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# Application of group method of data handling technique in assessing deformation of rock mass

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**Abstract.** Strength and deformation characteristics of a rock play a remarkable role in designing any geotechnical structure connected to rock mass. This study aims to propose a practical intelligence system, namely the group method of data handling (GMDH) for indirect rock deformation prediction. Direct measurement of rock deformation in laboratory is time consuming, difficult and costly. In the current study, several rock index tests were conducted, together with unconfined compressive strength tests, on collected granitic block samples. In this study, in accordance to the first set objective, four empirical equations were proposed based on predictors, including Schmidt hammer rebound number, p-wave velocity, porosity and point load strength index, aiming to predict rock deformation. The results of these analyses confirmed that there is a need to develop new multiple-input models in predicting rock deformation. To this end, a GMDH model was designed to forecast rock deformation. Aiming to obtain a fair comparison, a pre-developed artificial neural network (ANN), as a benchmark model of intelligence systems, was also developed to predict rock deformation. Then, through the use of some well-known performance indices, the GMDH and pre-developed ANN models were assessed and their results were compared to select the best predictive model amongst them. Results confirmed that the GMDH is a powerful and robust technique to the reliable prediction of rock deformation.

**Keywords:** rock deformation; indirect measure; predictive model; group method of data handling; artificial neural network

## 1. Introduction

Determination of rock engineering properties is crucial in the design process of geotechnical structures. In this regard, unconfined compressive strength (UCS) and modulus of elasticity (E), are two important properties which can be obtained through UCS test. The UCS test is standardized by the ISRM suggested method (Ulusay and Hudson 2007). In recent years, many studies have shown that UCS and E can be predicated indirectly. The interest of the international

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scientific community in investigating methods which can estimate UCS through indirect measurements, is due to the fact that performing direct UCS test is costly and time consuming; therefore, in practice, indirect tests like point load or p-wave velocity are performed for UCS estimation. Such tests, also known as rock index tests, are relatively easy to perform and quick (Zhou and Yang 2007, Momeni *et al.* 2015, Liang *et al.* 2016, Yang *et al.* 2018, Fang *et al.* 2019). Hence, numerous efforts have been made to relate index tests to UCS or E (Kahraman *et al.* 2005, Diamantis *et al.* 2009, Khandelwal and Singh 2009, Moradian and Behnia 2009, Yilmaz and Yuksek 2009, Beiki *et al.* 2013, Khandelwal 2013, Armaghani *et al.* 2016a). These studies proposed regression-based correlations. For example, Yilmaz and Yuksek (2009) developed a correlation between E and porosity for 121 samples of gypsum. According to their study, the coefficient of determination ( $\mathbb{R}^2$ ) for their proposed correlation was 0.83. Moradian and Behnia (2009) estimated E values of intact sedimentary rocks using the p-wave velocity test. They developed an exponential correlation between E and p-wave velocity test with  $\mathbb{R}^2$  equal to 0.92. It should be highlighted that publishing new data is always of added value, as rock behavior varies from location.

Nevertheless, with the advent of artificial intelligence techniques, many studies have highlighted their capability in solving various problems in different areas of civil engineering(Momeni et al. 2015, Zhou et al. 2016, Armaghani et al. 2018, Momeni et al. 2018, Armaghani et al. 2019, Asteris and Kolovos 2019, Asteris and Nikoo 2019, Hajihassani et al. 2019, Huang et al. 2019, Xu et al. 2019, Zhou et al. 2019, Asteris and Mokos 2019, Armaghani et al. 2019, Apostolopoulou et al. 2019, Asteris et al. 2019, Duan et al. 2020, Marto et al. 2014, Rezaei et al. 2016, Khandelwal et al. 2017, Bunawan et al. 2018, Apostolopoulou et al. 2020, Asteris and Mokos 2020). More specifically, in the field of rock strength and deformation, a review of available studies showed that the intelligent-based techniques outperform conventional regression-based methods. Momeni et al. (2015) reported that in contrary to regression analysis, in neural networks the estimated UCS value does not have to be a mean value. Since 2010 numerous studies implemented soft computing techniques for indirect prediction of either UCS or E (Tiryaki 2008, Majdi and Beiki 2010, Yesiloglu-Gultekin et al. 2013, Mohamad et al. 2014, 2018, Bejarbaneh et al. 2018). It should be highlighted that the proposed predictive models are considered reliable if they are trained with a large database of high-quality data. Armaghani et al. (2016) showed that E of granite can be estimated indirectly using improved artificial neural networks (ANN). They used Schmidt hammer rebound number, SRn, p-wave velocity, Vp, porosity, n, and point load index  $(I_{50})$  test results for indirect prediction of E. According to their study, when the contact nature between input and output parameters are unknown and non-linear, soft computing methods outperform regression-based techniques. In fact, in soft computing and artificial intelligence techniques, a predictive model is trained using available database and after training, the predictive model can be utilized for estimating the parameter of interest. Beiki et al. (2013) developed a predictive model of UCS using genetic programming. The input parameters used in their developed model was density, n as well as Vp. The reported  $\mathbb{R}^2$  of their study was 0.67. Dehghan et al. (2010) used two techniques i.e. regression and ANN for predicting elasticity modulus of carbonate rocks. The model inputs in their study include Vp, Is50, SRn, and n. According to their study, predictive model of E, with  $R^2$  equal to 0.77 serve as a feasible tool for estimation of E. In another study, Abdi et al. (2018) estimated the strength parameters (UCS and E) of sedimentary rocks using neural network and multiple linear regression (MLR) analysis. They obtained 196 various types of rock samples including limestone, conglomerate, sandstone, and marl. The inputs of their recommended model were dry unit weight, Vp, n, and water absorption.

According to their conclusion, ANN works better compared to MLR. In another research, Torabikaveh et al. (2015) recommended an ANN-based predictive model of E. They used density, n and  $V_p$  as the model inputs. The size of their dataset was 105 rock samples which were obtained from two different dam sites. According to their conclusion, there was no significant correlation between the predicted and determined E in the regression-based methods. Bejarbaneh et al. (2018) utilized fuzzy logic and neural network systems for predicting E of sandstone. They collected 96 specimens from roller-compacted concrete (RCC) dam located in the Malaysian state of Sarawak. According to their results both systems works good enough in predicting E. They used Vp, SRn, and Is50 results to train their predictive model of E. The R<sup>2</sup> equal to 0.812 and 0.719 for ANN and FIS, respectively, shows the workability of their proposed model. It is worth mentioning that no significant mutual correlations between E and other input parameters were reported in their study. This highlights the superiority of ANN over regression techniques in predicting E. Singh et al. (2012) implemented adaptive neuro fuzzy inference system (ANFIS) for predicting the elasticity modulus of rocks. According to their study, water absorption  $I_{550}$  value and density were their model inputs. However, the R<sup>2</sup> equal to 0.66 suggests that their proposed ANFIS-based predictive model is not highly reliable. Gokseoglu and Zorlu (2004) utilized fuzzy inference system (FIS) for the prediction of E. They used block punch index, Brazilian tensile strength and Is50 tests values as well as Vp for training their FIS-based predictive model of E. According to their study, the R<sup>2</sup> of their proposed model was 0.79. Yilmaz and Yuksek (2008) proposed a ANN-based predictive model of E with R<sup>2</sup> equal to 0.91. Their model input includes slake durability index, Is<sub>50</sub> values, effective *n* and *SRn*. Yilmaz and Yuksek (2009) performed another study with the aid of ANFIS for estimation of E. In their study, water content, Is50 value, Vp, and SRn were used as model inputs and the elasticity modulus was set as the model output. The obtained  $R^2$  equal to 0.95 recommends the high reliability level of their model. Based on the aforementioned discussion, it can be seen there is no specific criteria for selecting input parameters of the intelligent based predictive models of E. In fact, input parameters are the choices in the hand of designers and they mostly depends on the engineering judgment.

In the present paper, a new model in the field of rock deformation prediction, namely the group method of data handling (GMDH) is introduced. Literature shows that the models that work on the basis of self-organizing networks, containing active neurons (GMDH), are of a higher effectiveness in terms of making more accurate and less labour-intensive predictions. In addition, the paper evaluates the precision of another predictive technique i.e., ANN in predicting rock deformation or E. Then, after evaluating the performance predictions of the aforementioned models, the best one amongst them is selected and introduced to solve the problem. In the following parts, the principles of predictive intelligence techniques are given. Then, after some explanations regarding the experimental framework and after conducting simple regression and MLR models, model developments of ANN and GMDH in predicting rock deformation will be described in detail. Finally, the proposed models are evaluated through the use of well-known performance indices and the best predictive model amongst them will be introduced for the estimation of E.

# 2. Principles of predictive models

# 2.1 ANN

McCulloch and Walter (1943) were first researchers prepared a binary threshold logic(decision) unit by modelling neural net, and utilized it for modeling an artificial neuron behavior. In this



Fig. 1 Mesh grid of topographic model



Fig. 2 The PSRWT tunnel route constructed in Malaysia

model, weighted sum of incoming signals assigned to artificial nodes which form a network. In the next step, these signals pass through a particular activation function and deliver a more profitable output. In other word, ANNs have an extremely parallel structure containing network of units, neurons or nodes which have sequential layer organization and are computationally interconnected. Moreover, network behavior is affected by pattern of neurons connections, which determine class of network as well (Ch and Mathur 2012). Mentioned earlier, train a network in order to improve network performance, is conceivable so that structure and connection weights of network modify iteratively and minimize the error of every output layer node. A squared error function shows by E, calculate the result output error as:

$$E = \frac{1}{2} \sum_{i=1}^{p} (t^{(i)} - y^{(i)})^2$$
<sup>(1)</sup>

where t, y and P stand for target value, produced actual value, and the number of training patterns, respectively.

Back-propagation (BP) learning algorithm is a gradient-based learning procedure usually used for network learning task, particularly regarding multilayer feed-forward nets (Simpson 1990, Mohamad *et al.* 2012, Tonnizam Mohamad *et al.* 2012, Asteris and Plevris 2017, Asteris *et al.* 2017). A twofold procedure includes both forward and backward stages consist of any training period in BP learning algorithm. Input signals move forwards through the network in forward stage and expel error signal for each output-layer node. Then rates of resulting error cross backward along the network and correct weights and biases of network in subsequent step (Koopialipoor *et al.* 2018a, Zhou *et al.* 2019a). In course of network architecture, ANNs classify as two functional groups: feed-forward and feedback. One popular variant of multilayer feed-forward networks is Multilayer Perceptron (MLP) that its successive layers of processing units (neurons) using weighted links to exchange information (signals) and activation functions to process them (Haykin 1999, Priddy and Keller 2005).

## 2.2 GMDH

The GMDH algorithm represents a model as sets of neurons organized in different layers. In each layer, different neuron pairs are connected to each other by a quadratic polynomial. This way, they generate new neurons in the next layer. With the use of this type of representation, inputs can be mapped to outputs. The identification problem, as formally defined, refers to the exploration of a function  $\hat{f}$  that can be employed approximately in place of the actual one, f, for the aim of predicting output  $\hat{y}$  for a given input vector  $X = (x_1, x_2, x_3, ..., x_n)$  as close as possible to its actual output y. As a result, given M observation of multi-input and single-output data pairs

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) (i = 1, 2, \dots, M)$$
<sup>(2)</sup>

Currently, a GMDH-based neural network can be trained in a way to forecast the output values  $\hat{y}$  for any given input vector  $X = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$ , that is

$$\hat{y}_i = \hat{f} (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) (i = 1, 2, \dots, M)$$
(3)

A key challenge is determination of a GMDH type neural network in a way to minimize the square of difference between the actual output and the predicted one, that is

$$\sum_{i=1}^{M} [\hat{f} (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) - y_i]^2 \to min$$
(4)

The general connections between input parameters and output can be stated using a complicated discrete form of the Volterra functional series in the following form

$$y = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots$$
(5)

where, is generally recognized as the Kolmogorov-Gabor polynomial (Sanchez *et al.* 1997). This complete form of mathematical description can be denoted by a system of partial quadratic polynomials that is consisted of only two variables (neurons) in the form of

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i^2 + a_4 x_j^2 + a_5 x_i x_j$$
(6)



Fig. 3 Conducted tests of (a) point load strength and (b) Schmidt hammer

In the GMDH type neural networks design, two core concepts are involved: the parametric and the structural identification problems. Structure of a GMDH system with four model inputs is shown in Fig. 1.

When GMDH is being designed, it should be remembered that all polynomials of the neurons that are present in each of the network layers are similar to each other and the network is designed based on similar processes. Indeed, the second-order polynomial is the fundamental structure of the GMDH network, which was pioneered by Ivakhnenko (1968). Generally, different types of polynomial, which include tri-quadratic, quadratic, bilinear, and the  $3^{rd}$  order have been used in design of the self-organized systems. In comparison with the quadratic type of polynomial, with the use of the tri-quadratic and  $3^{rd}$  order polynomial, more complex networks will be built. The bilinear polynomial generates a structure of lower complexity. The quadratic polynomial includes six weighting coefficients that can successfully solve different engineering problems (Najafzadeh and Barani 2011). According to related literature, the selection of one polynomial among various types is highly dependent on two parameters, i.e., the minimum error of objective function and the complexity of the polynomial type. Literature is consisted of numerous studies (Najafzadeh *et al.* 2013, Koopialipoor *et al.* 2018b) containing more detailed information in regard to the GMDH model and its applications.

## 3. Study area, laboratory tests and establishing database

The face of the Pahang-Selangor raw water transfer (PSRWT) tunnel in Malaysia was taken into consideration to gather rock block samples required for this research. The tunnelling project was aimed at transferring water from Pahang to Selangor states in Malaysia. The PSRWT tunnel is



Fig. 4 All 88 datasets of model inputs and output used in modeling stage of this study

Mode	Predictor	<b>Empirical Equation</b>	$\mathbb{R}^2$
Power	SRn	$E = 0.0006 \ SRn^{3.0304}$	0.616
	Vp	$E = 3E-05 Vp^{1.7149}$	0.604
	Is50	$E = 36.043 Is 50^{0.7613}$	0.706
	n	$E = 36.182 n^{-0.724}$	0.536
Exponential	SRn	$E = 3.9654 e^{0.0613 SRn}$	0.601
	Vp	$E = 14.804e^{0.0003 Vp}$	0.579
	Is50	$E = 36.924e^{0.24}$ Is50	0.637
	n	$E = 204.08e^{-2.584}$ n	0.577
Linear	SRn	E = 4.8774 <i>SRn</i> - 150.18	0.584
	Vp	E = 0.0257 Vp - 48.162	0.586
	Is50	E = 19.835 Is50 + 24.676	0.669
	n	E = -215.69 n + 166.68	0.618
Logarithmic	SRn	$E = 237.57 \ln (SRn) - 833.96$	0.582
	Vp	$E = 135.6 \ln (Vp) - 1071.5$	0.581
	Is50	$E = 59.43 \ln (Is50) + 26.633$	0.662
	n	$E = -62.85\ln(n) + 19.396$	0.621

Table 1 Different modes of regression with their empirical equations in predicting E of the rock samples

crossed under the main mountain range between Pahang and Selangor. The mountain range forms the backbone of Peninsular Malaysia and has an elevation ranging from between 100 and 1400 m. In the tunnel, the main rock type is granite, with a typical rock strength of 150-200 MPa. After primary investigations, the construction company decided to excavate three sections of the tunnel using tunnel boring machine (TBM) with lengths of 11.7 km (for TBM 1), 11.7 km (for TBM 2), and 11.3 km (for TBM 1). The route of PSRWT tunnel constructed in Malaysia is shown in Fig. 2. A number of samples of granite blocks were gathered from the face of the PSRWT tunnel in

the areas of different TBMs for the purpose of developing a model that can predict rock deformation values. The samples were transferred to laboratory, and after coring and cutting, the end of each sample was flattened perpendicular to the sample axis. In addition, smoothing and polishing processes were done on their sides; then, they were completely examined in order to make sure they are free of fissures, cracks, veins, and other flaws since these negative factors may cause the real rock properties to be undesirably altered. The rock properties typically refer to physical properties of the rock, its index strengths, and its fundamental strengths. In case of all the collected samples, the physical property, i.e., p-wave velocity velocity, which shows the compactness degree of rock samples was measured. Moreover, for each sample, the point-load test apparatus and L-type Schmidt's hammer were used to measure the Is50 and SRn values, respectively. Furthermore, each one of the collected samples was examined in terms of its porosity level. Additionally, granitic samples were tested regarding their UCS and E value, which are parameters of high reliability for the evaluation of the compressive strength of the rock. More specifically, the rock stress-strain curve was applied to prediction of the E value. In addition, two linear variable differential transformers (LVDT) were implemented in order to measure the axial strain, and also the tangent method was applied to the measurement of the E value. The present paper applies the procedures suggested by ISRM (Ulusay and Hudson 2007) to all experiments. Fig. 3 shows conducted tests of Schmidt hammer and point load strength during laboratory investigations.

A database with 88 datasets was prepared in the modelling of ANN and GMDH predictive techniques. In this database, Vp, n,  $Is_{50}$  and SRn were used as input parameters while rock deformation (*E*) was set as output of ANN and GMDH systems. Ranges of (37-61), (3065-7943 m/s), (0.89-6.54 MPa), (0.1-0.57%) and (22-183.3 GPa) were obtained for SRn, Vp,  $Is_{50}$ , n, and E, respectively. In addition, Fig. 4 shows all 88 datasets of model inputs and output of this study used in modelling stage.

## 4. Model analysis

In this section, first, simple regression analysis is conducted in order to identify the relationships between dependent and independent parameters. Then, after evaluation of the results, this study presents ANN and GMDH model developments in detail for rock deformation prediction.

#### 4.1 Simple and multiple regression analysis

The simple regression analysis was used for the aim of establishing the empirical equations between predictors (*SRn*, *Vp*, *Is*<sub>50</sub>, and *n*,) and E of the rock samples. An analysis was done on the connections between E and the independent variables and then a number of exponential, linear, power, and logarithmic equations were suggested by simple regression (see Table 1). The evaluation of these empirical equations was performed by comparing  $R^2$  results. The  $R^2$  values can be computed using the following equation

$$R^{2} = 1 - \frac{\sum_{i} (x_{imeas} - x_{ipred})^{2}}{\sum_{i} (x_{imeas} - \bar{x})^{2}}$$
(7)



Fig. 5 Scatter plots of model predictors with rock deformation values

where,  $x_{imeas}$  and  $x_{ipred}$  are the determined and predicted values, respectively.  $\bar{x}$  represents the average determined values. Amount of R<sup>2</sup> equal to 1 will be obtained for a perfect predictive model. Ranges of 0.5-0.7 for R<sup>2</sup> of the empirical equations in predicting E show that all predictors have a meaningful relationship with rock deformation.

More specifically, according to  $R^2$  values, power, power, power and logarithmic were obtained as the best modes for SRn, Vp, Is<sub>50</sub>, and n, respectively in predicting E. The developed empirical equations using SRn, Vp, Is<sub>50</sub>, and n, are presented in Eqs. (8) - (11), respectively.

$$E = 0.0006 \ SR_n^{3.0304} \tag{8}$$

$$E = 0.00003 \ Vp^{1.7149} \tag{9}$$

$$E = 36.043 \ Is50^{0.7613} \tag{10}$$

$$E = -62.85 \ln(n) + 19.396 \tag{11}$$

Fig. 5 depicts scatter correlations between SRn, Vp,  $Is_{50}$ , n, and E of the rock samples.  $R^2$  results of these predictors are as 0.616, 0.604, 0.706 and 0.621, respectively which as acceptable and meaningful. This will be highlighted when considering the point that these performances are obtained based on only one predictor. However, it seems that these results are inadequate when a high level of accuracy is of interest and need. Therefore, in order to have a higher accuracy level for prediction of rock deformation, all model predictors can be used in developing multi-inputs techniques such as MLR.

By developing MLR model, a multi-linear relationships can be found between model inputs and model output (E of the rock). In fact, in this model, the effects of all model inputs will be considered on the results of model output. The mentioned model has been used in several related studies in order to

Number of Node	R <sup>2</sup>		Rank		Rank Summation
	Train	Test	Train	Test	
1	0.801	0.651	2	4	6
2	0.783	0.849	1	9	10
3	0.864	0.743	3	5	8
4	0.879	0.801	4	8	12
5	0.894	0.744	6	6	12
6	0.918	0.743	7	5	12
7	0.891	0.545	5	2	7
8	0.982	0.768	9	7	16
9	0.976	0.638	8	3	11

Table 2 Train and test results of ANN models for all 9 hidden nodes

Table 3 The results of parametric study on the number of layers

GMDH model No.	No. of Layer	R <sup>2</sup>		Rank		Rank Summation
		Train	Train	Train	Test	
1	2	0.764	0.764	2	5	7
2	3	0.743	0.743	1	1	2
3	4	0.774	0.774	3	2	5
4	5	0.810	0.810	5	3	8
5	6	0.852	0.852	9	8	17
6	7	0.822	0.822	6	9	15
7	8	0.803	0.803	4	7	11
8	9	0.844	0.844	8	6	14
9	10	0.829	0.829	7	4	11

Table 4 The results of parametric study on the number of neurons

GMDH model No.	No. of Neuron	R <sup>2</sup>		Rank		Rank Summation
		Train	Test	Train	Test	
1	2	0.893	0.839	3	1	4
2	4	0.871	0.855	2	2	4
3	6	0.852	0.865	1	3	4
4	8	0.895	0.902	4	4	8
5	10	0.923	0.934	5	6	11
6	12	0.985	0.961	10	9	19
7	14	0.977	0.968	8	10	18
8	16	0.981	0.955	9	8	17
9	18	0.951	0.944	7	7	14
10	20	0.928	0.910	6	5	11



Fig. 6 Plotted all 18 testing datasets for determined E together with predicted by GMDH and ANN models

solve engineering problems (Khandelwal and Monjezi 2013, Gordan *et al.* 2016, Mohamad *et al.* 2017a). The constructed MLR equation for predicting E of the rock sample is presented as follows:

$$E = -71.25 \times n + 1.49 \times SRn + 0.006 \times Vp + 7.62 \times I_{550} - 15.54$$
(12)

The performance prediction of the proposed MLR equation in predicting E is 0.773 which is good and acceptable. However, this result is based on statistical techniques and it seems that if intelligent systems like ANN and GMDH are used, more accurate results will be obtained.

#### 4.2 ANN Model development

This section describes ANN model development process in predicting E of the rock samples. First, all datasets regarding training or testing feature must be separated in order to develop and evaluate the model. Nelson and Illingworth (1991) based on their studies suggested that (20%-30%) of whole datasets should be considered as testing datasets. Accordingly, we allocated 20% (18 datasets) of whole datasets to testing datasets. Since prosperous applying of Levenberg–Marquardt (LM) training algorithm has noticed in many researches (Hajihassani *et al.* 2015, Mohamad *et al.* 2017b), it was also used in this study in order to design ANN. Moreover, an ANN with one hidden layer is capable to approximate any continuous function, reportedly (Alavi Nezhad Khalil Abad *et al.* 2016, Jahed Armaghani *et al.* 2016). Characterize the number of hidden nodes fulfilled by Hornik *et al.* (1989) suggested formula that is based on *Ni* as number of input layers in form of  $\leq 2 \times Ni + 1$ . Replacing Ni = 4 in this equation indicate that the problem of rock deformation can be solved by a range of 1 to 9 for hidden nodes. Therefore, ANN models were constructed to predict rock deformation using hidden nodes of 1-9. Table 2 presents train and test R<sup>2</sup> results of ANN models for all 9 hidden nodes in predicting rock deformation values. As a result, in general, train results are increasing by increasing hidden nodes. However, test results do not

Model	Set	$\mathbb{R}^2$	RMSE	VAF (%)	a20-index
ANN	Train	0.982	5.209	98.225	0.971
	Test	0.768	17.855	67.180	0.667
GMDH	Train	0.985	4.801	98.478	1
	Test	0.961	7.190	95.069	1

Table 5 The computed performance indices for ANN and GMDH models

have the same behavior of train results and there is a considerable difference between train and test values based on  $\mathbb{R}^2$ . Therefore, a rank value was assigned to each set of train and test. The best value (9) will go for the highest  $\mathbb{R}^2$  and the worst value (1) will go for the lowest  $\mathbb{R}^2$ . Then, the column of Table 2 is a summation of rank train and rank test columns in Table 2. According to rank summation results, node number 8 showed the best ANN model performance for E prediction with  $\mathbb{R}^2$  of 0.982 and 0.768 for training and testing datasets, respectively. It is confirmed that the optimum ANN model to predict rock deformation would have the architecture of  $4 \times 8 \times 1$ . More details of the selected ANN model will be discussed later.

# 4.3 GMDH model development

For designing a GMDH model, the most important parameters, namely the number of GMDH layers, the number of neurons, as well as the selection pressure, are needed to be considered. The selection pressure enables the system to choose the optimum fits in each step and moves them to the following layers. Such process is iterated until the predefined criteria, i.e., normal system error, is accomplished. As a result, a parametric research was carried out in order to examine the impacts of this parameter by a trial-and-error method. Findings indicated that the best value for the selection pressure parameter was 60%. Thus, this value was applied to the rest of modelling process. Another parametric research was required for designing another efficient parameter (number of layers) upon the GMDH model. To this end, based on recommendations of some previously-conducted studies (Koopialipoor et al. 2018b), possible numbers of layers were set ranging from 2 to 10. Then, accordingly, 9 GMDH models were constructed to predict E of the rock samples and their results based on R<sup>2</sup> of training and testing datasets are shown in Table 3. Remember that this parametric research was carried out with the use of 6 neurons and the selection pressure of 60%. Based on the last column of Table 3, GMDH model number 5 with 6 layers shows the best performance (summation rank of 17) among all GMDH models. Results of 0.852 for train and 0.865 for test were obtained for the GMDH model using 6 layers.

At the final phase of the GMDH modelling, the number of neurons is needed to be specified by another parametric research. Accordingly, previously-conducted studies (e.g., Koopialipoor *et al.* 2018b) were reviewed and then values ranging from 2 to 20, with incremental step 2, were set as the number of neurons in the parametric study. Table 4 presents the obtained results of GMDH models on the basis of  $R^2$  for train and test stages together with their rank values. As the table clearly shows, the GMDH model number 6 (with summation rank of 19) consisting of 12 neurons offered the optimal performance.  $R^2$  values of 0.985 and 0.961 for train and test stages of the proposed GMDH model, respectively revealed that GMDH is an applicable, practical and powerful predictive technique in estimation rock deformation values. Note that values of 6, 12, and 60% were considered for the layer number, neuron number, and selection pressure,

respectively, in the developed GMDH model. In the following, the performance of the selected GMDH model will be explained in detail.

### 5. Assessment of the developed models

This section provides more details regarding assessment of the selected ANN and GMDH models in forecasting rock deformation. For the selection of the optimum predictive models, four performance indexes, i.e., R<sup>2</sup>, variance account for (VAF), the *a20-index*, and root mean square error (RMSE) were taken into account. Numerous researchers have extensively applied the abovementioned indexes to the evaluation of the model performance in the previous related works (Asteris *et al.* 2019, Armaghani *et al.* 2020, Han *et al.* 2020, Zhang *et al.* 2020). In the following, the formulas for calculation of RMSE, VAF, and *a20-index* are presented

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_{ipred} - x_{imeas})^2}$$
(13)

$$VAF = \begin{bmatrix} 1 - \frac{var (x_{imeas} - x_{ipred})}{var (x_{imeas})} \end{bmatrix} \times 100$$
(14)

$$a20 - index = \frac{m20}{N} \tag{15}$$

where,  $x_{imeas}$  and  $x_{ipred}$  stand for the determined and predicted values, respectively, N denotes the total number of datasets, and m20 signifies the number of samples with values of rate measured/predicted values (ranging from 0.8 to 1.2). For the  $R^2$ , VAF, and RMSE indexes, the values of 1, 100%, and 0 are needed, respectively, if seeking for a perfect predictive model. Remember that for such model, the *a20-index* should be equal to 1. The computed results of the mentioned indices for ANN and GMDH models are tabulated in Table 5. According to this table and considering results of both sets of train and test, it is clear that the GMDH model was more successful during model development compared to the ANN model. Because, its results during testing stage are significantly better than those obtained by the ANN technique. Although results of training sets of ANN and GMDH models are similar to each other, by developing a GMDH model, the system's results and predicted rock deformation values will be very close to their determined values. This revealed that the GMDH predictive technique is more powerful and applicable compared to ANN model in predicting rock deformation values. Values of (0.768, 17.855, 67.180% and 0.667) and (0.961, 7.190, 95.069% and 1) were obtained for R<sup>2</sup>, RMSE, VAF (%) and a20-index of testing set of ANN and GMDH models, respectively. These results show that if a higher degree of accuracy is required for rock deformation prediction, the GMDH model is able to provide a predictive technique with lowest error compared to the ANN model. To have a better view regarding testing results, all determined 18 values of E together with the predicted E by GMDH and ANN techniques are plotted in Fig. 6. As clearly displayed, E values predicted by GMDH are closer to the determined E values compared to those predicted by ANN predictive model. The developed GMDH model can be used for prediction of rock deformation prior evaluation and design of geotechnical projects. It is important to mention that the accuracy level obtained by GMDH model in this study to estimate rock deformation is higher than the previous relevant studies reviewed in introduction section.

#### 6. Conclusions

A reliable prediction of rock deformation is of crucial importance to civil engineering applications such as foundations and slope stability. The present study aims to propose an intelligence predictive model i.e., GMDH for the prediction of rock deformation. For this purpose, a series of UCS tests, together with several rock index tests, were conducted on the collected rock block samples and then a database with 88 datasets was compiled. In the initial analysis stage of this study, several empirical equations were suggested to predict E using SRn, Vp,  $I_{550}$ , and n. With R<sup>2</sup> ranging between 0.5-0.7, these equations were assessed as relatively inadequate in predicting rock deformation. In addition, R<sup>2</sup> of 0.773 for the proposed MLR equation revealed that the said equation is acceptable in terms of accuracy, however, it seems that there is a need to develop new model and, thus, the GMDH model was proposed to predict E of the rock. To have a comparison purpose, a pre-developed ANN, as a benchmark in intelligent techniques, was also constructed to predict rock deformation. After modelling process of both GMDH and ANN models and considering four performance indices, namely R<sup>2</sup>, RMSE, a20-index and VAF, it was found that the results of the training stage of ANN and GMDH models are very close to each other with slightly better performance of the GMDH model. However, regarding the case of the testing stage, a completely different conclusion was drawn; the GMDH model provided a significant improvement during the testing stage compared to the ANN model. R<sup>2</sup>, RMSE, a20-index and VAF results of (0.768, 17.855, 0.667 and 67.180%) and (0.961, 7.190, 1 and 95.069%) were obtained for testing stage of ANN and GMDH models, clearly indicating that the GMDH model can perform better compared to the ANN model in predicting E. The model development process of this study can be used by designers and researchers in solving similar problems, of course with the appropriate caution.

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