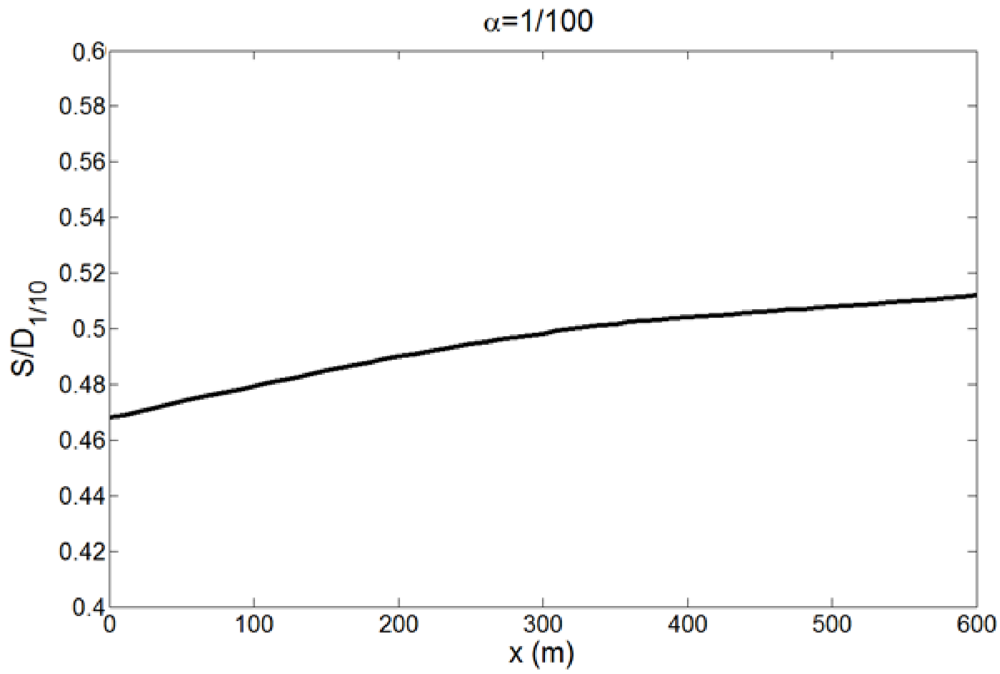

Fig. 9 $S/D_{1/10}$ versus x in finite water depth for slope α =1/100

4.3 Random waves plus currents

The effect of the current on random wave-induced scour around a vertical pile is investigated in this section. This can be achieved by changing U_c in U_{cw} (see Eq. (38)). When a current is added to the random waves, the truncation point $\hat{H}_t$ which is related to the current velocity, will change corresponding to the sea state (see Eqs. (36), (38), (40)). The wave height distribution of $\hat{H}$ will alter accordingly.

Fig. 13 shows the truncated wave height distribution versus $\hat{H}$ at location 1 (see Table 1). For U_{cwrms} = 0, the distribution is the same as that for random wave alone shown in Fig. 7. As U_{cwrms} increases, $\hat{H}_t$ decreases significantly. The value of $\hat{H}_t$ decreases to zero as U_{cwrms} increases to 0.10. This indicates that scouring will occur by adding even a small current to the existing weak waves which are unable to generate scour.

Fig. 14 shows $S/D_{1/3}$ (i.e., S/D for the (1/3)rd highest waves) and $S/D_{1/10}$ versus U_{cwrms} in combined random waves plus current for α =1/100 at the four locations along the seabed (see Table 1). It should be noted that the results for $S/D_{1/10}$ for U_{cwrms} = 0 (random waves alone) are the same as those presented in Fig. 9. The increase of $S/D_{1/10}$ (and $S/D_{1/3}$) from location 1 to location 4 for U_{cwrms} = 0 shows that the scour depth increases as the water depth decreases. It is clearly seen in Fig. 14 that the effect of the current is to increase the scour depth compared with that for random waves alone, and the effect is enhanced as U_{cwrms} increases. More specifically, for all the locations, $S/D_{1/3}$ and $S/D_{1/10}$ for U_{cwrms} = 0.4 are approximately 2.4 times and 1.9 times larger, respectively, than those for random wave alone. The reason is that by adding a current to random waves for co-directional current and waves a stronger horseshoe vortex is formed in front of the pile, causing a larger scour depth.

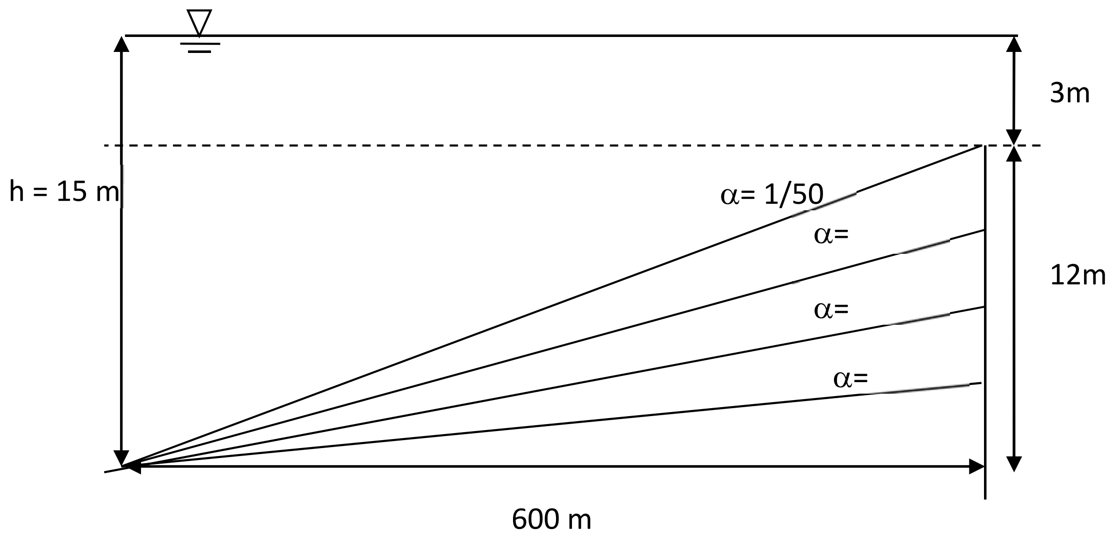

Fig. 10 Sketch of the seabed conditions for four slopes α = (1/50, 1/100, 1/150, 1/250). The total horizontal length of the sloping bed is 600 m, and the water depth at the seaward location is 15 m

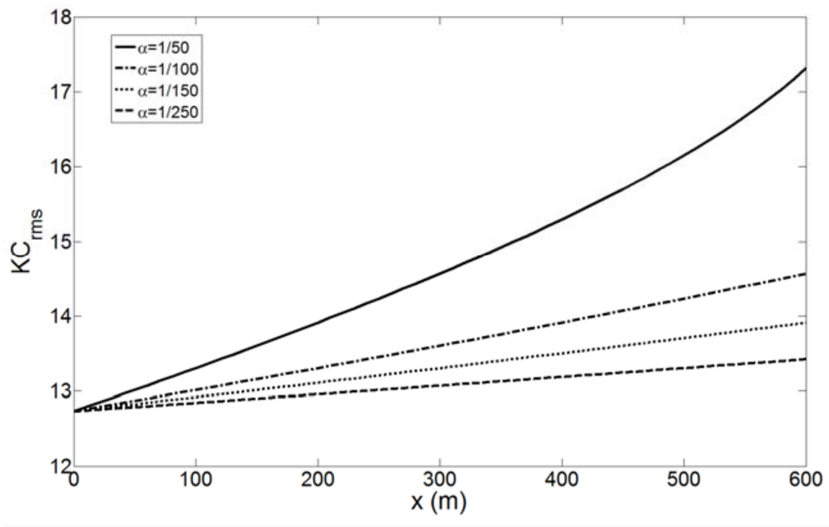

Fig. 11 KC_{rms} versus x for α = (1/50, 1/100, 1/150, 1/250)

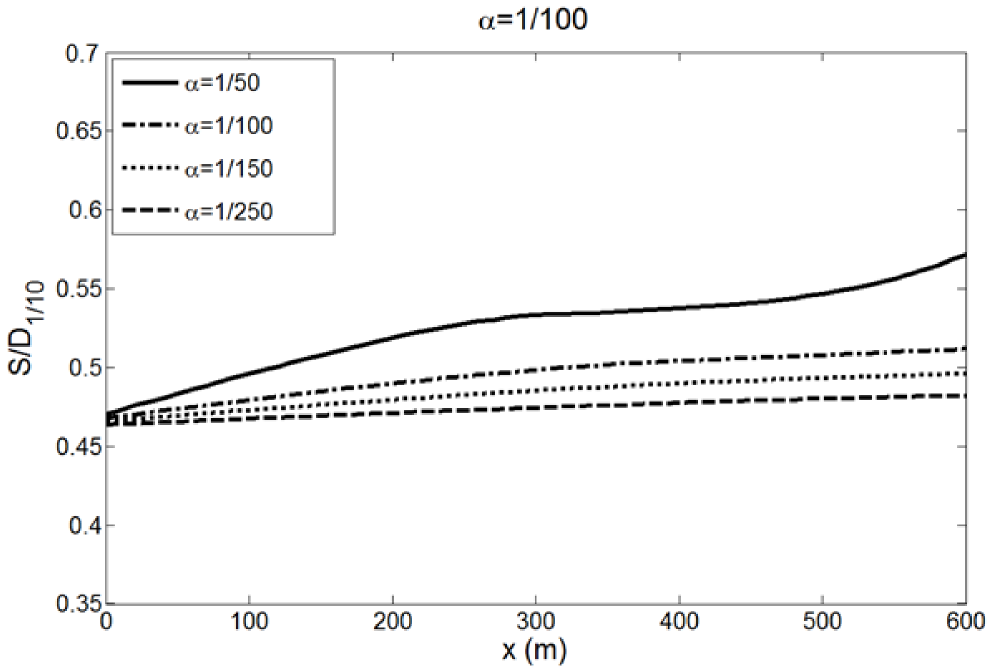

Fig. 12 $S/D_{1/10}$ versus x for α = (1/50, 1/100, 1/150, 1/250)

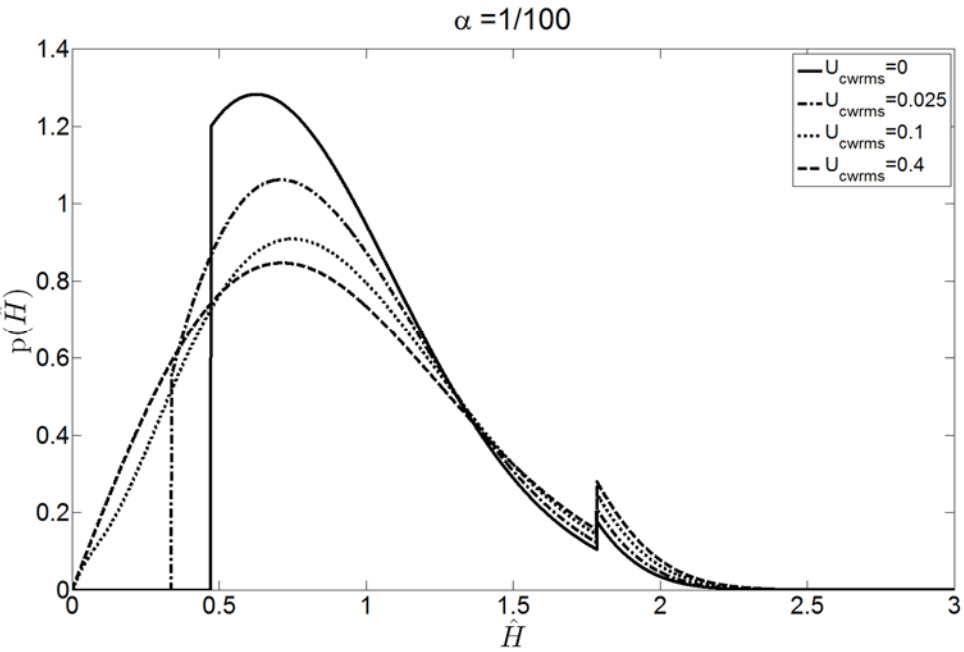

Fig. 13 Truncated *pdf* of $\hat{H}$ for four different values of U_{cwrms} at location 1 (x = 0, h = 15 m) for slope α =1/100

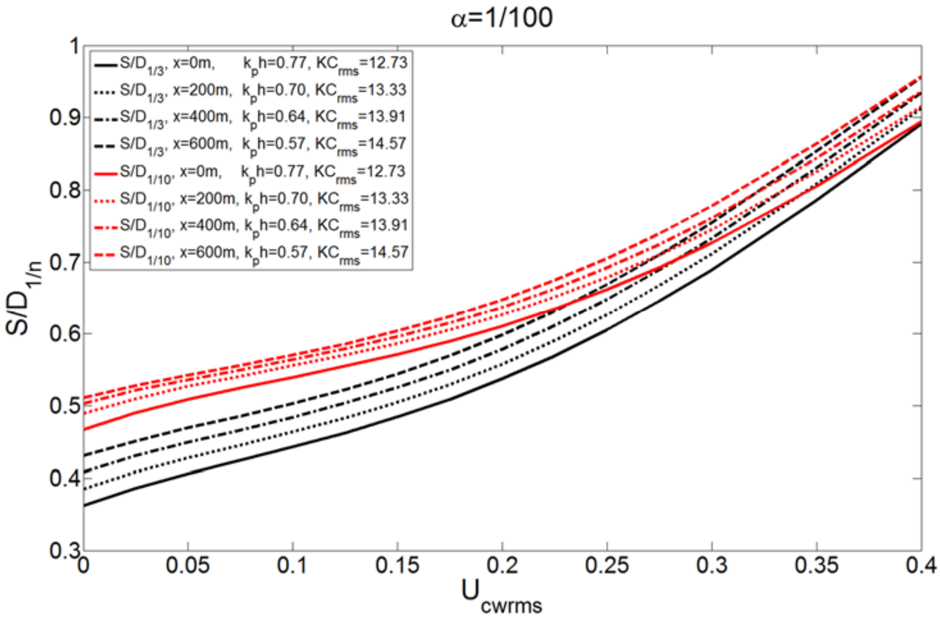

Fig. 14 $S/D_{1/3}$ and $S/D_{1/10}$ for n= (3, 10) versus U_{cwrms} at four locations for slope α =1/100

Fig. 14 also shows the comparisons between $S/D_{1/3}$ and $S/D_{1/10}$ with respect to U_{cwrms}. For a given current, the $(1/10)^{th}$ highest waves plus current cause more scour than the $(1/3)^{rd}$ highest waves plus current. However, the difference becomes smaller as U_{cwrms} increases, i.e., both $S/D_{1/3}$ and $S/D_{1/10}$ approach to each other as U_{cwrms} approaches to 0.4. The reason for this is: For small U_{cwrms}, the random waves dominate the contribution to scour; as U_{cwrms} increases, the effect of the current also contributes and becomes important, when U_{cwrms} is larger than approximately 0.4. Nevertheless, $S/D_{1/10}$ is generally larger than $S/D_{1/3}$ at a given location for $0 \le U_{cwrms} \le 0.4$. This is physically sound because the flow with a stronger wave activity should create a larger scour depth.

4.4 Alternative view of random-wave induced scour: Approximate method

An alternative pragmatic view of the scour process below pipelines and around a single vertical pile under random waves is that of Sumer and Fredsøe (1996, 2001) referred to in Section 2. They looked for which parameters of the random waves to represent the scour variable, finding by trial and error that the use of H_{rms} and T_p in an otherwise deterministic approach gave the best agreement with data.

This alternative view of the scour process will now be considered using the results of the present stochastic method. The question is how well the mean scour depth caused by the $(1/n)^{th}$ highest waves, $E\left[S(H)\,|\,H > H_{1/n}\right]$ (see Eq. (41)), can be represented by using the mean of the $(1/n)^{th}$ highest waves in the scour depth formula for regular waves, i.e., $S\left(E\left[H_{1/n}\right]\right)$.

An alternative KC number for random waves in the approximate method can be defined as

$$KC_{1/n} = \frac{E\left[U_{1/n}\right]T_p}{D} = \frac{2\pi E\left[A_{1/n}\right]}{D} \tag{44}$$

Based on the narrow-band assumption, $E\left[U_{1/n}\right]$ and $E\left[A_{1/n}\right]$ can be defined as

$$E\left[A_{1/n}\right] = \frac{E\left[H_{1/n}\right]}{2\sinh k_p h} \tag{45}$$

$$E\left[U_{1/n}\right] = \omega_p E\left[A_{1/n}\right] = \frac{\omega_p E\left[H_{1/n}\right]}{2\sinh k_p h} \tag{46}$$

where $E\left[A_{1/n}\right]$, $E\left[U_{1/n}\right]$ and $E\left[H_{1/n}\right]$ are the mean values of the $(1/n)^{\text{th}}$ largest values of the near-bed orbital displacement amplitude, velocity and wave height, respectively.

The scour depth around a vertical pile for random waves alone can be obtained by replacing *KC* with $KC_{1/n}$ in Eq. (1), given by

$$\frac{S}{D} = C(1 - \exp[-q(KC_{1/n} - r)]) \quad \text{for} \quad KC_{1/n} \geq r \tag{47}$$

with *C*, *q* and *r* as given in Eq. (2).

For random waves plus current, the approximate model can be obtained by replacing *KC* with $KC_{1/n}$ in Eqs. (36)- (40)

$$\hat{s} \equiv \frac{S}{CD} = 1 - \exp\left[-q(KC_{1/n} - r)\right] \; ; \; \hat{H} \geq \hat{H}_t = \frac{r}{KC_{1/n}} \tag{48}$$

where

$$q = 0.03 + 0.75\left(E\left[U_{cw1/n}\right]\right)^{2.6} \tag{49}$$

$$r = 6\exp\left(-4.7\, E\left[U_{cw1/n}\right]\right) \tag{50}$$

$$E\left[U_{cw1/n}\right] = \frac{U_c}{U_c + E\left[U_{1/n}\right]} \tag{51}$$

For the case of random waves alone, the results of the stochastic to approximate method ratio of the scour depth for the four slopes are shown in Fig. 15, denoted by $R_{1/10}$ for $n = 10$. The reason for choosing $n = 10$ is based on earlier comparison between the stochastic method (Myrhaug and Rue (2003) and Myrhaug *et al.* (2009)) and the corresponding experimental data for random wave-induced scour around a vertical pile reported by Sumer and Fredsøe (2001), where the stochastic method predictions for $n = 10$ overall gives upper bound values compared with the experimental data. It is interesting to note that the approximate method gives almost the same values as that of the stochastic method for all slopes. It appears that the approximate method can replace the stochastic method for random waves alone.

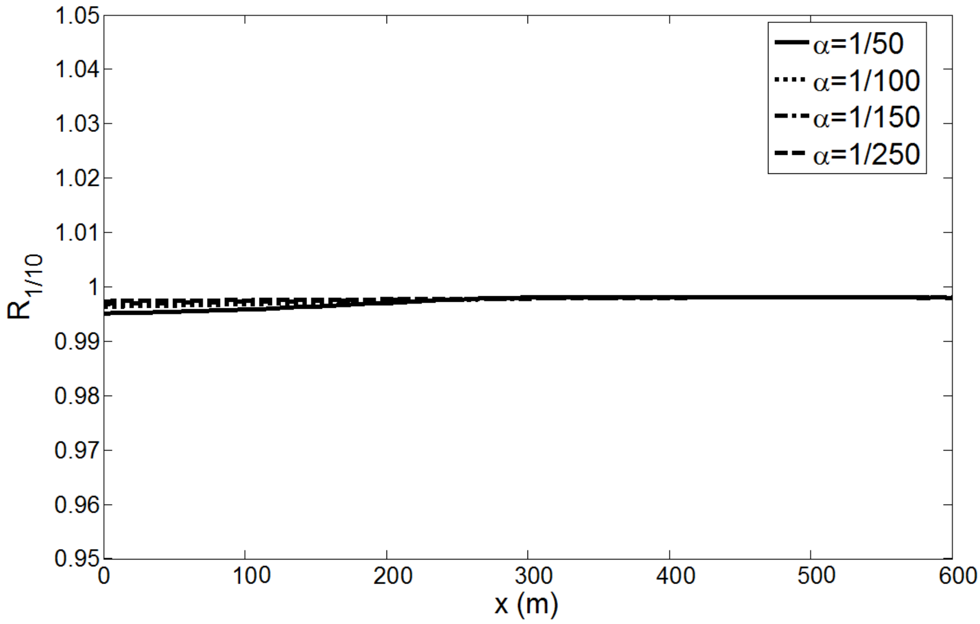

Fig. 15 Random waves alone: The stochastic to approximate method ratio $R_{1/10}$ (i.e., n = 10) versus x for four slopes α = (1/50, 1/100, 1/150, 1/250)

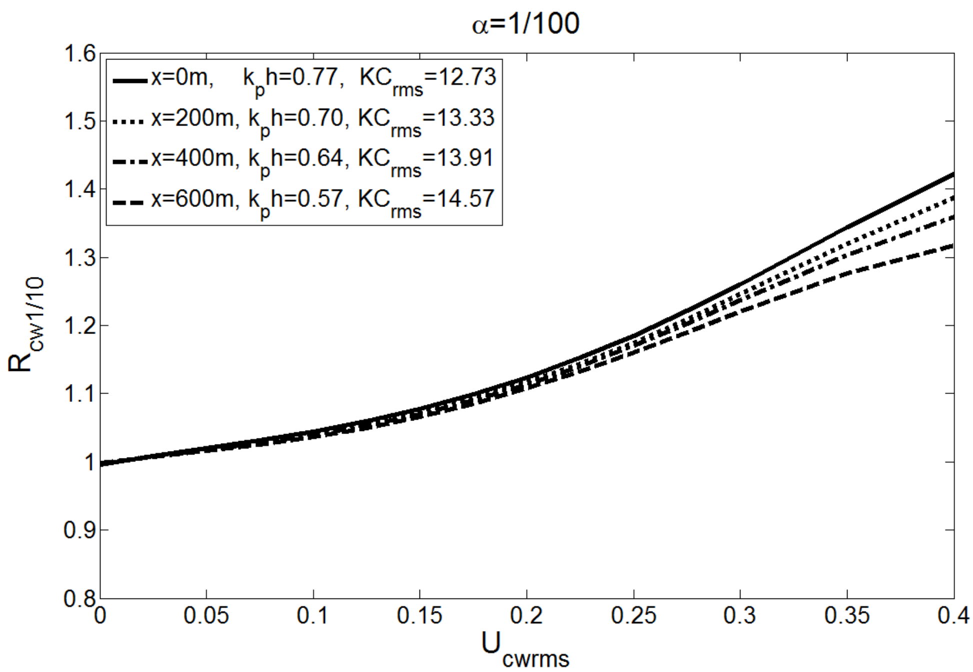

Fig. 16 Random waves plus current: The stochastic to approximate method ratio $R_{cw1/10}$ (i.e., n = 10) versus U_{cwrms} at four locations for slope α =1/100

For random waves plus current, Fig. 16 shows the result of the stochastic to approximate method ratio of the scour depth, denoted by $R_{cw\ 1/10}$ for n = 10 and α =1/100. It appears that for all locations, the values of $R_{cw\ 1/10}$ are larger than one. The difference between the stochastic method and the approximate method increases as U_{cwrms} increases, suggesting that the effect of current increases the difference between two methods. Overall, it appears that the stochastic method cannot be replaced by the approximate method for estimating the scour depth for random waves plus a current.

4.5 Shields parameter

As described in Section 2, the scour prediction model in Eq. (1) is valid for live-bed scour, for which $\theta > \theta_{cr}$, where θ is the undisturbed Shields parameter defined in Eq. (4).

When the bed is sloping, the gravity gives a force component on the grain which may increase or decrease the threshold shear stress required from the flow. The threshold Shields parameter, $\theta_{\alpha cr}$, for initiation of motion of the grains at a bed sloping at an angle α to the horizontal in upsloping flows is related to the value θ_{cr} for the same grains on a horizontal bed by (see e.g., Soulsby (1997 Section 6.4))

$$\frac{\theta_{\alpha cr}}{\theta_{cr}} = \cos\alpha\left(1 + \frac{\tan\alpha}{\tan\phi_i}\right) \tag{52}$$

where ϕ_i is the angle of repose of the sediment.

The non-dimensional maximum Shields parameter for individual linear random waves near a horizontal bed, $\hat{\theta} = \theta / \theta_{rms}$, is equal to the non-dimensional maximum bottom shear stress for individual linear random waves, $\hat{\tau} = \tau_w / \tau_{wrms}$. Here θ_{rms} is defined as

$$\theta_{rms} = \frac{\tau_{wrms} / \rho}{g(s-1)d_{50}} \tag{53}$$

where, by definition

$$\frac{\tau_{wrms}}{\rho} = \frac{1}{2}c\left(\frac{A_{rms}}{z_0}\right)^{-d} U_{rms}^2 \tag{54}$$

and θ is defined as in Eq. (4). By using this and following Myrhaug and Holmedal (2002, Eq. (21)), $\hat{\theta}$ is given as

$$\hat{\theta} = \hat{U}^{2-d} \tag{55}$$

where $\hat{U} = U/U_{rms}$, and $\hat{U} = \hat{H}$ according to Eq. (23).

For random waves it is not obvious which value of the Shields parameter to use to determine the conditions corresponding to live-bed scour. However, it seems to be consistent to use corresponding statistical values of the scour depth and the Shields parameter, e.g. given by

$$E\left[\hat{\theta}\left(\hat{U}\right)\middle|\hat{U} > \hat{U}_{1/n}\right] = n\Gamma\left(2-\frac{d}{2},\ln n\right) \tag{56}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function (see Abramowitz and Stegun (1972, Ch. 6.5, Eq. (6.5.3)). This is used in conjunction with Eq. (52) when the bed is sloping.

5. Tentative approaches to related cases for random waves alone

5.1 Scour around vertical piles with a square cross-section

Sumer *et al*. (1993) carried out laboratory tests in regular waves with slender vertical piles with square cross-sections at two orientations to the flow; 90° and 45° orientations (see Fig. 17). They found that the empirical formula for the equilibrium scour depth *S* around the pile with the cross-section *D* (see Fig. 1 for definition of *D*) is given by Eq. (1) with the following values of the coefficients *C, q* and *r*:

Square pile, 90° orientation

$$(C,q,r) = (2,0.015,11) \tag{57}$$

Square pile, 45° orientation

$$(C,q,r) = (2,0.019,3) \tag{58}$$

Thus the method is the same as for the scour depth around a circular pile; the only difference is the values of the coefficients *C, q* and *r*. No data exist for scour in random waves in this case. It should be noted that the scour depth predictions on a horizontal bed for linear random waves were presented in Myrhaug and Rue (2003).

5.2 Scour around group of slender vertical piles

The scour around group of slender vertical piles with circular cross-sections in regular waves was investigated in laboratory tests by Sumer and Fredsøe (1998). The equilibrium scour depth *S* data for live-bed conditions and the gap *G* to diameter ratio *G/D* = 0.4 were presented in Fig. 14 and Table 2 of their paper. It appears that their data can be well represented by Eq. (1) with the following values of the coefficients (the pile arrangements are shown in Fig. 18):

Two-pile side-by-side arrangement

$$(C,q,r) = (1.5,0.09,2) \tag{59}$$

Two-pile 45° staggered arrangement

$$(C,q,r) = (1.3,0.037,3) \tag{60}$$

4 ×4 square pile arrangement

$$(C, q, r) = (4.5, 0.023, 12) \tag{61}$$

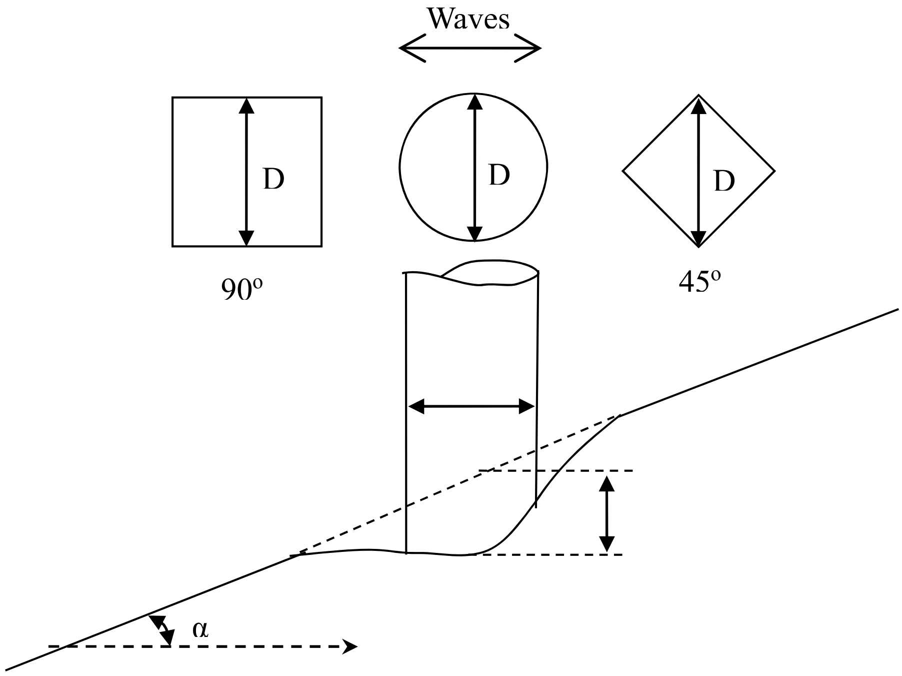

Fig. 17 Definition sketch of the scour depth (*S*) around vertical piles with diameter (*D*) and different shapes on a mild slope (*α*)

Thus the method is the same as for the scour depth around a circular pile; the only difference is the values of the coefficients *C, q* and *r*. No data exist for scour in random waves in this case. It should be noted that the scour depth predictions on a horizontal bed for linear random waves was presented in Myrhaug and Rue (2005).

5.3 Effects of sand-clay mixtures on scour around vertical piles

The scour around vertical piles in sand-clay mixtures under regular waves was investigated in laboratory tests by Dey *et al*. (2011). They found that the empirical formula for the equilibrium scour depth *S* around the pile with diameter *D* is given by Eq. (1) with the coefficients *C*, *q* and *r*; *C* and *q* have different values associated with the proportions of clay in sand-clay mixtures (see Dey *et al.* (2011) for details), and $r = 6$. Thus the method is the same as for the scour depth on mild slopes in the present paper; the only difference is the values of the coefficients *C* and *q*. It should be noted that the scour depth predictions on a horizontal bed for long-crested and short-crested nonlinear random waves were presented in Myrhaug and Ong (2013a).

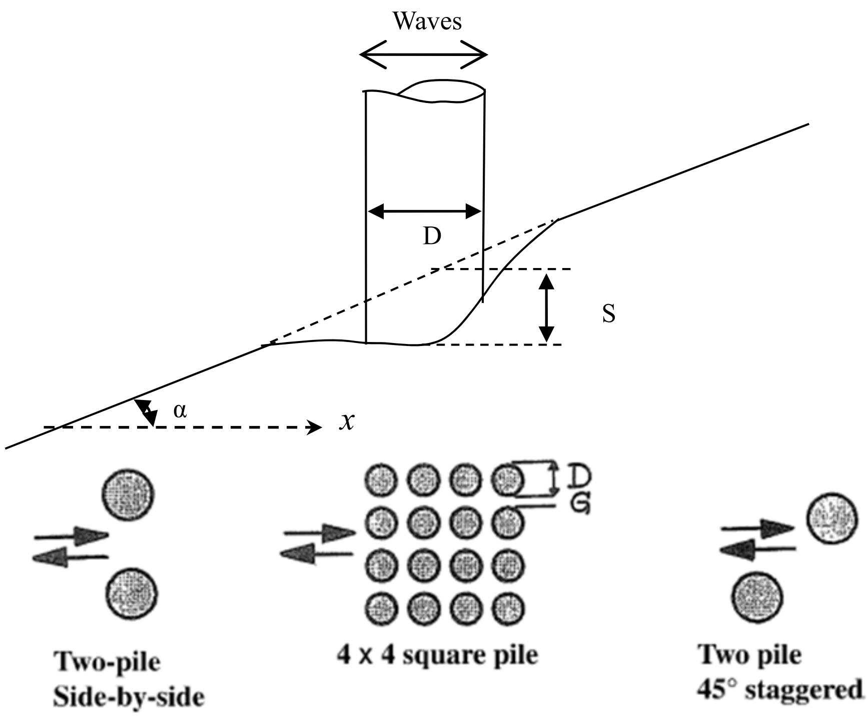

Fig. 18 Definition sketch of pile arrangements for *G/D* = 0.4 and scour-depth (S) around vertical pile on a mild slope (α) (reproduced from Sumer and Fredsøe (1998))

6. Conclusions

A practical method for estimating the scour depth around vertical piles exposed to random waves alone and random waves plus current on mild slopes for wave-dominated flow conditions with $0 \le U_c / (U_c + U_{rms}) \le 0.4$ is provided.

The main conclusions are:

1. The Battjes and Groenendijk (2000) wave height distribution for mild slopes is applied to describe the random wave condition on mild slopes including the effect of breaking waves. A method for transformation of the wave spectrum from deep water to finite water depth is presented. In addition, a truncated wave height distribution is derived for calculating the random wave-induced scour around a vertical pile.
2. For random waves alone, the present results reveal that the effect of a mild slope increases the scour depth compared with that at the seaward location. Moreover, a larger bed slope causes more scour at a fixed location.

3. The present results show that the effect of a current increases the random wave-induced scour depth. This effect becomes more pronounced as the current increases. The scour depth for random waves plus current ranges up to about 1.9 to 2.4 times that for random waves alone.
4. It appears that the approximate method can replace the stochastic method for random waves alone; however the approximated method cannot be applied for random waves plus current.
5. Tentative approaches to related random wave-induced scour cases on mild slopes are also suggested, such as scour around a single vertical piles with square cross-sections, scour around group of vertical piles, and effects of sand-clay mixtures on scour around vertical piles.

Although the methodology is simple, it should be useful as a first approximation to represent the stochastic properties of the scour depth around vertical piles under both random waves alone and random waves plus current on mild slopes for wave-dominated flow conditions. However, comparisons with data are required before a conclusion regarding the validity of this method can be given. In the meantime the method should be useful as an engineering tool for the assessment of scour and in scour protection work of vertical piles on mild slopes.

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