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Frequency analysis of wave run-up on vertical cylinder in transitional water depth

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Abstract. Wave run-up is an important issue in offshore engineering, which is tightly related to the loads on the marine structures. In this study, a series of physical experiments have been performed to investigate the wave run-up around a vertical cylinder in transitional water depth. The wave run-ups of regular waves, irregular waves and focused waves have been presented and the characteristics in frequency domain have been investigated with the FFT and wavelet transform methods. This study focuses on the nonlinear features of the wave run-up and the interaction between the wave run-up and the cylinder. The results show that the nonlinear interaction between the waves and the structures might result wave run-up components of higher frequencies. The wave run-ups of the moderate irregular waves exhibit 2nd order nonlinear characteristics. For the focused waves, the incident waves are of strong nonlinearity and the wavelet coherence analysis reveals that the wave run-up at focal moment contains combined contributions from almost all the frequency components of the focused wave sequence and the contributions of frequency components up to 4th order harmonic levels are recommended to be included.

Keywords: wave run-up; transitional water depth; vertical cylinder; model tests; wavelet transform

1. Introduction

Cylindrical offshore structures such as an offshore wind turbine and a spar platform have been attracting researchers' attention for their wide practical applications. For the safety of those marine structures, great efforts have been done to assess the wave loads accurately. With severe sea state, a large wave run-up may largely increases the risk of slamming and green water, and thereby results in heavy loads on structures. Therefore, investigating the wave run-up is essential for accurate evaluation of wave loads.

A large amount of studies on the wave run-up around a cylinder have been performed in the last few decades. Earlier works are mostly relied on theoretical researches, which are based on the diffraction model. MacCamy and Fuchs (1954) originally studied the wave run-up around a cylinder by applying the diffraction theory and proposed an estimation formula for the wave run-up with small wave steepness kA and scattering parameter ka. Kriebel (1987) proposed a more accurate estimation formula by extending the linear diffraction theory to second order. Although part of nonlinear effect has been considered, it cannot yet meet the requirement of engineering

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design.

Besides, varied kinds of estimation formulas were proposed based on velocity stagnation head theory (Hallermeier 1976). The wave run-up is theoretically interpreted as a process that the wave height increases with kinetic energy converting into potential energy as incident waves lash against structures. Niedzwecki and Duggal (1992) introduced a collection factor m for the nonlinear waves. Hence, the semi-empirical formula is given as

$$A_r = \eta_{\max} + mu^2 / 2g \tag{1}$$

where u is the water particle velocity at the wave crest η_{max}, both are obtained from proper wave theory.

Lykke Andersen *et al*. (2011) took the distribution of m values into account for large steepness waves or waves near breaking. The distribution of m values had been studied extensively (De Vos *et al*. 2007, Mase *et al*. 2001, Niedzwecki and Duggal 1992), which made the formula a handy tool to assess the wave run-up.

Researches on wave run-up focus primarily on the regular waves and irregular waves, most of which are moderate. In fact, the wave run-up under the severe sea environment, which is tightly related to the slamming problem, is more destructive. The run-ups of extreme waves can't be simply predicted using the conclusions of regular waves or irregular waves due to its strong nonlinearity. Li *et al*. (2012) investigated the interactions between multi-directional focused wave and vertical bottom-mounted cylinder and presented the significant effect of the wave parameters.

It is worth noting that the assessment formulas based on the velocity stagnation head theory do not take into account the diffraction effect, and the existing researches based on diffraction theory are commonly regarded as not sufficient in predicting the wave run-up. It is not clear as to the effect of which harmonic component of the incident wave should be included for more accurate prediction of the wave run-up, particularly for the extreme waves. Morris-Thomas and Thiagarajan (2005) investigated the harmonic components of the wave run-ups on a fixed vertical cylinder in monochromatic progressive waves and found that the third-harmonic becomes important when kA >0.2. Shu-xue *et al*. (2010) made harmonics analysis of the extreme wave run-up through the addition and subtraction of the crest and trough focusing waves.

In this study, the frequency characteristics of the wave run-up on a vertical circular cylinder in transitional water depth have been investigated by using the FFT and the wavelet transform methods. A series of wave sequences, including regular waves, irregular waves and focused waves, are generated to inspect the wave run-ups. It may provide a reference in selecting an appropriate order for the numerical model.

2. Experimental set-ups and wave parameters

2.1 Experimental set-ups

Experiments were conducted in the wave basin of State Key Laboratory of Ocean Engineering at Shanghai Jiao Tong University. The wave basin measures 50 m in length, 30 m in width and 6 m in depth. An elevating artificial floor is equipped, which makes the water depth adjustable between 0 m to 5 m. A flap paddle is used to generate various kinds of waves and an absorption wave beach stands at the downstream of the basin to eliminate wave reflection.

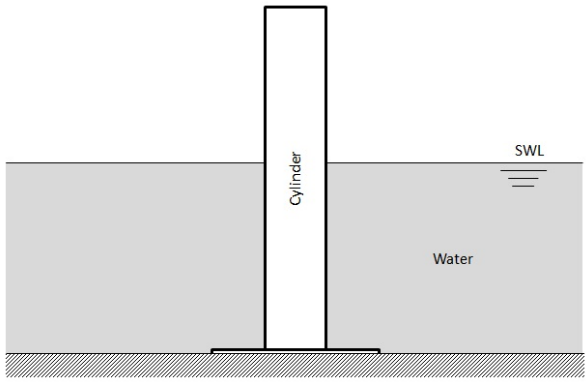

Fig. 1 Sketch of a vertical circular cylinder

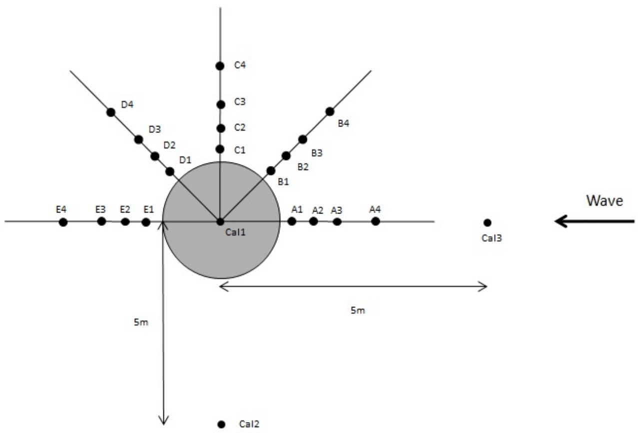

Fig. 2 Locations of wave gauges

For all the experiments, a scaling ratio of 1:40 was selected to scale the wave characteristics and model parameters to those of model scale. A vertical circular cylinder with cross section diameter 0.15 m was fixed on the artificial floor (Fig. 1). In this work, the draft of the cylinder is 0.5 m and the freeboard is 0.4 m.

To inspect the extreme nonlinear wave run-up around the cylinder, five sets of wave gauges were installed along the different radial directions, i.e., 180°, 135°, 90°, 45° and 0° away from wave direction, respectively. And each set contains four wave gauges, which are 0.0234 m, 0.0344 m, 0.0563 m and 0.1000 m away from the wall. Besides, three wave gauges were applied for the wave calibration. One of them was located at the center of the basin and replaced by the cylinder model in formal tests, the other two were located 5 m up-wave of model and 5 m to the side of the model, respectively.

2.2 Wave parameters

A series of regular waves, irregular waves and focused waves were calibrated prior to the formal tests. The regular incident waves were designed with the varied periods 8.0 s, 10.0 s, 12.0 s, 14.0 s in full scale. In the experiment, the wave amplitudes of regular wave sequences were made as large as possible to inspect the wave run-up under steep waves.

Two irregular waves of different wave spectral peak periods (12.0 s and 14.0 s) were generated with the target significant wave height 8.0 m. The target spectrum of the irregular waves is a JONSWAP spectrum, with a spectral shape parameter γ = 2.5. The duration of irregular waves is more than 3 hours in full scale for each case.

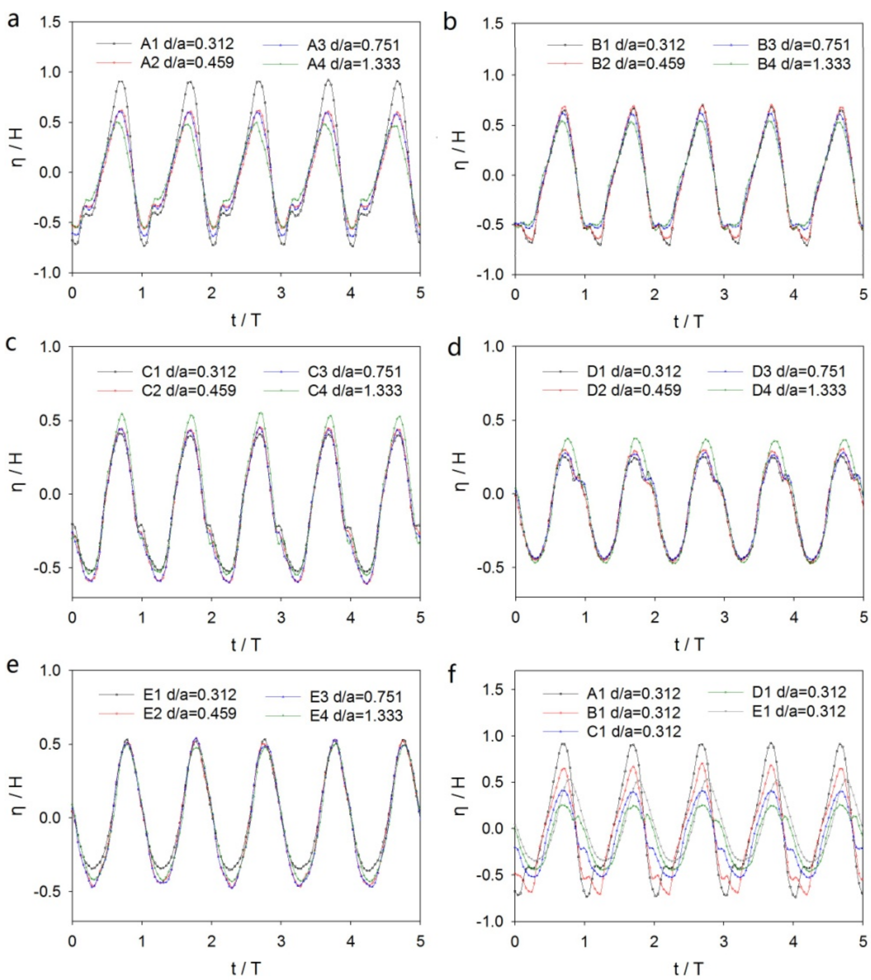

Fig. 3 Normalized wave elevations around the cylinder (*T*=1.265 s, *H*=0.185 m)

For the focused waves, two focused waves were generated based on the model proposed by Kriebel and Alsina (2000), which embedded a transient wave within a realistic background random sea. The two focused waves are of different crest periods (5.0 s and 4.0 s), which are the overlap periods between zero up-crossing and zero down-crossing. The focused waves were controlled to focus at the position where the cylinder located at afterwards. The heights of the focused waves were made as large as possible (i.e., breaking waves).

It is noted that the water depth in the experiment is 20 m in full scale, and the h/L of all the above incident waves are between 1/20 and 1/2. Hence, the water depth condition in this paper is the transitional water depth.

3. Results and discussion

3.1 Wave run-up around the cylinder

In order to inspect the wave run-ups surrounding the cylinder, the regular wave case T = 1.265 s is selected as an example. Fig. 3 presents the wave elevations around the cylinder which have been normalized by the measured wave height H=0.185 m. The normalized surface elevations at A1, B1, C1, D1 and E1 are 0.91, 0.67, 0.42, 0.26 and 0.51, respectively, and that of undisturbed incident wave is 0.59. It is observed that the wave run-up along the 180° direction is the most significant, particularly near the cylinder. Instead, it is the weakest along the 45° direction. The surface elevation along the 0° direction is larger than that along the 45° direction. Besides, since the wave length is considerably larger than the diameter of the cylinder, the phase of wave elevations around the cylinder is almost synchronous, i.e., the crest and trough of them are nearly appear at the same time.

Due to the blocking of the cylinder, the water particles will accumulate in front of the cylinder and run up along the surface with their kinetic energy converting into the potential energy. As a consequence, the surface elevation at A1 gains the largest rise, and that of B1 is the second highest since B1 is located away from the center line but in the unmasked region as well. The surface elevation at D1 is very low as the blocking of cylinder forms an obscured area. However, the surface elevation at E1 is larger due to the superposition of waves from both sides. In fact, wave elevation at A1 is directly related to some strong nonlinear issues such as green water and slamming, etc. Hence, more attention should be paid to the wave run-up at A1.

3.2 Regular wave tests

Fig.4 presents the normalized time series of undisturbed incident waves, with time divided by corresponding wave period and surface elevation divided by corresponding wave height. The wave profiles in transitional water depth are with even troughs and sharp crests, which indicates that the wave sequences in transitional water depth are always of nonlinear characteristics. To obtain an accurate inspection of the nonlinearity, spectral analyses have been conducted for the incident waves.

Fig. 5 presents the normalized wave spectra of the regular incident waves. The fundamental harmonic component (1st order) and the 2nd order harmonic component account for the majority of the wave energy. The zero-order component is also visible for each incident regular wave sequence and the 3rd order harmonic component or above can be almost ignored. Thus, a 2nd order stokes model might give a better description of transitional water depth waves.

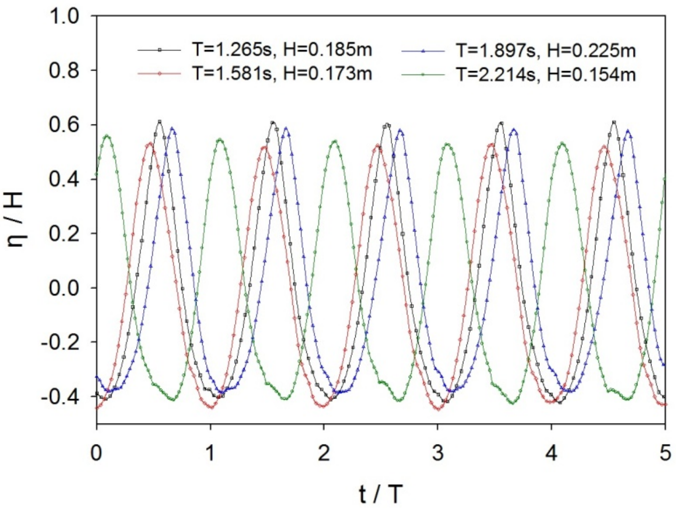

Fig. 4 Normalized time series of undisturbed incident waves

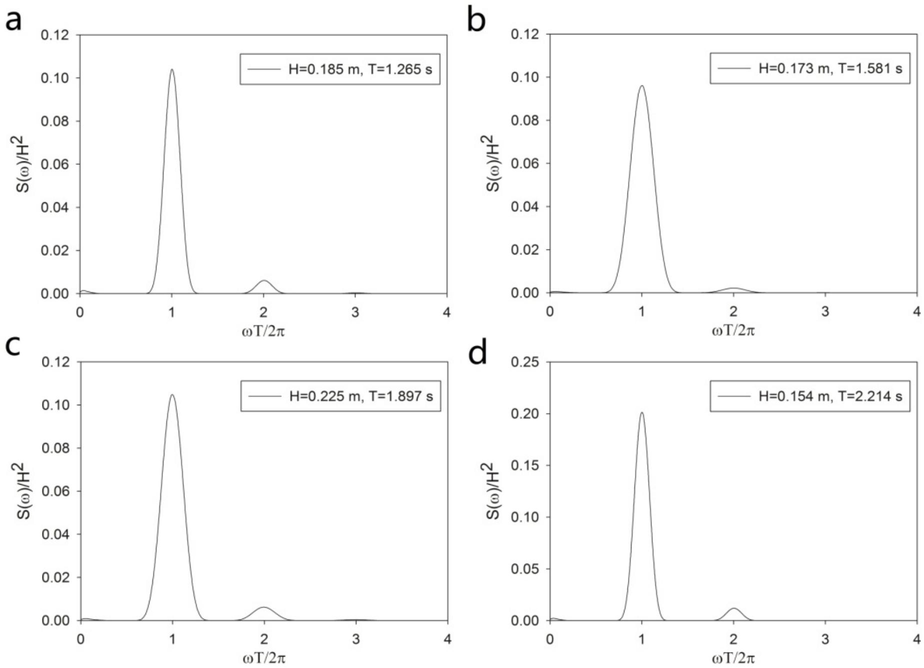

Fig. 5 Normalized incident wave spectra

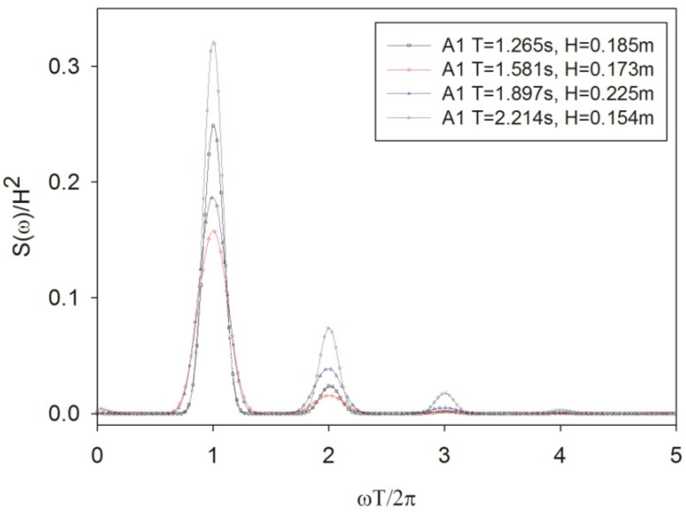

Fig. 6 Normalized spectra of wave run-up at location A1

Fig. 6 shows the normalized spectra of wave run-up at location A1. The wave run-ups contain higher order components and the energy share of 2nd or above is relatively more compared to the incident waves. It means that the wave run-up is a complex process with strong nonlinearity and part of the wave energy transfers towards higher order during this process. Moreover, the wave run-up sequences contain obvious 3rd harmonic components which are not visible in undisturbed wave sequences. The reason may be that the nonlinear interaction between the wave sequences and the cylinder results in higher frequency components of wave run-ups.

In brief, the regular waves in transitional water depth here are of vertical asymmetry and present 2nd order nonlinear characteristics. The interaction between waves and structure may results in higher order frequency components, which should be considered in the numerical model.

3.3 Irregular wave tests

To investigate the interaction between the waves and the cylinder structure and learn more about the changes of the frequency components during wave run-up process, spectral analyses were conducted for the wave elevation sequences here.

The spectral analysis results of undisturbed irregular waves and wave run-ups are presented in Table 1. The significant wave heights here are defined as twice the power spectrum density of wave elevations. The results show that the peak periods of irregular waves change slightly but the significant wave elevation increases significantly.

The normalized spectra of undisturbed waves and wave run-ups are shown in Fig. 7, in which the frequencies and the power spectrum densities are divided by the peak frequencies and the square of the significant wave heights of corresponding undisturbed waves, respectively. It is shown that the wave energy increases at an extensive frequency area compared with undisturbed waves. Specifically, a sharp increase is clearly shown at the peak frequency and a small peak at twice the peak frequency is also observed. This illustrates that the wave run-ups of the moderate irregular waves contain 2nd order nonlinear characteristics.

Table 1 Results of irregular wave cases

Undisturbed waves		Wave elev. at A1	
H_s (m)	T_p (s)	H_s (m)	T_p (s)
0.166	1.87	0.211	1.84
0.173	2.28	0.215	2.28

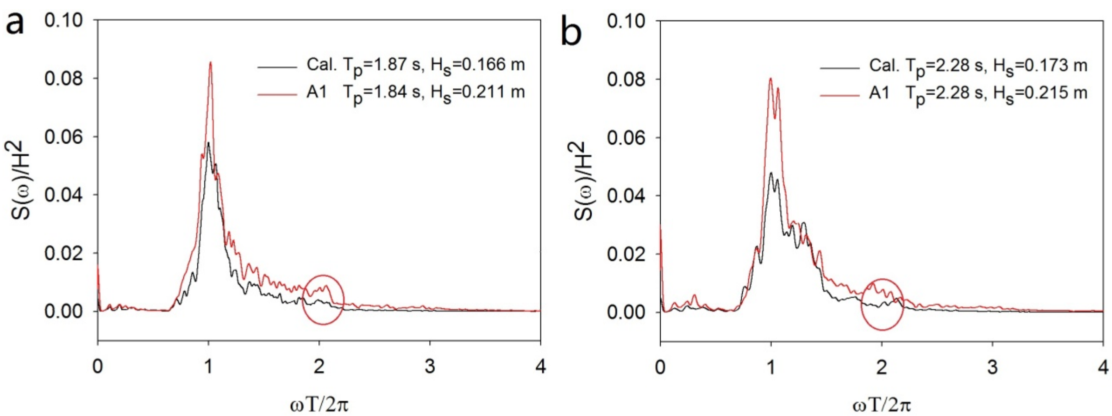

Fig. 7 Normalized spectra of undisturbed waves and wave run-ups

3.4 Focused wave tests

For the focused wave tests, part of the incident wave sequences and wave elevations at A1 are displayed in Fig. 8. In order to investigate the results in frequency domain, spectral analyses were conducted for the whole focused wave sequences. Both spectral and statistical results of the incident waves and the wave elevations at A1 are presented in Table 2. The power density spectra normalized by the incident wave parameters are shown in Fig. 9. Due to the strong nonlinearity of local focused waves, the nonlinearity characteristics nearly extend to 4th order level and the overall high order component energy increases more significantly compared with the moderate irregular wave cases.

Table 2 Results of focused wave cases

Undisturbed waves					Wave run-ups at A1		
H_s (m)	T_p (s)	H_{max} (m)	η_{max} (m)	T_{crest} (s)	H_s (m)	T_p (s)	η_{max} (m)
0.092	2.17	0.323	0.227	0.754	0.129	2.2	0.35
0.134	1.89	0.369	0.274	0.637	0.176	1.89	0.37

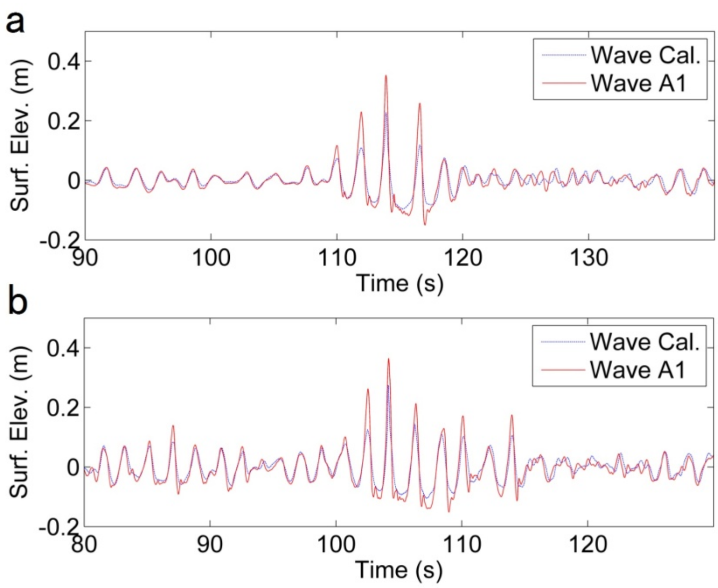

Fig. 8 Calibrated focused waves and wave run-ups at A1 of crest period 0.791s (a) and 0.632s (b)

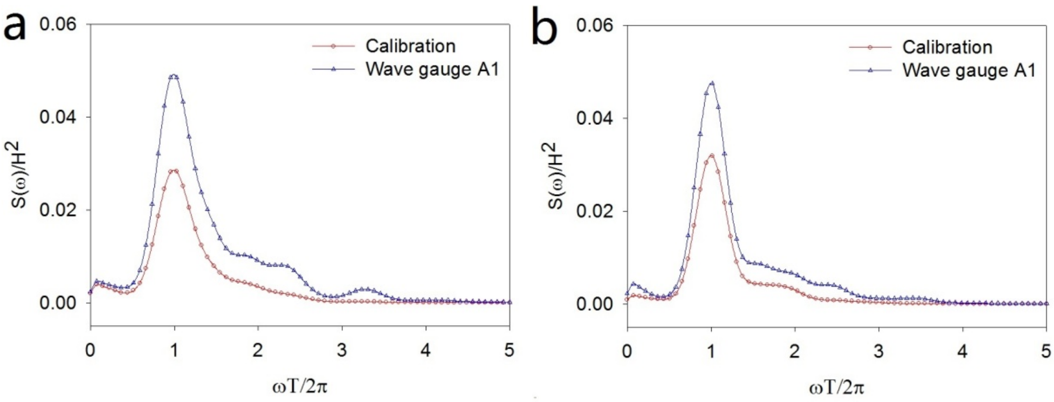

Fig. 9 Normalized spectra of crest period 0.791s (a) and 0.632s (b)

To characterize local features in both time and frequency domains, the wavelet analyses and coherence analyses were applied. The wavelet analysis method has been widely used in the analysis of non-steady processes and the coherence analysis based on wavelet transform has been successfully applied to reveal the correlation between an sudden impact force and a freak wave (Kwon *et al*. 2005). A Morlet wavelet with central frequency $f_c = 0.8125$ Hz was used as a mother wavelet function in this work and the scale factors in wavelet transform were transformed into the frequency values according to the relation

$$f = \frac{f_c f_s}{a} \tag{2}$$

where f_s is the sampling frequency and a is the scale value.

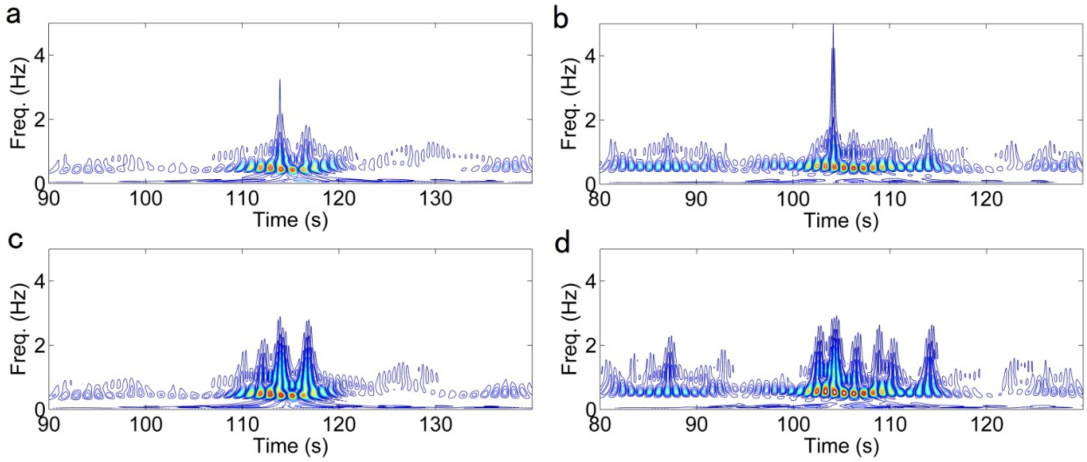

Fig. 10 Wavelet spectra of undisturbed waves (a, b) and wave run-ups (c, d). ((a, c) crest period 0.791s; (b, d) crest period 0.632s)

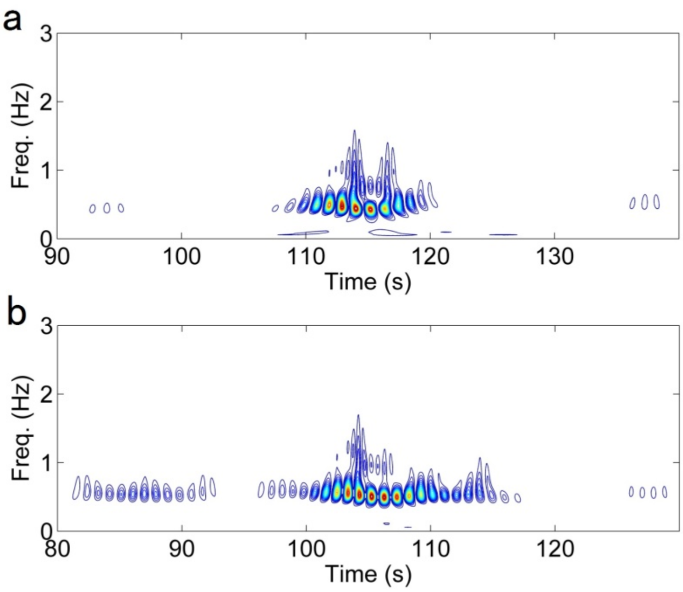

Fig. 11 Cross wavelet spectra with crest period 0.791s (a) and 0.632s (b)

Fig. 10 presents the wavelet spectra of undisturbed waves and wave run-ups. The majority of wave energy is distributed around the wave peak frequency during the entire time history. It is clear that more frequency components appear before and after the focal moment. Meanwhile, the wave energy of wave peak frequency becomes much larger. At the focal moment, the wave run-ups do not necessarily cover wider frequency range compared with the undisturbed waves, but the wave energy of the major frequency region is much larger. The frequency components of wave energy at any time have been presented clearly through the wavelet analysis method, which fully demonstrates the superiority of the wavelet analysis in analyzing those transient phenomena.

The cross wavelet spectra have been presented in Fig. 11. The region where large values exist indicates the significant contributions from both undisturbed waves and wave run-up series. Hence, this region should be given great concern when investigating the wave run-ups of focused waves.

Fig. 12 displays the relative phase information between the undisturbed wave sequences and wave run-up sequences as a function of scale and time. The scales at vertical coordinates have been transformed into the frequencies. The phase information is obtained from the smoothed estimation of the wavelet cross spectrum. The green color represents the two signals are in-phase while the yellow color means the contrary. The blue vertical lines in the figure represents the focal moments of the corresponding focused wave sequences. It is clear that, at the focal moments, the wave run-ups are in-phase with the corresponding focused wave sequences within large frequency range. Therefore, unlike the high-impact force which results from the high frequency components (Kwon *et al*. 2005), the extreme wave run-up at focal moment contains combined contributions from almost all the frequency components of the focused wave sequences. For this reason, the effect from the high frequency components of wave sequences must be included when considering the wave run-up problems under the focused waves.

The wavelet coherence spectra are the most intuitive representation of the correlation between two signals. The wavelet coherence can be interpreted as the local squared correlation coefficient in the time-scale plane. In Fig. 13, the focal moments and the 1st to 4th harmonic frequency levels regarding to the peak spectral frequency are represented with the dotted lines. The coherence values at the focal moments are very close to 1 within 1st to 4th harmonic levels. It means that the frequency components of the 1st to 4th order all make contribution to form such an extreme wave run-up. Hence, it is recommended that the contributions of frequency components up to 4th order harmonic levels should be included when estimating the wave run-ups under nonlinear focused waves.

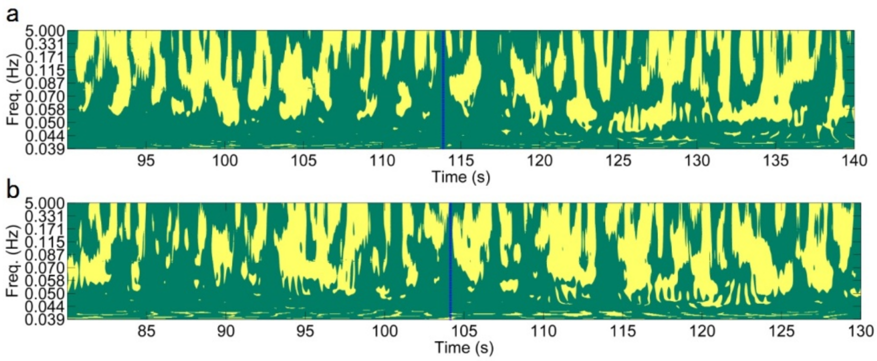

Fig. 12 Angle of cross wavelet transform with crest period 0.791s (a) and 0.632s (b)

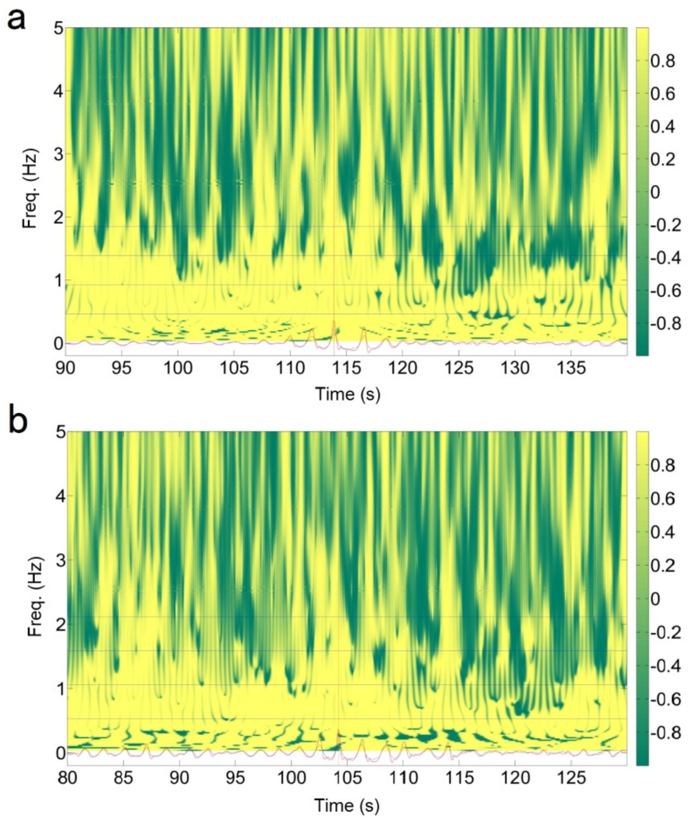

Fig. 13 Wavelet coherence spectra with crest period 0.791s (a) and 0.632s (b)

It should be noted that there exists a small distance between the wave gauge A1 and the cylinder surface for technical difficult issues in the experiment, and stronger nonlinear characteristics of the wave run-up on the cylinder surface is possible. Besides, different nonlinear levels should be considered for waves with different nonlinear characteristics. Further research is needed to make a comprehensive investigation on the wave run-up.

4. Conclusions

Frequency analysis of regular waves, irregular waves and focused waves in transitional water depth and their effects on the wave run-ups around a vertical cylinder were conducted with the FFT and the wavelet transform methods. Some conclusions are drawn as follows.

- The nonlinear interaction between the waves and the structures results in higher frequency components of wave run-ups.
- 2nd order nonlinear characteristics are observed in the wave run-ups of the moderate irregular waves.

- The contributions of frequency components up to the 4th order harmonic levels need to be included when estimating the wave run-ups under focused waves.

The frequency analysis in this paper presents the characteristics of wave run-ups in transitional water depth. It can provide us a reference for the selection of the order of theoretical model. Moreover, the wavelet coherence method applied in this work has been proved to be an efficient tool to analyze the correlation between the two transient phenomena.

Acknowledgments

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