Ocean Systems Engineering, *Vol. 5, No. 4 (2015) 279-299*
DOI: http://dx.doi.org/10.12989/ose.2015.5.4.279

Analytical study of the failure mode and pullout capacity of suction anchors in sand

Haixiao Liu[*1,2], Jinsong Peng[1] and Yanbing Zhao[1]

[1]*State Key Laboratory of Hydraulic Engineering Simulation and Safety, Tianjin University, Tianjin 300072, China*
[2]*Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai Jiao Tong University, Shanghai 200240, China*

(Received May 25, 2015, Revised September 17, 2015, Accepted November 27, 2015)

Abstract. Suction anchors are widely adopted and play an important role in mooring systems. However, how to reliably predict the failure mode and ultimate pullout capacity of the anchor in sand, especially by an easy-to-use theoretical method, is still a great challenge. Existing methods for predicting the inclined pullout capacity of suction anchors in sand are mainly based on experiments or finite element analysis. In the present work, based on a rational mechanical model for suction anchors and the failure mechanism of the anchor in the seabed, an analytical model is developed which can predict the failure mode and ultimate pullout capacity of suction anchors in sand under inclined loading. Detailed parametric analysis is performed to explore the effects of different parameters on the failure mode and ultimate pullout capacity of the anchor. To examine the present model, the results from experiments and finite element analysis are employed to compare with the theoretical predictions, and a general agreement is obtained. An analytical method that can evaluate the optimal position of the attachment point is also proposed in the present study. The present work demonstrates that the failure mode and pullout capacity of suction anchors in sand can be easily and reasonably predicted by the theoretical model, which might be a useful supplement to the experimental and numerical methods in analyzing the behavior of suction anchors.

Keywords: suction anchor; failure mode; pullout capacity; inclined loading; analytical model; sand

1. Introduction

Suction anchors are widely adopted and play an important role in mooring systems. However, for engineers and researchers, further work should still be done on how to reliably predict the failure mode and the ultimate pullout capacity of suction anchors. Relative studies and the analytical model for predicting the failure mode and the ultimate pullout capacity of suction anchors in clay were introduced in detail in the earlier work (Liu *et al*. 2013). The present study emphasizes on the analytical model for predicting the failure mode and the ultimate pullout capacity of suction anchors in sand, which is much different to that in clay.

Compared to clay, there are fewer studies in sand. The pullout capacity of suction anchors in sand, including the horizontal, vertical and inclined capacities, was studied by some researchers.

*Corresponding author, Professor, E-mail: liuhx@tju.edu.cn

http://www.techno-press.org/?journal=ose&subpage=7 ISSN: 2093-6702 (Print), 2093-677X (Online)

However, these studies were performed mainly by experiments (Bang *et al*. 2006a, Jiao *et al*. 2009, Kim *et al*. 2009, Bang *et al*. 2011, Jang and Kim 2013) and finite element analysis (Zeinoddini *et al*. 2010, Ahmed and Hawlader 2014). The theoretical analysis is uncommon. Based on published model test results of piles, a simple method was proposed by Zhang *et al*. (2005) to calculate the ultimate lateral resistance of piles in cohesionless soils through analyzing the distribution of lateral soil resistance along the width of the pile. By using the method of limiting equilibrium, analytical solutions were developed by Bang *et al.* (2006b, c) to estimate horizontal and vertical pullout capacities of suction anchors, in which an assumed soil failure wedge was required. A failure envelope was first proposed by Jones *et al*. (2007) and then developed by Bang *et al*. (2011) to analyze the inclined pullout capacity of suction anchors, in which a "fitting analysis" should be done. Based on the fitting curve, the inclined pullout capacity of the anchor could be determined from the ultimate horizontal and vertical pullout capacities. However, the "fitting analysis" was strongly dependent on experimental results. Since the published theoretical methods on the inclined pullout capacity of suction anchors in sand need to be used combining with experimental data, further work is necessary to develop an independent theoretical method that can efficiently evaluate the failure mode and pullout capacity of suction anchors.

During failure of the suction anchor, the anchor exhibits a state of being pulled out. The direction of being pulled out or the movement direction of the suction anchor is important for understanding the failure mode of the anchor, which directly determines the ultimate pullout capacity of the anchor, as already demonstrated in clay in the earlier work (Liu *et al*. 2013). There is an interesting phenomenon that, when the suction anchor is pulled along a certain direction which exceeds a critical angle, the anchor will be pulled out vertically, i.e., the vertical failure mode arises. This phenomenon was found in clay in the numerical simulation by Randolph and House (2002) and Aubeny *et al*. (2003) and in model tests by EI-Sherbiny (2005). The analytical method developed by Liu *et al*. (2013) can reasonably predict the critical angle in clay, over which the anchor will be vertically pulled out. Recently, Gao *et al*. (2013) performed experimental studies on the anti-uplift behavior of suction anchors in sand, and found that the anchor tended to move upwards when pulled along a certain direction which was much less than 90°. It should be noted that this phenomenon reflects the failure mechanism of the suction anchor under inclined loading. Further knowledge of the phenomenon is beneficial to studying the failure mode of the suction anchor.

In the present work, based on a rational mechanical model for suction anchors and the failure mechanism of the anchor in the seabed, an analytical model that can predict the failure mode and the ultimate pullout capacity of suction anchors in sand is developed. As will be shown in the following, the analytical model is basically different to that in clay. Since sand is cohesionless, the expression of the forces has a direct relationship with the depth of sand, and the shear force on the anchor surface is related to the distribution of the bearing stress and varies with the variation of the mooring angle. Detailed parametric analysis is performed to explore the effects of different parameters on the failure mode and the ultimate pullout capacity. Being verifications of the analytical model, the results of finite element simulation and experiments are adopted to examine the theoretical predictions.

2. Mechanical model for suction anchors

A rational mechanical model of suction anchors is the premise for an analytical model that can predict the failure mode and the ultimate pullout capacity of the anchor. An assumption is made in the present work that the external force is applied at the optimal position of the attachment point of the anchor, at which the maximum pullout capacity of the anchor can only be achieved and the failure mode of the anchor is pure translation without rotation. Note that this assumption is based on experimental observation and numerical simulation (Keaveny *et al*. 1994, Tjelta 2001, Aubeny *et al*. 2003, Clukey *et al*. 2003). The analytical method that can evaluate the optimal position of the attachment point of the suction anchor in clay can be found in the earlier work (Liu *et al*. 2013). The similar work for the suction anchor in sand will be presented at the end of the present work.

2.1 Equilibrium equation in the failure mode

The geometry of the suction anchor is illustrated in Fig. 1(a), in which H is the height of the anchor, D is the diameter of the anchor, and H_p is the distance from the seafloor to the anchor bottom, i.e., the penetration depth of the anchor. The mechanical model of the anchor in the failure state is shown in Fig. 1(b), where T_a is the mooring tension at the attachment point, T_m and T_n represent two components of T_a which are along and normal to the failure direction, respectively, θ is the load angle to the horizontal at the attachment point which can also be called the mooring angle ($0 \le \theta \le \pi/2$), β is the movement direction to the horizontal of the anchor and can also be called the failure angle ($0 \le \beta \le \pi/2$), F_s is the shear force in the failure direction on the anchor surface, F_b is the end bearing in the horizontal direction, H_{bot} is the horizontal shear force acting on the bottom of the anchor (including the annulus of the anchor and the bottom of the soil plug inside the anchor), and W' is the total submerged weight of the anchor and the soil plug inside the anchor.

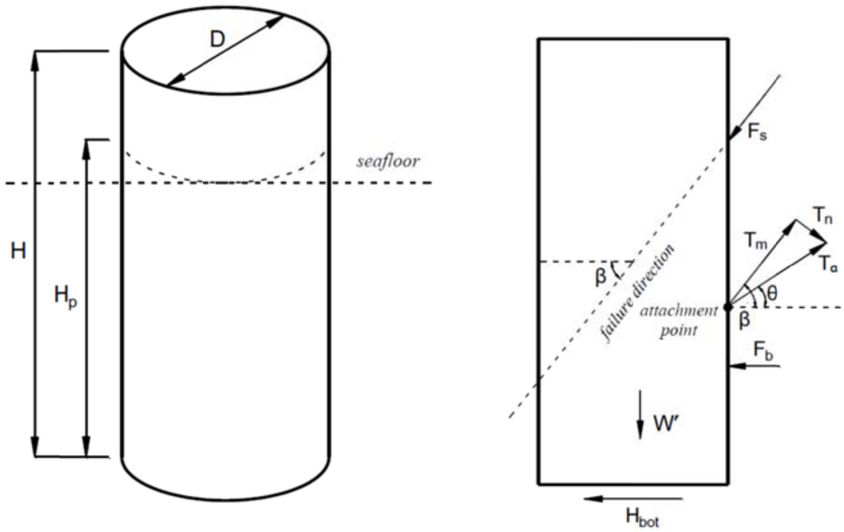


Fig. 1 Geometry and mechanical model of the suction anchor

The equilibrium equation in the failure direction can be expressed as

$$T_m = F_b \cos\beta + F_s + W' \sin\beta + H_{bot} \cos\beta \tag{1}$$

Then the ultimate mooring tension at the attachment point T_a, or the ultimate pullout capacity of the anchor, can be expressed as

$$T_a = \frac{1}{\cos(\beta - \theta)}[F_b \cos\beta + F_s + W' \sin\beta + H_{bot} \cos\beta] \tag{2}$$

The vertical and horizontal pullout capacities are

$$\begin{cases} V_{er} = T_a \sin\theta \\ H_{or} = T_a \cos\theta \end{cases} \tag{3}$$

2.2 Expressions of the forces

Different to clay, the shear force acting on the suction anchor in sand is relevant to the distribution of the bearing stress on the anchor surface. A reasonable expression of the bearing stress distribution is essential to calculate the shear force and the end bearing on the anchor. It is assumed the distribution of the bearing stress in the horizontal direction at a certain depth (Bang and Cho 2001, Zhang *et al.* 2005, Bang *et al.* 2006a), as shown in Fig. 2, where σ_ψ is the bearing stress which is normal to the outside surface of the anchor, and ψ is the angle of the bearing stress to the horizontal axis.

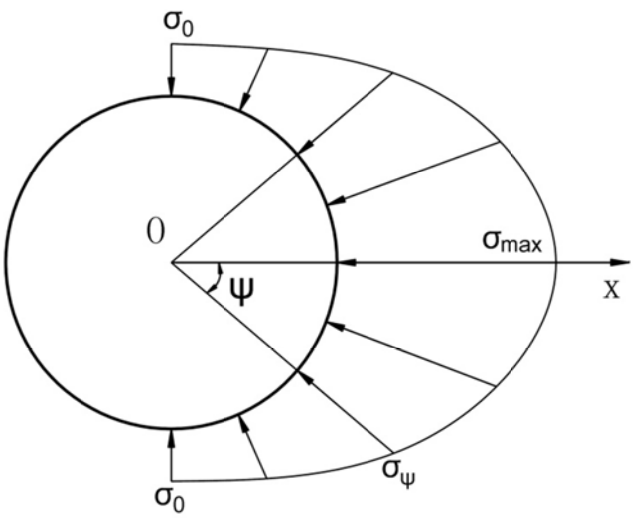


Fig. 2 Distribution of the bearing stress in the horizontal direction of the anchor

Based on the published work (Bang and Cho 2001, Bang *et al*. 2006a, Bang *et al*. 2011), σ_ψ can be expressed as

$$\sigma_\psi = (\sigma_{\max} - \sigma_0)(1 - \frac{2\theta}{\pi})^2 \cos\psi + \sigma_0,\ -\frac{\pi}{2} \le \psi \le \frac{\pi}{2} \tag{4}$$

where, $\sigma_0 = K_0 \gamma' z$ is the earth pressure at rest, in which K_0 is the coefficient of earth pressure at rest, γ' is the submerged soil weight, and z is the depth from the seafloor; $\sigma_{\max} = k_{\max}\gamma' z$ is the earth pressure at $\theta = 0$ and $\psi = 0$, in which $k_{\max}$ is a coefficient, which will be discussed in the subsequent section.

The end bearing F_b can be obtained by integrating the bearing stress, that is

$$\begin{aligned} F_b &= \int_0^{H_p} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sigma_\psi \cos\psi \frac{D}{2} d\psi dz \\ &= \frac{1}{2}\gamma' D H_p^{\ 2} \left[\frac{\pi}{4}(K_p^2 - K_0)(1 - \frac{2\theta}{\pi})^2 + K_0 \right] \end{aligned} \tag{5}$$

The shear force F_s can be obtained by integrating the frictional stress on the surface of the anchor, that is

$$\begin{aligned} F_s &= \int_0^{H_p} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\tau_\psi}{\cos\beta} \frac{D}{2} d\psi dz \\ &= \frac{1}{2}\gamma' D H_p^{\ 2} \left[\frac{(1 - 2\theta/\pi)^2}{1 + \cos\beta}(K_p^2 - K_0) + \beta K_0 / \sin\beta \right] \tan\delta \end{aligned} \tag{6}$$

where $\tau_\psi = \sigma_\psi \tan\delta$ is the frictional stress on the anchor surface in the horizontal direction, in which δ is the interface friction angle.

In the situation of $\theta = 0$, which means that the suction anchor is under horizontal loading, the horizontal shear force $H_{bot,\,0}$ on the bottom of the anchor can be expressed as

$$H_{bot,\,0} = \gamma' H_p (A_{plug} \tan\varphi + A_{annu} \tan\delta) \tag{7}$$

where, φ is the internal friction angle; A_{plug} and A_{annu} are the bottom areas of the soil plug inside the anchor and the annulus area of the anchor, respectively.

Since the horizontal shear force at the loading angle θ, H_{bot}, decreases with increasing value of θ, an inclination factor $\lambda = 2\theta / \pi$ is introduced and H_{bot} can be expressed as (Liu *et al*. 2013)

$$H_{bot} = \gamma' H_p (1-\lambda)(A_{plug} \tan\varphi + A_{annu} \tan\delta) \tag{8}$$

3. Analytical model for predicting the failure mode and ultimate pullout capacity

3.1 Failure mechanism of the anchor

Correctly understanding the failure mechanism of the suction anchor is the basis for establishing a rational analytical model of suction anchors. During failure of the suction anchor, the anchor exhibits a state of being pulled out. The direction of being pulled out or the movement direction of the suction anchor, i.e., the failure angle β, not only reflects the failure mode but also determines the ultimate pullout capacity of the anchor. If the value of β is clearly known, the ultimate pullout capacity of the anchor can then be obtained through Eq. (2).

An analytical method for predicting the movement direction of the drag anchor with an arbitrarily fluke section both in cohesive and noncohesive soils was proposed by Liu *et al*. (2012), where an important principle was assumed, i.e., among all possible movement directions of the anchor, the real movement direction must be the direction in which the soil resistance was easiest to be overcome by the drag force. In other words, the direction that needs the least drag force to overcome the soil resistance is the real movement direction. The least-force principle can be generalized to be the failure mechanism of the structure embedded in soils. It was formerly adopted in developing the analytical model for predicting the failure mode and the ultimate pullout capacity of suction anchors in clay (Liu *et al*. 2013), and now is still adopted in the present work to calculate the failure angle β.

3.2 Analytical procedure for determining the failure direction and pullout capacity

The least-force principle is applied along with the mechanical model for suction anchors. As shown in Fig. 1, β is the key angle that needs to be determined and is assumed to possibly vary in the range $[0, \pi/2]$, where 0 and $\pi/2$ represent the horizontal and vertical directions, respectively. According to the least-force principle, the real failure direction can be determined if the variation of T_a with the independent variable β in the range $[0, \pi/2]$ is clearly known.

The variation of T_a with β in $[0, \pi/2]$ can be known by investigating the values of T_a at several special points, in other words, the minimum value of T_a can only be reached at these special points. The special points include three types: (a) the boundary points, i.e., $\beta = 0$ and $\beta = \pi/2$; (b) the points that meet the equation $dT_a/d\beta=0$, i.e., the first derivatives at the points are zero; (c) the points that the first derivatives are non-existent. To identify the last two types of point, the equation $dT_a/d\beta=0$ must be solved. The first derivative of T_a with β can be obtained according to Eq. (2) as

$$\frac{dT_a}{d\beta} = \frac{1}{\cos^2(\beta-\theta)} T(\beta) \tag{9}$$

where $T(\beta)=(dF_s/d\beta)\cos(\beta-\theta)+F_s\sin(\beta-\theta)-F_b\sin\theta+W'\cos\theta-H_{bot}\sin\theta$, in which $\beta>0$ and $\theta<\pi/2$.

For the suction anchor, the real failure angle and the corresponding ultimate pullout capacity can be obtained by performing the following analytical procedure (Liu *et al*. 2013):

(1) Accurately express the forces acting on the anchor as described Fig. 1(b).

(2) According to Eq. (9), solve the equation $dT_a/d\beta=0$ and obtain the exact points whose first derivatives are zero.

(3) By analyzing the expression of the first derivative of T_a, obtain the exact points whose first derivatives are non-existent.

(4) Calculate and compare the values of T_a at the three types of point according to Eq. (2). The point or angle with the minimum value of T_a is then the real failure angle β_r, and the corresponding T_a is the ultimate pullout capacity of the anchor.

In the present analysis, the failure angle β_r is determined at first. Then the ultimate pullout capacity can be calculated with the known value of β_r. Similar to clay, the failure mode of the suction anchor in sand falls into three types according to the failure angle, i.e., horizontal failure, inclined failure and vertical failure. The failure mode will vary from horizontal failure to inclined failure and finally vertical failure as the mooring angle increases. The present method can clearly analyze the failure mode of the suction anchor in sand under a certain loading condition.

3.3 Discussion on the parameters

In this section, some important parameters involved in the analytical model are discussed, which directly determine the failure mode and the ultimate pullout capacity of the anchor. These parameters include the coefficient of the maximum bearing stress $k_{\max}$, the internal friction angle φ, the coefficient of earth pressure at rest K_0, and the interface friction angle δ. Clearly understanding the effects of these parameters is important to correctly using the analytical model.

(1) Coefficient of the maximum bearing stress $k_{\max}$

The maximum bearing stress $\sigma_{\max}$ is determined by $k_{\max}$, as illustrated in Fig. 2, which can be expressed as $\sigma_{\max}=k_{\max}\gamma' z$. Zhang *et al*. (2005) discussed the expression of $\sigma_{\max}$. Several available methods from Brinch Hansen (1961), Broms (1964), Reese *et al*. (1974) and Fleming *et al*. (1992) were introduced to determine the ultimate lateral resistance in cohesionless soils. By comparing with existing experimental data, $\sigma_{\max}=k_{\max}\gamma' z$ was proposed by Zhang *et al*. (2005) to calculate the ultimate lateral resistance, and $k_{\max}=K_p^2$ was proposed as the most reasonable expression, where K_p is the coefficient of passive earth pressure. In the present work, $k_{\max}=K_p^2$ is adopted following the work by Zhang *et al*. (2005), in which K_p directly relates to the internal friction angle φ as $K_p=\tan^2(45°+\varphi/2)$.

(2) Internal friction angle φ

The value of φ reflects the strength of sand.

φ=26° and 39° were used in the analytical studies of suction anchors on the horizontal pullout capacity by Bang and Cho (2001) and on the vertical pullout capacity by Bang *et al*. (2006c), respectively. The sands used by Kim *et al*. (2009) and Gao *et al*. (2013) in the model tests on suction anchors had the internal friction angles of 33° and 36.8°, respectively. $\varphi = 30° \sim 45°$ was used by Zeinoddini *et al*. (2010) in the finite element analysis on the inclined load-bearing capacity of suction caissons in sand. In conclusion, the value of φ can be in the range $26° \sim 45°$.

(3) Interface friction angle δ

The interface friction angle depends on the properties of the soil and the suction anchor.

Kulhawy *et al*. (1983) suggested the value of the interface friction angle. For the rough steel, $\delta = 0.7\varphi \sim 0.9\varphi$, and for the smooth steel, $\delta = 0.5\varphi \sim 0.7\varphi$. Allersma *et al*. (2000) adopted $\delta = 22°$ in centrifuge testing and numerical modelling of suction piles. $\delta = 2\varphi / 3$ and $\delta = 0.6\varphi$ were assumed by Achmus *et al*. (2013) and Ahmed and Hawlader (2014) in the finite element analysis of load-bearing behavior of suction anchors, respectively. $\delta = 19°$ was used by Lehane *et al*. (2014) in the finite element analysis of the extraction of suction caissons. Thieken *et al*. (2014) took the values of 20.6° and 20.8° for δ in the finite element simulation of the load-bearing behavior of suction anchors. In general, the value of δ can vary in the range $18° \sim 32°$.

(4) Coefficient of earth pressure at rest K_0

The coefficient of earth pressure at rest relates the vertical stress to the horizontal stress after the soil consolidation without lateral deformation.

In calculating the ultimate inclined uplift capacity of suction caissons under inclined loadings, which included the effect of the initial stress state, $K_0 = 0.5 \sim 1.0$ was adopted by Deng and Carter (2000). $K_0 = 1 - \sin\varphi$ was used in the analytical studies on the ultimate lateral resistance to piles by Zhang *et al*. (2005) and on the vertical pullout capacity of suction anchors by Bang *et al*. (2006c), and was also used by Achmus *et al*. (2013) in the finite element analysis on the load-bearing behavior of suction anchors. Lehane *et al*. (2014) adopted $K_0 = 0.4$ in the finite element analysis of the extraction of suction caissons. Thieken *et al*. (2014) took the values of 0.47 and 0.49 for K_0 in the finite element simulation of the load-bearing behavior of suction anchors. In general, the value of K_0 can vary in the range $0.3 \sim 1.0$.

4. Parametric study

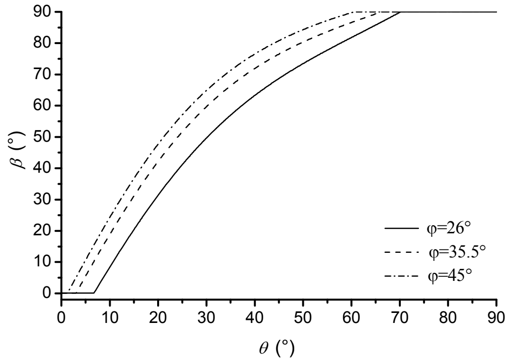
In order to further understand the failure mode and pullout capacity of suction anchors in sand, a parametric study is performed. Five cases are hypothesized to investigate the effects of the internal friction angle φ, interface friction angle δ, coefficient of earth pressure at rest K_0, submerged soil weight γ', and anchor height-to-diameter ratio H / D. Based on the earlier discussion on the parameters, $\varphi = 26° \sim 45°$, $\delta = 18° \sim 32°$ and $K_0 = 0.3 \sim 1.0$ are adopted in the parametric study. $\gamma' = 2.94\,kN/m^3 \sim 11.76\,kN/m^3$ is used based on the work by Zhang (2011). The height-to-diameter ratio H/D is assumed to vary in the range 1~6. In general, the penetration depth H_p is very close to the anchor height H, and hence $H_p = H$ is assumed in the parametric study. The parameters for hypothetical cases are listed in Table 1.

Following the analytical procedure for determining the failure direction and pullout capacity as described in Section 3.2, the effects of different parameters on the failure direction and ultimate pullout capacity of suction anchors are calculated. Fig. 3 shows the variation of the failure angle with increasing mooring angle, Fig. 4 shows the variation of the ultimate pullout capacity with increasing mooring angle, and Fig. 5 shows the relation between horizontal and vertical pullout capacities, which are denoted by H_{or} and V_{er}, respectively.

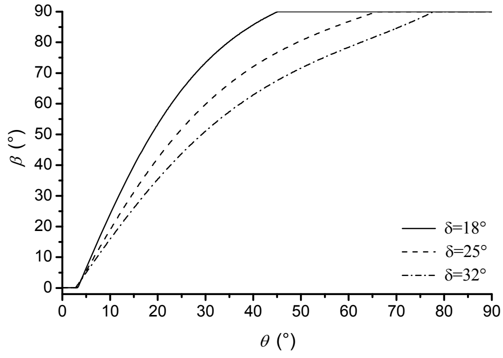
As observed from Fig. 3, all curves generally present three stages. At the first stage, β keeps 0° while θ increases within a small-value range. When θ reaches a critical value, which is defined as the start point of the second stage and denoted by θ_d, β starts to rapidly increase with increasing value of θ. When θ increases to another critical value, which is defined as the start point of the third stage and denoted by θ_u, β reaches 90° and keeps a constant.

Table 1 Parameters for hypothetical cases

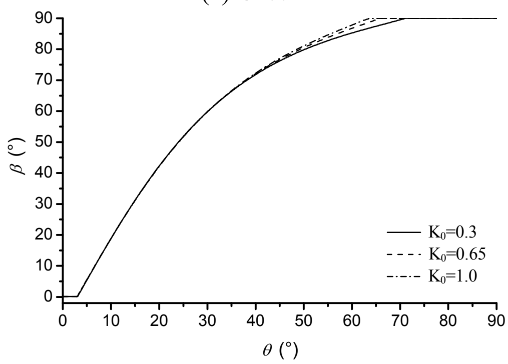
<table>
<tr><th>Case</th><th>φ (°)</th><th>δ (°)</th><th>K_0</th><th>γ' (kN/m^3)</th><th>H/D</th></tr>
<tr><td rowspan="3">1</td><td>26.0</td><td rowspan="3">25</td><td rowspan="6">0.65</td><td rowspan="9">7.35</td><td rowspan="12">3.5</td></tr>
<tr><td>35.5</td></tr>
<tr><td>45.0</td></tr>
<tr><td rowspan="3">2</td><td rowspan="12">35.5</td><td>18</td></tr>
<tr><td>25</td></tr>
<tr><td>32</td></tr>
<tr><td rowspan="3">3</td><td rowspan="9">25</td><td>0.30</td></tr>
<tr><td>0.65</td></tr>
<tr><td>1.00</td></tr>
<tr><td rowspan="3">4</td><td rowspan="6">0.65</td><td>2.94</td></tr>
<tr><td>7.35</td></tr>
<tr><td>11.76</td></tr>
<tr><td rowspan="3">5</td><td rowspan="3">7.35</td><td>1.0</td></tr>
<tr><td>3.5</td></tr>
<tr><td>6.0</td></tr>
</table>



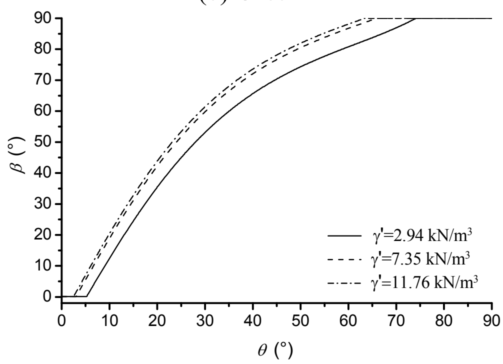
(a) Case 1



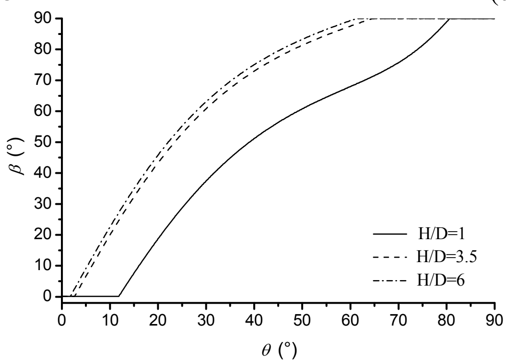
(b) Case 2



(c) Case 3



(d) Case 4



(e) Case 5

Fig. 3 Effects of different parameters on the failure direction of suction anchors

Fig. 3 clearly demonstrates that, the anchor will fail in the horizontal direction if $\theta \le \theta_d$, in the vertical direction if $\theta \ge \theta_u$, and in the inclined direction if $\theta_d < \theta < \theta_u$.

Fig. 3 also demonstrates that, θ_d decreases as the value of the parameters (φ, γ' and H/D) increases, while the influence of the parameters (δ and K_0) on θ_d is not significant. With increasing value of the parameters (φ, K_0, γ' and H/D), the anchor tends to fail vertically at a smaller θ_u, while θ_u increases with increasing value of δ. It can also be observed that at a certain value of θ ($\theta_d < \theta < \theta_u$), the failure angle increases with increasing value of the parameters (φ, K_0, γ' and H/D). This means that φ, K_0, γ' and H/D have more influence on V_{er} than H_{or} of the anchor. However, an opposite relationship between β and θ can be found for δ, which means that it has less influence on V_{er} than H_{or} of the anchor.

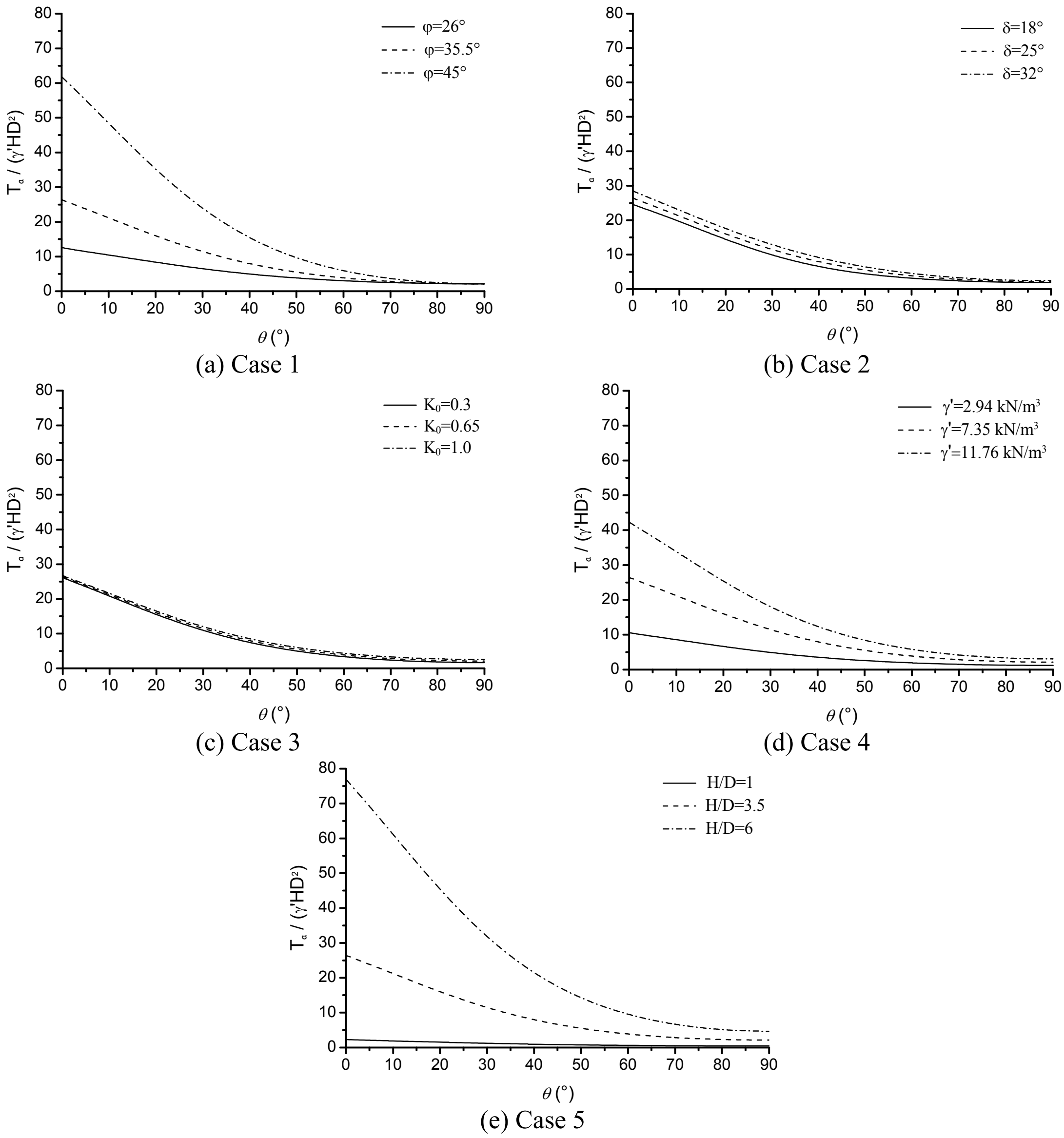


Fig. 4 Effects of different parameters on the ultimate pullout capacity of suction anchors

Fig. 4 demonstrates that, T_a is a monotonic function of θ. With increasing value of the mooring angle from 0° to 90°, the ultimate pullout capacity of the anchor decreases from the maximum to the minimum. With increasing value of the parameters, the ultimate pullout capacity of the anchor generally increases. However, the effect of different values of K_0 on the ultimate pullout capacity is not evident.

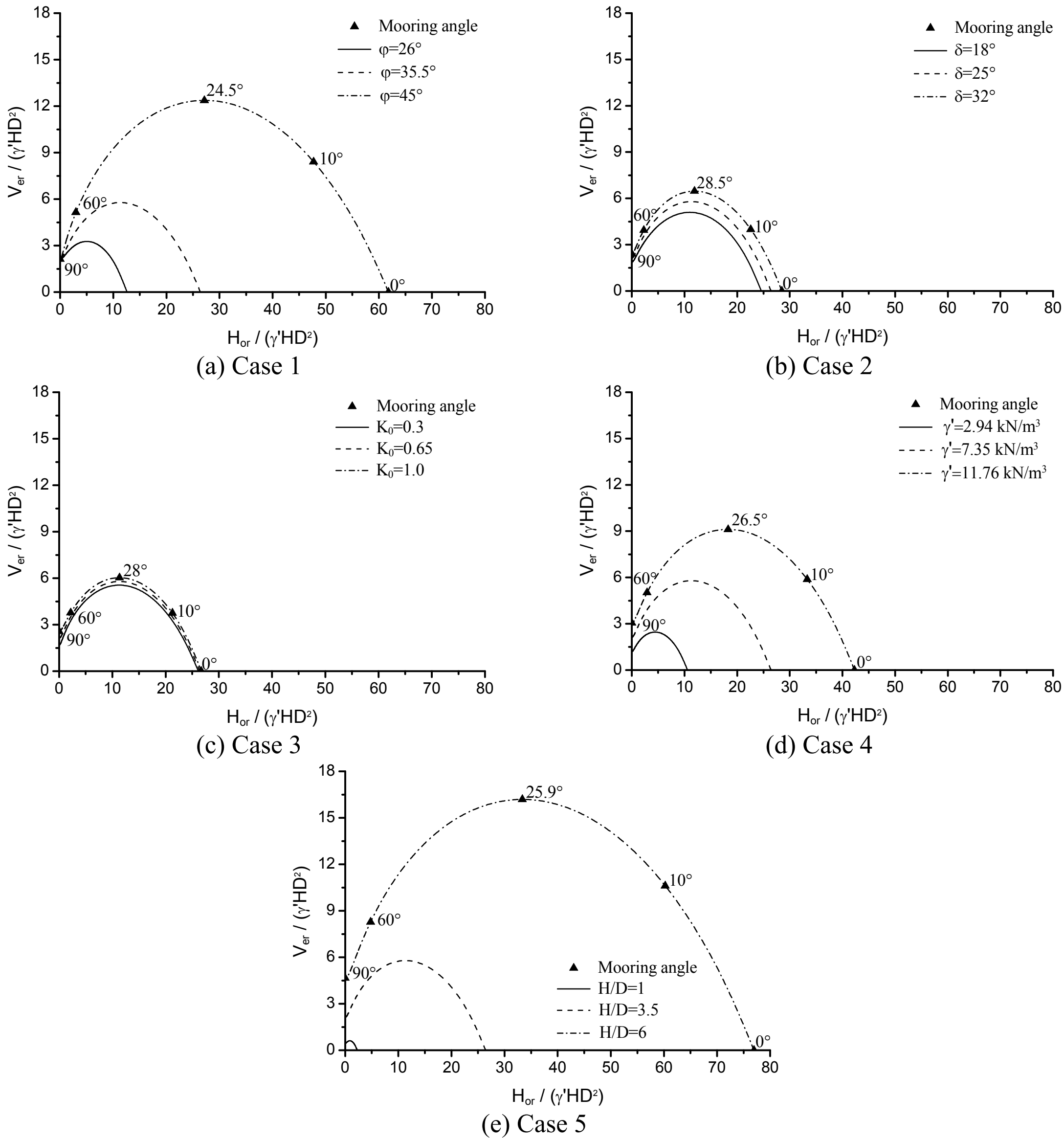


Fig. 5 Effects of different parameters on the relation between horizontal and vertical pullout capacities of suction anchors

Fig. 5 demonstrates that, with increasing value of the parameters, the ultimate pullout capacity of the anchor increases at all values of θ except at $\theta = \pi/2$ in Case 1. This is because according to Eqs. (2), (3) and (6), $V_{er} = \beta K_0 \gamma' D H^2 (\tan\delta)/(2\sin\beta) + W'$ at $\theta = \pi/2$, i.e., φ is not an influential factor of the vertical pullout capacity of the anchor at $\theta = \pi/2$. The effect of K_0 on the ultimate pullout capacity of the anchor is not so significant as the other parameters. It can be observed that, with the increase of H_{or} from zero (corresponding to $\theta = \pi/2$), V_{er} increases to a peak, and then gradually decreases to zero (corresponding to $\theta = 0$). Since sand is cohesionless, the expression of the forces has a direct relationship with the depth of sand, and the shear force on the anchor surface is related to the distribution of the bearing stress and varies with the variation of the mooring angle. These make the relation curve between H_{or} and V_{er} exhibit a bump, which is a significant phenomenon confirmed by experimental observations. It can be noted that, the bump usually arises when the mooring angle is in the range (24°, 30°).

5. Comparative study

As presented in the earlier section, the present model can produce the results of the failure mode (β-θ curve) and the ultimate pullout capacity (T_a-θ and V_{er}-H_{or} curves) of the suction anchor. However, it cannot produce the load-displacement curve of the suction anchor, on which many researchers worked out. The only studies that can be adopted to examine the present model include the experiments by Kim *et al*. (2009) and Gao *et al*. (2013) and the finite element analysis by Deng and Carter (2000).

5.1 Comparison with experimental results of Kim et al. (2009)

A series of centrifuge model tests in sand were conducted by Kim *et al*. (2009) to determine the horizontal, vertical and inclined loading capacities of suction piles. The prototype suction pile had a diameter of 3m, a length of 6 m and a thickness of 0.1 m. Values of φ and γ' were 33° and $10.1\ kN/m^3$, respectively. The model tests were conducted at five mooring angles, including 0°, 22.5°, 45°, 67.5° and 90°. In the present work, the centrifuge test results of the pullout capacity are used to examine the analytical model. According to the earlier parametric study, $K_0 = 0.65$ and $\delta = 0.7\varphi$ are used in the analytical model. The comparative results are presented in Fig. 6.

It is observed that, among all measured data, four points agree well with the analytical predictions. The relative errors of the ultimate pullout capacity from the analytical predictions to the experimental data corresponding to the five mooring angles (0°, 22.5°, 45°, 67.5° and 90°) are 4.4%, 24.7%, 4.4%, 0.08% and - 4.6%, respectively. The average of the five relative errors is 5.8%. The shape of the analytical curve is consistent with the distribution of the experimental data, i.e., there is a bump in the relation curve between the vertical and horizontal pullout capacities of the suction anchor. It proves that the present analytical method is able to reflect the basic characteristic of pullout capacity of the suction anchor in sand.

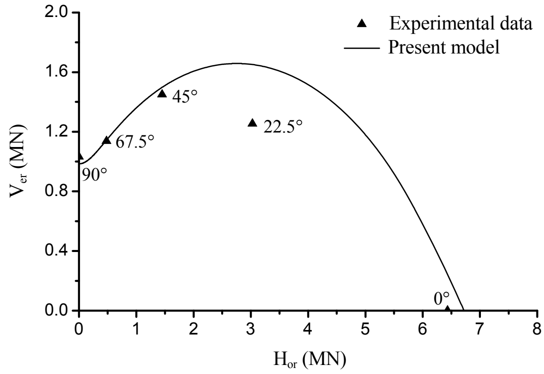


Fig. 6 Comparison with experimental results of Kim *et al.* (2009)

5.2 Comparison with experimental results of Gao et al. (2013)

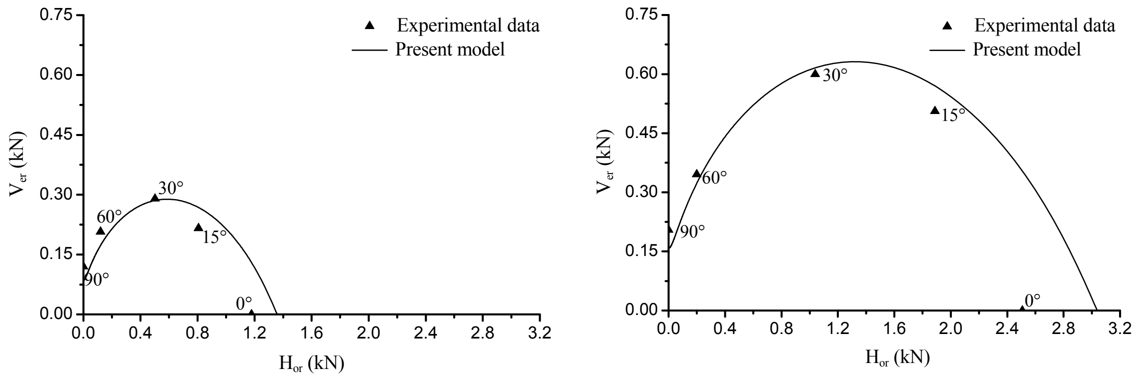
A series of model tests were conducted by Gao *et al*. (2013) to explore the anti-uplift behavior of suction anchors in sand, where the effects of the height-to-diameter ratio and the mooring angle were considered. The model suction anchor had a diameter of 101 mm and a thickness of 2 mm. Three lengths including 202 mm, 404 mm and 606 mm were designed. Values of φ and γ' were 36.8° and $9.73\ kN/m^3$, respectively. Five mooring angles were applied in the tests, including 0°, 15°, 30°, 60° and 90°. According to the parametric study, $K_0 = 0.65$ and $\delta = 0.7\varphi$ are used in the analytical model. Since the analytical model is based on the assumption that the external force is applied at the optimal attachment point of the anchor, the model anchor with the length of 202 mm did not obey the assumption and then is excluded in the comparative study. The comparative results are presented in Fig. 7.

It is observed that, the analytical predictions generally agree well with the experimental data. In the case of $H/D = 4$, the relative errors of the ultimate pullout capacity from the analytical predictions to the experimental data corresponding to the five mooring angles (0°, 15°, 30°, 60° and 90°) are 15.3%, 12.8%, -1.7%, -26.8% and -28.3%, respectively. The average of the five relative errors is -5.7%. In the case of $H/D = 6$, the relative errors of the ultimate pullout capacity from the analytical predictions to the experimental data corresponding to the five mooring angles (0°, 15°, 30°, 60° and 90°) are 21.1%, 6.6%, 3.3%, -11.3% and -22.4%, respectively. The average of the five relative errors is -0.5%. These confirm again the ability of the present model to reflect the basic characteristic of pullout capacity of the suction anchor in sand.

5.3 Comparison with finite element analysis of Deng and Carter (2000)

Finite element analysis was conducted by Deng and Carter (2000) to study the inclined uplift capacity of suction anchors in sand. The Mohr-Coulomb elastoplastic soil model was used in the numerical analysis. Values of φ and K_0 were 32° and 0.8, respectively. The mooring angles of 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80° and 90° were investigated. Based on the analysis, a simplified method was proposed for the estimation of inclined uplift capacity of suction anchors, in which two reduction factors were introduced. In order to satisfy the assumption in the present model that the external force is applied at the optimal attachment point of the anchor, only the case of $H/D = 1.5$ is selected in the comparative study. According to the earlier parametric study, $\delta = 0.7\varphi$ is used in the analytical model. Based on the work by Verruijt (2006), values of γ' range in $10.31 kN/m^3 \sim 11.55 kN/m^3$ for dense sand, and then $\gamma' = 10.93 kN/m^3$ is used in the comparative study. The comparative results are presented in Fig. 8, in which the horizontal and vertical pullout capacities are normalized by the ultimate horizontal pullout capacity H_0 and the ultimate vertical pullout capacity V_0, respectively.

It is observed that, the analytical predictions are relatively larger than the finite element results. Several reasons may lead to the divergences. On the one hand, two reduction factors were introduced by Deng and Carter (2000) to modify the finite element results.



(a) $H/D = 4$ (b) $H/D = 6$

Fig. 7 Comparison with experimental results of Gao *et al*. (2013)

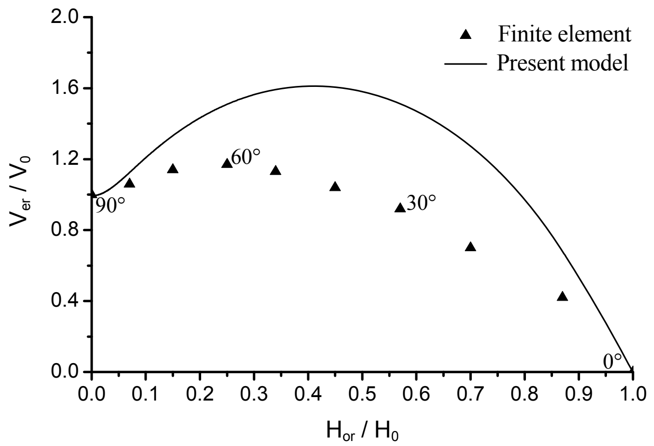


Fig. 8 Comparison with finite element results of Deng and Carter (2000)

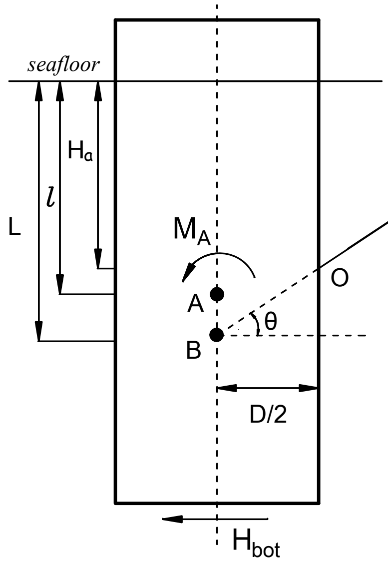


Fig. 9 Mechanical model for predicting the optimal attachment point

On the other hand, as seen from Fig. 8, the ratio of the maximum vertical pullout capacity ($V_{\max}$) to the ultimate vertical pullout capacity (V_0) is 1.61 in the analytical predictions, while is 1.17 in the finite element results. It is noted that with almost same values of parameters in the case of Section 5.1, values of the ratio $V_{\max}/V_0$ in the experimental data and in the analytical predictions are 1.41 and 1.68, respectively. Hence, it is reasonable to think that the finite element analysis produced relatively small results.

6. Method for predicting the optimal attachment point

As introduced earlier, experimental observation and numerical simulation have confirmed that, if the external force is applied at the optimal position of the attachment point of the suction anchor, the failure mode of the anchor is pure translation without rotation and the anchor possesses the maximum pullout capacity. Based on the knowledge of the optimal attachment point, a theoretical method is proposed in this section that can simply evaluate the optimal position of the attachment point of the suction anchor in sand.

The mechanical model for predicting the location of the optimal attachment point is shown in Fig. 9, in which Point O denotes the optimal attachment point, H_a is the distance between the seafloor and Point O, l is the distance between the seafloor and Point A, which is at the symmetrical axis (dashed line) of the anchor and at the depth of the centroid of the soil profile, and L is the distance between the seafloor and Point B, which is the intersection of the applied mooring force and the symmetrical axis of the anchor. Considering that the end bearing F_b and the shear force F_s approximately pass through Point A, only T_a and H_{bot} are considered in the resultant overturning moment. The location of the optimal attachment point can be calculated by making the resultant overturning moment to Point A equal zero, that is

$$\sum M_A = 0 \tag{10}$$

Eq. (10) can be further written as

$$\left(L-l\right)T_a\cos\theta - \left(H_p - l\right)H_{bot} = 0 \tag{11}$$

By utilizing the geometrical relationship between Point O and Point B, i.e., $\tan\theta = 2(L-H_a)/D$, the location of the optimal attachment point can be expressed as

$$H_a = l + \frac{H_{bot}}{T_a\cos\theta}\left(H_p - l\right) - \frac{D}{2}\tan\theta \tag{12}$$

where $l = 2H_p/3$.

Eq. (12) demonstrates that, the mooring angle θ is the major factor that influences the location of the optimal attachment point. If the mooring angle is relatively large, the location of the optimal attachment point is near the top of the suction anchor. Contrarily, if the mooring angle is less than 45°, the optimal attachment point is in the range of 0.55-0.70 times of the insertion depth. These conclusions basically agree with the relative studies (Deng and Carter 2000, Bang *et al*. 2006b, Zeinoddini *et al*. 2010), in which the optimal attachment point is in the range of 0.60-0.80 times of the insertion depth.

7. Conclusions

In the present work, based on a rational mechanical model for suction anchors and the failure mechanism of the anchor in the seabed, i.e., the least-force principle, an analytical model is developed to predict the failure mode and ultimate pullout capacity of suction anchors in sand under inclined loading. The external force is assumed to apply at the optimal position of the attachment point of the anchor, at which the maximum pullout capacity of the anchor can only be achieved and the failure mode of the anchor is pure translation without rotation. To examine the present model, the results from experiments and finite element analysis are employed to compare with theoretical predictions, and a general agreement is obtained.

The results of the parametric study show that the failure mode of the anchor can be easily judged according to the relation curve between the mooring angle θ and the failure angle β. The anchor will fail in the horizontal direction if $\theta \le \theta_d$, in the vertical direction if $\theta \ge \theta_u$, and in an inclined direction if $\theta_d < \theta < \theta_u$. θ_d decreases as the value of the parameters (φ, γ' and H/D) increases, while the influence of the parameters (δ and K_0) is not significant. With the increase of the parameters (φ, K_0, γ' and H/D), the anchor tends to fail vertically at a smaller θ_u, while θ_u increases as δ increases.

The ultimate pullout capacity is a monotonic function of the mooring angle. With increasing value of the mooring angle from 0° to 90°, the ultimate pullout capacity of the anchor decreases from the maximum to the minimum. With increasing value of the parameters, the ultimate pullout capacity of the anchor generally increases. The parameters (φ, δ, γ' and H/D) have significant influence on the ultimate pullout capacity of the anchor, while the effect of K_0 is not evident. A significant phenomenon is observed in the present study, i.e., the relation curve between H_{or} and V_{er} exhibits a bump, which usually arises when the mooring angle is in the range (24°, 30°). This special phenomenon that only arises in sand for suction anchors has been confirmed by experimental observations.

An analytical method is also proposed to predict the location of the optimal attachment point of suction anchors under various loading conditions. The results indicate that if the mooring angle is relatively large, the position of the optimal attachment point is near the top of the suction anchor. If the mooring angle is less than 45°, the optimal attachment point is in the range of 0.55-0.70 times of the insertion depth of the anchor. These conclusions basically agree with the relative studies.

Acknowledgments

Financial support from the National Natural Science Foundation of China (Grant nos. 51539008 and 51179124) and the Key Project of Tianjin Research Program of Application Foundation and Advanced Technology (14JCZDJC39900) is greatly acknowledged.

References

Achmus, M., Akdag, C.T. and Thieken, K. (2013), “Load-bearing behavior of suction bucket foundations in sand”, *Appl. Ocean Res.*, **43**, 157-165.

Allersma, H.G.B., Brinkgreve, R.B.J., Simon, T. and Kirstein, A.A. (2000), “Centrifuge and numerical modelling of horizontally loaded suction piles”, *Int. J. Offshore Polar.*, **10**(3), 222-228.

Ahmed, S.S. and Hawlader, B.C. (2014), “Finite element modeling of inclined load capacity of suction caisson in sand with Abaqus/Explicit”, *Proceedings of the 24th International Offshore and Polar Engineering Conference*, Busan, Korea, June.

Aubeny, C.P., Han, S.W. and Murff, J.D. (2003), “Inclined load capacity of suction caissons”, *Int. J. Numer. Anal. Method. Geomech.*, **27**(14), 1235-1254.

Bang, S. and Cho, Y. (2001), “Ultimate horizontal loading capacity of suction piles”, *Proceedings of the 11th International Offshore and Polar Engineering Conference*, Stavanger, Norway, June.

Bang, S., Jones, K., Kim, Y.S., Kim, K.O. and Cho, Y. (2006b), “Horizontal pullout capacity of embedded suction anchors in sand”, *Proceedings of the 25th International Conference on Offshore Mechanics and Arctic Engineering*, Hamburg, Germany, June.

Bang, S., Jones, K., Kim, Y.S., Kim, K.O. and Cho, Y. (2006c), “Vertical pullout capacity of embedded suction anchors in sand”, *Proceedings of the 16th International Offshore and Polar Engineering Conference*, San Francisco, California, USA, May.

Bang, S., Jones, K.D., Kim, K.O., Kim, Y.S. and Cho, Y. (2011), “Inclined loading capacity of suction piles in sand”, *Ocean Eng.*, **38**(7), 915-924.

Bang, S., Leahy, J.C., Cho, Y. and Kwon, O. (2006a), “Horizontal bearing capacity of suction piles in sand”, *J. T. Res. Board*, Transportation Research Board of the National Academies, **No. 1975**, 21-27.

Brinch Hansen, J. (1961), “The ultimate resistance of rigid piles against transversal forces”, *Danish Geotechnical Institute*, **12**, 5-9.

Broms, B.B. (1964), "Lateral resistance of piles in cohesive soils", *J. Soil Mech. Foundations Div.*, **90**(2), 27-64.

Clukey, E.C., Aubeny, C.P. and Murff, J.D. (2003), "Comparison of analytical and centrifuge model tests for suction caissons subjected to combined loads", *Proceedings of the 22nd International Conference on Offshore Mechanics and Arctic Engineering*, Cancun, Mexico, June.

Deng, W. and Carter, J.P. (2000), "Inclined uplift capacity of suction caissons in sand", *Proceedings of the 32nd Annual Offshore Technology Conference*, Houston, Texas, USA, May.

EI-sherbiny, R.M. (2005), *Performance of suction caisson anchors in normally consolidated Clay*, Ph.D. Dissertation, The University of Texas at Austin, Texas.

Fleming, W.G.K., Weltman, A.J., Randolph, M.F. and Elson, W.K. (1992), *Piling engineering*. Surrey University Press, London.

Gao, Y.F., Qiu, Y., Li, B., Li, D.Y., Sha, C.M. and Zheng, X. (2013), "Experimental studies on the anti-uplift behavior of the suction caissons in sand", *Appl. Ocean Res.*, **43**, 37-45.

Jones, K.D., Bang, S. and Cho, Y. (2007), "Pullout capacity of embedded suction anchors in sand", *Ocean Eng.*, **34**(16), 2107-2114.

Jang, Y.S. and Kim, Y.S. (2013), "Centrifugal model behavior of laterally loaded suction pile in sand", *KSCE J. Civil Eng.*, **17**(5), 980-988.

Jiao, B.T., Lu, X.B., Zhao, J., Wang, A.L., Shi, Z.M. and Zeng, X.H. (2009), "Experimental study on the bearing capacity of suction caissons in saturated sand", *Proceedings of the 19th International Offshore and Polar Engineering Conference*, Osaka, Japan, June.

Keaveny, J.M., Hansen, S.B., Madshus, C. and Dyvik, R. (1994), "Horizontal capacity of large-scale model anchors", *Proceedings of the 13th International Conference on Soil Mechanics and Foundation Engineering*, New Delhi, India, January.

Kim, K.O., Kim, Y.S., Cho, Y., Bang, S. and Jones, K. (2009), "Centrifuge model tests on suction piles in sand under inclined loading", *Proceedings of the 19th International Offshore and Polar Engineering Conference*, Osaka, Japan, June.

Kulhawy, F.H., Trautmann, C.H., Beech, J.F., O'Rourke, T.D., Mcguire, W., Wood, W.A. and Capano, C. (1983), *Transmission line structure foundations for uplift-compression loading*, Report, Electric Power Research Institute, USA.

Lehane, B.M., Elkhatib, S. and Terzaghi, S. (2014), "Extraction of suction caissons in sand", *Geotechnique*, **64**(9), 735-739.

Liu, H.X., Wang, C. and Zhao, Y.B. (2013), "Analytical study of the failure mode and pullout capacity of suction anchors in clay", *Ocean Syst. Eng.*, **3**(2), 79-95.

Liu, H.X., Zhang, W., Liu, C.L. and Hu, C. (2012), "Movement direction of drag anchors in seabed soils", *Appl. Ocean Res.*, **34**, 78-95.

Reese, L.C., Cox, W. R. and Koop, F.D. (1974), "Analysis of laterally loaded piles in sand", *Proceedings of the 6th Offshore Technology Conference*, Houston, May.

Randolph, M.F. and House, A. (2002), "Analysis of suction caisson capacity in clay", *Proceedings of the34th Annual Offshore Technology Conference*, Houston, USA, May.

Thieken, K., Achmus, M. and Schro¨der, C. (2014), "On the behavior of suction buckets in sand under tensile loads", *Comput. Geotech.*, **60**, 88-100.

Tjelta, T.I. (2001), "Suction piles: their position and applications today", *Proceedings of the 11th International Offshore Conference*, Stavanger, Norway, June.

Verruijt, A. (2006), *Soil Mechanics*, Merwehoofd, Papendrecht, Netherlands.

Zeinoddini, M., Mousavi, A. and Hobbi, D. (2010), "Inclined load bearing capacity of suction caissons embedded in sands", *Int. J. Earth Sci. Eng.,* **3**(4), 563-571.

Zhang, L.Y., Silva, F. and Grismala, R. (2005), "Ultimate lateral resistance to piles in cohesionless soils", *J. Geotech. Geoenviron.*, **131**(1), 78-83.

Zhang, W. (2011), *Penetration mechanism and kinematic behavior of drag anchors*, Ph.D. Dissertation, Tianjin University, Tianjin, China.

Nomenclature

A_{annu} annulus area of the anchor

A_{plug} bottom area of the soil plug inside the anchor

D diameter of the anchor

F_b end bearing of the anchor

F_s shear force on the anchor surface in the failure direction

H height of the anchor

H_0 ultimate horizontal pullout capacity

H_a distance from the seafloor to the optimal attachment point

H_{bot} horizontal shear force acting on the bottom of the anchor

$H_{bot,0}$ horizontal shear force acting on the bottom of the anchor under horizontal loading

H_{or} horizontal component of the ultimate pullout capacity

H_p distance from the seafloor to the anchor bottom

K_0 coefficient of earth pressure at rest

$k_{\max}$ coefficient of the maximum bearing stress

K_p coefficient of passive earth pressure

l distance between the seafloor and the point at the depth of the centroid of the soil profile

L distance between the seafloor and the point which is the intersection of the mooring force and the symmetrical axis of the anchor

M_A resultant overturning moment to the point at the depth of the centroid of the soil profile

T_a mooring tension at the attachment point

T_m component of T_a along the failure direction

T_n component of T_a normal to the failure direction

V_0 ultimate vertical pullout capacity

V_{er} vertical component of the ultimate pullout capacity

$V_{\max}$ maximum vertical pullout capacity of the anchor

W' total submerged weight of the anchor and the soil plug inside the anchor

z depth from the seafloor

β failure angle

β_r real failure angle

σ_0 earth pressure at rest

σ_ψ bearing stress corresponding to the angle of the bearing stress to the horizontal axis

$\sigma_{\max}$ maximum bearing stress when applied to horizontal loading

δ interface friction angle

φ internal friction angle

γ' submerged soil weight

λ inclination factor

ψ angle of the bearing stress to the horizontal axis

τ_ψ frictional stress on the anchor surface in the horizontal direction

θ mooring angle

θ_d critical value of the mooring angle when the failure angle starts to increase

θ_u critical value of the mooring angle when the failure angle reaches 90°