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Review of static soil-framed structure interaction

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Abstract. A wide literature review on Static Soil-Structure-Interaction (SSI) is done to highlight the key impacts of soil complexity on structural members of framed structures. Attention is paid to the developed approaches, i.e., conventional and Finite Element Method (FEM), to emphasize on deficiencies and merits of the proposed methods according to their applicability, accuracy and power to model and idealization of the superstructures as well as the soil continuum. Proposed hypothesis are much deeply discussed herein for better understanding which is normally neglected in literature review papers due to the large number of references and limit of space.

Keywords: review paper; static soil-structure-interaction; winkler model; finite element analysis; shallow foundation

1. Introduction

Analysis of the framed structures has been subjected to significant changes during the last decades. Since these structures are resting on soil media the term of soil-structure interaction analysis came into computations and altered researchers' view. Many new methods and hypothesis are reported either to improve the former methods or introducing new approaches, mostly focused on previous deficiencies presented by scholars.

A major concern of soil-structure interaction is flexibility of the foundation and compressibility of the soil mass due to the settlements of the deforming supports which in turn alters superstructure behavior as the significant result of footings differential settlements imposes load changing to the structural elements and alters positive and negative moments in the beam elements as well as transferring the vertical forces to the interior columns.

All the results reported by researchers so far witness the effect of incorporation of flexible foundation or in other words soil-structure interaction while buildings are in analysis and design stages resisting against either static or dynamic loadings. Hence importance of soil continuum-footing-superstructure interaction is proved to be a key issue when it comes to a real analysis of any structure.

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The current work presents methods of static Soil-Structure Interaction with especial consideration on framed structures-foundation-soil interaction and effort is made to critically cover generally related research works regarding such a complex field. Two well-known hypotheses, mainly decomposed of classical and numerical methods, are covered with especial attention to the current state of knowledge in the field of SSI and presents major aspects of this interdisciplinary field to highlight the importance of soil non-linearity and its subsequent effects on superstructure and infrastructure members such as beams, columns and footings. Discussion is confined to shallow types of footing while deep foundations are beyond the scope of the present review.

The first part contains classical approaches which are mostly in relation with Winkler hypothesis and elastic half space idealization. Although these approaches are not robust enough for SSI analysis as most of pioneers only focused on single beams resting on Winkler medium while no reliable approach were ever presented to include a comprehensive analysis of super structure performance, however authors decided to review classical methods to demonstrate the progressing trend of this knowledge which has led to better understanding of foundation design and soil idealization.

Secondly the numerical approach of FEM is discussed with noticing all aspects of SSI in the context of FEM. This part basically represents the most well-known and universally accepted approach for SSI in which idealization of structural and soil medium elements have been considerably noticed in addition to interface elements for modeling discontinuities. The last section of this paper has been allocated to review some well-known soil constitutive laws that have been modified and employed for soil analysis. This is because selection of a constitutive model has considerable effect on the soil behavior and its interaction with foundation.

2. Classical approaches

2.1 Winkler model

Modeling of structures and the underneath soil came into the existence when drastic differences among practical cases and theory were observed by the design engineers and researchers. Although prediction of deviation from reality was not difficult, however it was the supporting soil which was not easy to be modeled or formulated due to its complex nature. The representation of the soil mass has been always major concern for engineers and researchers for a non-accurate model is selected investigation of soil-structure interaction would be a time consuming matter.

Winkler foundation model is found as a straightforward and simple solution for interaction of foundation and underlying soil (Fig. 1). This simple idealization is almost the first analytical solution which deals with the complexity of the soil as well as behavior of the footing resting on it.

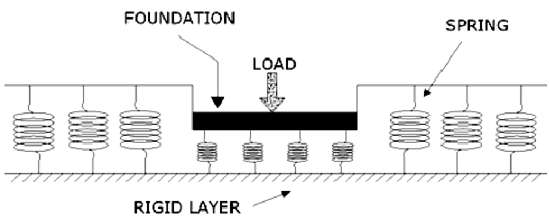


Fig. 1 Winkler foundation

Extensive studies were carried out to modify this solution since it is dependent of a coefficient called sub-grade modulus which brings uncertainty into the calculation since this parameter does not represent any characteristic of the soil, Stavridis (2002). Sub-grade modulus of reaction as a key factor of Winkler hypothesis has been focused in order to be more accurately interpreted through a mathematical model namely Winkler-Type Simplified Continuum (WTSC), Horvath (1983a). This model follows previously assumed boundary conditions consist of zero displacement at the base of elastic soil layer as well as equality of vertical stress and applied force at each point on the surface. The sub-grade modulus is defined as the function of layer thickness H and Young modulus E. In addition H is function of width of footing provided below

$$H = I_k b \tag{1}$$

I_k and b stand for influence factor and width of footing respectively. Therefore considered modulus is function of dimensions of loaded area indirectly, Horvath (1983a). The value of sub-grade coefficient for different soil types can be obtained from literature, Chowdhury and Dasgupta (2008).

The soil-line method which counts for the foundation flexibility takes advantage of Winkler model for analysis of superstructure and the foundation as a single compatible unit and that is the advantage of soil-line method to the conventional method which is based on rigid foundation, however neither the conventional method nor soil-line method result in moment redistribution which originates from differential settlements. Lee and Harrison (1970) studied the effect of soil-foundation relative flexibility on internal column forces of a single- and a two-bay single story frame structures supported by raft foundation, modeled as beam on Winkler elastic medium, through two analytical model slope-deflection method and contact pressure method.

Variation of internal forces due to inclusion of foundation relative flexibility is reported by Lee and Brown (1972) where the differential settlements have resulted in increase of axial loads in inner columns and decrease in outer ones of a three-bay multi-story frame. The maximum negative moments corresponding to rigid foundation are found in the middle of raft or strip foundation while this location is reported to shift to somewhere between outer and the first inner columns as the flexibility rises.

Panayotounakos and his fellow researchers developed a matrix solution suitable for analysis of multistory plane frames resting on elastic foundation subjected to different types of loading. Soil medium is idealized through application of translational and rotational springs neglecting shearing effects and axial forces in the so called basic structure representing footing. Inclusion of elastic foundation on superstructure was found remarkable Panayotounakos *et al.* (1987).

Generally speaking Winkler hypothesis idealizes the soil mass through spreading independent, closely spaced, linearly elastic springs due to which considerable improvement as well as uncertainties are raised. The following lists the merits and demerits of this approach;

(1) It is simple and more realistic compared to fixed base idealization (conventional analysis),

(2) Sub-grade modulus is function of nature of the soil as well as dimensions of loaded area but not the contact pressure and it is the same for all the area compressed by external load Pavlovic' and Tsikkos (1982),

(3) Winkler model has shown acceptable accuracy for calculation of deflections and stresses of the cases in which footings under load deflect toward the foundation, i.e., there is no lift-off, Pavlovic' and Tsikkos (1982).

Furthermore there are some crucial deficiencies which lead to many further modifications:

(1) Winkler hypothesis can predict contact pressure more accurately for relatively flexible footings idealized by beam or representing mat foundation and this may reduce the applicability of the model, Lee and Harrison (1970), Fig. 2. Besides, in case of flexible footing this approach is incapable of predicting actual deformation pattern acted upon under uniform load, Fig. 3, Horvath (1983b),
(2) Winkler solution models the sub-grade behavior as linear and elastic while the nature of soil is different,
(3) Behavior of soil is idealized only with sub-grade modulus and none of soil parameters count,
(4) Soil is a continuum and continuity of the soil structure corresponds transverse shear stress which is neglected by Winkler model due to the presence of independent springs, (Colasanti and Horvath 2010, Teodoru and Musat 2010). Due to the nature of the independent springs, displacement discontinuity occurs between the loaded part and unloaded area of sub-grade, i.e., there is no cohesive bond among soil particles, Teodoru and Musat (2010), and this is in addition to the localization of the applied loads to the point of application, e.g. displacement.
(5) Winkler model does not give bending moment and shear forces induced inside the beam, Vallabhan and Das (1991).

Many efforts have been made so far to modify, improve or even develop new methods apparent to the Winkler approach and almost all tended to come up with some solutions for Winkler deficiencies as stated above. These methods are mostly known as beam on elastic foundation idealizing the footing by beam and the soil as an elastic material however there are very few researches in this category that deals with nonlinearity of soil and will be reviewed in this section.

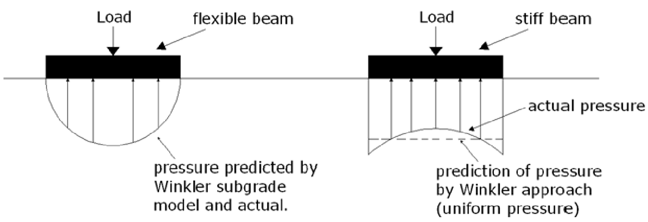
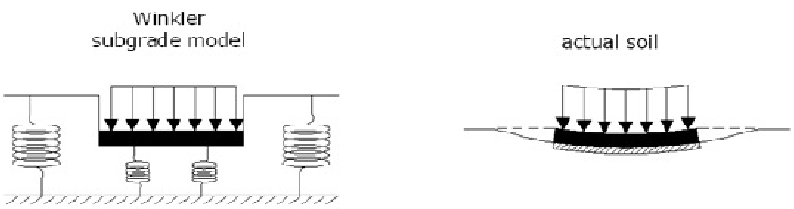
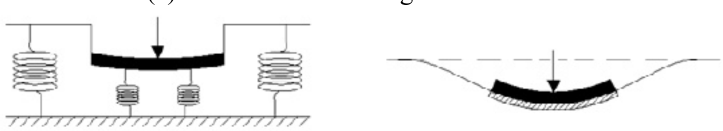


Fig. 2 Typical contact pressure; (a) Flexible beam, (b) Stiff beam



(a) Uniform load/non-rigid beam or mat



(b) Concentrated load/non-rigid beam or mat
Fig. 3 Winkler model and actual behavior of soil: (a) uniform load and (b) point load

Pavlovic has presented a method, called quasi-Winkler foundation, which is capable of handling the interactions with loss of contact between footing resting on a so called Winkler foundation and here is the time that a tensile reaction of the supporting medium is required. When the foundation is acted upon by point loads beam is divided into two parts, one is the part deflects toward the soil and the one experiences uplift which makes this portion to detach from the soil surface and subsequently does not engage in soil behavior. For this reason Eq. (2b) does not involve any of soil properties and only follows the mathematical deflection pattern. The governing equations for each part are recalled herein respectively, Pavlovic' and Tsikkos (1982)

$$EI\frac{d^4y}{dx^4} + Ky = 0 \tag{2a}$$

$$\frac{d^4y}{dx^4} = 0 \tag{2b}$$

Where y is downward deflection and K is sub-grade modulus multiplied by width of footing. Above equations are solved through differential methods and implementation of boundary conditions. Comparing the quasi-Winkler method and Winkler approach, Pavlovic and Tsikkos have concluded that their method is more suitable for cases with point couple exerted away from end supports and also for cases in which beams deflect away from soil-footing interface. The latter case may take place, for instance, when internal reaction of structure and foundation are mobilizing against wind load where the Winkler method underestimates deflections and stresses. Both hypotheses perform well in the general cases where there is a downward deflection to soil mass while quasi-Winkler foundation does not perform well under general load conditions but point loads, Pavlovic' and Tsikkos (1982). However applicability of this method to only point load seems to be a draw back as far as accuracy of the model for settlement prediction is concerned since replacing the point load by uniform load leads to larger values and therefore more reliable for practicing foundation engineers, Masih (1994).

Nonlinear behavior of soil has made researchers to consider it in soil-structure interaction since results have shown considerable differences among linear and non-linear analysis of stress levels in soil medium. While importance of soil nonlinearity is considered as a key aspect of SSI in many analysis it is usually found that performance of the superstructure members are mostly linear while soil enters nonlinearity even under small portions of stress.

Along the application of Winkler foundation Al-Mahaidi and his fellow researchers have come up with a linear single-parameter soil whose sub-grade modulus is not a constant. Soil mass is modeled by employing very closely located nonlinear springs representing a nonhomogeneous and nonlinear foundation. This methodology is used under an iterative analysis for beams located on nonlinear soil mass on which the resting beam does not need to be discretized into more than one element. Differential equation like Eq. (3) is employed and presented as follows, Al-Mahaidi *et al.* (1990)

$$EI\,\frac{d^4y}{dx^4} + K(x)\,y = w(x) \tag{3}$$

Where $w(x)$ is load function. It is clear that sub-grade modulus, load and solution functions are dependent of location of considered point along x direction, Fig. 4. Maclaurine series are selected as the solution of earlier mentioned functions for differential equation cannot be earned as a

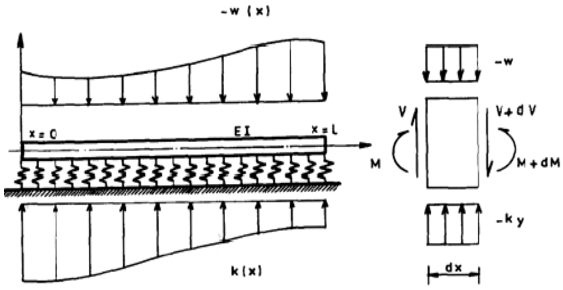


Fig. 4 Beam resting on continuous springs

punctilious solution for these non-constants.

Dependency of beam deflection function on x has been considered as primary remedy for accounting the significant mechanism of vertical shear while this dependency is said to imitate the response of actual sub-grade whose springs are coupled.

Moreover, key uncertainties and inaccuracies of Winkler model, like shear effects, have been pacified to some extent through application of beam-column analogy proposed by Horvath (1993). The developed model by Horvath is a combination of Pasternak's model and beam column analogy responding to vertical load. The resultant equation represents a beam under constant tension of magnitude C_{p2}, Table 1, resting on independent springs representing an isotropic homogeneous sub-grade. Two other modified versions of Winkler's model are presented in the Table, namely Filonenko-borodich and Hetenyi's foundations for comparison. In order to minimize the soil effects originated beyond the modeled beam Horvath built up his model on two assumptions:

(1) Transverse shear stress is constant at both ends of the beam which leads the vertical force to be zero at these locations,

(2) Continuity of displacement and its first derivative at both ends which induces the reaction force acting against the external load as a consequence of settlement at ends. It suits to mention that this assumption is found to give more realistic results.

Due to the absence of full incorporation of soil effect in structural performance of the beam, shear forces as well as horizontal displacement of the beam are inaccurate and cannot be used for design purposes. Nevertheless the significant improvement of the presented method by Horvath is considerable. Study on a tensionless elastic beam resting on an elastic soil medium, of Winkler type, furnishing essential foundation design requirements like contact pressure, deflection, corresponding internal shear and moment forces has been carried out through the proposed method comprised of Newton's method and Finite element approach. This numerical method is also capable of finding lift-off regions, Fig. 5, Kaschiev and Mikhajlov (1995).

Linearity and non-linearity of the solution presented by Kaschiev and Mikhajlov depends on the deformation pattern, $y(x)$, of the beam acted upon by external forces and this is basically because of dependency of sub-grade modulus, $\beta(y)=k$, on beam deflection sign.

$$\beta(y) = k \qquad y(x) > 0, \tag{4a}$$

$$\beta(y) = 0 \qquad y(x) \le 0 \tag{4b}$$

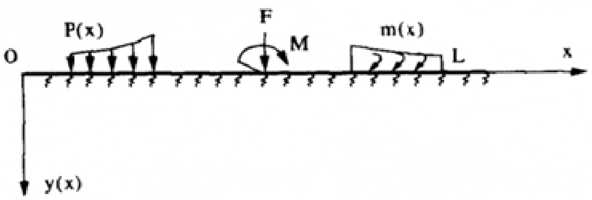


Fig. 5 Beam resting on Winkler tensionless foundation

Differential equation of considered beam deflection function is as follows

$$L(y)=E\frac{d^2I(x)}{dx^2}.\frac{d^4y(x)}{dx^4}+\beta y(x)-P(x)-\frac{dm(x)}{dx}+\sum_{i=1}^{p}F_i\delta(x-x_i^f)-\sum_{i=1}^{q}M_i\frac{d}{dx}\delta(x-x_i^m)=0 \quad (5)$$

In which $I(x)$ and E stand for moment of inertia and Young modulus, $P(x)$ and $m(x)$ describe distributed load and distributed moment. F_i, M_i and X_i represent concentrated forces, concentrated moments and points acted upon by these forces respectively.

Newton's method is employed to solve the nonlinear problem and finite element for idealizing and discretizing the beam capable of fast and with at most second-degree converging especially for every considered initial deformation function, Kaschiev and Mikhajlov (1995). Since the equation proposed by Kaschiev and Mikhajlov incorporates a general load pattern, therefore it seems more applicable than Pavlovic's equation and Al-Mahaidi's equation.

Application of photoelastic method has been reported parallel to utilization of theoretical approach to study interaction of framed building and half-plane multilayered soil medium. The first approach has been assigned to investigate the plane stress analysis of the mentioned interaction represented by semi-circular 2-D space located on rigid and smooth support and the theoretical solution is decomposed into simulation of semi-infinite multi-layered medium by analytical approach and FEM for superstructure idealization. Theoretical and experimental results have shown consistency in analyzing contact pressure getting some effects by rigidity of idealizing beam due to what the performance of foundation tends to follow the punching pattern. Winkler model is also employed for comparison whose results have shown considerable errors in case of simulating multi-layered soil medium, for both displacement and contact pressure estimation. It is also found that the pattern of stress distribution, except next to the interface surface, is not influenced by the way contact pressure is distributed which, similar to super-structural bending stresses, is affected by rigidity of superstructure, Chandrashekhara and Antony (1993).

Pandey *et al.* (1994) employed Winkler approach, on the basis of no discontinuity, along with utilization of finite element method for comparison. Structural elements were modeled by beam element and brick finite element was used to represent the soil. Displacements were found marginally close by which structural forces were found affected under footing displacement. Outer columns of modeled frame had been subjected to more increase in forces compared to those of fixed base and bending moments and shear forces were considerable enough in beams to be subjected to section modification. Weigel *et al.* (1989) and Pandey *et al.* (1994) concluded that differential settlements may be the origin of stress reversal in structural elements. The maximum settlement was found in flexible linear-elastic frame while the rigid one experienced minimum settlement. In addition, compared to flexible frame, major moment changes happened in rigid frame.

Eventually Horvath and Colasanti (2011a) and Horvath and Colasanti (2011b) introduced the modified sub-grade hybrid model named modified Kerr/Horvath-Colasanti (MK-R) model, Table 1, which brings the up-to-date solution to the fundamental Winkler's flaw, i.e., lack of interaction between adjacent springs. This model is capable of replacing the multi-layered soil medium by an equivalent elastic single layer comprised of two upper and lower spring layers separated by a perfectly flexible membrane under constant tension T through which the interspring shearing is produced. The proposed p-W equation which defines the behavior of the model is presented in Table 1. The proposed model is applicable to software implementation for SSI analysis, Colasanti and Horvath (2010).

Based on what was reviewed for Winkler's model and all the associated modified versions, it should be mentioned that although these types of simplified models look very straightforward for manual calculation, however they are not capable of simulating the exact behavior of the soil under loading. As was mentioned at the beginning of this review the complexity of soil associated with the nonlinearity is somehow ignored where other soil parameters related to mechanics of the soil behavior can get included to modify and well estimating the foundation behavior idealization. Furthermore this type of analysis has never been put into SSI analysis effectively such that designers can rely on, hence the key aspects of interaction analysis with regarding to framed superstructures are not well studied. In addition Winkler model, based on to-date research works, is incapable of estimating load distribution through incremental influence area along the depth under gradual increase of the external load. Therefore the conventional modified models are not comprehensive enough to be employed as robust analysis.

2.2 Continuum model

Elastic continuum model is almost a physical representation of the infinite soil media which is homogeneous, isotropic and linear elastic subjected to a concentrated point load. The behavior of such an elastic medium is described through the deflection line of the soil surface. Special attention is paid to mat foundation by Horvath through an approach built up on theory of elasticity namely Reissner Simplified Continuum (RSC) whose governing differential equation included the elastic parameters of soil mass, i.e., elastic modulus, shear modulus and the thickness of considered soil layer (Table 2), Horvath (1983b). Assumptions can be listed as:
(1) Horizontal and shear stresses are set to zero (σ_x, σ_y and τ_{xy}=0),
(2) Transverse shear stress (τ_{xz} and τ_{yz}) is constant and does not vary along the depth,
(3) Variation of vertical normal stress (σ_z) is linear along the depth.

It is observed that RSC performs more precisely compared to WTSC model especially for cases with zero Poisson's ratio. Edge stresses are found to be more accurately computed for rigid and strip footings compared to those in WTSC, hence substantial modification in outcomes for corresponding moments are expected Horvath (1983b). To represent the subsoil or more specifically restraining the stress variation in the soil continuum, the presented sub-grade models are known as one- or two- parameter sub-grade model. Vlasov sub-grade model presented in Table 2, is a two parameter model whose factor (γ) deals with the variation of stress and displacement in the continuum medium and is dependent of loading, Teodoru (2009). However this parameter has got to be Estimated, (Vallabhan and Das 1991, Teodoru and Musat 2010). This is in addition to the complex solution of this model through minimization of potential energy and due to why this model is hardly employed by engineers, Vallabhan and Das (1991). Variation of γ and $\varphi(z)$ along the depth is depicted in Fig. 6. The followings are the assumptions by this method:

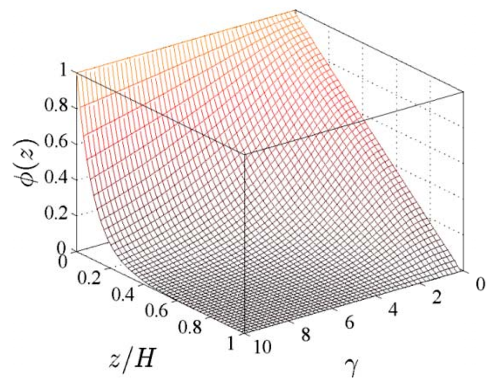


Fig. 6 Variation of shape function vs. γ and z/H. Teodoru and Musat (2010)

(1) Vertical displacement varies from maximum to zero along the depth, i.e., deflection mode shape $\varphi(z)$, Fig. 6.
(2) Horizontal displacement is ignored.
(3) Vertical compatibility is considered only at the interface of soil and beam.

A modified version of Vlasov approach is developed taking advantage of finite difference method to come up with a solution of potential energy derivations.

The beneficial advantage of this model is inclusion of modular ratio (E_b/E_s) whose variation has the most effect on displacement. E_b and E_s stand for modulus of elasticity of beam and soil respectively. Comparisons between the modified version of Vlasov sub-grade and numerical methods such as finite element method has shown close agreement between these two approaches, Vallabhan and Das (1991). Onu (2008) also studied the Vlasov foundation model considering varying thickness of soil layer along the beam foundation bringing the shear deformation effect into the picture which was suggested for foundation structural members with small length/depth ratio supporting large point loads, e.g. column loads.

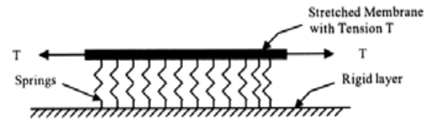
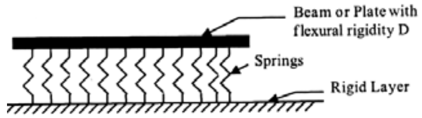
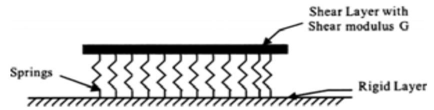
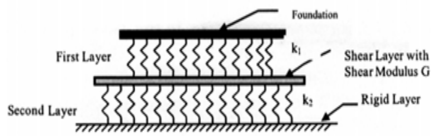
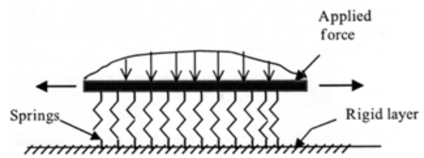
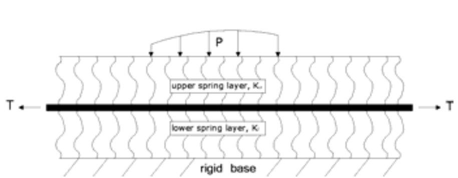
Another modification of Vlasov hypothesis is carried out taking advantage of numerical method into the account as an alternative for implementing on static analysis of beams resting on elastic soil medium. Equations are well documented in literature and here is a brief explanation of developed method in which the two aforementioned Vlasov parameters are introduced in matrix form and formulated as follows, Teodoru and Musat (2010)

$$([K_e] + [K_e^s] + [K_e^t])\{d_e\} = [S_e] - [R_e] \tag{6}$$

Where K_e, K_e^s and K_e^t stand for stiffness matrix of respectively flexural beam element, first sub-grade parameter (equivalent Winkler spring constant) and second sub-grade parameter (shear parameter). d_e, S_e and R_e represent vector of nodal degrees of freedom, vector of nodal loads and vector of equivalent nodal loads exerted by uniformly distributed load. Results from the modified approach by Teodora and Musat are found to be overestimating those obtained by finite element method with small difference, Teodoru and Musat (2010).

Stavridis studied an analytical approach through which interaction of soil and structure or more specifically two of the major aspects of soil-structure interaction namely support reactions and settlements, which lead to stress and pressure in the soil mass, can be computed through combination of continuum model for underlying layered soil and finite element for superstructure

Table 1 Modified versions of winkler model

Hypothesis	Modified version	Beam response equation	Foundation idealization	Remarks
Winkler	*Filonenko-borodich*	$p = kw - T\nabla^2 w$, for rectangular, strip and circular foundation		Discrete springs are coupled through a thin elastic membrain acted upon a tension (T). $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
	Hetenyi	$p = kw - D\nabla^2 w$		Connectivity of individual springs is modeled by a plate or beam that undergoes flexural deformation. $D = (E_p h_p^3)/(12(1-\mu_p)^2)$, flexural rigidity of elastic plate. h_p: thickness of plate
	Pasternak	$p = kw - G\nabla^2 w$		Transverse shear interaction of springs is idealized by employing a beam or plate capable of bearing vertical shear force only. G: shear modulus of beam or plate.
	Kerr	$(1 + k_2/k_1)P = (G/k_1)\nabla^2 P + k_2 w - G\nabla^2 w$		Two layers of springs with different stiffness are joined by a shear layer below foundation.
	Beam-column analogy by Horvath	$E_b I_b (d^4 w(x)/dx^4) - C_{p2}(d^2 w(x)/dx^2) + C_{p1} w(x) = q(x)$		Connectivity of springs is achieved by parameter C_{p2}. E_b, I_b: flexural rigidity of beam (constant) $q(x)$: external applied load $C_{p1} = E/H$: spring stiffness $C_{p2} = GH/2$ H: depth of rigid base.
	Modified kerr/Horvath-Colasanti	$p - (\frac{T}{K_u + K_l})\nabla^2 p = (\frac{K_u K_l}{K_u + K_l})W - (\frac{TK_u}{K_u + K_l})\nabla^2 W$		T: tension = $4G_sH/9$ (kN/m) K_u: upper layer spring stiffness (kN/m^3) = $4E_s/H$ K_l: lower layer spring stiffness (kN/m^3) = $4E_s/3H$ P: external uniform load (kN/m^2) W: surface displacement E_s, G_s *and* υ_s: equivalent average Young's modulus, shear modulus and Poisson's ratio H: soil depth

idealization. Generally speaking the analytical procedure of this method can be categorized as follows, Stavridis (2002):
(1) Discretization of foundation,
(2) Computation of soil stiffness matrix,
(3) Determination of support reactions,
(4) Calculation of stiffness of supports,
(5) Determination of nodal displacements of foundation,
(6) Calculation of stresses and soil pressure beneath the footing.

Comparison with Winkler method for a beam footing has shown more disagreements for stiffer beams and less for more flexible ones. As a merit of this analytical approach, unlike the Winkler approach there is no modulus of sub-grade reaction representing the soil property.

3. Finite element method (modeling and simulation)

Although Winkler method has been modified by many researchers, keeping in mind the great recuperation of SSI compared to fixed-based conventional analysis, finite element method as the most well-known numerical approach has performed well in minimizing the repugnancies and gaps between theory and reality and still growing larger as an advanced solution of intricate problems. The last three decades have witnessed a tremendous growth in the numerical methods through which it became possible to obtain more realistic and satisfactory solution for any complex problem such as soil-structure interaction. Among the numerical methods the most versatile, prominent and successful procedure has been finite element method (FEM).

Table 2 Modified versions of continuum hypothesis

Hypothesis	Modified version	Beam response equation	Further expression
Continuum	*Vlasov foundation*	$p = kw - 2t(d^2w/dx^2)$	Long slab with limited width is considered resting on elastic continuum under it. *k*; equivalent subgrade reaction modulus $= (E_\circ/(1-\nu_\circ)^2)\int_0^H (dh/dz)^2\,dz$ *t*: shear foundation parameter $= (E_\circ/4(1-\nu_\circ))\int_0^H h^2\,dz$ $E_\circ = E/(1-\nu)^2$ $\nu_\circ = \nu/(1-\nu)$
	Riessner Simplified Continuum (RSC)	$C_1w - C_2\nabla^2 w = p - C_3\nabla^2 p$	Analyzes mat foundation acted upon by static load and resting on an elastic continuum. $C_1=E/H$ $C_2=GH/3$ $C_3 = GH^2/12E$

Generally speaking when any discussion is opened regarding the application of Finite Element Method (FEM) in modeling and analyzing the SSI, debate is automatically drawn into different aspects of numerical modeling such as, Viladkar *et al.* (1994a) and Agrawal and Hora (2010):
(1) Type of soil below the foundation at various depth,
(2) Constitutive relation of the soil media,
(3) Constitutive relation of the superstructure,
(4) Size, shape and types of footing/foundation,
(5) Interface between adjacent footings,
(6) Relative stiffness between soil and foundation,
(7) Types of loading,
(8) Transition boundary,
(9) Boundary condition,
(10) Far domain idealization.

Godbole *et al.* (1990) have focused on analyzing both flexible and rigid strip foundation resting on sand modeled through coupling of finite and infinite elements for near and far field of the soil mass respectively, following hyperbolic constitutive model, in plane strain condition. Furthermore, rigidity of foundation is represented by stiffness of soil-footing system which is analyzed through both finite element method and coupled formulation for comparison where close agreement between the two selected formulations was found but less CPU time in coupled finite-infinite idealization. Presence of infinite element has led to underestimation of settlements in flexible footing while the rest of aspects like normal stresses and their distribution trend were found similar. The maximum contact pressure beneath the rigid foundation is found under edges and minimum at the center while it is opposite for flexible strip foundation. Under linear behavior the relative stiffness of footing-soil system is presented by Noorzaei *et al.* (1993)

$$K_{fs} = \frac{K_{footing}}{K_{soil}} = \frac{E_f . I_f}{(1-v_f^2)} \times \frac{(1-v_s^2)}{(E_s)_{avg} . L_f^4} \tag{7}$$

Where E_f and I_f are modulus of elasticity and inertial moment of footings respectively. vf and vs are Poisson ratio of footing and soil correspondingly and Lf is the length of raft foundation.

Similar methodology for idealization and discretization of soil mass beneath the combined footing has been employed to analyze the interaction of plane-framed super structure and considered sandy soil while a bending element is presented for footing idealization, Viladkar *et al.* (1991). The proposed bending element simulates the performance of the footing for transverse shear deformations as well as axial-flexural interaction. Compare to the conventional element, the considered bending element is a 1-D and three-noded one capable of deforming due to shear and the angle of rotation which is defined as a function of effective transverse rotation. The proposed bending element for idealization of footing performance can overcome the deficiencies of previously proposed solutions for SSI.

It is worth mentioning that the far domain of soil is idealized through application of infinite element employing $1/r$ type of decay (see Table 3) and the location of transition boundary is gained by implementing same model but fully finite elements are used to justify the location. This location is found to affect the magnitudes of settlements which are found to be 3 to 4 times higher in case of nonlinear compared to linear analysis. Mozos and Luco (2011) used boundary springs instead of infinite element where smaller dimensions of soil medium can be modeled therefore it brings more accuracy and lower time cost, however with high performance computers the time

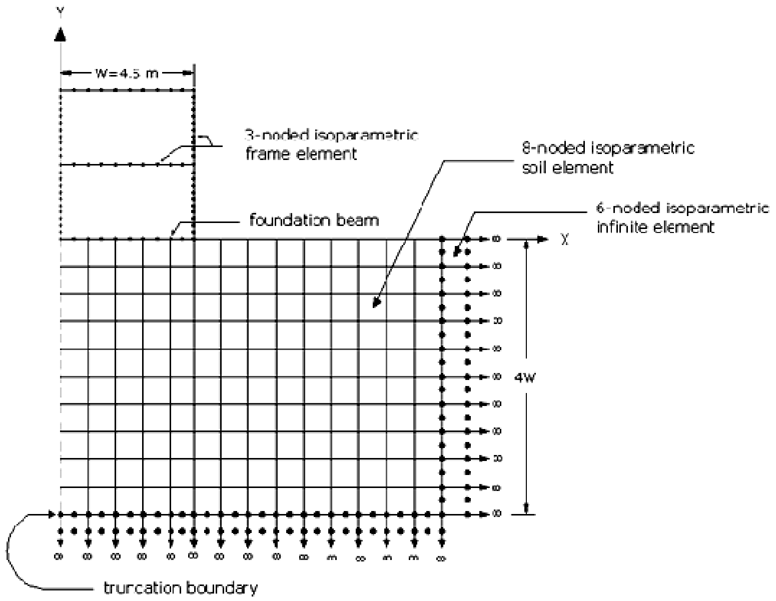


Fig. 7 Coupled finite/infinite idealization of plane frame-soil system (Agrawal and Hora 2010)

cost is not a problem even if the case under study is large in dimension. Viladkar *et al.* (1994c) reported the non-linear interactive settlements 1.5 to 2 times larger than that of linear analysis where non-linear analysis has caused minimal differential settlements as well as redistribution of contact pressure which has leaded to higher values of moments in z direction perpendicular to the plane and reduction in axial forces in columns. Agrawal and Hora (2010) obtained analogous results for a two-bay two-storey plain frame, Fig. 7.

Settlements are found to be 2.25 times and contact pressure at the footing edge 1.12 times as much as those of linear analysis. Reduction and increment of axial force has been reported for interior and outer columns respectively for both Linear-Interactive Analysis (LIA) and Non-Linear Interactive Analysis (NLIA) as a result of differential settlement. Outer ends of first and second floor beams have experienced higher values of bending moments due to the force transferring originated from differential settlement which causes increase of negative and positive bending moments in foundation beam. Similar result has been reported by Dalili *et al.* (2011) for the NLIA settlements of a plane-frame shear-wall structure which has highlighted the disparity between the non-interactive analysis and NLIA. The total settlements of such a frame are reported to be 1.3 to 1.4 times that of non-interactive analysis. Inclusion of soil nonlinearity is found to have remarkable influence on the frame deflection as well as stress distribution of the shear wall.

Foye *et al.* (2008) highlighted the short-term settlement of clay which arises from distortion due to presence of water, i.e., undrained condition. Proposed nonlinear solution was suitable for settlement of square, rectangular and strip footings subjected to axial load and not accurate enough for soil-structure interaction application.

Moving forward along developments made in the subdiscipline of interaction among soil, footing and superstructure contemplating the accuracy of physical modeling of a fully idealized system through finite/infinite techniques wherewithal to consider all the aspects of such an analysis has been carried out in Noorzaei *et al.* (1991) and Viladkar *et al.* (1992) to examine the effect of thickness of slabs and footing in a 3-D analysis of framed superstructure.

Viladkar *et al.* (1992) used special plate bending element to model the footing and slabs. The considered 8-noded isoparametric parabolic plate bending element has five degrees of freedom per node comprised of displacement along three main directions (i.e., *x*, *y* and *z*) and two rotations around x and z axis. Formerly formulation of this element had been based on three degrees of freedom (DOF) per node but the modified version of this element is used with five DOF. Detail mathematical formulation of this element is available in Viladkar *et al.* (1992). It is found that increasing the thickness of raft foundation leads to degradation of settlement at the center while it grows up at the corners, i.e., reduction of differential settlement, and this is in addition to the further redistribution. Enlarging the raft thickness has caused rigid behavior of the frame, less sway, reduction of the contact pressure, lowering the bending moments and redistribution of axial forces in columns, Noorzaei *et al.* (1991). Wood (1978) suggested large raft stiffness for those supporting shear walls to have realistic settlement estimations. Drucker-Prager yield criteria was employed by Noorzaei *et al.* (1995a) and Noorzaei *et al.* (1995b) considering strength hardening of the soil for a plane-frame analysis and the corresponding collapse loads are obtained through incremental load factors. The LIA and Elasto-Plastic Interactive Analysis (EPLIA) are found to result in close magnitudes of settlement at lower loads while the differences are more pronounced for higher values of loads. Furthermore the NLIA has overestimated the two other analyses. Footing settlements are found to be higher at the center of the foundation. In addition, variation of contact pressure is seen to be the same in all three interactive analyses. It was observed that widening of plastic zone starts from edges of footing and stretches to the center as long as load is incremented. The extension of plastic zone is detected to have significant influence on internal members of superstructure, Noorzaei *et al.* (1995a). Other soil yield criterion has also been employed by Viladkar *et al.* (1995) for elasto-plastic soil-frame analysis. Jahromi *et al.* (2007) and Jahromi *et al.* (2009) proposed an alternative approach called domain decomposition by which soil and superstructure are separated into two sub-domains where the coupling procedure is presented through iterative Dirichlet- Neumann algorithm.

Comparison was made between monolithic and domain decomposition techniques and although superiority of interface relaxation algorithms was reported however the proposed partitioning technique was introduced as a powerful method for non-linear soil-structure interaction analysis facilitated by an enhanced convergence method. Bending moments of the plane frame analyzed by decomposition method were found to be considerably higher than those of non-interactive analysis.

Rao *et al.* (1995) have carried out a comparison between two well known strategies for analysis of soil behavior namely plain-strain and half-space to justify application of former one instead of 3-D analysis. Presence of three adjacent framed structures was considered to study the internal induced forces of middle frame with consideration of interaction of soil and structure. Selection of model, plain and space analysis, was found to have minimum effect on sagging moments in the frame while plain strain model has overestimated it over that of half-space model. Application of plain-strain has been suggested when the super-structural forces are of interest with lower frame-soil stiffness and higher soil-footing stiffness. Maximum differential settlements of any subframe of an actual space-frame structure in 3-D analysis has been estimated by Brown and Si (1986) to be within 20% of that in a 2-D analysis.

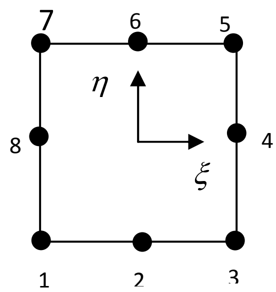
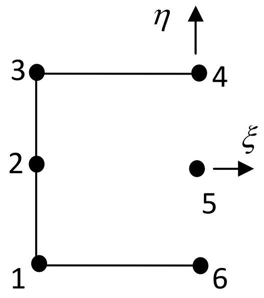
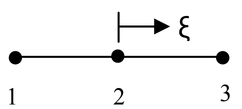
Combination of boundary element method and finite element method, namely the method of successive stiffness, has been put into practice for analyzing a 3-D framed structure supported by raft infrastructure resting on an inhomogeneous layered soil by Almeida and de Paiva (2004). The considered superstructure is modeled by finite element method comprised of shell element and beams modeled in 3-D and the so called half-space continuum stratified soil is idealized by boundary element approach. As a very practical methodology in imposing far domain condition Almeida and Paiva also have assumed null displacement and tractions for lateral surface, meanwhile equilibrium and compatibility conditions are satisfied for interfaces between each pair of layers like similar displacement and stress at the interface of two layers whose side boundaries are undisturbed.

Non-homogeneity of the continuum or in another word influence of each layer on the overall behavior of the soil is introduced through influence matrix, eqn. 9, decomposed of stiffness of top and bottom layers relating corresponding traction forces and displacements of the neighbor layers.

$$\begin{Bmatrix} P_t^i \\ P_b^i \end{Bmatrix} = \begin{bmatrix} [K_{tt}^i] & [K_{tb}^i] \\ [K_{bt}^i] & [K_{bb}^i] \end{bmatrix} \begin{Bmatrix} U_t^i \\ U_b^i \end{Bmatrix} \tag{8}$$

Subscripts t and b stand for top and bottom layer respectively while superscript i presents number of the layer and U and P are nodal displacements and tractions of corresponding layers.

Table 3 Two-dimensional serendipity type of finite, infinite and thin layer elements

Types of element	Element figure	Shape functions
Eight-node serendipity element		For corner nodes : $N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i+1)$ For mid-span nodes : $N_i = \frac{\xi_i^2}{4}(1+\xi\xi_i)(1+\eta^2)+\frac{\eta_i^2}{2}(1+\eta\eta_i)(1-\xi^2)$
Six-node infinite element		$N_1 = \frac{\xi\eta(1-\eta)}{(1-\xi)}$ $N_4 = \frac{(1+\xi)\eta(1-\eta)}{2(1-\xi)}$ $N_2 = -\frac{2\xi(1-\eta^2)}{(1-\xi)}$ $N_5 = \frac{(1+\xi)(1-\eta^2)}{(1-\xi)}$ $N_3 = \frac{-\xi\eta(1+\eta)}{(1-\xi)}$ $N_6 = \frac{-(1+\xi)\eta(1-\eta)}{2(1-\xi)}$
Three-node beam element		$N_1 = -\frac{1}{2}\xi(1-\xi)$ $N_2 = (1-\xi^2)$ $N_3 = \frac{1}{2}\xi(1+\xi)$

The membrane performance and plate behavior of the raft foundation is idealized by superposing triangular membrane element with triangular plate element with consequently six degrees of freedom per node including three rotations and three displacements. Beams and columns are modeled as linear elements neglecting influence of torsion. This is in addition to slabs assumed as diaphragms and the considered torsion center in each floor as a single point to which the force from columns and beams are exerted. Variation of soil rigidity along the soil depth as well as presence of non-deformable layer beneath a multi-storey framed building are studied and it is observed that these two conditions are not only more analogous to reality but also considerably affecting the vertical displacements and moments. In addition to the computational improvements the proposed method has performed well in investigation of soil-raft foundation-framed structure interaction, Almeida and de Paiva (2004). Similar results are reported by Son and Cording (2011) in comparing different framed structures supported by stiff and soft soil and influence of structural type as well as soil conditions are highlighted in relation to the excavation-induced ground settlement. Son and Cording concluded that stiffer soil leads to more structural damage while the softer one can modify ground settlements. Brick-infilled frame structures were found to have similar responses in facing soft and stiff soil while brick-bearing and open-frame structures are found to be more susceptible to damage depending on the soil type when subjected to similar ground settlement profile.

Natarajan and Vidivelli (2009) studied effect of column spacing on interaction of soil-raft-space frame where the soil was considered as homogeneous and isotropic since soil non-homogeneity had least effect on differential settlement. Increase of column spacing was found to have an incremental influence on settlement of considered raft foundation. Furthermore, effectiveness of soil modulus of elasticity on foundation settlement has been highlighted.

3.1 Interface element

In a precise physical modeling of two adjacent media especially where there is stress transformation, simulation of the interfacial behavior of the contact area is indeed a necessity because of the presence of discontinuous deformation which has been taken into consideration by means of interface elements. As a matter of fact continuity conditions, i.e., displacements and stresses, are key characteristic of any interface modeling that should be ensured by mechanical interface modeling. Opening, sliding and closing are displacement discontinuities that may occur at contact area. In a comparison between classical displacement model and finite element model Aivazzadeh and Verchery (1986) used constant stress triangle element representing classical approach and a four-nodded rectangular model with twenty degrees of freedom, i.e., two displacement and three stresses for each node, concluded that finite element method leads to more precise results in consideration of shear and normal stress distribution along the width of interface.

Interface elements in a general categorization have been grouped into two major types generally known as nodal interfaces and continuum elements, Wang and Wang (2006). The former category is basically consisting node-to-node spring elements and zero-thickness element while the latter involves thin-layer interface elements whose basic theory is discussed in literature, Potts and Zdravkovic (1999). Both types have been employed in geomechanics for solid-to-solid interface modeling and soil-structure interaction which is discussed in the following.

Zero-thickness and thin-layer interface elements are very well-known interface elements in the context of interfacial modeling with finite element method. The major reasons for that are first their simplicity and next their applicability to implement various constitutive models, such as linear and non-linear, whichever suits the best for the case under study. Utilization of these joint elements depending on the case with consideration of their limitation, for instance, to soft contact behavior or large shear deformation has opened a wide study field in the last three decades and a wide range review of related research works has been conducted hereafter.

Desai *et al.* (1984) used isoparametric eight-nodded finite element for interface element with uncoupled normal and shear stiffness and applicable to structural and geological interfaces. Parametric study was conducted in order to get the optimum thickness/width ratio and different deformation modes of stick, slip, debonding and rebonding were considered at the interface. Best range for thickness/width ratio was found 0.01 to 0.1 which of course does not avoid kinematic inconsistency, Coutinho *et al.* (2003). This element was then successfully used by Karampatakis and Hatzigogos (1999) and Karabatakis and Hatzigogos (2002) to study the creep behavior of soil in a time-dependent study. Later Desai and Rigby (1995) used disturbed state concept to improve constitutive behavior of interface elements through coupled effect of normal and shear response.

Sharma and Desai (1992) developed an interface element similar to Desai *et al.* (1984) but with six-nodded finite element which works as a solid element as well as zero-thickness element when thickness tends to zero. Transformation of stiffness matrix is initially applied to elasticity matrix rather than the final local stiffness matrix which is an advantage to zero-thickness element. Their six-nodded thin layer interface element was found suitable to compute stress and strains of thin finite zones which is used for evaluation of progressive damage. In contrast to Desai *et al.* (1984) and Sharma and Desai (1992) in a study conducted by Mayer and Gaul (2007) zero thickness element was found more suitable for solid-to-solid contact since it has no interfacial thickness, therefore contact stiffness is not dependent of element thickness, and also because of the traction field which is computed with the same order of displacement field while in thin-layer element it is approximated one order lower.

An axisymmetric interface element was formulated by Yuan and Chua (1992) for circular foundations whose stiffness element is repeated here for ease of refer. This interface was a four-node quadrilateral element with two degrees of freedom per node, therefore stiffness matrix has eight rows and columns.

$$K_i = \frac{L}{4}\begin{bmatrix} C_1k_s & 0 & C_2k_s & 0 & -C_2k_s & 0 & -C_1k_s & 0 \\ 0 & C_1k_n & 0 & C_2k_n & 0 & -C_2k_n & 0 & -C_1k_n \\ C_2k_s & 0 & C_3k_s & 0 & -C_3k_s & 0 & -C_2k_s & 0 \\ 0 & C_2k_n & 0 & C_3k_n & 0 & -C_3k_n & 0 & -C_2k_n \\ -C_2k_s & 0 & -C_3k_s & 0 & C_3k_s & 0 & C_2k_s & 0 \\ 0 & -C_2k_n & 0 & -C_3k_n & 0 & C_3k_n & 0 & C_2k_n \\ -C_1k_s & 0 & -C_2k_s & 0 & C_2k_s & 0 & C_1k_s & 0 \\ 0 & -C_1k_n & 0 & -C_2k_n & 0 & C_2k_n & 0 & C_1k_n \end{bmatrix} \quad (9)$$

In which, $C_1 = R_1 + R_2/3, C_2 = (R_1 + R_2)/3$ and $C_3 = R_2 + R_1/3$. L is length of the element, K_n and K_s are normal and tangential stiffness respectively and represents global nodal coordinate as shown in Fig. 8.

Validkar presented an isoparametric interface element to investigate the interface characteristics of the soil medium and foundation beam element. It should be noted that the formulated element is numerically compatible with three-nodded beam bending element, representing foundation, with

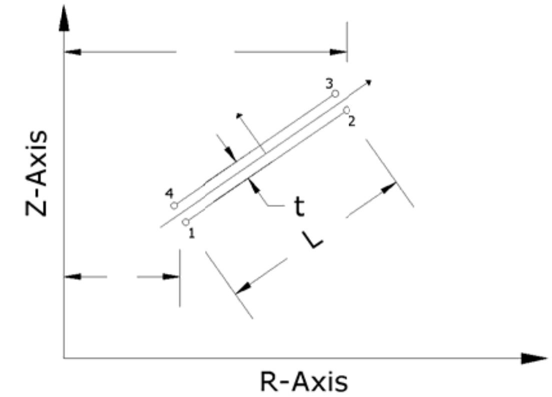


Fig. 8 Axisymetric interface element

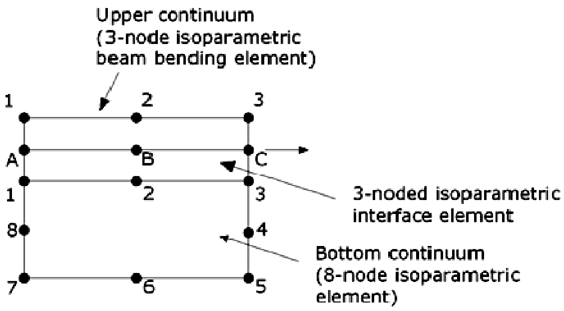


Fig. 9 Geometry of the combination of three elements

three degrees of freedom and considered eight-nodded plane-strain element to model the soil mass with two degrees of freedom per node, Noorzaei *et al.* (1993), Noorzaei *et al.* (1994) and Viladkar *et al.* (1994b).

The presence of such an interface element is a necessary need in order to come up with the practical and real nature of behavior of structures subjected to lateral load (unsymmetrical system). Fig. 9 is retrieved from Viladkar's work depicting the geometry of the combination of three earlier mentioned elements. Nodal displacements of the interface element are dependent of nodal displacements of both upper element idealizing footing and lower ones modeling bottom continuum. Hence all the relative displacements may be presented as the following

$$\Delta = (\Delta u_A, \Delta v_A, \Delta\theta_A, ..., \Delta u_C, \Delta v_C, \Delta\theta_C)^T = (u_1^t - u_1^b, v_1^t - v_1^b, \theta_1^t, ..., u_3^t - u_3^b, v_3^t - v_3^b, \theta_3^t)^T \quad (10)$$

Subsequently nodal strain-displacement relationship is defined as the function of above relative displacements. Stress at any point of the element is formulated in their work as:

$$\begin{Bmatrix} \varepsilon_{s1} \\ \varepsilon_n \\ \varepsilon_{s2} \end{Bmatrix} = [B]_J \{\delta\} \quad (11)$$

$$\{\sigma\} = \{\tau_s, \sigma_n, M\}^T = [D]\{\varepsilon\} \tag{12}$$

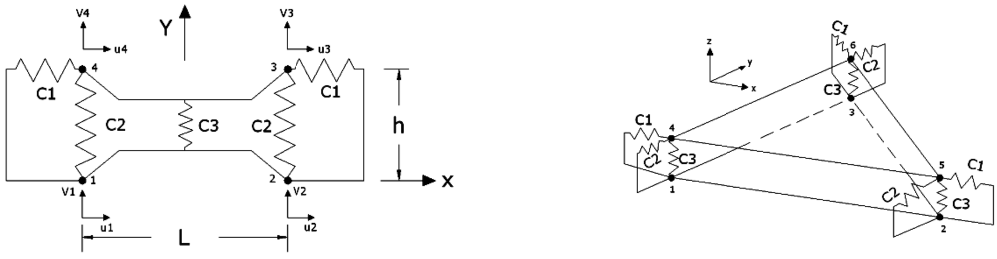
$$[D] = \begin{bmatrix} K_{ss1} & 0 & 0 \\ 0 & K_{nn} & 0 \\ 0 & 0 & K_{ss2} \end{bmatrix} = \begin{bmatrix} K_{ss} & 0 & 0 \\ 0 & K_{nn} & 0 \\ 0 & 0 & K_{ss} \end{bmatrix} \tag{13}$$

$$[K_{IJ}] = \int [B]_J^T [D][B]_J \, dv \tag{14}$$

In which ε_{s1}, ε_{s2} and ε_n are tangential strains and normal strain function of v, θ and u respectively. B is the shape function matrix and δ is the vector of nodal displacements. Tangential stress, normal stress and moment are represented by τ_s, σ_n and M respectively.

Stress-strain relationship is a very crucial aspect of the interface element due to the difficulty of determination of normal and tangential stiffness (K_{nn} and K_{ss} respectively) whose units is force per unit volume, e.g. kn/m^3, and they describe the variation rate of normal and tangential stresses against displacement, i.e., (kn/m^2)/m. Tangential stress is formulated as the function of unit weight of water, atmospheric pressure, initial stiffness, shear stress, adhesion at the interface, angle of friction of interface and a modulus number while an arbitrary value of normal stiffness is used although they have found that it is not always true, where some suggest to assign normal stiffness magnitude equal to that of soil material since the interface is modeled as a part of soil medium, Ng *et al.* (1997), as long as conventional elements like thin-layer are selected for interface simulation. Noorzaei *et al.* (1993) and Viladkar *et al.* (1994b) found Variation of normal stiffness so influential on the general behavior of superstructure since higher values yield less structural sway and lower values change the behavior of structure to more realistic one. By and large, presence of interface element caused redistribution of shear stress in footing as well as higher amount of sway, 1.3 times more, compared to the case excluding this type of element. In addition, because of considering soil non-linearity the sway has been found to be 1.6 times higher than a linear interactive case and total displacement to be double. In a comparison conducted by Swamy *et al.* (2011a) and Swamy *et al.* (2011b) between an interactive soil-space framed structure and non-interactive analysis, considered static response was found relatively similar as member end actions for beams and columns didn't differ considerably for both cases and presence of interface was suggested wherever pressures and settlements, i.e., soil settlement and differential settlement, are to be studied. Two compatibility cases namely uncoupling, i.e., slip/frictionless interface, and coupling, i.e., complete bonding, between raft foundation and soil were analyzed by Swamy and his fellow researchers. It is found that coupling effect can lead to increase of differential settlements as well as horizontal differential stresses.

Another interface element has been developed by Lei (2001) for contact friction analysis utilizing principle of virtual work which led to unknown nodal contact stresses rather than unknown nodal forces since failure and slip state of contact areas are often explained in terms of stresses not force. Therefore contact status would be evaluated in an iterative analysis according to comparison of previous and current constraint leading to computation of load vector in each iteration. Six-nodded quadrilateral element was employed to represent the element topology. Zheng *et al.* (2004) made improvements in node-pair type interface elements by mixing nodal contact forces with nodal displacements. They mostly focused on numerical stability which was accomplished by modified definition of contact state of node-pairs that also led to stable convergence. Rigid displacement was treated by condensing degrees of freedom of non-contact



(a) (b)

Fig. 10 Coutinho's interface element

nodes to interfaces which causes solution efficiency in some geomechanical difficulties.

Coutinho *et al.* (2003) developed an interface link similar to Herrmann zero-thickness element, well presented in Kaliakin and Li (1995) with four corner nodes, two degrees of freedom per each, and five fictitious springs connecting each pair of mating nodes and a vertical one located in the middle of element Fig. 10a. As no elastic link was assigned to element extremities element was free of kinematic inconsistency which is defined as disharmonious nodal displacement when the element is submitted to tangential force. The proposed element has been extended to three dimensional mathematical modeling consisting two triangular facing each other with three nodes on corners and three nodal degrees of freedom, Fig. 10(b).

Kaliakin and Li (1995) and Schellekens and De Borst (2005) studied major fundamental deficiencies of two and three dimensional interface elements, spurious stress oscillation which leads to inaccurate stress predictions. Inappropriate integration schemes, large dummy stiffness parameters and insufficient mesh fineness are key factors of such a drawback. Kaliakin and Li (1995) came up with a topologically similar interface element to that of Coutinho to overcome the kinematic deficiency of inaccurate tangential force prediction by considering a macro-element composed of two adjoined rectangular four-nodded element and condensing the two middle nodes, Fig. 11. This element was then used for interaction analysis of soil and strip footing and showed good accuracy in simulating soil-footing separation but not able to correctly account rebounding attributed to cyclic loading.

Following standard assembly procedure and finite element formulation, Kaliakan formulated stiffness matrix of the interface element as following

$$K = \frac{L}{48}\begin{bmatrix} 7d_{11} & 0 & -d_{11} & 0 & d_{11} & 0 & -7d_{11} & 0 \\ & d_{22} & 0 & -d_{22} & 0 & d_{22} & 0 & -7d_{22} \\ & & 7d_{11} & 0 & -7d_{11} & 0 & d_{11} & 0 \\ & & & 7d_{22} & 0 & -7d_{22} & 0 & d_{22} \\ & & & & 7d_{11} & 0 & -d_{11} & 0 \\ & & sym. & & & 7d_{22} & 0 & -d_{22} \\ & & & & & & 7d_{11} & 0 \\ & & & & & & & 7d_{22} \end{bmatrix} \tag{15}$$

$d_{11}$and d_{22} represent constitutive parameters tangential and normal stiffness per unit length of the interface. Evgin *et al.* (2003) conducted an experimental study on soil-structure, i.e., sand-steel

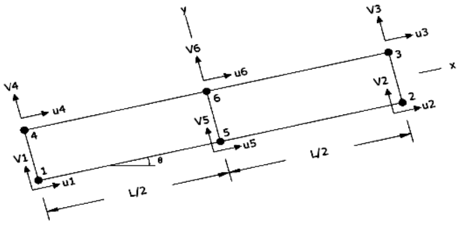


Fig. 11 Kaliakan’s interface element

interface. Shear and axial stress were found both to have a nonlinear path during shearing. Hu and Pu (2003) employed damage model, where the material behavior is assumed to be composed of two reference states namely intact and fully adjusted, i.e., known as disturbed state concept well presented in Desai and Ma (1992) and is macroscopic-observation based, to develop a nine-parameter constitutive model capable of simulating both strain softening and normal dilatancy assigned for interfacial behavior of rough interface between soil and solid surface where a four-nodded thin layer element was utilized as the element topology. Special attention was given to various asperities of the soil-steel interface and the critical value was found as a criterion to formation of shear zone, as narrow as five millimeters, and strain localization. Strain softening and normal dilatancy as two major aspects of interface deformation behavior have been considered in another constitutive model in Zhou and Lu (2009) by means of a 2D bi-potential surface elasto-plastic model with advantage of no plastic potential function assumption. According to the proposed formulation by Zhou the two potential functions and two plastic states depend on normal and shear stress variation and lead to a four-parameter plastic flexibility matrix whose components can be obtained through shear tests.

Hu and Pu (2004) studied the interfacial behavior of two medium, solid surface and soil, which was found to be dependent of roughness of the interface. This was done by considering different roughness for the contact surface of plate and eventually the critical magnitude of roughness was obtained as the criterion of type of failure. They found that in case of smooth surface where the roughness is less than its critical value the mode of failure is elastic perfectly plastic and if the roughness exceeds the critical value the failure mode would be accompanied with strain softening and bulk dilatancy. Based on their results Hu and Pu developed a constitutive model, called damage model, for soil-structure interaction simulating rough interface capable of idealization of strain softening as well as strong dilatancy. Thin layer element was employed to model soil-structure interface with hyperbolic relationship for shear stress and shear strain. They categorized performance of interface behavior into two states. The first part follows the elastic perfectly plastic behavior, before critical state is approached, and the second part commences when there is no further volume change under incremental shear deformation for a constant normal stress. The second state was presented by rigid plastic constitutive model with the interface strength being described by Mohr-Coulomb failure criteria. Liu *et al.* (2006) introduced an eleven-parameter constitutive model applicable for shearing phenomena between sandy soil and structure. The critical state soil mechanics concept was employed which states that the there is no dilatancy and stress ratio variation while large shear deformation is ongoing and this state is so called critical state. This state is associated with current soil void ratio as well as its respective critical magnitude by which the soil behavior can be defined at each stage. Liu and his co-researchers studied plane strain interface behavior in which the normal and tangential deformations, considered as uncoupled, are the key parameters of elasto-plastic constitutive matrix following non-associated flow rule where dilatancy is assumed to be function of stress ratio. The proposed model performed well in prediction of tangential displacement, normal stress and hardening/softening phases.

Mao (2005) presented a finite element formulation for geotechnical interfaces without a need to define additional constitutive equations which could be satisfied only by Mohr-Coulomb stress-strain relation for elasto-plastic interfacial behaviors associated with soil-structure interactions such as strip footing case. In contrast with other well-known interface formulae Mao developed a formulation in which the displacement compatibility was complied through independent degrees of freedom considered separately while computing for strain-displacement matrix. This means that the tractions of the neighboring faces on the contact boundary are computed through stresses of their respective media and they are not equal.

Wang and Wang (2006) modeled an interface element for joints with finite thickness subjected to large shear deformation, e.g., rock interface with fillings in, and equipped with anisotropic Mohr-Coulomb yield criterion. Unlike other interfaces Wang's continuous interface element was characterized with different strain expressions associated with shear strain concentration along the interface and the normal strain parallel to the interface was included in strain-displacement equation similar to that of regular solid elements, consider in Eq. (16) retrieved from Wang and Wang (2006) for better understanding.

$$\begin{Bmatrix} \dot{\varepsilon}_{nt} \\ \dot{\varepsilon}_{nn} \\ \dot{\varepsilon}_{tt} \end{Bmatrix} = \frac{1}{d} \begin{bmatrix} N_3 & 0 & N_4 & 0 & -N_1 & 0 & -N_2 & 0 \\ 0 & N_3 & 0 & N_4 & 0 & -N_1 & 0 & -N_2 \\ d\frac{\partial N_3^s}{\partial t} & 0 & d\frac{\partial N_4^s}{\partial t} & 0 & d\frac{\partial N_1^s}{\partial t} & 0 & d\frac{\partial N_2^s}{\partial t} & 0 \end{bmatrix} \times \{\dot{U}\} \tag{16}$$

Where d is the thickness of interface, N_i denotes the regular four-nodded quadrilateral shape functions, t is the length of the element, $\{\dot{U}\}$ represents the vector of rates of displacements. $\dot{\varepsilon}_{nt}$ and $\dot{\varepsilon}_{tt}$ are shear strains, the later is parallel to interface, and $\dot{\varepsilon}_{nn}$ stands for shear strain normal to interface. Wang's strain-displacement formulation may be compared with that of continuous element introduced in Potts and Zdravkovic (1999).

According to the proposed numerical algorithm by Wang, distortion as the result of large shear deformation was circumvented by updating interface elements through new nodal coordinates in every load step, after distortion, to which reconstruction of new interface elements depend on and the updated joint element could change to either triangular or quadrilateral interface element, as far as topology of the interface is concerned, based on the new nodal locations of the upper and lower surfaces and corresponding angles. The model was then used for pullout test assigning various interface thicknesses which was found to be very effective in controlling the performance of proposed element. Later on, large shear deformation between soil and strip foundation was studied by Sheng *et al.* (2007) through a developed displacement method called augmented Lagrangian method which was a combination of the two classical method Lagrange multiplier method and penalty method where the latter was used as a control to contact constraints and former as a solution method of virtual work. Comparing the considered quasistatic example which was an elastic strip footing resting on homogeneous single-layered soil subjected to inclined and eccentric load with Meyerhof's bearing capacity equation, Daicho Sheng and his fellow researchers

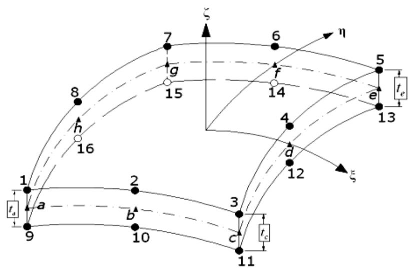


Fig. 12 Samadhiya's interface element

concluded that smoothness of the footing-soil contact is a key parameter to bearing capacity of soils with self-weight.

Principle of virtual work as a very basic methodology of interface elements was also used by Samadhiya *et al.* (2008) to develop a three-dimensional interface element for undulating discontinuities in rocks. The interface was capable of accounting for dilatancy which was considered as a function of normal strains and stiffness as re-presented here for better understanding, Eq. (17).

$$[D] = \begin{bmatrix} E_{s1} & sym. & \\ 0 & E_{s2} & \\ \dfrac{-E_n \tan\beta|\varepsilon_{s1}|}{\varepsilon_{s1}} & \dfrac{-E_n \tan\beta|\varepsilon_{s2}|}{\varepsilon_{s2}} & E_n \end{bmatrix} \tag{17}$$

$[D]$ is stress-strain matrix and parameter β represents angle of inclination of asperities with the plane of interface element. Dilatancy comes into the picture as soon as slippage occurs. An advantage of Samadhiya's joint element is that the proposed formulation takes into account the variable thickness of thin shear zone since the generalized thickness at any point is also defined as shape functions similar to nodal coordinates, Eq. (18). Details of proposed 16-node thin layer element are shown in Fig. 12. This element was suitable for static loading and no cyclic performance was considered in the constitutive behavior, i.e., it is not capable of bounding and rebounding.

$$t = N_a t_a + N_b t_b + N_c t_c + N_d t_d + N_e t_e + N_f t_f + N_g t_g + N_h t_h \tag{18}$$

N_i stands for nodal shape functions.

4. Soil constitutive model

Elastic-plastic behavior of the soil has been modeled through different mathematical equations each of which involves parameters that can significantly affect the resultant displacement and stresses induced in the soil body. As a matter of fact in the SSI domain as was mentioned earlier, displacement and differential settlement of the supporting soil are key aspects of such interaction analysis that play an important role in redistribution of forces in the superstructure elements. Therefore an overview of soil constitutive models is conducted hereafter.

Constitutive models of the soil may generally be divided into pressure-dependent and pressure-independent models. The former is associated with drained analysis (effective stress analysis) and contains Mohr-Coulomb, Drucker-Prager, hyperbolic and Cam-Clay models and the latter one is based on undrained analysis (total stress analysis) and includes Tresca and Von-Mises models. Torkamani (1990) has discussed the Tresca's yield condition as an associated flow rule but this criterion has been hardly used in the literature for soil analysis, Zhu (2004) and Zhu and Michalowski (2005).

Mohr-Coulomb is an elastic perfectly plastic model whose elastic domain is defined through Young's modulus and Poisson's ratio and the failure criteria is determined by friction and cohesion. In case the non-associative flow rule is ruling the realistic irreversible volume changes the dilatancy angle parameter is employed. The flow rule is in fact in charge for calculation of plastic strains through a plastic potential function that can be the same as the yield function (associated flow rule) or it is different form yield function then it is called non-associated flow rule. Mathematical expression of Mohr-Coulomb has been stated as, Valliappan (1981) and Michalowski and Shi (1995)

$$\sigma_1 - \sigma_2 = 2C\cos\phi - (\sigma_1 + \sigma_3)\sin\phi \tag{19}$$

Also in $\sigma - \tau$ plane (normal stress and shear strength components)

$$\tau = C + \sigma \tan\phi \tag{20}$$

Assuming isotropic material σ_1 , σ_2 and σ_3 stand for principal stresses. C and ϕ represent cohesion and angle of friction respectively. If ϕ is taken to be zero then the model reduces to Tresca's criterion. Although strength behavior of the soil is well expressed by Mohr-Coulomb but it fails to model the strain hardening and softening of the material. This model is appropriate for analysis of shallow foundations, stability of dams, slopes and embankments. The Drucker-Prager model is also another failure criterion formulated as a modified version of Von-Mises criterion and suitable for C-ϕ soil that can be represented as, Salencon *et al.* (1977) and Boulbibane and Ponter (2005)

$$\alpha J_1 + J_2^{1/2} - k = 0 \tag{21}$$

$\alpha = \dfrac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}$ and $k = \dfrac{6C\cos\phi}{\sqrt{3}(3-\sin\phi)}$. J_1 and J_2 are deviatoric stresses. Lee and Eun (2009) employed this yield criterion to study the bearing capacity of multiple footings resting on sand.

Another non-linear stress-dependent model is hyperbolic model also known as Duncan-Chang model. This model is mostly preferred over Mohr-Coulomb since the required parameters can be easily obtained and soil behavior can be well detected. Hyperbolic model is an associated flow rule so it fails to describe dilatancy. Further, in loading/unloading cases it does not perform effectively to distinguish a hypo-elastic model. However, hyperbolic model is appreciated for practical modeling in literature and has been employed many times as soil constitutive law. Non-linearity of soil corresponds to tangential modulus of soil which is represented as, Noorzaei *et al.* (1993)

$$E_T = [1 - \frac{R_f (1-\sin\phi)(\sigma_1 - \sigma_3)}{2(C\cos\phi + \sigma_3 \sin\phi)}]^2 K.P_a (\frac{\sigma_3}{P_a})^n \tag{22}$$

Where R_f is failure ratio, K is modulus number, P_a stands for atmospheric pressure and n represent an exponent which determines the variation of initial tangent modulus. σ_1 and σ_3 are major and minor principal stresses. Tangential modulus of soil according to Eq. (22) is a pressure dependent parameter and in an iterative analysis it is updated based on the previous iteration stress level.

It should be mentioned that all criteria discussed above, while in their basic expression, fail to mathematically simulate the hardening and softening characteristics of the soil. In fact a practical elasto-plastic yield criterion should not only be capable of handling yield surface but also hardening/softening behavior of the soil. Here is the time that the work-hardening hypothesis comes into picture to compute the increase of yield criterion. In another world hardening behavior is accompanied by increase of yield stress and enlarging the yield surface while the softening behavior is accompanied by dilatancy and subsequent decrease of yield stress curve. The non-associated single-hardening model developed by Kim and Lade (1988) and Lade and Kim (1988) has been reported to perform well for modeling plastic behavior of sand, Dakoulas and Sun (1992). Kim and Lade introduced a potential function which counts for volume changes and it is function of stress invariants. The hardening parameter can be computed by either plastic work or plastic strain. Nanda and Kuppusamy (1992) developed a hardening model to study the influence of drained anisotropic performance of clay on the bearing capacity and settlement which can get changed considerably compared to isotropic case. This model is of kinematic type and allows for multiple yield surfaces that makes it applicable for complex loading conditions. An elasto-plastic constitutive model suitable for modeling strain hardening/softening of sands is developed by Guo and Li (2008) to study the load-settlement relationship of sand, supporting shallow foundation, under these characteristic behaviors. The plastic work has been used as the hardening parameter to describe the strain hardening which is followed by softening behavior. Such model can reflect nonlinearity and dilatancy of the soil. Since the value of plastic work monotonically increases, whether the soil is in strain hardening or softening state, therefore a new equation has been formulated, function of hardening parameter and plastic work, which can demonstrate occurrence of softening behavior. Guo and Li employed this model to calculate the bearing capacity of soils that tend to strain-softening behavior.

Bearing capacity and settlement of saturated and unsaturated sandy soils are evaluated by Oh and Vanapalli (2008) through an elastic-perfectly plastic model. Inclusion of matric suction has shown considerable influence on modulus of elasticity, settlement and bearing capacity of sands whose differential settlement is predominant due to its heterogeneous nature. Their predictions were found to be underestimating settlements while reliable bearing capacity values were exploited from finite element analysis. However Oh and Vanapalli did not discuss the influence of suction on volumetric behavior, yielding stress and shear stress of unsaturated soils. Sheng *et al.* (2008) extended the volume-stress-suction relation of saturated soils to be applicable for unsaturated condition. Therefore a model capable of smooth transition between saturated/unsaturated condition was developed known as a continuous volumetric stress-strain model. The defined yield surface as well as shear strength were also defined as function of suction, bearing in mind that suction also should be treated as another additional stress variable. Although the volumetric stress-strain model tries to introduce a smooth mathematical expression for saturated/unsaturated condition but due to the non-convexity of yield surface of unsaturated soil the finite element implementation of this model is complicated which is usually avoided in numerical calculations.

A famous constitutive law for soft clay is Cam-Clay model. This model can track the non-reversible volumetric changes associated with voids that exist in the body of the soil filled by fluids like air and water. Moreover, it accounts for strength, dilatancy and critical states of the soil. The critical states describe the status of the soil in which shearing or distortion continues without any corresponding changes of stress or volume. This is why these type of constitutive law as well as other similar developed models like McDowell (2002) are said to be within the so called critical state framework. The models within the context of critical state framework are utilized for soils whose structures get reconstituted in the laboratory through sample preparation method. The critical state line, defined in $v - \text{In}\ p'$ (specific volume-effective mean stress) space, basically is the criterion for hardening and softening behavior. Rigorously speaking, for Cam-Clay model the maximum shear stress is at the location where critical state line intersects the peak of yield curve in stress space which is not true for granular materials like soil, McDowell and Hau (2004). This line as Leong discussed should account for suction for unsaturated soils, Leong *et al.* (2003).

Other types of this model also have been developed namely modified Cam-Clay, Zdravkovic *et al.* (2003), Grammatikopoulou *et al.* (2006) and Ivandic and Soldo (2009) and structured Cam-Clay model, Liyanapathirana *et al.* (2009). The structured Cam-Clay model has four additional parameters that account for the structure of the soil and among these four the additional void ratio sustained by soil structure and the size of initial yield surface were are found influential to bearing capacity of the clay, Carter (2006). According to the literature in most of studies related to soil bearing capacity and footing settlement, these models are reported as successful constitutive laws for prediction of nonlinear behavior of over-consolidated clays prior to failure.

Another elasto-plastic strain hardening model which is an extension to Drucker-Pracker's frictional model is called cap model. Applicability of cap models family has been further extended to sand, clay, rocks and also concrete. A major ability of this model is expressing the inelastic coupling between volumetric and deviatoric behaviors of soil. This model, similar to other ones, has been subjected to mathematical modifications due to the non-smoothness of the cap surface which has led to numerical difficulties, Swan and Seo (2000) and Dolarevic and Ibrahimbegovic (2007). Swan suggested the modified version to be used for ductile soil behaviors. DorMohammadi and Khoei (2008) modified the cap model by introducing isotropic and kinematic material functions as the hardening rule of the model along with associated flow rule for plastic deformation. Dormohammadi's cap model has successfully obtained close results from numerical analysis and experimental tests conducted on sand. Kohler and Hofstetter (2008) employed non-associated cap model for partially saturated sands and implement several parameters like net stress and matric suction into this model. Kohler discussed applicability of the modified cap model to switch from saturated to unsaturated state.

6. Conclusions

The wide review of the proposed methods for soil-foundation modeling in addition to the profound approach of FEM for SSI analysis was carried out to study the current state of them with regard to their precision as well as applicability of them to analysis of all major aspects of SSI. Hence the following conclusions should be highlighted for future practical engineering:

(1) Winkler and continuum hypothesis, although a simplified approach, yet has many advantages

over traditionally fixed base idealization analysis and leads to better understanding of foundation behavior yet not comprehensive enough for soil idealization.
(2) Accurate structural design essentially needs forces to be accurately estimated which initially comes from realistic modeling and idealization, therefore inclusion of soil-structure interaction at the very beginning steps is obligatory.
(3) Employing various superstructure elements in finite element method has led to force variation in structural members hence the more elements are modeled and involved in a frame idealization, either plane or space one, the more realistic are the forces and deformations.
(4) Application of interface elements has shown good improvement for evaluating the interfacial forces and has led to better idealization of interfacial condition between different mediums.
(5) Interaction analysis between structure and foundation has become a complicated matter mostly due to the non-linear behavior of the soil, hence to achieve a realistic stress-strain relationship the finite element method, known as an incremental iterative method and through which material nonlinearity and non-homogeneity can be incorporated, has been widely employed and desired. Several famous constitutive laws that correspond to nonlinearity of soil behavior were presented to highlight the importance of model designation which is crucial to the interpretation of numerical analysis.

Finally it suits to mention that among the current reviewed approaches the most applicable one is found to be FEM through which not only a precise superstructure modeling becomes practical but also a comprehensive analysis can be conducted for soil aspects which in turn brings a powerful idealization of such a nonlinear medium into account.

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