Mesoscopic numerical analysis of reinforced concrete beams using a modified micro truss model

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Abstract. Concrete is a heterogeneous material consisting of coarse aggregate, mortar matrix and interfacial zones at the meso level. Though studies have been done to interpret the fracture process in concrete using meso level models, not much work has been done for simulating the macroscopic behaviour of reinforced concrete structures using the meso level models. This paper presents a procedure for the mesoscopic analysis of reinforced concrete beams using a modified micro truss model. The micro truss model is derived based on the framework method and uses the lattice meshes for representing the coarse aggregate (CA), mortar matrix, interfacial zones and reinforcement bars. A simple procedure for generating a random aggregate structure is developed using the constitutive model at meso level. The study reveals the potential of the mesoscopic numerical simulation using a modified micro truss model to predict the nonlinear response of reinforced concrete structures. The modified micro truss model correctly predicts the load-deflection behaviour, crack pattern and ultimate load of reinforced concrete beams failing under different failure modes.

Keywords: mesoscopic analysis; micro truss; reinforced concrete; framework method.

1. Introduction

Though concrete is widely used for construction purposes, it is quite difficult to predict its behaviour due to its inherent heterogeneous nature. As pointed out by Zaitsev and Wittmann 1981, the hierarchy of material characterization of concrete plays an important role in predicting the response of concrete structures and they posed four hierarchical levels for concrete, namely, nano, micro, meso and macro levels. It is generally accepted that the overall behaviour of concrete can be predicted by the meso and macro level models (Asai *et al.* 2003). In meso scale modelling of concrete, concrete is considered as a three phase system made of coarse aggregate (CA), mortar matrix and interfacial zones, and they can be used to simulate the heterogeneity of concrete. Even though for routine analysis and design of concrete structures, a macro level model in which concrete is assumed as a homogeneous material is used, it does not represent the true nature of concrete.

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In the recent years, there has been a growing interest in simulating the fracture process in concrete using the mesoscopic models. The following are some of methods used for the mesoscopic analysis of concrete: numerical concrete (Roelfstra 1989), random particle model (Bažant *et al.* 1990), lattice model (Schlangen and Van Mier 1992, Schlangen 1993, Prasad *et al.* 1994, Schlangen and Garboczi 1997, Chiaia *et al.* 1997, Arslan *et al.* 2002, Van Mier *et al.* 2002, Lilliu and Van Mier 2003, Ince *et al.* 2003, Karihaloo *et al.* 2003, Asai *et al.* 2003, Leite *et al.* 2004, Prasad and Sagar 2006), nonlinear finite element analysis (Kwan *et al.* 1999), tridimensional random particle model (Cusatis 2001), mesoscopic mechanical model (Zhu and Tang 2002) and rigid body spring model (Nagai *et al.* 2004, 2005). Though much research has been done to simulate the fracture process in concrete, only few attempts have been made to simulate the macroscopic response of concrete structures using the mesoscopic models (Asai *et al.* 2003).

In the present study, an attempt has been made to predict the behaviour of reinforced concrete beams using mesoscopic analysis. The reinforced concrete structural element is replaced by a simpler model known as micro truss model (Salem 2004). Micro truss model, inspired by the truss models developed in the early 1900s (Park and Paulay 1975), is based on the framework method (Hrennikoff 1941) and uses a lattice type mesh. The random aggregate structure is developed over the micro truss model and the constitutive model at meso level are used for analysis. The method is validated by comparing the results for a simply supported shallow beam failing under different failure modes (under-reinforced failure, over-reinforced failure and shear failure).

2. Introduction to modified micro truss model

2.1. Background to micro truss model for reinforced concrete structural elements

In the present paper, the micro truss is based on the framework method proposed by Hrennikoff (1941), in which the structure is replaced by an equivalent pattern of truss elements. One of the patterns proposed by Hrennikoff for modelling the plane stress problems is shown in Fig. 1.

In Fig. 1, A_h , A_d and A_v are the cross sectional areas of the horizontal, diagonal and vertical truss members, respectively, which are evaluated as

$$A_h = \frac{3}{8}(3 - k^2)at$$
 (1)

$$A_d = \frac{3}{16} \frac{(1+k^2)^{3/2}}{k} at$$
(2)

$$A_{v} = \frac{3}{8} \frac{(3k^{2} - 1)}{k} at$$
(3)

where t is the thickness of the plate. The preceding expressions are accurate for materials with Poisson's ratio equal to 1/3. The choice of these expressions for reinforced concrete is justified by the fact that Poisson's ratio for concrete varies from 0.2 to 0.4, i.e., from the elastic to the failure state (Kotronis *et al.* 2003). The pattern of truss elements shown in Fig. 1 is used to develop the micro truss model. Fig. 2 shows the micro truss model for a reinforced concrete beam.

In Fig. 2, it can be seen that the horizontal and vertical members resist the normal stress in the respective direction and the diagonal members resist the shear stress. Hence, this model can capture



Fig. 1 Pattern of truss elements for plane stress problems



Fig. 2 Micro truss model for a reinforced concrete beam

both the flexure and shear types of failure in reinforced concrete structures. Further it can be seen that the steel bars can be easily simulated in this model. By incorporating both material and geometrical nonlinearity, the micro truss model can be used to predict the nonlinear response of concrete structures (Kotronis *et al.* 2003, Salem 2004).

In this study, the micro truss model is modified by developing the random aggregate structure over this pattern of truss elements and is used for mesoscopic analysis.

2.2 Mesoscopic constitutive model for concrete

Coarse aggregate, mortar matrix and interfacial zones are modelled as linearly elastic or ideally brittle materials. The Young's modulus (*E*) and tensile strength (σ_t) of these three phases recommended by Prasad and Sagar (2006) are used in this study, as listed in Table 1. The stress-strain relation for steel is described by an elastic-perfectly plastic curve, whose initial slope is the elastic modulus of the material. Full compatibility between the steel and concrete at their interface is assumed.

2.3 Development of random aggregate structure

In order to simulate the heterogeneity of concrete using meso scale models, different methods can be used (Schalangen and Garboczi 1997):

| Table 1 | Elastic | properties | of phases |
|---------|---------|------------|-----------|
|---------|---------|------------|-----------|

=

| Phase | E (MPa) | σ_t (MPa) |
|------------------|---------|------------------|
| Coarse aggregate | 70000 | 10.0 |
| Mortar | 25000 | 5.0 |
| Interface | 25000 | 1.25 |

- By randomly specifying the material properties, which follow some type of distribution, to the elements in the model.
- By using a mesh having random geometry and specifying same material properties to all elements.
- By using the actual or generated meso structure of concrete. The model is projected over the meso structure and the location of elements representing the coarse aggregate (CA), mortar matrix and interfacial zones are found out and the corresponding material properties are given.
- By using a combination of random geometry and meso structure of concrete.

In this study, a simplified method to develop the random aggregate structure is used. As the first step, a micro truss model for the structure is developed. The length of both the horizontal and vertical elements in the micro truss was taken as 1 mm (a = 1 mm, k = 1) in this study. Any one of the nodes in the mesh is selected at random and is assumed to be a part of CA. All the elements connected to this node and the other nodes of these connected elements are selected. A high probability (say 90%) is assigned to all such nodes to become a part of CA, using a random number generator, based on a uniform distribution. In other words, the adjacent nodes of the selected node will have 90% probability of becoming a part of the same CA. All the elements attached to the set of nodes, which are made to be a part of CA in the above step, are selected. A lower probability (say 60%) is assigned to the end nodes of these elements to be in the same CA, and the procedure is repeated till no additional nodes become part of the CA, or when the size of the CA becomes larger than 20 mm (maximum size of CA usually used for making concrete). Then another node is selected at random and the procedure is repeated. In this manner, all the nodes which are part of CA are obtained, and all the remaining nodes are considered to be in mortar.

Elements having both end nodes in CA are assigned the property of CA. Properties of mortar are assigned to those elements, in which both the end nodes are in mortar. Elements having one node in CA, and the other in mortar are assigned the properties of interface. By finding out the number of CA of different sizes, the grain size distribution (grading) can be obtained. Similarly, the percentage of elements representing CA in the generated meso structure can be found out. By suitably adjusting the value of probabilities at different levels, it is possible to get a meso structure for the required percentage of CA and grading. A typical random aggregate structure developed using the above procedure is shown in Fig. 3.

2.4 Basic steps in modified micro truss analysis

The analysis was done using ANSYS 10.0, a finite element analysis software. Macro programmes were written for developing the micro truss model, generating random aggregate structure and performing the analysis. Geometric nonlinearity is significant in the micro truss model because geometrical changes are higher for elements of small sizes. Hence, geometric nonlinearity was taken into consideration. The essential steps for analysis are given below.

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Fig. 3 Generated random aggregate structure

- 1. The micro truss model for the structure is developed, and the geometric properties for the horizontal, vertical and diagonal truss elements are assigned.
- 2. The random aggregate structure (meso structure) is developed. The grain size distribution and percentage of CA in the generated meso structure are found out, and compared with the required values. If the specifications are not met, new meso structure is developed, and the process is repeated till a satisfactory model is obtained.
- 3. The maximum load in the analysis is taken as 20% higher than the expected ultimate load, and it is applied in a number of load steps.
- 4. Initially, the load corresponding to the first load step is applied.
- 5. The structure is analyzed for the applied load and key parameters such as the load value, displacements and axial stress in all the elements are noted.
- 6. Those members in which the stress exceeds the corresponding tensile strength (σ_i) are deactivated (that is, the stiffness of the member is made nearly equal to zero, and hence the member is effectively removed).
- 7. By removing the deactivated truss members from the original model, the crack pattern can be obtained at any load step.
- 8. The load corresponding to the next load step is applied and the steps from 5 to 7 are repeated till the analysis shows a divergence in the computed results. The load corresponding to the last converged load step is taken as the ultimate load.

3. Validation of modified micro truss model

For verification of the modified micro truss model, the results obtained from the analysis are compared with the experimental ones. In order to show that this model can simulate all types of failure, a shallow beam failing under different failure modes (under-reinforced failure, over-reinforced failure and shear failure) reported by Revathy and Menon (2005) was considered. Also, the limited amount of experimental studies conducted by the authors on shallow beams are also used to validate the method.

3.1 Validation of micro truss model using available experimental results

The experimental programme reported by Revathy and Menon (2005) involved testing of reinforced concrete beams of identical cross section. The under-reinforced beams and over-reinforced beams were 2000 mm long, 200 mm deep and 150 mm wide. The beams were simply supported with a span of 1800 mm and loaded symmetrically in four-point bending. The beam for shear test was simply supported with a clear span of 900 mm and loaded at the mid span. The reinforcement details of the test beams are shown in Fig. 4. Percentage of CA used in the mix was 58.3% and the compressive strength of concrete was 20 MPa. The yield stress of the longitudinal steel was 415 MPa and for stirrups it was 250 MPa. The elastic modulus of steel was 2×10^5 MPa and for concrete it was 25980 MPa.

Since the modified micro truss model is statistical in nature, several trials should be performed to get an insight to the true behaviour of concrete. In the present study, three trials were performed. In each trial, a new random aggregate structure was developed and the analysis was carried out. The symmetry of the structure and loading was advantageously used in modelling in order to save computing time and effort. Hence only half of each test beam was used in analysis. The load deflection curves and the crack pattern from the micro truss model for all the beams are shown in Figs. 5-7. The ultimate loads of the beams reported in Revathy and Menon (2005) and those obtained from the analysis are given in Table 2.

3.2 Experimental programme conducted by the authors, and analysis using micro truss model

The experimental programme consisted of casting and testing reinforced concrete beams of the same dimensions ($100 \text{ mm} \times 150 \text{ mm} \times 1200 \text{ mm}$) designed to fail in under-reinforced (UR), over-



Fig. 4 Reinforcement details for the beams (Revathy and Menon)

| Tabl | le 2 | U | ltimate | load | of | the | beams (| (\mathbb{R}) | levathy | / and | M | lenon |) |
|------|------|---|---------|------|----|-----|---------|----------------|---------|-------|---|-------|---|
|------|------|---|---------|------|----|-----|---------|----------------|---------|-------|---|-------|---|

| | Ultimate Load (kN) | | | | | | | |
|------------------|--------------------|---------------------|---------|---------|---------|-----------|--|--|
| Type of beam | Experiment - | Mesoscopic Analysis | | | | | | |
| | | Trial 1 | Trial 2 | Trial 3 | Average | Error (%) | | |
| Under-reinforced | 38 | 42 | 44.4 | 39.13 | 41.84 | 10.11 | | |
| Over-reinforced | 93 | 97.6 | 88 | 88 | 91.2 | -1.94 | | |
| Shear-test | 92 | 80 | 98 | 102 | 93.33 | 1.45 | | |

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(b) Crack pattern at ultimate load

Fig. 5 Results of the under-reinforced beam (Revathy and Menon)



Fig. 6 Results of the over-reinforced beam (Revathy and Menon)



(b) Crack pattern at ultimate load

Fig. 7 Results of the shear-test beam (Revathy and Menon)



Fig. 8 Reinforcement details for test beams

Table 3 Ultimate load of shallow beams

| | | | Ultimate L | oad (kN) | | | | | |
|--------------|--------------|---------------------|------------|----------|---------|-----------|--|--|--|
| Type of beam | Experiment - | Mesoscopic Analysis | | | | | | | |
| | | Trial 1 | Trial 2 | Trial 3 | Average | Error (%) | | | |
| UR | 56 | 57.75 | 62 | 56 | 58.58 | 4.61 | | | |
| OR | 96 | 92.68 | 103.04 | 94.72 | 96.81 | 0.84 | | | |
| SM | 34 | 36.38 | 33.12 | 35 | 34.83 | 2.44 | | | |

reinforced (OR) and shear mode (SM) of failure. The effective span of the beam was 1.1 m and the beams were made of concrete with a compressive strength of 27.5 MPa and steel with a yield strength of 430 MPa. The percentage of CA used in the mix was 54.6%. The beams designed to fail as under-reinforced and over-reinforced beams was tested under two point loads, and the beam designed to fail in shear was subjected to a central point load. An effective cover of 20 mm for tension steel was provided for all beams and the reinforcement details of the beam are shown in Fig. 8. The ultimate loads obtained from the experiment and analysis are given in Table 3. The load-deflection plots and crack patterns for the beams are shown in Figs. 9-11.

The modified micro truss model for the shallow beams give results that are closely consistent with the test results. The load-deflection curves, crack pattern and the ultimate load obtained from the analysis matches well with those obtained from the experiment. The ductile behaviour of the under-reinforced beam and the brittle mode of failure for the over-reinforced beam and shear-test beam are well captured by the modified micro truss model. The statistical nature of the analysis helps to obtain some insight into the scatter of the results due to the heterogeneous nature of concrete.

Though the modified micro truss model can be used to predict the response of concrete structures, the computation time is large due to the enormous increase in the number of degrees of freedom required. Hence this method is not recommended for routine analysis. But the approach proposed herein can be used in situations where experimental results are not readily available or when experiments are not economically feasible. Virtual testing of entire structure is possible using the method of modelling proposed. Further, since this model can simulate the heterogeneity of concrete, it can be potentially used to simulate the size effect in concrete structures.





(c) Crack pattern at ultimate load from mesoscopic analysis

Fig. 9 Results of the UR beam



(a) Load-deflection plot at mid span



(b) Crack patterns from experiment



(c) Crack pattern at ultimate load from mesoscopic analysis

Fig. 10 Results of the OR beam



P_U = 34 kN

(b) Crack patterns from experiment



(c) Crack pattern at ultimate load from mesoscopic analysis

Fig. 11 Results of the SM beam

4. Conclusions

This paper presents a mesoscopic analysis of concrete structures using a modified micro truss model. It is based on the framework method and uses the lattice mesh to simulate the reinforced concrete. A simple methodology is developed to generate the random aggregate structure and the constitutive model at meso level is used for analysis. The method is validated by comparing the results obtained for some typical problems with the experimental ones. It is found that the proposed method of modelling can capture the nonlinear behaviour of the beams failing in different failure modes reasonably well. Further, it can be used to trace the crack pattern. Though the method requires more computation time than a regular nonlinear finite element analysis for reinforced concrete structures, it can be used in situations in which experimental studies are either not possible or expensive.

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