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Moving load response in a rotating generalized thermoelastic medium

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Abstract. The steady state response of a rotating generalized thermoelastic solid to a moving point load has been investigated. The transformed components of displacement, force stress and temperature distribution are obtained by using Fourier transformation. These components are then inverted and the results are obtained in the physical domain by applying a numerical inversion method. The numerical results are presented graphically for a particular model. A particular result is also deduced from the present investigation.

Keywords: rotation; generalized thermoelasticity; fourier transform; temperature distribution.

1. Introduction

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory thermoelasticity in which the parabolic type heat conduction equation is based on fourier's law of heat conduction. This newly emerged theory which admits finite speed of heat propagation is now referred to as the hyperbolic thermoelasticity theory, Chandrasekharaiah (1998), since the heat equation for rigid conductor is hyperbolic-type differential equation.

There are two important generalized theories of thermoelasticity. The first is due to Lord Shulman (L-S) (1967). The second generalization to the coupled theory of thermoelasticity which is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Muller (1971), in a review of the thermodynamics of thermoelastic solid, proposed an entropy production inequality, with the help of which he consider restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay (G-L) obtained another version of the constitutive equations (1972). These equations were also obtained independently and more explicitly by Suhubi (1975). This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equations. The classical Fourier law violated if the medium under consideration has a centre of symmetry. Theory of thermoelasticity without energy dissipation is

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another generalized theory and was formulated by Green and Naghdi (1993). It includes the " thermal-displacement gradient" among its independent constitutive variables and differs from the previous theories in that it does not accommodate dissipation of thermal energy.

The dynamical response of solid material subjected to moving loads is of great interest to a number of engineering fields, such as civil engineering, ocean engineering, earthquake engineering and tribiology. For example ground motion and stresses are induced in saturated soils by fast moving vehicular loads or surface blast waves due to explosives.

Various researchers investigated the dynamic response of half space subjected to a moving point load. Sneddon (1951) was the first to discuss the two dimensional problem of a line load moving with constant sub-sonic speed over the surface of a homogenous elastic half space. Some of the similar problems of the sub-sonic, transonic and supersonic were discussed by other researchers (Cole and Huth 1958, Fung 1968, Fryba 1999). A homogenous three dimensional elastic half space subjected to forces moving with a constant speed was studied by Eason (1965) using the double Fourier transformation method. Payton (1967) considered the transient problem for a line load applied suddenly and then moving with a constant speed on the surface of an elastic half space.

Frydrychowicz and Singh (1981) analyzed temperature and stress distribution for the case of uniform load of finite width moving at sub-sonic velocity over the surface of an uncoupled thermoelastic half space. Brock and Rodgers (1997) studied the steady-state response of thermo-elastic half space due to thermal/mechanical loads. Lykotrafitis and Georgiadis (2003) discussed three dimensional study state thermoelastic dynamic problem of moving sources over a half space. Sharma, Sharma and Gupta (2004) investigated the steady-state response of an applied load moving with constant speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space.

Some researchers in past have investigated different problem of rotating media. Chand *et. al.* (1990) presented an investigation on the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half space. Many authors (Schoenberg 1973, Clarke and Burdness 1994, Destrade 2004) studied the effect of rotation on elastic waves. Ting (2004) investigated the interfacial waves in a rotating anisotropic elastic half space by extendingthe Stroh (1962) formalism. Sharma and his co-workers (2006, 2007a, 2007b, 2008) discussed effect of rotation on different type of waves propagating in a thermoelastic medium. Othman and Song (2008) presented the effect of rotation in magneto thermoelastic medium. Ailawalia, Narah and Kumar (2009) discussed effect of rotation due to various sources at the interface of elastic half space and generalized thermoelastic half space.

In the present investigation we have obtained the expressions for displacement, force stress and temperature distribution in a rotating generalized thermoelastic medium due to a moving load by using Fourier transform. Such types of moving load problems in the rotating medium are very important in many dynamical systems. A particular case has also been derived. No attempt has been made so for to study the effect of rotation due to a moving load in generalized thermoelastic medium.

2. Formulation of the problem

A homogeneous generalized thermoelastic medium rotating uniformly with angular velocity $\vec{\Omega} = \Omega\hat{n}$ is considered where $\hat{n}$ is a unit vector representing the direction of the axis of rotation. All quantities considered are functions of the time variable t and of the coordinates x and z. The displacement equation of motion in the rotating frame has two additional terms (Schoenberg and Censor 1973): centripetal acceleration, $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ due to time varying motion only and $2\vec{\Omega} \times \vec{u}$ where $\vec{u} = (u_1, 0, u_3)$ is the dynamic displacement vector and angular velocity $\vec{\Omega} = (0, \Omega, 0)$. These terms do not appear in non-rotating media.

We consider a normal point load moving in an infinite generalized thermoelastic medium. To analyze the displacement, force stresses and temperature distribution at the interface of the medium, the continuum is divided into two half-spaces defined by

i. half-space I, $|x| < \infty$, $\infty < z \le 0$. $|y| < \infty$

ii. half-space II, $|x| < \infty$, $0 < z \le \infty$. $|y| < \infty$

A rectangular coordinate system (x, y, z) having origin on the surface $z = 0$ and z-axis pointing vertically into the medium is considered. We assume a pressure pulse $P(x + Ut)$, which is moving with a constant velocity U in the negative x-direction. Since the load has constant magnitude and move with a constant speed, after a sufficiently long time the solid response may become stationary in the reference system that is fixed to the load. In this paper we study possible pattern of this stationary response. The deformation of the medium subjected to a moving point load has been studied in particular for two theories of thermoelasticity viz. L-S theory (1967) and G-L theory (1972).

3. Basic equations

The field equations and constitutive relations in generalized linear thermoelasticity with rotation and without body forces and heat sources are given by

$$(\lambda + \mu)\nabla(\nabla.\vec{u}) + \mu\nabla^2\vec{u} - \upsilon\left(1 + \vartheta_0\frac{\partial}{\partial t}\right)\nabla T = \rho\left[\frac{\partial^2\vec{u}}{\partial t^2} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega}\frac{\partial\vec{u}}{\partial t}\right] \tag{1}$$

$$K^*\left(n^* + t_1\frac{\partial}{\partial t}\right)\nabla^2 T = \rho C_E\left(n_1\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \upsilon T_0\left(n_1\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u}) \tag{2}$$

$$t_{ij} = \lambda e\delta_{ij} + 2\mu e_{ij} - \upsilon\left(1 + \vartheta_0\frac{\partial}{\partial t}\right)T\delta_{ij} \tag{3}$$

where

λ, μ are Lame's constants, ρ is the density, $\vec{u}$ is the displacement vector, t_{ij} is stress tensor. τ_0, ϑ_0 are thermal relaxation times and $\upsilon = (3\lambda + 2\mu)\alpha_t, e = div\vec{u}$.

4. Solution of equations

For two dimensional problem (xz-plane) all quantities depends only on space coordinates x, z and time t, so the equations of motion (1) and (2) reduces to

$$\rho\left[\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega\frac{\partial u_3}{\partial t}\right] = (\lambda + \mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u_1 - \upsilon\left(1 + \vartheta_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} \tag{4}$$

$$\rho\left[\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega\frac{\partial u_1}{\partial t}\right] = (\lambda+\mu)\frac{\partial e}{\partial z} + \mu\nabla^2 u_3 - \upsilon\left(1+\vartheta_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} \tag{5}$$

$$K^*\left(n^* + t_1\frac{\partial}{\partial t}\right)\nabla^2 T = \rho C_E\left(n_1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \upsilon T_0\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t} \tag{6}$$

Following Fung (1968), a Galilean transformation

$$x^* = x + Ut, z^* = z, t^* = t \tag{7}$$

is introduced, then the boundary conditions would be independent of t^* and assuming the dimensionless variables defined by

$$x_i' = \frac{\omega^*}{c_0}x_i, \quad u_i' = \frac{\rho c_0\omega^*}{\upsilon T_0}u_i, \quad t' = \omega^* t, \quad \tau_0' = \omega^*\tau_0, \quad \vartheta_0' = \omega^*\vartheta_0$$

$$T' = \frac{T}{T_0}, \quad t_{ij}' = \frac{t_{ij}}{\upsilon T_0}, \quad \Omega' = \frac{\Omega}{\omega^*} \tag{8}$$

where

$$\omega^* = \rho C_E c_0^2/K^*, \quad \rho c_0^2 = \lambda + 2\mu$$

in Eqs. (4)-(6), we obtain the equations of motion in dimensionless form.

Introducing displacement potentials q and ψ which are related to displacement components u_1 and u_3 as

$$u_1 = \frac{\partial q}{\partial x} + \frac{\partial\psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} - \frac{\partial\psi}{\partial x} \tag{9}$$

in the resulting dimensionless equations and applying the Fourier transform defined by

$$\tilde{f}(\xi, z) = \int_{-\infty}^{\infty} f(x,z)e^{i\xi x}dx \tag{10}$$

we get

$$\left[\frac{d^2}{dz^2} - \xi^2 + \Omega^2 + \xi^2 M_1^2\right]\tilde{q} + 2\Omega i\xi M_1\tilde{\psi} - (1-\vartheta_0 i\xi M_1)\tilde{T} = 0 \tag{11}$$

$$\left[\frac{d^2}{dz^2} - \xi^2 + \alpha_1\Omega^2 + \alpha_1\xi^2 M_1^2\right]\tilde{\psi} - 2\Omega\alpha_1 i\xi M_1\tilde{q} = 0 \tag{12}$$

$$\left[\frac{d^2}{dz^2} - \xi^2 + i\xi M_1\left(\frac{n_1 - i\xi\tau_0 M_1}{n^* - i\xi t_1 M_1}\right)\right]\tilde{T} + \frac{n_1 - \tau_0 n_0 i\xi M_1}{n^* - i\xi t_1 M_1}(\varepsilon i\xi M_1)\left[\frac{d}{dz^2} - \xi^2\right]\tilde{q} = 0 \tag{13}$$

Eliminating $\tilde{T}$ and $\tilde{\psi}$ from Eqs. (11) - (13) we obtain

$$[\Delta^6 - A\Delta^4 + B\Delta^2 - C]\tilde{q} = 0 \tag{14}$$

where

$$\Delta = \frac{d}{dz}, \quad a_1 = \frac{\rho c_0^2}{\mu}, \quad M_1 = \frac{U}{c_0}$$

$$\in = \frac{\upsilon^2 T_0}{\rho K^* \omega^*}, \quad c_1 = \xi^2 - i\xi M_1\left(\frac{n_1 - i\xi\tau_0 M_1}{n^{\bullet} - i\xi t_1 M_1}\right)$$

$$c_2 = \xi^2 - \Omega^2 - \xi^2 M_1^2, \quad c_3 = -i\xi\varepsilon M_1(1 - i\xi\vartheta_0 M_1)\left(\frac{n_1 - n_0\tau_0 i\xi M_1}{n^{\bullet} - i\xi t_1 M_1}\right)$$

$$c_4 = \xi^2 - \alpha_1\Omega^2 - \alpha_1\xi^2 M_1^2$$

$$A = c_1 + c_2 + c_3 + c_4$$

$$B = c_4(c_1 + c_2 + c_3) + c_1 c_2 + c_3\xi^2 - 4\alpha_1\Omega^2\xi^2 M_1^2$$

$$C = c_4(c_1 c_2 + c_3\xi^2) - 4\alpha_1 c_1\Omega^2\xi^2 M_1^2 \tag{15}$$

The solutions of Eq. (14) are

$$\tilde{q} = A_1 e^{-q_1 z} + A_2 e^{-q_2 z} + A_3 e^{-q_3 z} + A_4 e^{q_1 z} + A_5 e^{q_2 z} + A_6 e^{q_3 z} \tag{16}$$

$$\tilde{\psi} = a_1^* A_1 e^{-q_1 z} + a_2^* A_2 e^{-q_2 z} + a_3^* A_3 e^{-q_3 z} + a_1^{\bullet} A_4 e^{q_1 z} + a_2^{\bullet} A_5 e^{q_2 z} + a_3^{\bullet} A_6 e^{q_3 z} \tag{17}$$

$$\tilde{T} = b_1^* A_1 e^{-q_1 z} + b_2^* A_2 e^{-q_2 z} + b_3^* A_3 e^{-q_3 z} + b_1^{\bullet} A_4 e^{q_1 z} + b_2^{\bullet} A_5 e^{q_2 z} + b_3^{\bullet} A_6 e^{q_3 z} \tag{18}$$

where q_i^2 are the roots of Eq. (14) and a_1^*, b_1^* are coupling constants defined by

$$a_i^* = \frac{q_i^2 - (c_1 + c_2 + c_3)q_i^2 + (c_1 c_2 + c_3\xi^2)}{2i\xi\Omega_1 M_1(c_1 - q_i^2)}$$

$$b_i^* = i\xi M_1\left(\frac{n_1 - i\xi n_0\tau_0 M_1}{n^{\bullet} - i\xi t_1 M_1}\right)\left(\frac{\xi^2 - q_i^2}{q_i^2 - c_1}\right), \quad i = 1, 2, 3 \tag{19}$$

5. Boundary conditions

For a concentrated point force, we take $P(x + Ut) = F\delta(x^*)$, where $\delta(x^*)$ is Dirac-delta function and F is the magnitude of force applied along the interface of two media. In moving coordinates the boundary conditions at the interface $z = 0$ are,

$$\text{(i)}\ t_{33}(x, 0^+, t) = t_{33}(x, 0^-, t) - F\delta(x^*), \quad \text{(ii)}\ t_{33}(x, 0^+, t) = t_{33}(x, 0^-, t)$$

$$\text{(iii)}\ u_1(x, 0^+, t) = u_1(x, 0^-, t), \quad \text{(iv)}\ u_3(x, 0^+, t) = u_3(x, 0^-, t), \quad T = 0 \tag{20}$$

Using Eqs. (3), (8), and (9) in the boundary conditions (20), we obtain the boundary conditions in the dimensionless form. On suppressing the primes and applying the Fourier transform defined by

(10) on the dimensionless boundary conditions and using (16) - (18) in the resulting transformed boundary conditions, we get the transformed expressions for displacement, force stress, and temperature distribution in a rotating generalized thermoelastic medium as

$$\tilde{u}_1 = \tilde{F}\left(\sum_{m=1}^{3} b_m D_m e^{-q_m z} + \sum_{w=4}^{6} b_w D_w e^{q_w z}\right) \tag{21}$$

$$\tilde{u}_3 = \tilde{F}\left(\sum_{m=1}^{3} p_m D_m e^{-q_m z} + \sum_{w=4}^{6} p_w D_w e^{q_w z}\right) \tag{22}$$

$$\tilde{t}_{31} = \tilde{F}\left(\sum_{m=1}^{3} s_m D_m e^{-q_m z} + \sum_{w=4}^{6} s_w D_w e^{q_w z}\right) \tag{23}$$

$$\tilde{t}_{33} = \tilde{F}\left(\sum_{m=1}^{3} r_m D_m e^{-q_m z} + \sum_{w=4}^{6} r_w D_w e^{q_w z}\right) \tag{24}$$

$$\tilde{T} = \tilde{F}\left(\sum_{m=1}^{3} b_m^{\bullet} D_m e^{-q_m z} + \sum_{w=4}^{6} b_w^{\bullet} D_w e^{q_w z}\right) \tag{25}$$

6. Particular case

Neglecting angular velocity (i.e., $\vec{\Omega} = 0$) in Eq. (1), we obtain the transformed components of displacement, force stress and temperature distribution in a generalized thermoelastic medium due to moving load at the interface as

$$\tilde{u}_1 = \tilde{F}\left(\sum_{m=1}^{3} b'_m D_m^{(1)} e^{-q'_m z} + \sum_{w=4}^{6} b'_w D_w^{(1)} e^{q'_w z}\right) \tag{26}$$

$$\tilde{u}_3 = \tilde{F}\left(\sum_{m=1}^{3} p'_m D_m^{(1)} e^{-q'_m z} + \sum_{w=4}^{6} p'_w D_w^{(1)} e^{q'_w z}\right) \tag{27}$$

$$\tilde{t}_{31} = \tilde{F}\left(\sum_{m=1}^{3} s'_m D_m^{(1)} e^{-q'_m z} + \sum_{w=4}^{6} s'_w D_w^{(1)} e^{q'_w z}\right) \tag{28}$$

$$\tilde{t}_{33} = \tilde{F}\left(\sum_{m=1}^{3} r'_m D_m^{(1)} e^{-q'_m z} + \sum_{w=4}^{6} r'_w D_w^{(1)} e^{q'_w z}\right) \tag{29}$$

$$\tilde{T} = \tilde{F}\left(\sum_{m=1}^{2} b'^{\bullet}_m D_m^{(1)} e^{-q'_m z} + \sum_{w=4}^{6} b'^{\bullet}_w D_w^{(1)} e^{q'_w z}\right) \tag{30}$$

In Eqs. (21)-(25) the transformed displacement, force stress and temperature distribution components for the region $-\infty < z \le 0$, are obtained by inserting $D_4 = D_5 = D_6 = 0$ and in Eqs. (26)-(30) by inserting $D_4^{(1)} = D_5^{(1)} = D_6^{(1)}$. Similarly, for the region $0 \le z\ \infty$, the components are obtained by inserting $D_1 = D_2 = D_3 = 0$ in Eqs. (21)-(25) and $D_1^{(1)} = D_2^{(1)} = D_3^{(1)}$ in Eqs. (26)-(30).

7. Numerical results

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variations of temperature, displacement and stress fields in the context of L-S and G-S theories. For this purpose magnesium crystal like material is taken as the thermoelastic material for which we take the following values of physical constants (Dhaliwal and Singh (1980)) at $T_0 = 298K$

$$\lambda = 2.17 \times 10^{10}\mathrm{Nm}^{-2}, \quad \mu = 3.278 \times 10^{10}\mathrm{Nm}^{-2}, \quad \rho = 1.74 \times 10^{3}\mathrm{Km}^{3}$$
$$C_E = 1.04 \times 10^{3} J\mathrm{Kg}^{-1}\ \deg^{-1}, \quad \upsilon = 2.68 \times 10^{6}\mathrm{Nm}^{-2}\deg^{-1}$$
$$K^* = 1.7 \times 10^{2}\, W\mathrm{m}^{-1} s^{-1}\deg^{-1}$$

The computations are carried out for $U < c_0$ on the surface $z = 1.0$ at $t = 1.0$ The graphical results for normal displacement u_3, normal force stress t_{33} and temperature distribution T for $\Omega = 0.3$ and non dimensional thermal relaxation times $\tau_0 = 0.1$ and $\vartheta_0 = 0.2$ are shown in Figs. (1)-(3), for

(i) thermoelastic solid with rotation (L-S theory) by solid line (———)
(ii) thermoelastic solid without rotation (L-S theory) by dashed line (---------)
(iii) thermoelastic solid with rotation (G-L theory) by solid lines with centered symbols (*——*——*——)
(iv) thermoelastic solid without rotation (G-L theory) by dashed lines with centered symbols (*----*----*).

8. Special cases of thermoelastic theory

8.1 The equations of the coupled thermoelasticity (C-T theory) for a rotating media are obtained when

$$n^* = n_1 = 1, \quad t_1 = \tau_0 = \vartheta_0 = 0 \tag{31}$$

Eqs. (1) and (2) has the form

$$(\lambda + \mu)\nabla(\nabla.\vec{u}) + \mu\nabla^2\vec{u} - \upsilon\nabla T = \rho\left[\frac{\partial^2\vec{u}}{\partial t^2} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega}\frac{\partial\vec{u}}{\partial t^2}\right] \tag{32}$$

$$K^*\nabla^2 T = \rho C_E\frac{\partial T}{\partial t} + \upsilon T_0\frac{\partial e}{\partial t} \tag{33}$$

8.2 For Lord-Shulman (L-S theory), when

$$n^* = n_1 = n_0 = 1, \quad t_1 = \vartheta_0 = 0, \quad \tau_0 > 0$$

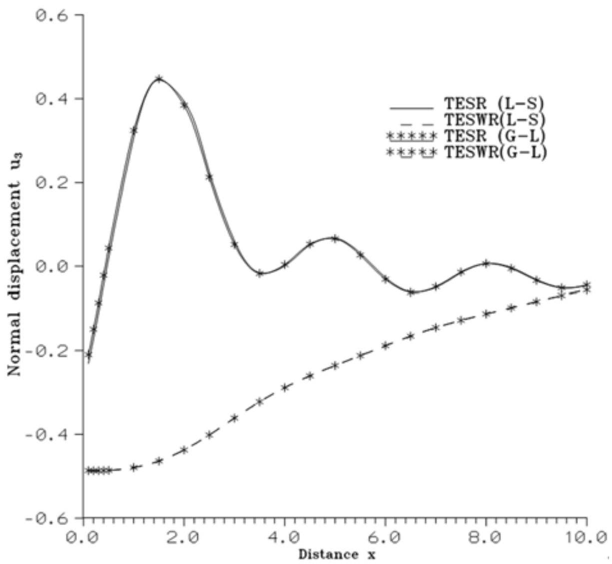

Fig. 1 Variation of normal displacement u_3 with horizontal distance x

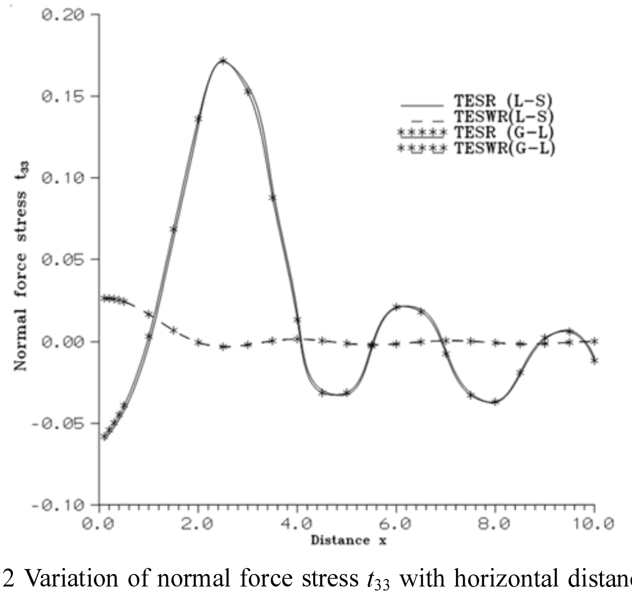

Fig. 2 Variation of normal force stress t_{33} with horizontal distance x

where τ_0 is the relaxation time. Eq. (1) is the same as Eq. (32) and Eq. (2) has the form

$$K^* \nabla^2 T = \rho C_E \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + \upsilon T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) e \tag{34}$$

8.3 For Green –Lindsay (G-L theory)

$$n^* = n_1 = 1, \quad n_0 = 0, \quad t_1 = 0, \quad \vartheta_0 \geq \tau_0 > 0 \tag{35}$$

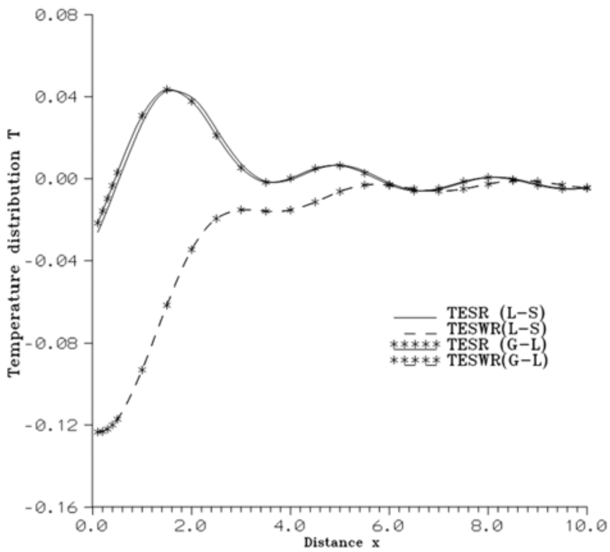

Fig. 3 Variation of temperature distribution T with horizontal distance x

where ϑ_0, τ_0 are the two relaxation times. Eq. (1) remains unchanged and Eq. (2) takes the form

$$K^*\nabla^2 T = \rho C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t}+\upsilon T_0\frac{\partial e}{\partial t}\right) \tag{36}$$

8.4 The equations of the generalized thermoelasticity for a rotating media, without energy dissipation (the linearlized GN theory of type II) are obtained when

$$n^* = n_1 = 0, \quad n_0 = 1, \; t_1 = \vartheta_0 = 0, \quad \tau_0 = 1 \tag{37}$$

Eq. (1) is the same as Eq. (32) and Eq. (2) takes the form

$$K^*\nabla^2 T = \rho C_E\frac{\partial^2 T}{\partial t^2}+\upsilon T_0\frac{\partial^2 e}{\partial t^2} \tag{38}$$

where n^* is constant which has the dimension of (1/sec) and
$n^*K^* = K' = C_E(\lambda+2\mu)/4$ is a characteristic constant of this theory.

9. Discussions

The values of all the quantities i.e., normal displacement, normal forces stress and temperature distribution are very close for L-S and G-L theories. These variations of normal displacement and normal force stress under the effect of rotation ($\Omega \neq 0$) are oscillatory to a large extent. When the rotation effect is neglected ($\Omega = 0$), the variations of normal displacement for both L-S and G-L theories increases linearly in the range $0 \le x \le 10$. Similarly in the absence of rotation the values of normal force stress lie in a very short range and are close to zero in the range $2 \le x \le 10$. These variations of normal displacement and normal force stress are shown in Figs. 1 and 2 respectively.

When the medium is rotating with some angular velocity, the values of temperature distribution are very less in magnitude. To compare the results between both the mediums, these values of temperature distribution have been multiplied by 10^4. The variations of temperature distribution are shown in Fig. 3.

10. Conclusions

The variations of all the quantities are similar in nature for L-S and G-L theories. As observed from the graphical results, rotation plays an important role on the deformation of the body.

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Appendix A

The field equations and constitutive relations for Lord Shulman (L-S) (1967) theory are

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+\mu\nabla^2\vec{u}-\upsilon\nabla T = \rho\left[\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2\vec{\Omega}\frac{\partial\vec{u}}{\partial t}\right]$$

$$K^*\nabla^2 T = \rho C_E\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\upsilon T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u})$$

$$t_{ij} = \lambda e\delta_{ij}+2\mu e_{ij}-\upsilon T\delta_{ij}$$

The field equations and constitutive relations for Green-Lindsay (G-L) (1972) theory are

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+\mu\nabla^2\vec{u}-\upsilon\left(1+\vartheta_0\frac{\partial}{\partial t}\right)\nabla T = \rho\left[\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2\vec{\Omega}\frac{\partial\vec{u}}{\partial t}\right]$$

$$K^*\nabla^2 T = \rho C_E\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\upsilon T_0\frac{\partial}{\partial t}(\nabla.\vec{u})$$

$$t_{ij} = \lambda e\delta_{ij}+2\mu e_{ij}-\upsilon\left(1+\vartheta_0\frac{\partial}{\partial t}\right)T\delta_{ij}$$

$$D_m = \frac{\Delta_m}{\Delta},\ D_w = \frac{\Delta_w}{\Delta},\ m = 1, 2, 3\text{ , and }w = 4, 5, 6$$

$$\Delta = -(f_1h_1+f_2h_2+f_3h_3+f_4h_4+f_5h_5+f_6h_6+f_7h_7+f_8h_8+f_9h_9$$
$$+f_{10}h_{10}+f_{11}h_{11}+f_{12}h_{12}+f_{13}h_{13}+f_{14}h_{14}+f_{15}h_{15})$$

$$\Delta_1 = \tilde{F}(E_1+E_2),\quad \Delta_2 = \tilde{F}(E_3+E_4),\quad \Delta_3 = \tilde{F}(E_5+E_6)$$

$$f_1 = k_2 d_6 - d_5 k_3, \quad f_2 = l_5 d_6 - d_5 l_6, \quad f_3 = l_5 k_3 - k_2 l_6, \quad f_4 = b_5 d_6 - d_5 b_6$$

$$f_5 = b_5 k_3 - k_2 b_6, \quad f_6 = b_5 l_6 - l_5 b_6, \quad f_7 = s_5 d_6 - d_5 s_6, \quad f_8 = s_5 k_3 - k_2 s_6$$

$$f_9 = s_5 l_6 - l_5 s_6, \quad f_{10} = s_5 b_6 - b_5 s_6, \quad f_{11} = r_5 d_6 - d_5 r_6, \quad f_{12} = r_5 k_3 - k_2 r_6$$

$$f_{13} = r_5 l_6 - l_5 r_6, \quad f_{14} = r_5 b_6 - b_5 r_6, \quad f_{15} = r_5 s_6 - s_5 r_6$$

$$g_1 = r_1 s_2 - s_1 r_2, \quad g_2 = b_2 r_1 - r_2 b_1, \quad g_3 = r_1 l_2 - l_1 r_2 \quad g_4 = r_1 k_2 - k_1 r_2$$

$$g_5 = r_1 d_2 - d_1 r_2, \quad g_6 = b_2 s_1 - s_2 b_1, \quad g_7 = l_2 s_1 - s_2 l_1, \quad g_8 = s_1 k_2 - k_1 s_2$$

$$g_9 = s_1 d_2 - d_1 s_2, \quad g_{10} = b_1 l_2 - l_1 b_2, \quad g_{11} = b_1 k_2 - k_1 b_2, \quad f_{12} = b_1 d_2 - d_1 b_2$$

$$g_{13} = l_1 k_2 - k_1 l_2, \quad g_{14} = l_1 d_2 - d_1 l_2, \quad g_{15} = k_1 d_2 - d_1 k_2$$

$$p_1 = s_2 k_3 - k_2 s_3, p_2 = s_2 l_3 - l_2 s_3, p_3 = s_2 k_3 - k_2 s_3, p_4 = s_2 d_3 - d_2 s_3$$

$$p_5 = b_2 l_3 - l_2 b_3, p_6 = b_2 k_3 - k_2 b_3, p_7 = b_2 d_3 - d_2 b_3, p_8 = l_2 k_3 - k_2 l_3$$

$$p_9 = l_2 d_3 - d_2 l_3, p_{10} = k_2 d_3 - d_2 k_3, \; p_{11} = s_1 b_3 - b_1 s_3, p_{12} = s_1 l_3 - l_1 s_3$$

$$p_{13} = s_1 k_3 - k_1 s_3, p_{14} = s_1 d_3 - d_1 s_3, p_{15} = b_1 l_3 - l_1 b_3, p_{16} = b_1 k_3 - k_1 b_3$$

$$p_{17} = b_1 d_3 - d_1 b_3, p_{18} = l_1 k_3 - k_1 l_3, p_{19} = l_1 d_3 - d_1 l_3, p_{20} = k_1 d_3 - d_1 k_3$$

$$h_1 = y_1 l_4 - y_2 b_4 + y_3 s_4 - y_4 r_4, \; h_2 = y_5 b_4 - y_1 k_1 - y_6 s_4 + y_7 r_4$$

$$h_3 = y_1 d_4 - y_8 b_4 + y_9 s_4 - y_{10} r_4, \; h_4 = y_2 k_1 - y_5 l_4 + y_{11} s_4 - y_{12} r_4$$

$$h_5 = y_8 l_4 - y_2 d_4 - y_{13} s_4 + y_{14} r_4, \; h_6 = y_5 d_4 - y_8 k_1 + y_{15} s_4 - y_{16} r_4$$

$$h_7 = y_6 l_4 - y_3 k_1 - y_{11} b_4 + y_{17} r_4, \; h_8 = y_3 d_4 - y_9 l_4 + y_{13} b_4 - y_{18} r_4$$

$$h_9 = y_9 k_1 - y_6 d_4 - y_{15} b_4 + y_{19} r_4, \; h_{10} = y_{11} d_4 - y_{13} k_1 + y_{15} l_4 - y_{20} r_4$$

$$h_{11} = y_4 k_1 - y_7 l_4 + y_{12} b_4 - y_{17} s_4, \; h_{12} = y_{10} l_4 - y_4 d_4 - y_{14} b_4 + y_{18} s_4$$

$$h_{13} = y_7 d_4 - y_{10} k_1 + y_{16} b_4 - y_{19} s_4, \; h_{14} = y_{14} k_1 - y_{12} d_4 - y_{16} l_4 + y_{20} s_4$$

$$h_{15} = y_{17} d_4 - y_{18} k_1 + y_{19} l_4 - y_{20} b_4, \; y_1 = g_1 b_3 - g_2 s_3 + g_6 r_3$$

$$y_2 = g_1 l_3 - g_3 s_3 + g_7 r_3, y_3 = g_2 l_3 - g_3 b_3 + g_{10} r_3, y_4 = g_6 l_3 - g_7 b_3 - g_{10} s_3$$

$$y_5 = g_1 k_3 - g_4 s_3 + g_8 r_3, y_6 = g_2 k_3 - g_4 b_3 + g_{11} r_3, y_7 = g_6 k_3 - g_8 b_3 + g_{11} s_3$$

$$y_8 = g_1 d_3 - g_5 s_3 + g_9 r_3, y_9 = g_2 d_3 - g_5 b_3 + g_{12} r_3, y_{10} = g_6 d_3 - g_9 b_3 + g_{12} s_3$$

$$y_{11} = g_3 k_3 - g_4 l_3 + g_{13} r_3, y_{12} = g_7 k_3 - g_8 l_3 + g_{13} s_3, y_{13} = g_3 d_3 - g_5 l_3 + g_{14} r_3$$

$$y_{14} = g_7 d_3 - g_9 l_3 + g_{14} s_3,\; y_{15} = g_4 d_3 - g_5 k_3 + g_{15} r_3,\; y_{16} = g_8 d_3 - g_9 k_3 + g_{15} s_3$$

$$y_{17} = g_{10} k_3 - g_{11} l_3 + g_{13} b_3,\; y_{18} = g_{10} d_3 - g_{12} l_3 + g_{14} b_3,\; y_{19} = g_{11} d_3 - g_{12} k_3 + g_{15} b_3$$

$$y_{20} = g_{13} d_3 - g_{14} k_3 + g_{15} l_3,\; n_1 = f_1 l_4 - f_2 k_1 + f_3 d_4,\; n_2 = f_1 b_4 - f_4 k_1 + f_5 l_4$$

$$n_3 = f_2 b_4 - f_4 l_4 + f_6 d_4,\; n_4 = f_3 b_4 - f_5 l_4 + f_6 k_1,\; n_5 = f_1 s_4 - f_7 k_1 + f_8 d_4$$

$$n_6 = f_2 s_4 - f_7 l_4 + f_9 d_4,\; n_7 = f_3 s_4 - f_8 l_4 + f_9 k_1,\; n_8 = f_4 s_4 - f_7 b_4 + f_{10} d_4$$

$$n_9 = f_5 s_4 - f_8 b_4 + f_{10} k_1,\; n_{10} = f_6 s_4 - f_9 b_4 + f_{10} l_4$$

$$E_1 = p_1 n_1 - p_2 n_2 + p_3 n_3 - p_4 n_4 + p_5 n_5 - p_6 n_6$$

$$E_2 = p_7 n_7 - p_8 n_8 - p_9 n_9 + p_{10} n_{10}$$

$$E_3 = -p_{11} n_1 + p_{12} n_2 - p_{13} n_3 + p_{14} n_4 - p_{15} n_5 + p_{16} n_6$$

$$E_4 = -p_{17} n_7 + p_{18} n_8 + p_{19} n_9 - p_{20} n_{10}$$

$$E_5 = g_6 n_1 - g_7 n_2 + g_8 n_3 - g_9 n_4 + g_{10} n_5 - g_{11} n_6$$

$$E_6 = g_{12} n_7 - g_{13} n_8 - g_{14} n_9 + g_{15} n_{10}$$

$$l_i = i\xi a_i^* - q_i \quad ,\; l_{4,5,6} = i\xi a_i^* + q_i,\; s_i = \frac{\mu}{\rho c_0^2}\left(2i\xi q_i + \left(q_i^2 + \xi^2\right)a_i^*\right)$$

$$s_{4,5,6} = \frac{\mu}{\rho c_0^2}\left[(q_i^2 + \xi^2)a_i^* - 2i\xi q_i\right],\; b_i = -(i\xi + a_i^* q_i),\; b_{4,5,6} = a_i^* q_i - i\xi$$

$$r_i = q_i^2 - \frac{\lambda \xi^2}{\rho c_0^2} - \frac{2i\mu\xi a_i^* q_i}{\rho c_0^2} - \left(1 - i\xi\vartheta_0 M_1\right)b_i^*,\; k_i = k_{4,5,6} = b_i^*,\; d_i = -q_i b_i^*$$

$$r_{4,5,6} = q_i^2 - \frac{\lambda \xi^2}{\rho c_0^2} + \frac{2i\xi\mu a^{\bullet}{}_{4,5} q_i}{\rho c_0^2} - \left(1 - i\xi\vartheta_0 M_1\right)b^{\bullet}{}_i,\; d_{4,5,6} = q_i b_i^*$$

Appendix B

$$D_m^{(1)} = \frac{\Delta_m^{(1)}}{\Delta^{(1)}},\; D_w^{(1)} = \frac{\Delta_w^{(1)}}{\Delta^{(1)}}$$

$$\Delta^{(1)} = 8(d_2' E_1' - d_1' E_2'),\;\; \Delta_1^{(1)} = 4\widetilde{F} k_2' b_3' (s_3' g_3' - l_3' g_4')$$

$$\Delta_2^{(1)} = -\frac{k_1'}{k_2'}\Delta_1^{(1)},\;\; \Delta_3^{(1)} = 4\widetilde{F} g_2' (d_2' p_{10}' - d_1' p_9')$$

$$E_1' = l_3'(g_1'p_1' + g_2'p_2') + s_3'(g_1'p_3' - g_2'p_4')$$

$$E_2' = s_3'(g_2'p_6' - g_1'p_5') - l_3'(g_1'p_7' + g_2'p_8')$$

$$p_1' = s_1'b_3' - b_1's_3',\ p_2' = r_1's_3' - s_1'r_3',\ p_3' = b_1'l_3' - l_1'b_3',\ p_4' = r_1'l_3' - l_1'r_3'$$

$$p_5' = b_2'l_3' - l_2'b_3',\ p_6' = r_2'l_3' - l_2'r_3',\ p_7' = s_2'b_3' - b_2's_3',\ p_8' = r_2's_3' - s_2'r_3'$$

$$p_9' = s_2'l_3' - l_2's_3',\ p_{10}' = s_1'l_3' - l_1's_3',\ g_1' = k_1'r_2' - r_1'k_2',\ g_2' = b_1'k_2' - k_1'b_2'$$

$$g_3' = l_1'd_2' - d_1'l_2',\ g_4' = s_1'd_2' - d_1's_2',\ r_{1,2}' = q_{1,2}' - \frac{\lambda\xi^2}{\rho c_0^2} - (1 - i\xi\vartheta_0 M_1)b_{1,2}'^{\bullet}$$

$$r_3' = -\frac{2i\xi\mu q_3'}{\rho c_0^2},\ \ s_{1,2}' = \frac{2i\xi\mu q_{1,2}'}{\rho c_0^2},\ s_3' = \frac{\mu}{\rho c_0^2}\left(q_3'^2 + \xi^2\right),\ b_{1,2}' - i\xi,\ b_3' = -q_3'$$

$$l_{1,2}' = -q_{1,2}',\ l_3' = i\xi,\ k_{1,2}' = k_{4,5}' = b_{1,2}'^{\bullet},\ d_{1,2}' = -q_{1,2}'b_{1,2}'^{\bullet},\ b_{1,2}'^{\bullet} = \frac{q_{1,2}'^2 - e_2'}{1 - i\xi\vartheta_0 M_1}$$

$$r_{4,5}' = r_{1,2}',\ r_6' = -r_3',\ s_{4,5}' = -s_{1,2}',\ s_6' = s_3',\ b_{4,5}' = b_{1,2}',\ b_6' = -b_3',\ d_{4,5}' = -d_{1,2}'$$

$$q_{1,2}'^2 = \frac{A_1 \pm \sqrt{A_1^2 - 4B_1}}{2},\ q_3'^2 = \xi^2\left(1 - \frac{\rho U^2}{\mu}\right),\ A_1 = e_1 + e_2' + e_3,\ B_1 = e_1e_2' + e_3\xi^2$$