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Representation of fundamental solution and vibration of waves in photothermoelastic under MGTE model

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Abstract. In this paper, Moore-Gibson-Thompson theory of thermoelasticity is considered to investigate the fundamental solution and vibration of plane wave in an isotropic photothermoelastic solid. The governing equations are made dimensionless for further investigation. The dimensionless equations are expressed in terms of elementary functions by assuming time harmonic variation of the field variables (displacement, temperature distribution and carrier density distribution). Fundamental solutions are constructed for the system of equations for steady oscillation. Also some preliminary properties of the solution are explored. In the second part, the vibration of plane waves are examined by expressing the governing equation for two dimensional case. It is found that for the non-trivial solution of the equation yield that there exist three longitudinal waves which advance with the distinct speed, and one transverse wave which is free from thermal and carrier density response. The impact of various models (i)Moore-Gibson-Thomson thermoelastic (MGTE)(2019), (ii) Lord and Shulman's (LS)(1967) , (iii) Green and Naghdi type-II(GN-II)(1993) and (iv) Green and Naghdi type-III(GN-III)(1992) on the attributes of waves i.e., phase velocity, attenuation coefficient, specific loss and penetration depth are elaborated by plotting various figures of physical quantities. Various particular cases of interest are also deduced from the present investigations. The results obtained can be used to delineate various semiconductor elements during the coupled thermal, plasma and elastic wave and also find the application in the material and engineering sciences.

Keywords: fundamental solution; Moore-Gibson-Thompson thermoelastic model; photothermoelastic isotropic; plane waves; steady oscillations

1. Introduction

Study of mechanical and thermal interaction within a solid medium is of emended significance in various scientific fields. There are few examples such as high energy particle accelerated devices, modern aeronautical and astronomical engineering and different system exploited in nuclear and industrial applications with the consideration of second sound effect in thermoelastic model plays a significant role in analysing elastic body with in a variety of scientific and technological fields. In contradiction with physical observation the infinite thermal propagation speed is observed through conventional uncoupled theories.The coupled thermoelasticity proposed by Biot (1956) in order to

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eradicate the classic uncoupled principle's inherent paradox. Generalized thermoelasticity theories are designed to solve the weaknesses and shortcomings inherent in classic dynamic thermoelasticity coupled theory. Lord and Shulman (1967) and Green and Lindsay (1972) developed generalized theory of thermoelasticity involving one and two relaxation parameters.

Green and Naghdi (1991, 1992, 1993) derived three models in thermoelasticity which are labelled as GN-I, II and III models. The linearized form of model-I reduces to classical heat conduction theory whereas linearized version of model-II and III permit propagation of thermal waves at finite speed.GN-II (1993) shows a feature which makes it different from other thermoelastic models as it does not allow dissipation of thermal energy. The model GN-III (1992) contains the thermal displacement gradient alongwith temperature gradient among the constitutive variables and admits the dissipation of energy. Tzou (1995) proposed the dual-phase heat conduction law which is a more common one with two different phase delays, one in the heat flow vector and the second in the temperature gradient, which takes into account the effects of the microstructure on the heat transmission mechanism, in order to evaluate the delayed reaction caused by the microstructure effects over time. One of the most recent advances in the theory of thermoelasticity is the three-phase lags suggested by Roychoudhari (2007). This model also has phase delays of thermal displacement gradients, in addition to the phase lags in the hot flux vector and temperature gradient. These two suggestions, involving different derivatives as the Taylor spectrum approaches the heat flow and temperature gradients, assume that the suggestion by Roychoudhari seeks to restore Green and Naghdi models if various Taylor approaches are taken into account.

Abbas and Abd-alla (2008) investigated the thermoelastic interactions in an infinite orthotropic elastic medium with a cylindrical cavity subjected to ramp-type heating applied to the boundary of the cavity.Abbas (2011) discussed the influence of reinforcement on the total deformation body by applying Green and Naghdi theory. Marin *et al.* (2014) studied the basic equations and conditions of the mixed initial boundary value problem in the context of micropolar thermoelastic diffusion, which is an extension of known Saint-Venant's principle from classical Elasticity. Zenkour and Abbas (2014) analysed the nonlinear transient thermal stress of temperature dependent infinite cylinders subjected to a decaying with time thermal loading. Abbas (2015) studied the natural frequencies, thermoelastic damping and frequency shift of a thermoelastic hollow sphere into the context of the generalized thermoelasticity theory with one relaxation time. Abbas *et.al.* (2016) examined the propagation of waves in thermoelastic plate in the context LS theory and obtained an analytical solution for the temperature, displacement components, and stresses using the eigenvalue approach. Abbas and Kumar (2016) studied the plane problem in initially stressed thermoelastic half-space with voids due to thermal source. Ghanmi and Abbas (2019) introduced the bioheat equation under fractional derivatives to study the thermal damage within the skin tissue during the thermal therapy.

The semiconducting materials were used widely in modern engineering, with the development of technologies. The study of wave propagation in a semiconducting medium will have important academic significance and application value. Of recent interest is the relevance of the excitation of short elastic pulses (high-frequency elastic waves) by photothermal means to several areas of applied physics including the photoacoustic microscope, thermal wave imaging, determination of thermoelastic material parameters, non-destructive evaluation of devices, monitoring of laser drilling, and laser annealing and melting phenomena in semiconductors. When a semiconductor surface is exposed to a beam of laser, some electrons will be excited. In this case, the photo-excited free carriers will be produced with non-radiative transitions, and a recombination between electron and hole plasma occurs. Many efforts are made to explore the nature of semiconductors in last few years. The technique adopted is photo acoustic and photo thermal technology.

Photoacoustic (PA) and photothermal (PT) science and technology have extensively developed new methods in the investigation of semiconductors and microelectronic structures during the last few years. PA and PT techniques were recently established as diagnostic methods with good sensitivity to the dynamics of photoexcited carrier (Mandelis 1987, Almond and Patel 1996, Mandelis and Michaelian 1997, Nikolic and Todorovic 1989). Photogeneration of electron-hole pairs, i.e., the carriers-diffusion wave or plasma wave, generated by an absorbed intensity modulated laser beam, may, play a dominant role in PA and PT experiments for most semiconductor materials. Depth dependent plasma waves contribute to the generation of periodic heat and mechanical vibrations, i.e., thermal and elastic waves. This mechanism of elastic wave generation is a specific of semiconductors. The electronic deformation mechanism is based on the fact that photogenerated plasma in the semiconductor causes deformation of the crystal lattice, i.e., deformation of the potential of the conduction and valence bands in the semiconductor. Thus, photoexcited carries may cause local strain in the sample. This strain in turn may produce plasma waves in the semiconductor in a manner analogous to thermal wave generation by local periodic elastic deformation.

The difference influences of the thermoelastic and electronic deformations in semiconductor media with disregard the coupling between the plasma and the thermoelastic equations have been analyzed by numerous researchers (McDonald and Wetsel 1978, Jackson and Amer 1980, Stearns and Kino 1985). Todorovic (2003a, b, 2005) presented the theoretical analysis to describe two phenomena that provide information about the properties of transport and carrier recombinations in the semiconducting medium. The changes in the propagations of thermal and plasma waves go back to the linear coupling between the thermal and the mass transport (i.e., thermodiffusion) have included. Sharma (2010) investigated the boundary value problems in generalized thermodiffusive elastic medium. Sharma and Sharma (2014) investigated the temperature fluctuations in tissues based on Penne’s bio-heat transfer equation. Hobiny and Abbas (2019) investigated the photothermal interactions in a two-dimensional semiconducting half-space under the coupled of thermo-elastic theory and plasma wave based on Green and Naghdi theory. Abbas *et al*. (2020) examined the effect of the variability of thermal conductivity in semi-conductor media with cylindrical cavity using the eigen value methods. Marin *et al.* (2021) analysed a new picture of the porothermoelastic model using the fractional calculus with thermal relaxation times. Sharma and Kumar (2021) developed a dynamic mathematical model of photothermoelastic (semiconductor) medium to analyse the deformation due to inclined loads. Sharma and Kumar (2022) examined photothermoelastic deformation in dual phase lag model due to concentrated inclined load. Kumar *et al.* (2022) investigated deformation due to thermomechanical carrier density loading in orthotropic photothermoelastic plate.

The Moore-Gibson-Thompson theory of thermoelasticity has received immense level of concern in recent years. This theory starting from a third-order differential equation and built in the context of some considerations related to fluid mechanics by Thompson (1972). Quintanilla (2019) presented a Moore-Gibson-Thompson thermoelasticity in which the heat conduction equation is described by MGT equation. This equation is obtained by incorporating relaxation parameter in the GN-III (1992) model. Conti *et al.* (2020) explored thermoelasticity of MGT type with history dependence in the temperature. Conti *et al.* (2020a) presented the analyticity of viscoelastic plate under MGT model of thermoelasticity. Quintanilla (2020) proposed a new thermoelastic model of MGT heat conduction equation with two temperature and examine some basic theorems. Pellicer and Quintanilla (2020) examined the uniqueness and instability of some thermomechanical problems based on MGT theory of thermoelasticity.

Bazarra *et al.* (2020) examined a thermoelastic problem numerically where the heat conduction law is modelled by using Moore-Gibson-Thompson equation. Marin (2020) presented mixed initial-boundary value problem in the context of the Moore-Gibson-Thompson theory of thermoelasticity for dipolar bodies. Abouelregal *et al.* (2021) presented a modified Moore–Gibson–Thompson photo-thermoelastic model for a rotating semiconductor half-space under magnetic field. Kumar *et al.* (2022) studied the deformation due to thermomechanical and carrier density loading in orthotropic photothermoelastic plate under Moore-Gibson-Thompson thermoelastic model. Sharma *et al.* (2013b) studied the wave propagation in anisotropic thermoviscoelastic medium in the context Green-Naghdi theories of type-II and type-III. The concept of fundamental solutions has significant role in investigation of various problem of mathematical physics, which are encountered in many mathematical, mechanical, physical and engineering applications. The applications of fundamental solutions to a recently developed area of boundary value method has provided a corporeal advantage, is that an integral representation of the solution to a boundary value problem (BVP) in terms of fundamental solution can be solved more easily by numerical methods with respect to differential equation having specific boundary and initial conditions. Several methods are known for constructing fundamental solutions of the system of differential equations, theory of elasticity and thermoelasticity, which are given in the books [Kupradze (1979), Nowacki (1962,1975)]. For a historical and bibliographical material on the fundamental solutions of partial differential equation is also available in the books [Hörmander (1983), Kythe (1996).]

Sharma *et al.* (2013a) investigated the propagation of plane waves and fundamental solution in a homogeneous isotropic electro-microstretch elastic solids. Sharma *et al.* (2014) investigated the propagation of plane waves and fundamental solution of homogeneous isotropic electro-microstretch viscoelastic solids. Svanadze (2017) constructed the fundamental solution and uniqueness theorems in the linear theory of thermoviscoelasticity for solids with double porosity. Kumar *et al.* (2020) constructed the fundamental solution of the system of differential equations in bio-thermoelasticity with dual phase lag in case of steady oscillations. Kumar *et al.* (2021) constructed the basic theorem in terms of elementary function which analyse the behaviour of non-local and dual phase lag model and determine the existence of longitudinal and transverse wave. El-Bary and Atef (2021) obtained the fundamental solution of generalized magneto thermo viscoelasticity with two relaxation times for perfect isotropic conduction. Kumar and Batra (2022) investigated the fundamental solution and propagation of plane waves in swelling porous thermoelastic medium involving mixtures of solid, fluid, and gas.

In this paper, the fundamental solution and propagation of plane waves in photothermoelastic under Moore-Gibson-Thompson model has been studied. The representation of fundamental solution of system of equations in the case of study oscillations is considered in terms of elementary functions. Some basic properties of the fundamental solution are also established. The phase velocity, attenuation coefficient, specific loss and penetration depth of plane waves for MGTE (2019), LS (1967), GN-II (1993) and GN-III (1992) models are computed and presented graphically with respect to frequency.

2. Basic equations

Let $x=(x_1,x_2,x_3)$ be the point of the Euclidean three- dimensional space E^3.

$|x| = \left(x_1^2 + x_2^2 + x_3^2\right)^{1/2}$, $D_x = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$ and let t denote the time variable.

Following (Todorovic 2003b, Quintanilla 2019), the basic equations for homogeneous isotropic photothermoelastic based on Moore-Gibson-Thompson heat equation in absence of body force, heat source and carrier photogeneration sources are

$$(\lambda + \mu)u_{j,ij} + \mu u_{i,jj} - \gamma_t T_{,i} - \gamma_n N_{,i} = \rho \ddot{u}_i \,, \tag{1}$$

$$K\dot{T}_{,ii} + K^* T_{,ii} = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left(\rho C_e \ddot{T} + T_o \gamma_t \ddot{e}_{kk} - \frac{E_g}{\tau}\frac{\partial N}{\partial t}\right), \tag{2}$$

$$D_e N_{,ij} - \frac{\partial N}{\partial t} - \frac{N}{\tau} + \zeta \frac{T}{\tau} = 0. \qquad (\text{i, j, k} = 1, 2, 3) \tag{3}$$

where

λ *and* μ are Lame's constants, *T*- the temperature distribution, T_o the reference temperature, u_i components of displacement, ρ - the medium density, *K* thermal conductivity, K^*thermal conductivity rate, D_e the coefficients of carrier diffusion, C_e the specific heat, $N = n - n_o$, n_o equilibrium carrier concentration, E_g the semiconductor energy gap, $\gamma_n = (3\lambda + 2\mu)\alpha_n, \alpha_n$ is coefficient of electronic deformation, $\gamma_t = (3\lambda + 2\mu)\alpha_t, \alpha_t$ is the linear thermal expansion coefficient. $\zeta = \frac{\partial n_o}{\partial T}$ the thermal activation coupling parameter, τ_o the thermal relaxation time, τ - the photogenerated carrier lifetime, t - the time variable.

Following dimensionless parameters are taken as

$$\left(x_1', x_2', x_3', u_1', u_2', u_3'\right) = \eta_1 C_o \left(x_1, x_2, x_3, u_1, u_2, u_3\right) \,,\; (t', \tau_o', \tau') = \eta_1 C_o^2 (t, \tau_o, \tau) \,,\; T' = \frac{\gamma_t T}{\rho C_o^2} \,,\; N' = \frac{N}{n_o} \tag{4}$$

where

$$\eta_1 = \frac{\rho C_e}{K}, \; C_o^2 = \frac{\lambda + 2\mu}{\rho}$$

Eqs. (1)-(3) by considering Eq. (4) take the form (after removing primes)

$$g_1 grad\, div\, \boldsymbol{u} + g_2 \Delta \boldsymbol{u} - grad\, T - g_3\, grad\, N = \ddot{\boldsymbol{u}}, \tag{5}$$

$$\Delta\dot{T} + g_4 \Delta T = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left[\ddot{T} + g_5\, div\, \ddot{\boldsymbol{u}} - \frac{g_6}{\tau}\dot{N}\right], \tag{6}$$

$$g_8 \frac{T}{\tau} + \Delta N - g_7 \dot{N} - g_7 \frac{N}{\tau} = 0, \tag{7}$$

where

$$g_1 = \frac{\lambda+\mu}{\lambda+2\mu}, g_2 = \frac{\mu}{\lambda+2\mu}, g_3 = \frac{\gamma_n n_o}{\lambda+2\mu},$$
$$g_4 = \frac{K^*}{K\eta_1 C_o^2}, g_5 = \frac{T_o\gamma_t^2}{K\eta_1 C_o^2\rho}, g_6 = \frac{E_g n_o \gamma_t}{K\eta_1 \rho C_o^2},$$
$$g_7 = \frac{1}{\eta_1 D_e}, g_8 = \frac{\zeta\rho C_o^2}{\gamma_t D_e n_o \eta_1}.$$

3. Steady oscillation

For the case of steady oscillation, we assume the displacement vector, temperature distribution and carrier density distribution as

$$\left(\boldsymbol{u}(\boldsymbol{x},t), T(\boldsymbol{x},t), N(\boldsymbol{x},t)\right) = \mathrm{Re}\left[\left(\boldsymbol{u}, T, N\right)e^{-i\omega t}\right] \tag{8}$$

where ω is oscillation frequency and $\omega > 0$.

Using Eq. (8) into Eqs. (5)-(7), reduce the system of equation of steady oscillations as

$$g_1 grad\ div\ \boldsymbol{u} + \left(g_2\Delta + \omega^2\right)\boldsymbol{u} - grad\,T - g_3\, grad\, N = 0\ , \tag{9}$$

$$g_{10}\, div\,\boldsymbol{u} + \left(g_{11} + g_{12}\Delta\right)T + \frac{g_{13}}{\tau}N = 0 \tag{10}$$

$$\frac{g_8}{\tau}T + \left(\Delta + \frac{g_{14}}{\tau}\right)N = 0, \tag{11}$$

where

$g_9 = 1 - i\omega\tau_o$, $g_{10} = \omega^2 g_5 g_9$, $g_{11} = \omega^2 g_9$, $g_{12} = -i\omega + g_4$, $g_{13} = -i\omega g_6 g_9$,
$g_{14} = (i\tau\omega - 1)g_7$.

Introducing the matrix differential operator

$$\boldsymbol{F}(\boldsymbol{D}_x) = \left\|\boldsymbol{F}_{gh}\left(\boldsymbol{D}_x\right)\right\|_{5\times5}, \tag{12}$$

where

$$\boldsymbol{F}_{mn}\left(\boldsymbol{D}_x\right) = \left(g_2\Delta + \omega^2\right)\delta_{mn} + g_1\frac{\partial^2}{\partial x_m \partial x_n}, \quad \boldsymbol{F}_{m4}\left(\boldsymbol{D}_x\right) = -\frac{\partial}{\partial x_m}, \boldsymbol{F}_{4n}\left(\boldsymbol{D}_x\right) = g_{10}\frac{\partial}{\partial x_n},$$
$$\boldsymbol{F}_{44}\left(\boldsymbol{D}_x\right) = g_{11} + g_{12}\Delta$$
$$\boldsymbol{F}_{55}\left(\boldsymbol{D}_x\right) = \Delta + \frac{g_{14}}{\tau}, \boldsymbol{F}_{45}\left(\boldsymbol{D}_x\right) = \frac{g_{13}}{\tau},$$
$$\boldsymbol{F}_{54}\left(\boldsymbol{D}_x\right) = \frac{g_8}{\tau}.$$

δ_{mn} is kronecker delta function.

The system of Eqs. (9)-(11) can be rewritten as

$$\boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{U}(\boldsymbol{x}) = 0, \tag{13}$$

where

$\boldsymbol{U} = (\boldsymbol{u}, T, N)$ is a five components vector function on E^3.

we assume that

$$g_2 g_{12} \neq 0. \tag{14}$$

Definition. The fundamental solution of the system of Eqs. (9)-(11) (the fundamental matrix of operator $\boldsymbol{F}$) is the matrix $\boldsymbol{G}(\boldsymbol{x}) = \left\| G_{gh}(\boldsymbol{x}) \right\|_{5\times5}$ satisfying condition (Hörmandertal 1963)

$$\boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{G}(\boldsymbol{x}) = \delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}) \tag{15}$$

where δ is the Dirac delta, $\boldsymbol{I} = \left\| \delta_{gh} \right\|_{5\times5}$ is the unit matrix and $\boldsymbol{x} \in E^3$.

Now we construct $\boldsymbol{G}(\boldsymbol{x})$ in terms of elementary functions.

4. Representation of fundamental solutions

We consider the system of equations

$$g_1 grad\ div\ \boldsymbol{u} + \left(g_2\Delta + \omega^2\right)\boldsymbol{u} + g_{10} grad\, T = \boldsymbol{H}, \tag{16}$$

$$-div\,\boldsymbol{u} + (g_{11} + g_{12}\Delta)T + \frac{g_8}{\tau}N = L\ , \tag{17}$$

$$-g_3\, div\,\boldsymbol{u} + \frac{g_{13}}{\tau}T + \left(\Delta + \frac{g_{14}}{\tau}\right)N = M\ . \tag{18}$$

where $\boldsymbol{H}$ in Eq. (16) are two vector function on E^3 and L & M are scalar functions on E^3.

The system of Eqs. (16)-(18) can be written in the form

$$\boldsymbol{F}^{tr}(\boldsymbol{D}_x)\boldsymbol{U}(\boldsymbol{x}) = \boldsymbol{Q}(\boldsymbol{x}), \tag{19}$$

where $\boldsymbol{F}^{tr}$ is the transpose of matrix $\boldsymbol{F}, \boldsymbol{Q} = (\boldsymbol{H}, L, M)$ and $\boldsymbol{x} \in E^3$.

Applying the operator div to Eq. (16), we obtain

$$\left(\Delta + \omega^2\right)div\,\boldsymbol{u} + g_{10}\Delta T = div\,\boldsymbol{H}, \tag{20}$$

Eqs. (20), (17) and (18) may be written in the form

$$N(\Delta)\boldsymbol{S} = \boldsymbol{\psi}, \tag{21}$$

where
$\boldsymbol{S} = (div\boldsymbol{u}, T, N)$ *and* $\boldsymbol{\psi} = (\psi_1, \psi_2, \psi_3) = (divH, L, M)$
and

$$N(\Delta) = \|N_{mn}\|_{3\times3} = \begin{Vmatrix} \Delta + \omega^2 & g_{10}\Delta & 0 \\ -1 & g_{11} + g_{12}\Delta & \dfrac{g_8}{\tau} \\ -g_3 & \dfrac{g_{13}}{\tau} & \Delta + \dfrac{g_{14}}{\tau} \end{Vmatrix}_{3\times3}, \tag{22}$$

Eq. (21) implies

$$\Gamma_1(\Delta)\boldsymbol{S} = \tilde{\boldsymbol{\psi}} \tag{23}$$

also $\tilde{\boldsymbol{\psi}} = (\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_3)$ and $\tilde{\psi}_n = \frac{1}{g_{12}}\sum_{m=1}^{3} N^*_{mn}\psi_m$, $\Gamma_1(\Delta) = \frac{1}{g_{12}}\det N(\Delta)$; $n = 1,2,3$. N^*_{mn} is the cofactor of the elements N_{mn} of the matrix N.

From Eqs. (21) and (23), we notice that

$$\Gamma_1(\Delta) = \prod_{m=1}^{3}(\Delta + \lambda_m^2)\ , \tag{24}$$

where λ_m^2 , m=1,2,3 are the roots of the equation $\Gamma_1(\Delta)$ or $\Gamma_1(-\kappa)$ =0 (w.r.t. κ)

Now applying the operator $\Gamma_1(\Delta)$ to Eq. (16), yield

$$\Gamma_1(\Delta)\left(g_2\Delta + \omega^2\right)\boldsymbol{u} = \Gamma_1(\Delta)\left(-g_1 grad\ div\ \boldsymbol{u} - g_{10} grad\ T + \boldsymbol{H}\right),$$

$$\Gamma_1(\Delta)\left(g_2\Delta + \omega^2\right)\boldsymbol{u} = -g_1 grad\,\psi_1 - g_{10} grad\ \psi_2 + \Gamma_1(\Delta)\boldsymbol{H}, \tag{25}$$

Eq. (25) can be written as

$$\Gamma_1(\Delta)\Gamma_2(\Delta)\boldsymbol{u} = \boldsymbol{\psi}^* \tag{26}$$

where

$$\Gamma_2(\Delta) = \frac{1}{g_2}\det\begin{Vmatrix} \Delta & -\dfrac{\omega}{g_2} \\ \omega & 1 \end{Vmatrix}_{2\times2}, \tag{27}$$

and

$$\boldsymbol{\psi}^* = \frac{1}{g_2}\left\{-g_1 grad\,\psi_1 - g_{10} grad\ \psi_2 + \Gamma_1(\Delta)\boldsymbol{H}\right\}, \tag{28}$$

It can be seen that

$$\Gamma_2(\Delta) = (\Delta + \lambda_4^2)$$

where λ_4^2 is a root of the equation $\Gamma_2(-\kappa) = 0$ (w.r.t. κ)

On the basis of Eqs. (21) and (26), we obtain

$$\boldsymbol{\Theta}(\Delta)\boldsymbol{U}(\boldsymbol{x}) = \hat{\boldsymbol{\psi}}(\boldsymbol{x}), \tag{29}$$

where

$$\hat{\boldsymbol{\psi}}(x) = (\boldsymbol{\psi}^*, \tilde{\psi}_2, \tilde{\psi}_3),$$
$$\boldsymbol{\Theta}(\Delta) = \left\| \boldsymbol{\Theta}_{gh}(\Delta) \right\|_{5\times5},$$
$$\boldsymbol{\Theta}_{mm}(\Delta) = \Gamma_1(\Delta)\Gamma_2(\Delta) = \Gamma_1(\Delta)(\Delta + \lambda_4^2),$$
$$\boldsymbol{\Theta}_{gh}(\Delta) = 0, \boldsymbol{\Theta}_{55}(\Delta) = \boldsymbol{\Theta}_{44}(\Delta) = \Gamma_1(\Delta), m = 1,2,3,4 \quad g,h = 1,2,3,4,5 \quad g \neq h. \tag{30}$$

From Eqs. (23) and (28), we find

$$\psi^* = q_{11}(\Delta) grad\, div \boldsymbol{H} + \frac{1}{g_{12}}\Gamma_1(\Delta)\boldsymbol{H} + q_{21}(\Delta) grad\, L + q_{31}(\Delta) grad\, M, \tag{31}$$

$$\psi_2 = q_{12}(\Delta) div \boldsymbol{H} + q_{22}(\Delta)L + q_{32}(\Delta)M, \tag{32}$$

$$\psi_3 = q_{13}(\Delta) div \boldsymbol{H} + q_{23}(\Delta)L + q_{33}(\Delta)M, \tag{33}$$

where

$$q_{11}(\Delta) = \frac{1}{g_2 g_{12}}(-g_1 N_{11} - g_{10} N_{12}),\ q_{21}(\Delta) = \frac{1}{g_2 g_{12}}(-g_1 N_{21} - g_{10} N_{22}),$$

$$q_{31}(\Delta) = \frac{1}{g_2 g_{12}}(-g_1 N_{31} - g_{10} N_{32}),\ q_{12}(\Delta) = \frac{1}{g_{12}} N_{12},\ q_{22}(\Delta) = \frac{1}{g_2 g_{12}} N_{22},$$

$$q_{32}(\Delta) = \frac{1}{g_2 g_{12}} N_{32},\ q_{13}(\Delta) = \frac{1}{g_2 g_{12}} N_{13},\ q_{23}(\Delta) = \frac{1}{g_2 g_{12}} N_{23},\ q_{33}(\Delta) = \frac{1}{g_2 g_{12}} N_{33}.$$

From Eqs. (31)-(33), we have

$$\hat{\boldsymbol{\psi}} = \boldsymbol{R}^{tr}(\boldsymbol{D}_x)\boldsymbol{Q}(\boldsymbol{x}), \tag{34}$$

where

$\boldsymbol{R}^{tr}$ is the transpose of the matrix R and $\boldsymbol{R} = \left\| R_{gh} \right\|_{5\times5}$,

$$R_{mn}(\boldsymbol{D}_x) = \frac{1}{g_2}\Gamma_1(\Delta) + q_{11}(\Delta)\frac{\partial^2}{\partial x_m \partial x_n},\ R_{m5}(\boldsymbol{D}_x) = q_{13}(\Delta)\frac{\partial}{\partial x_m},\ R_{5n}(\boldsymbol{D}_x) = q_{31}(\Delta)\frac{\partial}{\partial x_n},$$

$$R_{4n}(\boldsymbol{D}_x) = q_{21}(\Delta)\frac{\partial}{\partial x_n},\ R_{55}(\boldsymbol{D}_x) = q_{33}(\Delta),\ R_{44}(\boldsymbol{D}_x) = q_{22}(\Delta).\ m,n = 1,2,3. \tag{35}$$

Also, from Eqs. (19), (29) and (34), we obtain

$$\boldsymbol{\Theta U} = \boldsymbol{R}^{tr}\boldsymbol{F}^{tr}\boldsymbol{U} \tag{36}$$

It implies that

$$\boldsymbol{\Theta} = \boldsymbol{R}^{tr}\boldsymbol{F}^{tr},$$
$$\boldsymbol{\Theta}(\Delta) = \boldsymbol{R}(\boldsymbol{D}_x)\boldsymbol{F}(\boldsymbol{D}_x), \tag{37}$$

We assume that

$$\lambda_m^2 \neq \lambda_n^2 \neq 0, m,n = 1,2,3,4 \quad m \neq n. \tag{38}$$

Let

$$Y(\boldsymbol{x}) = \left\|Y_{rs}(\boldsymbol{x})\right\|_{5\times 5}, Y_{mm}(\boldsymbol{x}) = \sum_{n=1}^{4} r_{1n}\zeta_n(\boldsymbol{x}),$$
$$Y_{vw}(\boldsymbol{x}) = 0, \tag{39}$$
$$m = 1,2,3,4 \ and\ v,w = 1,2,3,4,5,\ v \neq w.$$

where

$$\zeta_n(\boldsymbol{x}) = \frac{-\exp(i\lambda_n|\boldsymbol{x}|)}{4\pi|x|}, n = 1,2,3,4. \tag{40}$$

$$r_{ml} = \prod_{\substack{m=1\\ m\neq l}}^{4}\left(\lambda_m^2 - \lambda_l^2\right)^{-1}, l = 1,2,3,4, \tag{41}$$

$$r_{mv} = \prod_{\substack{m=1\\ m\neq v}}^{4}\left(\lambda_m^2 - \lambda_v^2\right)^{-1}, v = 3,4. \tag{42}$$

We will prove the following lemma:

Lemma: The matrix $\boldsymbol{Y}$ defined above is the fundamental matrix of operator $\boldsymbol{\Theta}(\Delta)$, which is

$$\boldsymbol{\Theta}(\Delta)\boldsymbol{Y}(\boldsymbol{x}) = \delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}), \tag{43}$$

Proof: To prove the lemma, it is sufficient to prove that

$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(\boldsymbol{x}) = \delta(\boldsymbol{x}), \tag{44}$$

We find that

$$r_{11}+r_{12}+r_{13}+r_{14}=0,$$
$$r_{12}(\lambda_1^2-\lambda_2^2)+r_{13}(\lambda_1^2-\lambda_3^2)+r_{14}(\lambda_1^2-\lambda_4^2)=0,$$
$$r_{13}(\lambda_1^2-\lambda_3^2)(\lambda_2^2-\lambda_3^2)+r_{14}(\lambda_1^2-\lambda_4^2)(\lambda_2^2-\lambda_4^2)=0,\ r_{14}(\lambda_1^2-\lambda_4^2)(\lambda_2^2-\lambda_4^2)(\lambda_3^2-\lambda_4^2)=1,$$
$$(\Delta+\lambda_m^2)\zeta_n(\boldsymbol{x})=\delta(\vec{x})+(\lambda_m^2-\lambda_n^2)\zeta_n(\boldsymbol{x}),\ \text{m,n=1,2,3,4.} \tag{45}$$

Now consider

$$\begin{aligned}\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(\boldsymbol{x}) &= (\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\\ &\quad(\Delta+\lambda_4^2)\sum_{n=1}^{4}r_{1n}\left[\delta+(\lambda_1^2-\lambda_n^2)\zeta_n\right],\\ &=(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)\sum_{n=2}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)\zeta_n,\\ &=(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)\sum_{n=2}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)\left[\delta+(\lambda_2^2-\lambda_n^2)\zeta_n\right]\\ &=(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)\sum_{n=3}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)(\lambda_2^2-\lambda_n^2)\zeta_n,\end{aligned}$$
$$=(\Delta+\lambda_4^2)\sum_{n=3}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)(\lambda_2^2-\lambda_n^2)\ (\lambda_3^2-\lambda_n^2)\left[\delta+(\lambda_3^2-\lambda_n^2)\zeta_n\right]\ =(\Delta+\lambda_4^2)\zeta_n=\delta\ , \tag{46}$$

We introduce the matrix

$$\boldsymbol{G}(\boldsymbol{x})=\boldsymbol{R}(\boldsymbol{D}_x)\boldsymbol{Y}(\boldsymbol{x}), \tag{47}$$

From Eqs. (31)-(33), (37) and (43), we obtain

$$\begin{aligned}\boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{G}(\boldsymbol{x}) &= \boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{R}(\boldsymbol{D}_x)\boldsymbol{Y}(\boldsymbol{x})\\ &=\boldsymbol{\Theta}(\Delta)\boldsymbol{Y}(\boldsymbol{x})=\delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}).\end{aligned} \tag{48}$$

Hence $\boldsymbol{G}(\boldsymbol{x})$ is the solution of Eq. (21).

Therefore we have proved the following theorem:

Theorem 1. If the condition (14) is satisfied, then the matrix $\boldsymbol{G}(\boldsymbol{x})$ (which is constructed using four elementary functions ζ_1,ζ_2,ζ_3 *and* ζ_4 in Eq. (40)) defined by Eq. (47) is a solution of system of Eqs. (9)-(11), where $\boldsymbol{R}(\boldsymbol{D}_x)$ and $\boldsymbol{Y}(\boldsymbol{x})$ are given by Eqs. (35) and (39) respectively.

Now we can establish the basic properties of $\boldsymbol{G}(\boldsymbol{x})$. Theorem 1 leads to the following results.

Corollary 1. If the condition (14) is satisfied, then the fundamental solution of the system

$$g_2\Delta\boldsymbol{u}+g_1\nabla div\,\boldsymbol{u}=0, \tag{49}$$

$$g_{12}\Delta T=0, \tag{50}$$

$$\frac{g_8}{\tau}T+\Delta N-g_7N=0, \tag{51}$$

is the matrix $\Phi=\left\|\Phi_{gh}\right\|_{5\times 5}$, where

$$\Phi_{mn}(\boldsymbol{x})=\left(g_2\Delta\delta_{mn}+g_1\frac{\partial^2}{\partial x_m\partial x_n}\right)\zeta^3(x)$$

$$,\Phi_{m4}(\boldsymbol{x})=0,\Phi_{4n}(\boldsymbol{x})=0,\Phi_{44}(\boldsymbol{x})=g_{12}\zeta^4(x),$$

$$\Phi_{55}(\boldsymbol{x})=g_7\zeta^4(x),\Phi_{45}(\boldsymbol{x})=0\ ,$$

$$\Phi_{54}(\boldsymbol{x})=\frac{g_8}{\tau}\zeta^4(x),$$

$$\Phi_{\boldsymbol{mn}}(\boldsymbol{x})=\mathrm{O}(|\boldsymbol{x}|^{-1})\ \text{ and }\ \Phi_{\boldsymbol{mn,r}}(\boldsymbol{x})=\mathrm{O}(|\boldsymbol{x}|^{-2})$$

hold in a neighbourhood of the origin, where m,n =1,2,3,4,5. and r =1,2,3.

On the basis of Theorem 1 and Corollary 1 we obtain the following

Theorem 2. If the condition (14) is satisfied, then the relations

$$\boldsymbol{G}_{\boldsymbol{mn}}(\boldsymbol{x})=\mathrm{O}(|\boldsymbol{x}|^{-1})\ \text{ and }\ \boldsymbol{G}_{\boldsymbol{mn,r}}(\boldsymbol{x})=\mathrm{O}(|\boldsymbol{x}|^{-2})$$

$$\boldsymbol{G}_{\boldsymbol{mn}}(\boldsymbol{x})-\Phi_{\boldsymbol{mn}}(\boldsymbol{x})=const+\mathrm{O}(|\boldsymbol{x}|)$$

$$\frac{\partial^q}{\partial x_1^{q_1}\partial x_2^{q_2}\partial x_3^{q_3}}\left[\boldsymbol{G}_{\boldsymbol{mn}}(\boldsymbol{x})-\Phi_{\boldsymbol{mn}}(\boldsymbol{x})\right]=\mathrm{O}(|\boldsymbol{x}|^{1-q})$$

hold in a neighbourhood of the origin, where $q=q_1+q_2+q_3, q\geq 1, q_r\geq 0, r=1,2,3$ *and* $m,n=1,2,3,4,5.$ Thus , $\Phi(\boldsymbol{x})$ is the singular part of the fundamental matrix $\boldsymbol{G}(\boldsymbol{x})$ in the neighbourhood of the origin. Taking into account inequality $\mathrm{Im}\,\lambda_m>0\,(m=1,2,3,4)$ we have

$$\zeta_n(\boldsymbol{x})=\exp(-\lambda_0|\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-1})\qquad and\qquad \zeta_{n,r}(\boldsymbol{x})=\exp(-\lambda_0|\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-2})$$

for $|\boldsymbol{x}|>>1,$ where $\lambda_0=\min\{\mathrm{Im}\,\lambda_j, j=1,2,3,4\}>0$ *and* $r=1,2,3.$ Consequently, on the basis of Theorem 1 each element of $\boldsymbol{G}(\boldsymbol{x})$ is represented in the form

$$\Phi_{mn}(\boldsymbol{x})=\sum_{s=1}^{4}\Phi_{mn}^{(s)}(\boldsymbol{x}),$$

where $\left(\Delta+\lambda_s^2\right)\Phi_{mn}^{(s)}(\boldsymbol{x})=0$ *for* $|x|\neq 0,$ and has the following property at the infinity

$$\Phi_{mn}^{(s)}(\boldsymbol{x})=\exp(-\lambda_0|\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-1}),\quad and\quad \Phi_{mn,r}^{(s)}(\boldsymbol{x})=\exp(-\lambda_0|\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-2}),$$

for $|\boldsymbol{x}|>>1, m,n=1,2,3,4,\ s=1,2,3,4$

and $r=1,2,3.$

5. Plane waves

We consider a plane wave propagation in a homogeneous isotropic photothermoelastic medium under Moore-Gibson-Thompson thermoelasticity. For two dimensional problem, we take

$$u_i = \left(u_1(x_1,x_3,t),0,u_3(x_1,x_3,t)\right),\quad T(x_1,x_3,t), N(x_1,x_3,t). \tag{52}$$

By Helmholtz decomposition theorem, we have

$$u_1 = \frac{\partial \Phi}{\partial x_1} - \frac{\partial \Psi}{\partial x_3} \; and \; u_3 = \frac{\partial \Phi}{\partial x_3} + \frac{\partial \Psi}{\partial x_1} \quad . \tag{53}$$

Eqs. (5)-(7), with the aid of Eqs. (52) and (53), take the form

$$\left(\Delta - \frac{\partial^2}{\partial t^2}\right)\Phi - T - g_3 N = 0, \tag{54}$$

$$\left(\Delta - \frac{1}{g_2}\frac{\partial^2}{\partial t^2}\right)\Psi = 0, \tag{55}$$

$$\left[g_5\left(1+\tau_o\frac{\partial}{\partial t}\right)\Delta\left(\frac{\partial^2}{\partial t^2}\right)\right]\Phi - \left[\Delta\frac{\partial}{\partial t} + g_4\Delta - \left(1+\tau_o\frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial t^2}\right]T - \left[\frac{g_6}{\tau}\left(1+\tau_o\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\right]N = 0, \tag{56}$$

$$g_8\frac{T}{\tau} + \left(\Delta - g_7\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)\right)N = 0, \tag{57}$$

We assume the solution for Eqs. (54)-(57) of the form

$$(\Phi, \Psi, T, N) = \left(\overline{\Phi}, \overline{\Psi}, \overline{T}, \overline{N}\right) e^{-i\left[\xi(l_1 x_1 + l_3 x_3) - \omega t\right]} \tag{58}$$

where $\omega = \xi c$ I the frequency, ξ is the wave number and c is the phase velocity. $\overline{\Phi}, \overline{\Psi}, \overline{T}, \overline{N}$ are undetermined amplitudes, that are dependent on time t and coordinates $x_m (m = 1,3)$. l_1 $and\, l_3$ are the direction cosines of the wave normal to the $x_1 x_3 -$ plane with the property $l_1^2 + l_3^2 = 1$.

Making use of Eq. (58) in Eqs. (54)-(57), we get

$$\left(-\xi^2 + \omega^2\right)\overline{\Phi} - \overline{T} - g_3\overline{N} = 0, \tag{59}$$

$$g_{10}\xi^2\overline{\Phi} + \left(g_{12}\xi^2 - \omega^2 g_9\right)\overline{T} - \frac{g_{13}}{\tau}\overline{N} = 0, \tag{60}$$

$$\frac{g_8}{\tau}\overline{T}+\left(-\xi^2+\frac{g_{14}}{\tau}\right)\overline{N}=0, \tag{61}$$

$$\left(\xi^2-\frac{\omega^2}{g_2}\right)\overline{\Psi}=0, \tag{62}$$

For non- trivial solution of the system of Eqs. (59)-(61), yields the following polynomial characteristic equation in ξ as

$$\left(\xi^6+R_1\xi^4+R_2\xi^2+R_3\right)=0, \tag{63}$$

where

$$R_1=\frac{\left(-g_{10}\tau-g_9\omega^2\tau-g_{12}\omega^2\tau-g_{12}g_{14}\right)}{g_{12}\tau},$$

$$R_2=\frac{\begin{pmatrix}-g_8g_{13}+\omega^4g_9\tau^2+g_{10}g_{14}\tau+g_9g_{14}\omega^2\tau\\-g_3g_{10}g_8\tau-g_{12}g_{14}\omega^2\tau\end{pmatrix}}{g_{12}\tau^2},\ R_3=\frac{\left(\omega^2g_8g_{13}-\omega^2\tau g_8g_{14}\right)}{g_{12}\tau^2}.$$

Solving Eq. (63), we obtain six roots of ξ, that is $\xi_1,\xi_2 and \xi_3$ correspond to positive x_3 direction and other three roots $-\xi_1,-\xi_2 and -\xi_3$ correspond to negative x_3 direction. Corresponding to roots $\xi_1,\xi_2 and\ \xi_3$, there exist three waves in descending order of their velocity, namely a longitudinal wave (P-wave) , thermal wave (T-wave) and plasma wave(PL-wave). From Eq. (62) we obtain two roots of ξ, that is $\pm\xi_4$ and corresponding to this root ,there exists a transverse wave (SV).It is noticed that these two values are unaffected by the thermal properties of the photothermoelastic medium.

We derive the expressions of phase velocity, attenuation coefficient, specific loss and penetration depth of these type of waves as

(i) Phase velocity

The phase velocities is given by

$$V_i=\frac{\omega}{\left|\mathrm{Re}(\xi_i)\right|},\ \ i=1,2,3. \tag{64}$$

where V_1,V_2,V_3 are the phase velocities of P,T and plasma waves respectively.

(ii) Attenuation coefficient

The attenuation coefficient are defined as

$$Q_i=\mathrm{Im}(\xi_i),\ \ i=1,2,3. \tag{65}$$

where $Q_1,Q_2\ and\ Q_3$ are the attenuation coefficients of P,T and plasma waves respectively.

(iii) Specific loss

The specific loss is defined as

$$R_i = \left(\frac{\Delta W}{W}\right) = 4\pi\left|\frac{\mathrm{Im}(R_i)}{\mathrm{Re}(R_i)}\right|, \quad i = 1,2,3. \tag{66}$$

where W is elastic energy and R_1, R_2 *and* R_3 are specific loss of P,T and plasma waves respectively.

(iv) Penetration depth

The penetration depth is defined as

$$S_i = \frac{1}{|\mathrm{Im}(\xi_i)|}, \quad i = 1,2,3. \tag{67}$$

where S_1, S_2 *and* S_3 are penetration depth of P,T and plasma waves respectively.

6. Particular cases

Photothermoelasticity under Moore–Gibson–Thompson model in which K, K^* *and* τ_o all are positive is limited to following cases

(i) If we take $K^* = 0$ in Eqs. (47) and (63), we obtained the corresponding result for Lord and Shulman's (LS) model.

(ii) If we take $\tau_o = \mathrm{K} = 0$ in Eqs. (47) and (63), we obtain the corresponding result for Green and Naghdi of type-II (GN-II) model.

(iii) If we take $\tau_o = 0$ in Eqs. (47) and (63), we obtain the corresponding result for Green and Naghdi of type-III (GN-III) model.

7. Numerical results and discussion

For the numerical calculations we take material constants for an isotropic Silicon (Si) material as $\lambda = 3.64\ N/m^2$, $\mu = 5.46\ N/m^2$, $\alpha_t = 0.00414\,K^{-1}$, $\alpha_n = -0.00198 m^3/kg$, $\rho = 2330 kg/m^3$, $T_o = 300K$, $K = 150\,w/mk$, $E_g = 1.11eV$, $C_e = 695\ j/kg\,K$, $\tau = 0.05s$, $D_e = 2.5 m^2/s$, $n_o = 10^2 m^{-3}$

The values of phase velocity, attenuation coefficient, specific loss and penetration depth of plane waves are determined by using MATLAB software. The variations of phase velocity, attenuation coefficient, specific loss and penetration depth with respect to frequency are shown in Figs. (1.1) to (1.12) respectively. Comparison has been made among the generalization theories presented by Moore-Gibson-Thomson thermoelasticity (MGTE), Lord and Shulman (LS), Green and Naghdi of type-II (GN-II) and Green and Naghdi of type-III (GN-III).

In all the figures solid line correspond to photothermoelastic MGTE model, dashed line corresponds to LS model, dotted line corresponds to GN-II model and dashed-dot line corresponds to GN-III model.

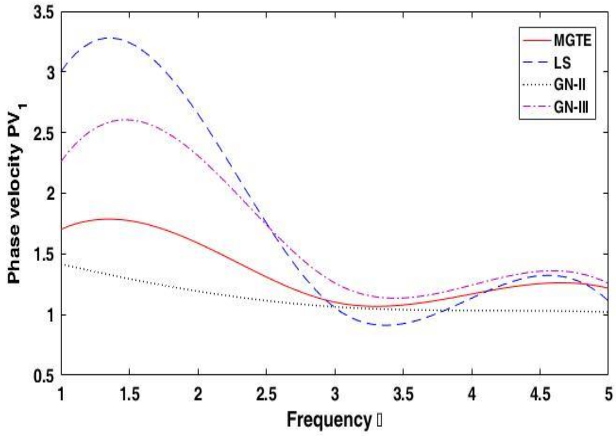

Fig. 1.1 Profile of phase velocity PV_1 vs. ω

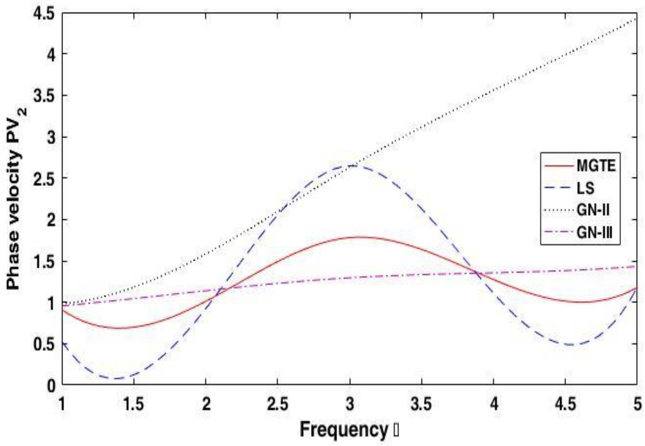

Fig. 1.2 Profile of phase velocity PV_2 vs. ω

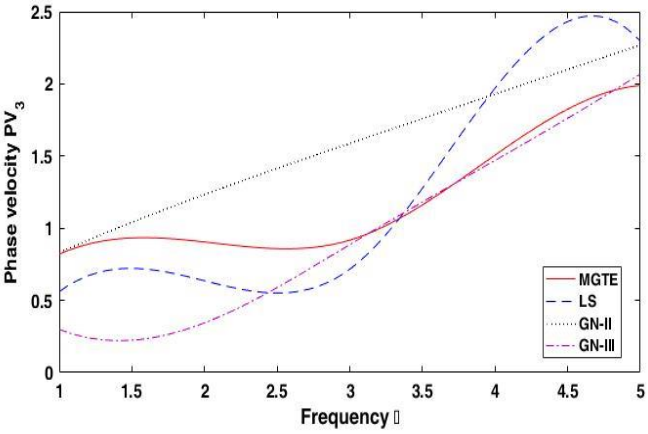

Fig. 1.3 Profile of phase velocity PV_3 vs. ω

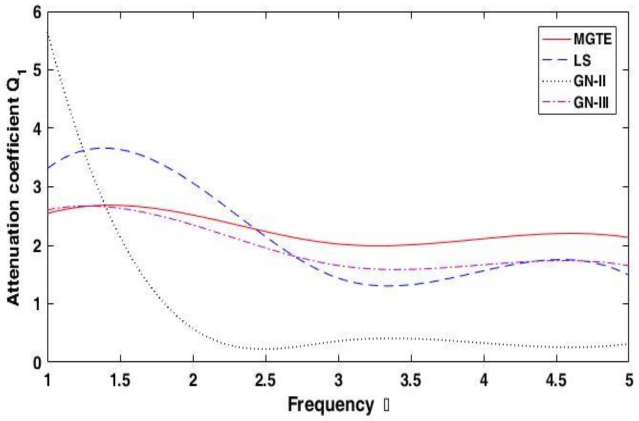

Fig. 1.4 Profile of attenuation coefficient Q_1 vs. ω

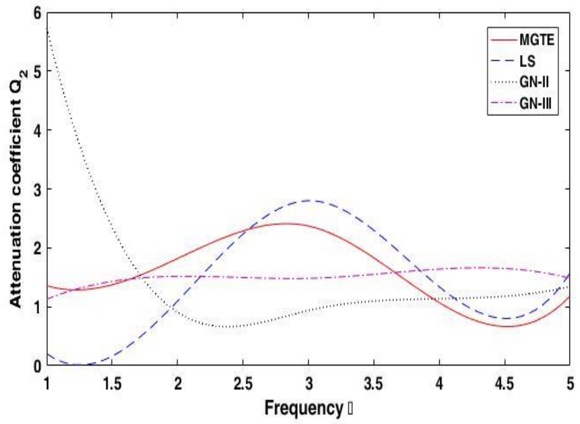

Fig. 1.5 Profile of attenuation coefficient Q_2vs. ω

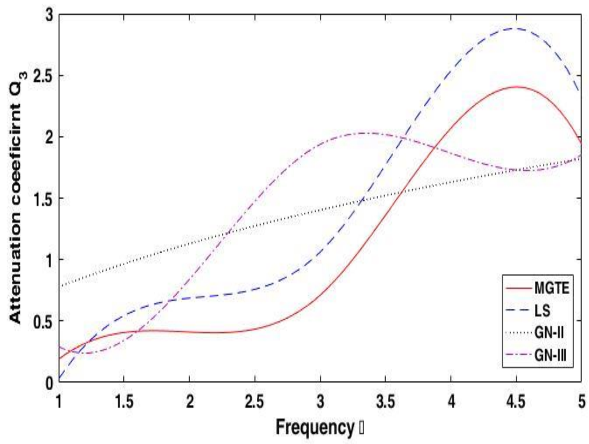

Fig. 1.6 Profile of attenuation coefficient Q_3 vs. ω

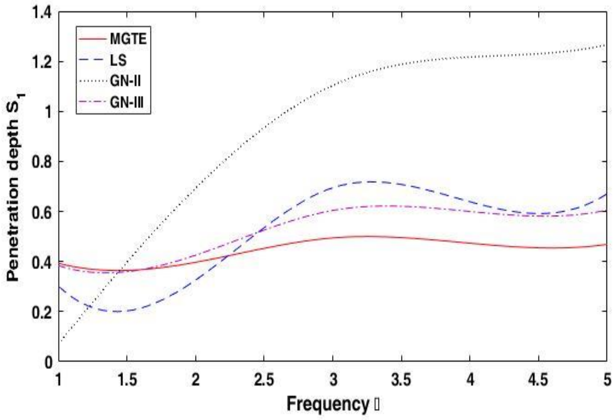

Fig. 1.7 Profile of penetration depth S_1 **vs.** ω

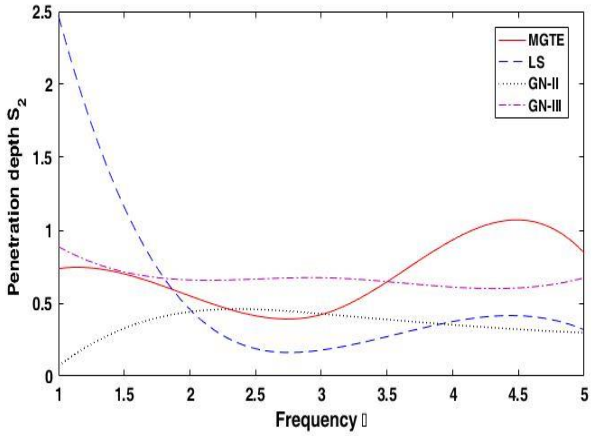

Fig. 1.8 Profile of penetration depth S_2 vs. ω

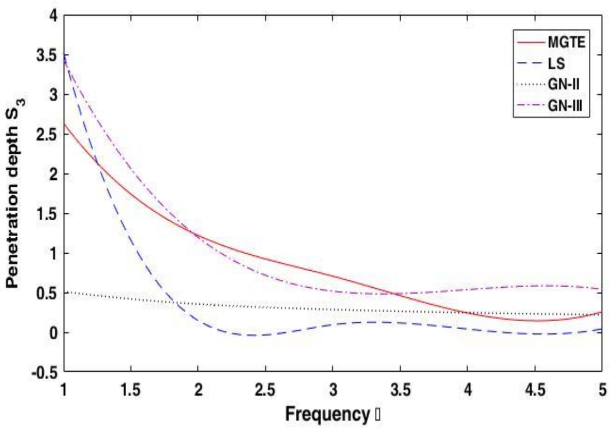

Fig. 1.9 Profile of penetration depth S_3 vs**.** ω

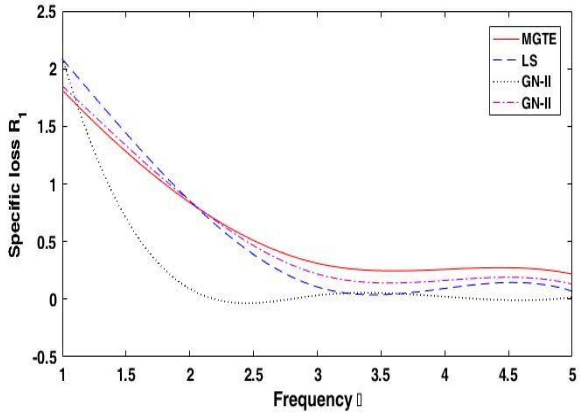

Fig. 1.10 Profile of specific loss R_1vs. ω

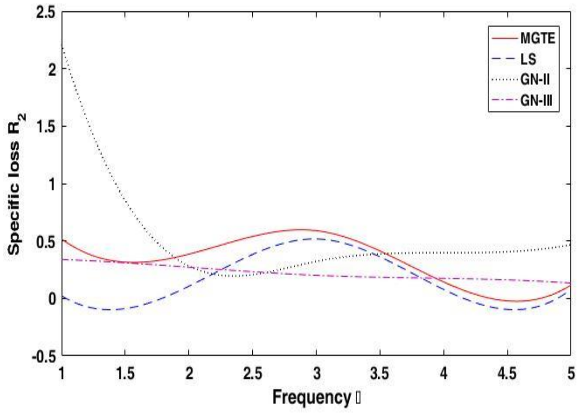

Fig. 1.11 Profile of specific loss R_2 vs ω

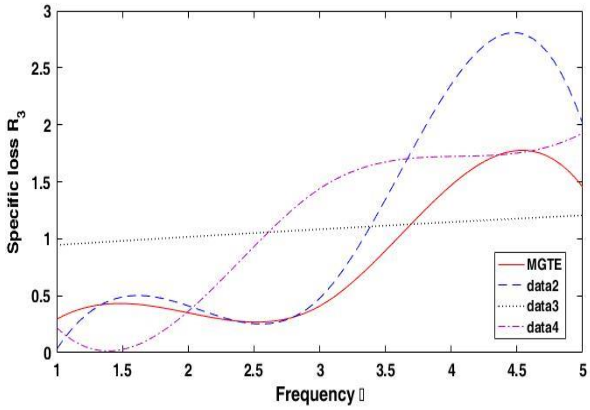

Fig. 1.12 Profile of specific loss R_3 vs. ω

Phase velocity

Fig. 1.1 depicts trend of phase velocity PV_1 vs. ω. Initially, the magnitude of PV_1 is maximum for LS model and minimum for GN-II model for the lower frequency. The behaviour and variation of PV_1 for MGTE, LS and GN-III is oscillatory. GN-III model minimize the value of PV_1 for extreme values of frequency. All the curves correspond to PV_1 are in decreasing trend for the whole range of ω.

Fig. 1.2 displays trend of phase velocity PV_2 vs. ω. In the initial range of frequency, the magnitude of PV_2 is higher for GN-III model and lower for LS model. The value of PV_2 is monotonically increasing with the increase in frequency due to GN-II model. The curves correspond to PV_2 is oscillatory in behaviour for LS and MGTE model.

Fig. 1.3 demonstrates trend of phase velocity PV_3 vs. ω. The magnitude of PV_3 is maximum for lower frequency due to GN-II model. The values of PV_3 is monotonically increasing for the whole range of frequency due to GN-II and GN-III models. The curves correspond to PV_3 is oscillatory in behaviour for LS and MGTE model.

Attenuation coefficient

Fig. 1.4 depicts trend of attenuation coefficient Q_1 vs. ω. In the initial range of frequency, the magnitude of Q_1 is maximum for GN-II model and minimum due to MGTE model. The behaviour and variation of Q_1 is opposite oscillatory for GN-III and LS model, in the range $2.5 \le \omega \le 4.5$. The values of Q_1 is monotonically decreasing in the range $0 \le \omega \le 2$ due to GN-II model as compare to other models.

Fig. 1.5 demonstrates trend of attenuation coefficient Q_2 vs. ω. For the extreme values of frequency, the magnitude of Q_2 is maximum for GN-II model and minimum due to LS model, whereas for intermediate values of frequency LS model intensify and GN-II model minimize the values of Q_2. The curves correspond to Q_2 are oscillatory in nature due to MGTE, LS and GN-III.

Fig. 1.6 displays trend of attenuation coefficient Q_3 vs. ω. In the initial range of frequency, the magnitude of Q_3 is maximum for GN-II model and minimum due to LS model. The behaviour and variation of Q_3 is opposite oscillatory for GN-III and LS model, for the whole range of frequency. The curves correspond to Q_3 for MGTE and LS model fluctuate in the same way.

Penetration depth

Fig. 1.7 displays trend of penetration depth S_1 vs. ω. Initially, the magnitude of S_1 is maximum for MGTE model and minimum due to GN-II model. The behaviour and variation of S_1 is monotonically increasing due to GN-II model for whole range of frequency. All the curves correspond to S_1 are in increasing trend for higher values of frequency.

Fig. 1.8 depicts trend of penetration depth S_2 vs. ω. Initially, the magnitude of S_2 is maximum for LS model and minimum due to GN-II model. The behaviour and variation of S_2 is monotonically decreasing for LS model in the range $0 \le \omega \le 2.5$ and opposite oscillatory for GN-III & MGTE model and LS & GN-II model.

Fig. 1.9 demonstrates trend of penetration depth S_3 vs. ω. Initially, the magnitude of S_3 is maximum for LS model and minimum due to GN-II model. The behaviour and variation of S_3 is decreasing for all models. The variation of S_3 is opposite oscillatory for GN-III and MGTE model for the whole range of frequency.

Specific loss

Fig. 1.10 demonstrates trend of specific loss R_1 vs. ω. Initially, the magnitude of R_1 is maximum for LS model and minimum due to MGTE model. The behaviour and variation of R_1 is decreasing for all models for the whole range of frequency. The variation of R_1 is monotonically decreasing for GN-II model in the range $0 \le \omega \le 2$.The curves due to MGTE,LS and GN-III travels in similar manner with small difference in their magnitude.

Fig. 1.11 displays trend of specific loss R_2 vs. ω. Initially, the magnitude of R_2 is maximum for GN-II model and minimum due to LS model. The behaviour and variation of R_2 is oscillatory for all models except GN-III model. The variation of R_2 is opposite oscillatory for GN-II and MGTE models in the range $\omega > 2$.

Fig. 1.12 depicts trend of specific loss R_3 vs. ω. Initially, the magnitude of R_3 is maximum for GN-II model and minimum due to LS model. The behaviour and variation of R_3 is oscillatory for all models except GN-II model. The variation of R_3 is opposite oscillatory for GN-III and LS models for the whole range of frequency. The curve due to MGTE model behave opposite oscillatory with GN-III model in the initial range of frequency.

8. Conclusions

Study of fundamental solution and plane wave vibrations are an important problem of mechanics of continua.The fundamental solution of system of equations in the generalized theories of photothermoelastic medium under Moore-Gibson-Thompson thermoelasticity for steady oscillation in terms of elementary functions has been constructed.

On the basis of fundamental solution of the Eqs. (9)-(11), it is possible to construct the surface (single layer and double layer) and volume potential in photothermoelastic with MGTE model and to establish their basic properties. To obtain the formulae of integral representation of regular solutions of the Eqs. (9)-(11), for the investigation of three dimensional BVP in the considered model by means of potential method and the theory of two dimensional singular integral equation. The method of fundamental solution is an elegant method for the solution of the basic differential equations. On the basis of the Theorem 1 and Theorem 2 discussed above, we can construct the regular solution of the 3 dimensional BVP of steady vibration by using potential methods and theory of singular integral equation in the considered model.

The propagation of plane wave in the considered medium under Moore-Gibson-Thompson thermoelasticity has also been studied. It is observed that there exist three longitudinal waves namely Longitudinal wave (P-wave), thermal wave(T-wave) and plasma wave(PL-waves) in addition to transverse wave SV wave which is not affected by thermal properties and photothermal effect of the materials.

Opposite behaviour of phase velocities PV_1 and PV_2 is observed for lower frequency due to all models. For higher frequency, PV_3 attain higher magnitude due to one relaxation time in comparison to other models. Attenuation coefficient correspond to P-wave and T-wave remains higher and lower for higher frequency respectively in MGTE model in comparison to other models. One relaxation time predominant attenuation coefficient corresponds to PL-wave in comparison to other assumed models. Without energy dissipation and one relaxation time has predominant impact on S_1 and $S_2$for higher and lower frequency respectively. S_3 has dominant impact of energy dissipation for lower frequency. Impact of MGTE on R_1 is observed stronger in comparison to R_2 and R_3 for higher frequency.

The result showed that several physical quantities like MGTE, photothermoelasticity significantly impact the system interaction. The comparison of the different models on attributes of the waves shows the dependence of physical field quantities. The result shows that MGTE model of photothermoelastic predicts a finite speed of wave propagation that makes a new model more consistent with the physical property of the material. The model explored in this work find application in various field such as structural engineering, theoretical seismology etc.

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