Ocean Systems Engineering, *Vol. 9, No. 1 (2019) 97–109*
DOI: https://doi.org/10.12989/ose.2019.9.1.097

Irregular frequency effects in the calculations of the drift forces

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(Received August 31, 2018, Revised March 18, 2019, Accepted March 20, 2019)

Abstract. Accurate calculation of the mean drift forces and moments is necessary when studying the higher order excitations on the floater in waves. When taking the time average of the second order forces and moments, the second order potential and motion diminish with only the first order terms remained. However, in the results of the first order forces or motions, the irregular frequency effects are often observed in higher frequencies, which will affect the accuracy of the calculation of the second order forces and moments. Therefore, we need to pay close attention to the irregular frequency effects in the mean drift forces. This paper will discuss about the irregular frequency effects in the calculations of the mean drift forces and validate our in-house program MDL Multi DYN using some examples which are known to have irregular frequency effects. Finally, we prove that it is necessary to remove the effects and demonstrate that the effectiveness of the formula and methods adopted in the development of our program.

Keywords: irregular frequency effect; drift force; hydrodynamics; body-wave interaction

1. Introduction

When we use the potential method to study the hydrodynamic responses of the floaters, we can break down the nonlinear problem into zeroth, first, second and higher orders. We can compute the corresponding potentials in order to find the forces and moments on the floaters. The second order wave forces contain the contribution from the first and second order potentials. However, in the first order problem, we may observe the irregular frequency effects at the relatively higher frequencies. The results of added mass, damping, forces and motion RAO may be affected. In this paper, we are interested in the irregular frequency effects in the calculations of the mean drift forces and demonstrate the necessity to remove the effects when higher wave frequencies are of interest.

Irregular frequencies result from the ill condition of the linear system in boundary integral problems. In other words, the matrix to be solved is almost singular at some frequencies. The solver can still provide some results because of the limited accuracy of numerical techniques and computer round-off errors. The irregular frequency effect in wave-body interaction was first found by John

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http://www.techno-press.org/?journal=ose&subpage=7 ISSN: 2093-6702(Print), 2093-677X(Online)

(1950). The harmful effects in hydrodynamic analysis were identified by Frank (1967). There are several applicable methods to resolve this phenomenon. One well known method is the extended-boundary-condition method, which was proposed by Wood, Paul (1972), mentioned in Angell, Hisao and Kleinman (1991). They enforced a fixed lid condition on the internal free surface. Ohmatsu (1975) validated the method for the 2D case. Kleinman (1982) revisited this method by applying a strict mathematical derivation. He proved the uniqueness in the potential formulation. Rezayat, Shipppy and Rizzo (1986) improved the method from Schenck (1968) and applied the ”lid” method in elastodynamics. Zhu (1994) followed Kleinman (1982), validating the effectiveness of his method. Lee (1996) further discussed this approach in a more general way, including the second order effect. Liu and Falzarano (2016) summarized the previous conclusions, discussed the uniqueness theorem and provided alternative formula to remove the irregular frequency effects. The numerical techniques are discussed in Liu and Falzarano (2017).

Researchers applied far-field and near-field method to study the drift forces. Herein, we are using the near-field method. The near field method was developed by Pinkster and Oortmerssen (1977). The basic idea of the near field method is to integrate the pressure on the wetted area of the floater and keep the terms up to the second order. Later, Pinkster (1980) summarized the computations of the second order mean drift forces. Lee and Newman (1991) re-derived the formula for the second order forces and moments. He did the study of the mean drift forces on the submerged spheroid. The more complete expressions of the drift forces were given in Lee (1995). Lee (2006) compared the results from methods based on pressure integration and momentum conservation and observed that more panels would lead to more accurate results especially in the range of higher wave frequencies. Chen (2007) studied the similar cases and applied more than 2000 panels for the hemisphere. Liu and Falzarano (2017) also re-derived the formula using the notation rule of the Green function and normal vectors in the development of the in-house program MDL Multi DYN. The authors thoroughly discussed about the calculations of the wave elevation around the waterline and came up with a method to improve the accuracy. MDL Multi DYN is a re-designed program for an arbitrary number of floaters. The in-house program MDL Multi DYN can be used for generating the input of the evaluation of wave energy converter discussed in Wang and Falzarano (2017), the evaluation of multiple ships with a small speed as discussed in Liu and Falzarano (2017). The boundary condition in the problem of ships with a forward speed is discussed in Liu and Falzarano (2017). The module of 2nd order forces has been extended by Xie, Liu and Falzarano (2019) to study the quadratic transfer function. The module to enforce artificial damping in the side-by-side problem is discussed in Liu and Falzarano (2019). MDL Multi DYN is currently for the study of the cases in deep water only. The module of finite depth Green function is under development by Xie, Liu and Falzarano (2017) and will be incorporated into the main program. As an improvement in the computational efficiency, Liu and Kang (2019) discusses about an alternative method to evaluate the near field component in Green functions.

In this paper, we discuss about the irregular frequency effects in the results of the mean drift forces. We will validate our results against the commercial software WAMIT version 6. The comparisons show that the results from our program MDL Multi DYN without the irregular frequency removal (IRR = 0) have close comparison with WAMIT. When using the irregular frequency removal (IRR = 1), the results from MDL Multi DYN are mostly close to the results of WAMIT. For some floaters, there may be some discrepancies in the range of the higher frequencies. The discrepancy is possibly due to the differences in the formula to remove the irregular frequency effects. The results from MDL Multi DYN seemingly follow the trend of the results more closely when IRR = 0. Finally, we demonstrate that it is necessary to remove the irregular frequency effects in the calculations of the mean drift forces, especially when the higher frequencies are of interest.

2. Methods

In this section, we discuss about the methods we use to remove the irregular frequency effects and the formula to evaluate the mean drift forces. Afterwards, we will use our in-house program MDL Multi DYN to run the cases which are known to have the irregular frequency effects and demonstrate the importance of removing the irregular frequencies in the evaluation of the mean drift forces.

2.1 Irregular frequency removal

In our in-house program MDL Multi DYN, we applied the extended-boundary-condition method to remove the irregular frequency effects. The basic idea of this method is to panelize the internal free surface of the floater with no-penetration boundary conditions. By applying the derived formula, this method ensures the uniqueness of the solution at the irregular frequencies. In this way, the irregular frequency effects are removed.

The module of irregular frequency removal in MDL Multi DYN is first discussed in Liu and Falzarano (2016). The theory background is reviewed in Liu and Falzarano (2016) and four methods to evaluate the log singularity is discussed in Liu and Falzarano (2017). The validations are conducted in the three papers and Liu and Falzarano (2017). The previous validations are about the added mass, damping, the 1st order forces and motion RAO to demonstrate the effectiveness of the module.

If using source formula to study the hydrodynamic problems, the corresponding equations to remove the irregular frequency effects will be as below

$$\begin{aligned}4\pi\frac{\partial\phi(\boldsymbol{x})}{\partial n_{\boldsymbol{x}}} &= -2\pi\sigma(\boldsymbol{x}) + PV\iint_{S_b}\sigma(\boldsymbol{\xi})\frac{\partial G(\boldsymbol{x};\boldsymbol{\xi})}{\partial n_{\boldsymbol{x}}}dS_{\boldsymbol{\xi}} + \iint_{S_i}\sigma'(\boldsymbol{\xi})\frac{\partial G(\boldsymbol{x};\boldsymbol{\xi})}{\partial n_{\boldsymbol{x}}}dS_{\boldsymbol{\xi}}\\ &= 4\pi V_n, \qquad \boldsymbol{x}\in S_b \end{aligned} \tag{1}$$

$$\begin{aligned}4\pi\frac{\partial\phi_-(\boldsymbol{x})}{\partial n_{\boldsymbol{x}}} &= -4\pi\sigma'(\boldsymbol{x}) + \iint_{S_b}\sigma(\boldsymbol{\xi})\frac{\partial G(\boldsymbol{x};\boldsymbol{\xi})}{\partial n_{\boldsymbol{x}}}dS_{\boldsymbol{\xi}} - PV\iint_{S_i}\sigma'(\boldsymbol{\xi})\frac{\partial G(\boldsymbol{x};\boldsymbol{\xi})}{\partial n_{\boldsymbol{x}}}dS_{\boldsymbol{\xi}}\\ &= 0, \qquad \boldsymbol{x}\in S_i \end{aligned} \tag{2}$$

where, S_b is the body boundary surface, S_i is the internal free surface inside the floater, V_n is the velocity normal to the body boundary surface, σ is the source strength on the body boundary surface and σ' is the source strength on the internal free surface. For more detailed information, please refer to Liu and Falzarano (2016).

2.2 Mean drift forces

In this section, we briefly review the theory about the mean drift forces. When we are calculating forces or moments, we will use the following equations

$$
\begin{aligned}
\vec{F} &= -\iint_S P\,\vec{n}\,dS \\
&= -\iint_{S_m} (P^{(0)} + \epsilon P^{(1)} + \epsilon^2 P^{(2)})(\vec{n}^{(0)} + \epsilon\vec{n}^{(1)} + \epsilon^2\vec{n}^{(2)})dS - \iint_{\Delta S} P\,\vec{n}\,dS \\
&= \vec{F}^{(0)} + \epsilon\vec{F}^{(1)} + \epsilon^2\vec{F}^{(2)}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\vec{M} &= -\iint_S P[(\vec{X} - \vec{X}_0) \times \vec{n}]dS \\
&= -\iint_{S_m} (P^{(0)} + \epsilon P^{(1)} + \epsilon^2 P^{(2)})[(\vec{X'} + \vec{\eta} + \vec{\alpha} \times \vec{X'} + \epsilon^2 H\vec{X'})\times \\
&\quad (\vec{n'}^{(0)} + \epsilon\vec{n'}^{(1)} + \epsilon^2\vec{n'}^{(2)})]dS \quad - \iint_{\Delta S} P[(\vec{X} - \vec{X}_0) \times \vec{n}]dS \\
&= \vec{M}^{(0)} + \epsilon\vec{M}^{(1)} + \epsilon^2\vec{M}^{(2)}
\end{aligned} \tag{4}
$$

where, S_m indicates the wet surface of the floater below $\bar{z} = 0$ when in equilibrium position. If we mark the waterline when the floater is in equilibrium position, then ΔS is the wet area between the wave elevation ζ and the marked waterline when the floater is moving. Mathematically, ΔS is the wet area between $z = \eta_3 + \eta_4 Y' - \eta_5 X'$ and wave elevation ζ.

When we substitute the perturbation relationships into the equations above, we will get the corresponding 0th, 1st, 2nd order forces and moments. Please note that the integral in the area ΔS is of order $o(\epsilon^2)$, as discussed in Liu and Falzarano (2017).

The expressions of the forces for the sea keeping problem are as below

$$
\begin{aligned}
\vec{F}^{(2)} = &\vec{\alpha}^{(1)} \times \vec{F}^{(1)} \\
&+ \iint_{S_m} \frac{1}{2}\rho\vec{\nabla}\Phi^{(1)} \cdot \vec{\nabla}\Phi^{(1)}\,\vec{n'}dS \\
&+ \rho g[-x_f A_{wp}\eta_4^{(1)}\eta_6^{(1)} - y_f A_{wp}\eta_5^{(1)}\eta_6^{(1)} \\
&- \frac{1}{2}Z_0 A_{wp}(\eta_4^{(1)^2} + \eta_5^{(1)^2})]\vec{k} \\
&+ \iint_{S_m} \rho[\frac{\partial}{\partial t}\vec{\nabla}\Phi^{(1)} \cdot (\vec{\eta}^{(1)} + \vec{\alpha}^{(1)} \times \vec{X'})]\vec{n'}dS \\
&- \int_{C_{wl}} dl\,\frac{1}{2}\,\rho g\zeta_r^{(1)^2}\vec{n'}\frac{1}{\sqrt{1 - n_3'^2}} \\
&+ \iint_{S_m} \rho\frac{\partial\Phi^{(2)}}{\partial t}\,\vec{n'}dS - \rho g(A_{wp}\eta_3^{(2)} + y_f A_{wp}\eta_4^{(2)} \\
&- x_f A_{wp}\eta_5^{(2)})\vec{k}
\end{aligned} \tag{5}
$$

$$
\vec{M}^{(2)} = -\rho g V[\eta_4^{(1)}\eta_6^{(1)}\frac{I_{XY}^A}{V} + \eta_5^{(1)}\eta_6^{(1)}\frac{I_{YY}^A}{V} + \eta_5^{(1)}\eta_6^{(1)}z_{CB}
$$

$$
\begin{aligned}
&+\frac{1}{2}(\eta_4^{(1)^2}-\eta_6^{(1)^2})y_{CB}\\
&+\frac{1}{2}(\eta_4^{(1)^2}+\eta_5^{(1)^2})y_f Z_0\frac{A_{wp}}{V}-x_{CB}\eta_4^{(1)}\eta_5^{(1)}+\eta_1^{(1)}\eta_6^{(1)}]\vec{i}\\
&+\rho g V[\eta_4^{(1)}\eta_6^{(1)}(z_{CB}+\frac{I_{XX}^A}{V})+\eta_5^{(1)}\eta_6^{(1)}\frac{I_{XY}^A}{V}\\
&-\frac{1}{2}x_{CB}(\eta_6^{(1)^2}-\eta_5^{(1)^2})\\
&+\frac{1}{2}(\eta_4^{(1)^2}+\eta_5^{(1)^2})x_f Z_0\frac{A_{wp}}{V}-\eta_2^{(1)}\eta_6^{(1)}]\vec{j}\\
&+\rho g V[-\eta_4^{(1)}\eta_6^{(1)}y_{CB}+\eta_5^{(1)}\eta_6^{(1)}x_{CB}+\eta_1^{(1)}\eta_4^{(1)}+\eta_2^{(1)}\eta_5^{(1)}]\vec{k}\\
&+\vec{\alpha}^{(1)}\times\vec{M}^{(1)}+\vec{\eta}^{(1)}\times\vec{F}^{(1)}\\
&-\int_{C_{wl}}dl\frac{1}{2}\rho g\zeta_r^{(1)^2}(\vec{X}'\times\vec{n}')\frac{1}{\sqrt{1-n_3'^2}}\\
&+\iint_{S_m}[\frac{1}{2}\rho(\vec{\nabla}\Phi^{(1)}\cdot\vec{\nabla}\Phi^{(1)})+\rho(\frac{\partial}{\partial t})\vec{\nabla}\Phi^{(1)}(\vec{\eta}^{(1)}\\
&+\vec{\alpha}^{(1)}\times\vec{X}')](\vec{X}'\times\vec{n}')dS\\
&+\iint_{S_m}\rho\frac{\partial\Phi^{(2)}}{\partial t}(\vec{X}'\times\vec{n}')dS\\
&+\rho g[V\eta_2^{(2)}-A_{wp}y_f\eta_3^{(2)}-(I_{YY}^A+Vz_{CB})\eta_4^{(2)}\\
&+I_{XY}^A\eta_5^{(2)}+Vx_{CB}\eta_6^{(2)}]\vec{i}\\
&+\rho g[-V\eta_1^{(2)}+A_{wp}x_f\eta_3^{(2)}+I_{XY}^A\eta_4^{(2)}\\
&-(I_{XX}^A+Vz_{CB})\eta_5^{(2)}+Vy_{CB}\eta_6^{(2)}]\vec{j}
\end{aligned}
\tag{6}
$$

In the equations, S_m indicates the wet surface of the floater below $z = 0$ when in equilibrium position. If we mark the waterline when the floater is in equilibrium position, then ΔS is the wet area between the wave elevation ζ and the marked waterline when the floater is moving, A_{wp} is defined as water plane area. Additionally, we define the term in the following formula

$$\zeta_r^{(1)}=\zeta^{(1)}-(\eta_3^{(1)}+\eta_4^{(1)}Y'-\eta_5^{(1)}X') \tag{7}$$

$$I_{XY}^A=-\iint_{S_m}Y'X'n_3'dS=\iint_{S_m}Y'X'dxdy \tag{8}$$

$$I_{XX}^A=-\iint_{S_m}X'X'n_3'dS=\iint_{S_m}X'X'dxdy \tag{9}$$

$$I_{YY}^A=-\iint_{S_m}Y'Y'n_3'dS=\iint_{S_m}Y'Y'dxdy \tag{10}$$

In the expressions of $\vec{F}^{(2)}$, $\vec{M}^{(2)}$, we may notice there exist the 2nd-order potential $\overline{\Phi}^{(2)}$ and the 6 DOF motions $\eta_j^{(2)}, j = 1 \sim 6$. However, up to this step, we have not solved for the terms except the

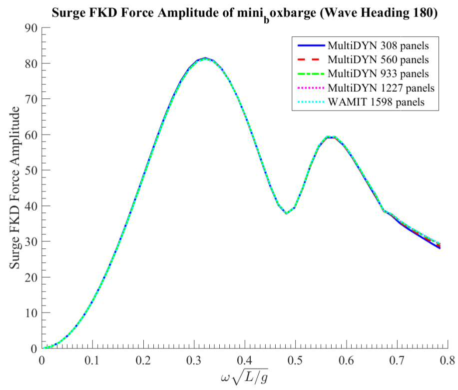

Fig. 1 CONVERGENCES TESTS IN SURGE

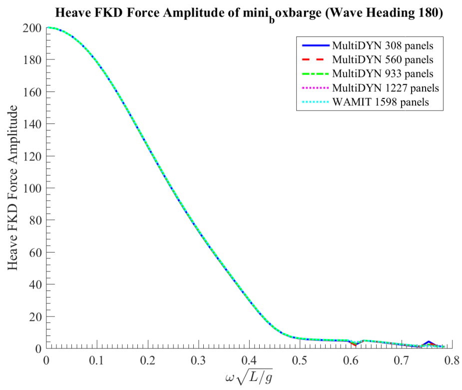

Fig. 2 CONVERGENCES TESTS IN HEAVE

2nd-order incident potential. To dig out more information from the 2nd-order forces and moments, we choose to take the time average of $\vec{F}^{(2)}$ and $\vec{M}^{(2)}$ so that $\overline{\Phi}^{(2)}$ and $\eta_j^{(2)}(j = 1 \sim 6)$ will vanish while the contribution from the 1st-order terms remains.

3. Results and discussion

When activating the module to remove the irregular frequencies, we conduct the comparisons of the results from MDL Multi DYN and WAMITv6 with or without the irregular frequency removal module. The cases include one box barge, two box barges, one hemisphere and one cylinder dock. ”Multi” stands for the results from our in-house program MDL Multi DYN. IRR0 indicates the module of irregular frequency removal is off. IRR1 means that the module is activated.

We firstly did the convergence tests for the cases we chose. For example, the results for the mini-boxbarge (Fig. 7) are shown in Figure 1 to 6. To make sure the panel near the waterline is small. We cut the panels using a surface and convert the triangular panels into quadrilateral. Herein, we generated the cases with 308, 560, 933, 1227 and 1598 panels. From the results, we can observe that when the number of panels is 933 or 1277, the results almost converged. Thus, we choose the case with panel number to be 1227. Similarly, the case of hemisphere has 4352 panels, the case of cylinder dock has 1408 panels. The meshes displayed below are for the purpose of the visualization only.

All the models are free in 6 DOF (Degrees Of Freedom). To simplify, the vertical center of gravity is 0, which means on the water free surface. The radius of gyration (k_{xx}, k_{yy}, k_{zz}) is set to be 1. The off-diagonal parts are all 0.

From the convergence tests, we may observe that more panels will make the irregular frequency effects less significant at some frequencies. With a more refined panel, we may observe a weak irregular frequency effect but we still recommend to remove the effect when studying the drift forces.

The dimensions are listed in Table 1, where L is the length, B is the breadth, T is the draft and R is the radius. NA is Not Available, which denotes the dimension does not apply. Num of panels is the number of panels for the case without a lid.

In Figs. 8 to 10, the results in the heave and pitch direction are consistent except near the resonant frequencies whether IRR = 0 or IRR = 1. In the surge direction, the result(IRR1) from WAMITv6

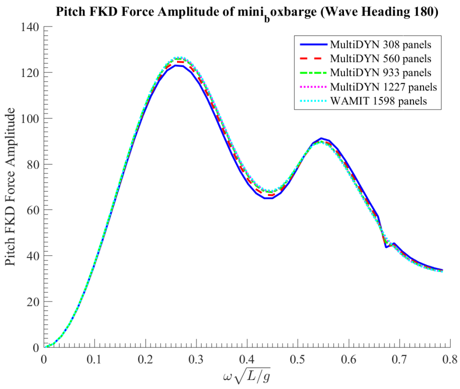

Fig. 3 CONVERGENCES TESTS IN PITCH

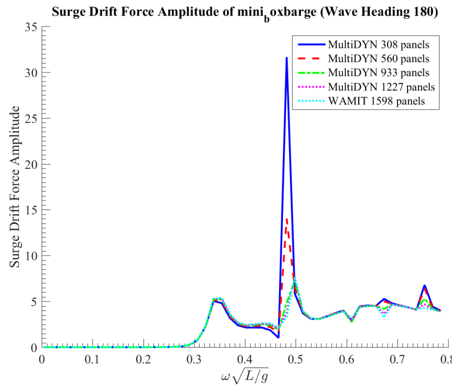

Fig. 4 CONVERGENCES TESTS IN SURGE

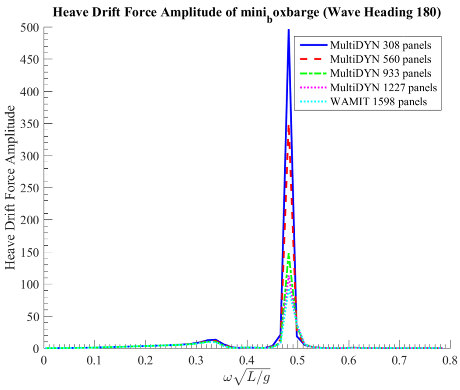

Fig. 5 CONVERGENCES TESTS IN HEAVE

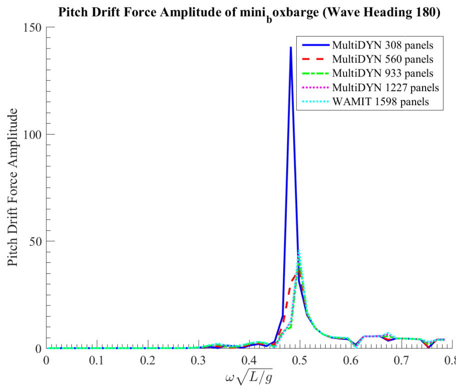

Fig. 6 CONVERGENCES TESTS IN PITCH

Table 1 Floater particulars

	L	B	T	R	Num of Panels
Miniboxbarge	20	10	5	NA	1227
Hemisphere	NA	NA	1	1	4352
Cylinder Dock	NA	NA	20	20	1408

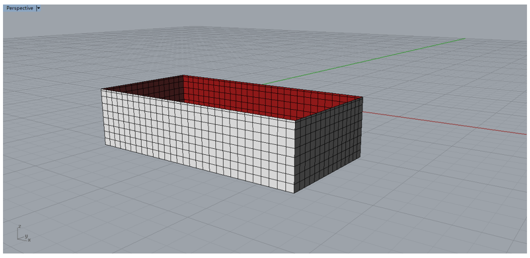

Fig. 7 OVERVIEW OF ONE MINI BOX BARGE

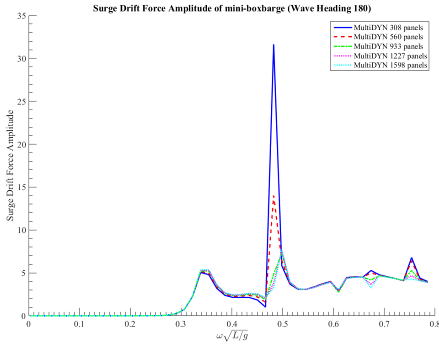

Fig. 8 SURGE DRIFT FORCE VS $\omega\sqrt{L/G}$

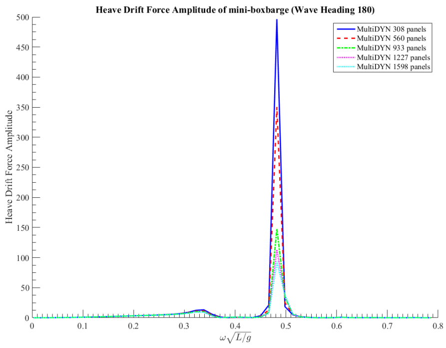

Fig. 9 HEAVE DRIFT FORCE VS $\omega\sqrt{L/G}$

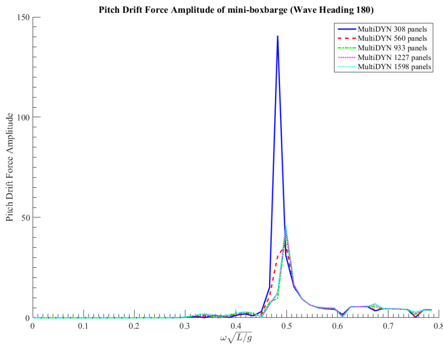

Fig. 10 PITCH DRIFT FORCE VS $\omega\sqrt{L/G}$

began to show some discrepancies while the result(IRR1) from MDL Multi DYN generally follows the trend of the curve when IRR equals 0.

In Figs. 12 to 14, we can observe a similar pattern with the single body case.

In Figs. 16 to 18, the results of MDL Multi DYN and WAMITv6 are in an excellent agreement. The pattern is similar to the results discussed in Chen (2007) and Lee (2006). A small discrepancy can be observed when the nondimensional wave frequency is around 2. Lee (2006) applied the higher order panel method in the simulation while in this paper, we have used a big number of panels to improve the results in the range of higher frequencies. In this special case, the higher order panel method is recommended because of the efficiency and accuracy.

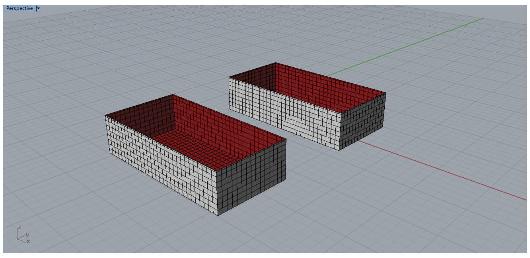

Fig. 11 OVERVIEW OF TWO MINI BOX BARGES

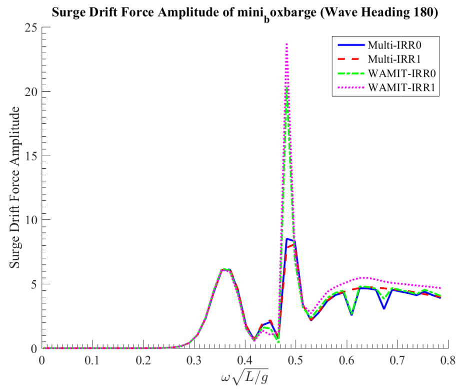

Fig. 12 SURGE DRIFT FORCE VS $\omega\sqrt{L/G}$

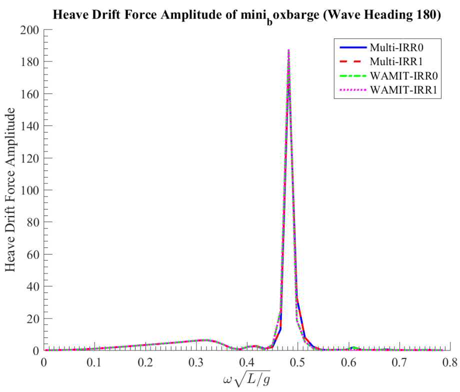

Fig. 13 HEAVE DRIFT FORCE VS $\omega\sqrt{L/G}$

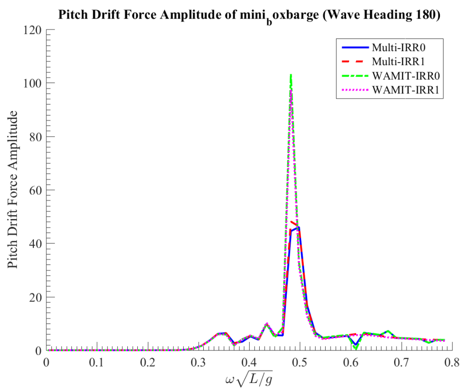

Fig. 14 PITCH DRIFT FORCE VS $\omega\sqrt{L/G}$

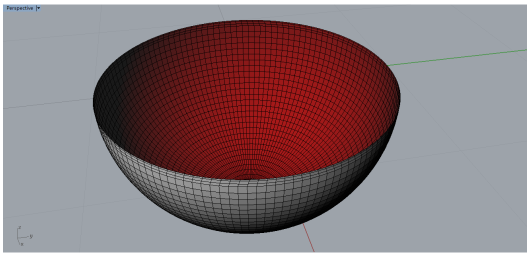

Fig. 15 OVERVIEW OF ONE HEMISPHERE

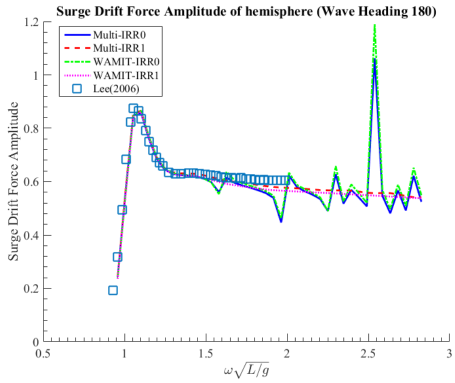

Fig. 16 SURGE DRIFT FORCE VS $\omega\sqrt{L/G}$

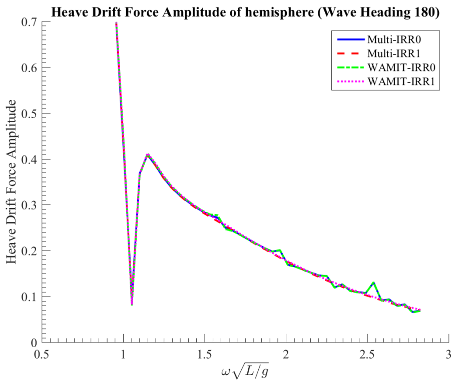

Fig. 17 HEAVE DRIFT FORCE VS $\omega\sqrt{L/G}$

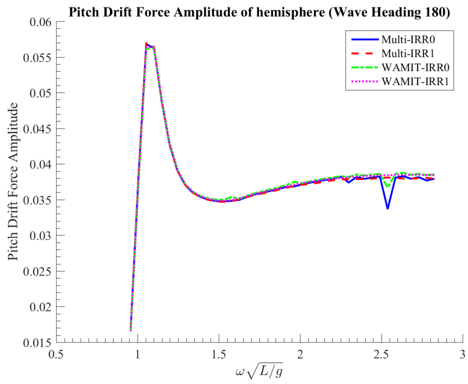

Fig. 18 PITCH DRIFT FORCE VS $\omega\sqrt{L/G}$

In Figs. 20 to 22, an excellent agreement between MDL Multi DYN and WAMITv6 is achieved in the heave and pitch directions whether IRR is 0 or 1. In the surge direction, the results when IRR = 0 are highly consistent. After IRR is set to be 1, the predicted curves are consistent in lower frequencies and seemingly have captured the trend of the results when IRR = 0 in the range of relatively higher frequencies. However, the discrepancy begins to appear. It may be because WAMITv6 uses a different boundary condition along the waterline or a different formula in removing the irregular frequencies.

To conclude, the results of the mean drift force from our in-house program MDL Multi DYN are highly consistent with WAMITv6 if we do not activate the irregular frequency removal module. If we have removed the irregular frequency effects, the results are consistent in the range of relatively lower frequencies. However, a discrepancy can be observed in the range of relatively higher frequencies. This may be due to different formula or numerical techniques for the irregular frequency removal problem. To further validate the predicted results, we may need to use the experimental data.

Additionally, we demonstrate that the effects of irregular frequency may be significant in the relatively higher frequencies in the mean drift forces or moments. In practice, we may need to conduct

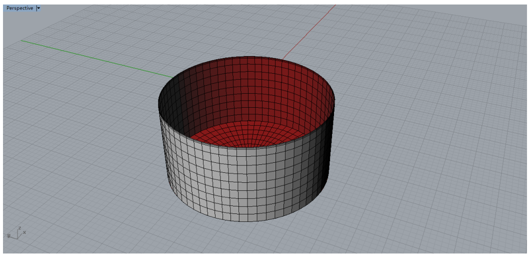

Fig. 19 OVERVIEW OF ONE CYLINDER DOCK

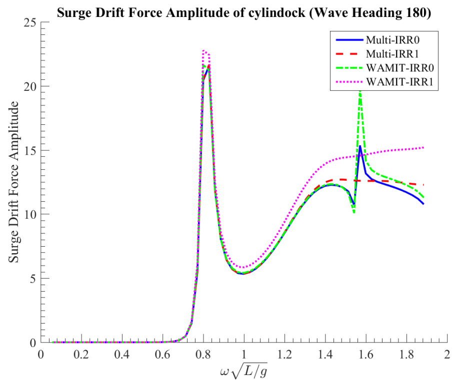

Fig. 20 SURGE DRIFT FORCE VS $\omega\sqrt{L/G}$

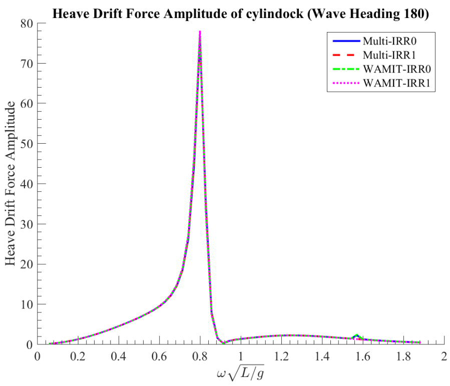

Fig. 21 HEAVE DRIFT FORCE VS $\omega\sqrt{L/G}$

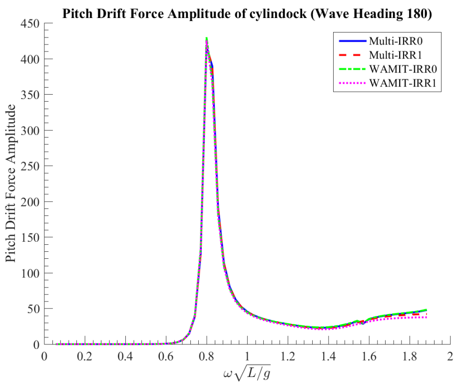

Fig. 22 PITCH DRIFT FORCE VS $\omega\sqrt{L/G}$

a comparative study for the cases with or without the irregular frequency removal module, to ensure that the conclusion is more reliable.

Acknowledgments

The research described in this paper was partially supported by the Office of Naval Research (ONR) —— ONR Grant N000-14-16-1-2281. The authors would like to thank Dr. Paul Hess from ONR EMSF Program and Andy Silver from NSWC-CD. The authors would also acknowledge partial funding supported by Alex Landsburg from the SNAME T&R Program.

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