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New approach to calculate Weibull parameters and comparison of wind potential of five cities of Pakistan

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Abstract. Wind energy can be utilized for the generation of electricity, due to significant wind potential at different parts of the world, some countries have already been generating of electricity through wind. Pakistan is still well behind and has not yet made any appreciable effort for the same. The objective of this work was to add some new strategies to calculate Weibull parameters and assess wind energy potential. A new approach calculates Weibull parameters; we also developed an alternate formula to calculate shape parameters instead of the gamma function. We obtained k (shape parameter) and c (scale parameter) for two-parameter Weibull distribution using five statistical methods for five different cities in Pakistan. Maximum likelihood method, Modified Maximum likelihood Method, Method of Moment, Energy Pattern Method, Empirical Method, and have been to calculate and differentiate the values of (shape parameter) k and (scale parameter) c. The performance of these five methods is estimated using the Goodness-of-Fit Test, including root mean square error, mean absolute bias error, mean absolute percentage error, and chi-square error. The daily 10-minute average values of wind speed data (obtained from energydata.info) of different cities of Pakistan for the year 2016 are used to estimate the Weibull parameters. The study finds that Hyderabad city has the largest wind potential than Karachi, Quetta, Lahore, and Peshawar. Hyderabad and Karachi are two possible sites where wind turbines can produce reasonable electricity.

Keywords: parameter estimation methods; statistical analysis; Weibull distribution; Weibull parameters; wind energy; wind speed

1. Introduction

Energy is the elementary demand for all humankind and the country. In Pakistan, the energy sector mainly depends upon natural gas, liquefied gasoline, and oil. Pakistan produces 67% of current electricity using thermal resources, 30% from hydro, 3% from Nuclear, and almost 0% from wind (Zameer and Wang 2018, Wakeel *et al*. 2016). In Pakistan, electricity production is divided into four categories (Thermal, Hydro, Nuclear, and Wind (Rehman and Deyuan 2018, Mohiuddin *et al*. 2016, Khan *et al*. 2018, Baloch *et al*. 2016).

In Pakistan, wind energy has not yet been significantly utilized to produce electricity as there was a deficiency of reliable and complete data on wind resources in Pakistan until recently. On the other hand, the population in third-world countries is growing more rapidly than in developed

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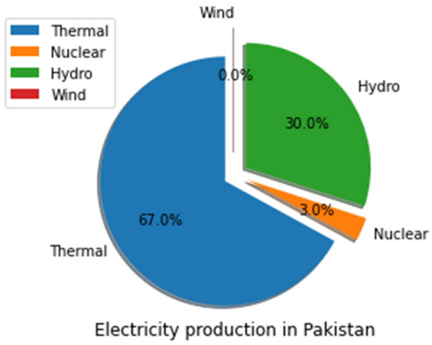


Fig. 1 Classification of Electrical energy produced through various modes (Rehman and Deyuan 2018)

countries; the increase in energy demand of third-world countries and developed countries is almost 213% per annum. This ratio shows that in the recent future, the situation in third world countries would be worst if proper measures are not taken to increase energy production is not taken timely (Khan *et al*. 2015). Wind energy, an alternative to conventional sources of energy, is a free, unlimited power source. However, the energy in the wind depends on its speed, which fluctuates continuously—the estimation of wind speed in five different cities of Pakistan by using many probability distribution functions. Various probability distribution functions (Weibull, Lognormal, Gamma, Rayleigh, and others) are employed in wind power studies (Carta *et al*. 2009, Safari and Gasore 2010, Arik *et al*. 2019).

2. Two parameters Weibull distribution

Waloddi Weibull (Swedish Physicist) first introduced the Weibull probability distribution. So far, this distribution is an excellent approximation to explain many natural phenomena (Lun and Lam 2000). In most cases, it is found that the wind data behaves like Weibull distribution; hence many Physicists and Engineers have employed this distribution to model wind distribution (Akdağ and Dinler 2009, Kadhem *et al*. 2017, Sarkar *et al*. 2017). Deep *et al*. mentioned that for estimating wind energy potential, availability factors and wind energy between cut-in and rated wind speeds should be properly estimated using Weibull models (Deep *et al*. 2020). Gugliani *et al*. showed that Weibull distribution presents a good fit for the wind speed data their assumption of using the Weibull distribution to model the monthly wind speed data was also confirmed (Gugliani *et al*. 2018, 2021).

Wind distribution can be modelled by other distribution e.g., Rayleigh, gamma, log normal, normal distributions (Zhou *et al*. 2010). These distributions could model wind speed well if wind distribution is positively skewed. Hence these distributions are not suitable for negatively skewed wind speed data. This is one of the reasons that people use combinations of these distribution to model wind speed data, e.g., normal-normal, gamma-normal, etc. Weibull distribution is suitable for modelling positively as well as negatively skewed wind speed data. The Weibull distribution does not give better results if wind speed data is bimodal. Therefore, Weibull distribution is used in combination with Weibull, normal, or gamma distributions.

2.1 Probability density function

Weibull distribution is the most frequently used Probability Density Function (abbreviated PDF) for modeling and predicting wind energy (Khan *et al*. 2015, Mahmood *et al*. 2020). The mathematical form of PDF is

$$f(v) = \frac{v}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}} \tag{1}$$

v is the wind speed, k is the dimensionless shape parameter, and c is the scale parameter, and it has the same dimension as wind speed (Chang 2011). A unique characteristic of Weibull distribution is the utilization of its parameter for different heights is they are known at a specific height.

2.2 Cumulative distribution function

The cumulative distribution function (CDF) is obtained by integrating the Weibull probability density function; hence, it is under Weibull pdf. The CDF represented by F(v) is expressed as (Saeed *et al*. 2019, Khahro *et al*. 2014).

$$F(v) = 1 - e^{-\left(\frac{v}{c}\right)^{k}} \tag{2}$$

3. Methods for calculation of Weibull parameters

Up till now, many different methods have been proposed for the calculation of parameters of Weibull distribution. In this paper, we used five methods of Maximum likelihood method (MLM), the Modified Maximum likelihood Method (MMLM), the Method of Moment (MOM), Energy Pattern factor Method (EPF), Empirical Method (EM) (Wadi and Elmasry 2021, Rocha *et al*. 2012).

3.1 Maximum likelihood method (*MLM*)

The Maximum likelihood method is one of the many statistical or parameter estimation techniques to determine parameters of a probability distribution (Akdağ and Dinler 2009, Chang 2011). The Maximum likelihood method has many sample properties that make approximation converges to the accurate values. Here we applied the MLM to calculate k and c (Usta 2016, Mohammadi *et al*. 2016, Salahaddin 2013, Khan *et al*. 2015, Arslan *et al*. 2014, Sumair *et al*. 2020). The equation can calculate the shape parameter k

$$k = \left[\frac{\sum_{i=1}^{n} v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} \ln(v_i)}{n}\right]^{-1} \tag{3}$$

Zero wind speeds are removed from the data as the logarithmic function is in the formula (Khan *et al*. 2015). Once k is determined, c can be calculated by the particular equation

$$c = \left(\frac{\sum_{i=1}^{n} v_i^k}{n}\right)^{1/k} \tag{4}$$

3.2 Modified Maximum likelihood Method (*MMLM*).

In both MLM and MMLM methods, the calculation of parameters k and c, the observed wind speed data must be converted into a frequency distribution. The value of '*k*' is found by numerical iteration. The Weibull parameters can be computed by following equations respectively (Chang 2011, Rocha *et al*. 2012, Teyabeen *et al*. 2018)

$$k = \left[\frac{\sum_{i=1}^{n} v_{\mathrm{i}}^{k} \ln(v_{\mathrm{i}}) f(v_{\mathrm{i}})}{\sum_{\mathrm{i=0}}^{\mathrm{n}} v_{\mathrm{i}}^{\mathrm{k}} f(v_{\mathrm{i}})} - \frac{\sum_{i=1}^{n} \ln(v_{\mathrm{i}}) f(v_{\mathrm{i}})}{f(v \geq 0)}\right]^{-1} \tag{5}$$

$$c = \left[\frac{1}{f(v \geq 0)} \sum_{i=1}^{n} v_i^k f(v_i)\right]^{\frac{1}{k}} \tag{6}$$

here $f(v_i)$ is the frequency of wind speed v_i i[th] bin and $f(v \geq 0)$ is the probability for all $v \geq 0$.

3.3 Moment method

The method of moment is historically one of the oldest methods. It is an alternative to the MLM. This method uses the second moment about the mean and the first moments about the origin to compute the Weibull parameters '*k*' and 'c'. The given equations calculate the Weibull parameters '*k*' and '*c*': (Chang 2011, Justus and Mikhail 1976, Sukkiramathi *et al*. 2020)

$$\bar{v} = c\Gamma\left(1 + \frac{1}{k}\right) \tag{7}$$

$$\sigma = c\left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right]^{\frac{1}{2}} \tag{8}$$

Where Γ is the gamma function.

3.4 Energy Pattern factor Method (*EPF*)

This method is easy and relies on averages of wind speed and its cube. First, the ratio of the average cube of wind speed $(\overline{v^3})$ and cube of average wind speed $(\bar{v}^3)$ is calculated, it is called energy pattern factor (E_{pf}) (Khahro *et al*. 2014, Mohammadi *et al*. 2016).

$$E_{pf} = \frac{\overline{v^3}}{\bar{v}^3} \tag{9}$$

Then the Weibull parameters (*k* and *c*) can be calculated as

$$k = 1 + \frac{3.69}{{E_{pf}}^2} \tag{10}$$

$$c = \frac{\bar{v}}{\Gamma\left(1 + \frac{1}{k}\right)} \tag{11}$$

3.5 Empirical method

In 1977, this method was given by Justus and Mikhail (1976). He calculated k and c by using standard deviation (σ) and average wind speed $(\overline{v})$. Weibull parameters can be calculated respectively by the following relation (Rocha *et al*. 2012, Bilir *et al*. 2015, Tizgui *et al*. 2017, Ullah 2013)

$$k = \left(\frac{\sigma}{\overline{v}}\right)^{-1.086} \tag{12}$$

$$c = \frac{\overline{v}}{\Gamma\left(1 + \frac{1}{k}\right)} \tag{13}$$

4. Statistical error analysis

The best-fitting distributions are calculated in this section using wind speed data from five separate Pakistan stations, namely Karachi, Hyderabad, Lahore, Quetta, and Peshawar (see Fig. 2). The wind speed measurements in 2016 were reported on a ten minutes' basis. These stations' coordinates and elevations are listed below. Table 1 shows the results (Seguro and Lambert 2000).

Table 1 Longitude and latitude for 5 cities

City	Longitude	Latitude	Elevation
Karachi	66.990501	24.860966	10 m
Hyderabad	68.3578°	25..960°	13 m
Lahore	74.3578°	31.5204°	217 m
Quetta	66.9750°	30.1798°	1679 m
Peshawar	71.5249°	34.0151°	331 m

Fig. 2 The map showing five cities of Pakistan whose wind speed distribution are studied

Every single metric only projects one series of model errors and, as a result, only illustrates one feature of the error characteristics. Consequently, model efficiency is often measured using various error statistics, including but not limited to MSEs and MAPE (Chai and Draxler 2014). The following tests are used to evaluate the efficacy of the methods described above: MSE, MABE, MAPE, and χ2. Their formulas are listed below.

$$\boldsymbol{RMSE} = \sqrt{\frac{\sum_1^n (\boldsymbol{W_i} - \boldsymbol{P_i})^2}{\boldsymbol{n}}} \tag{14}$$

$$\boldsymbol{MABE} = \frac{|\sum_1^n (\boldsymbol{W_i} - \boldsymbol{P_i})|}{\boldsymbol{n}} \tag{15}$$

$$\boldsymbol{MAPE} =: \frac{|}{\boldsymbol{n}} \left| \sum_1^n \frac{\boldsymbol{P_i} - \boldsymbol{W_i}}{\boldsymbol{P_i}} \right| \tag{16}$$

$$\chi 2 = \frac{\boldsymbol{1}}{\boldsymbol{n}} \sum_1^n \frac{(|\boldsymbol{W_i} - \boldsymbol{P_i}|)^2}{\boldsymbol{P_i}} \tag{17}$$

Where *P* is the relative probability, *W* is Weibull calculated probability is the class-boundary, *n* is the no. of probability class w.r.t class-boundary (B) (Akgül *et al*. 2016, Voinov *et al*. 2016). In addition to the above error criterion, Akaike Information Criterion (AIK) is calculated to decide the method selection for calculating Weibull parameters. Akaike’s information criterion (AIC) compares the relative quality of statistical models for a given data set. It allocates a single score to each model; based on these scores; the best model selection is carried out for a given dataset. The AIC selects the best model and does not give any information about its quality, it means it selects the best models from a set of bad models. The basic formula to calculate AIC depends on log-likelihood and is given by.

$$AIC = -2(log - likelihood) + 2K$$

Where *K* is the number of parameters.

Next, Tables 2-6 are the statistical analysis results and the errors estimated through *k* and *c* obtained from MMLM. Wind power in the wind is proportional to the cube of wind speed and swept area of turbine and expressed as

$$WP = \frac{1}{2} \rho A c_P v^3 \tag{18}$$

here, C_P = maximum power coefficient, A = Rotor swept area (m^2)

The wind speed data was recorded at the height of 10 m to calculate wind speed at the hub height of 50 m; the following formula was used to calculate wind speed at hub height. This wind speed is then used to calculate wind power.

$$\frac{v_1}{v_2} = \left(\frac{h_1}{h_2}\right)^{\alpha}$$

The Hellmann exponent is found to be 0.16 (Gugliani *et al*. 2021).

5. Results and discussion

In this study, the wind distribution of five different cities of Pakistan has been modeled using Weibull two parameters distribution. In addition, fitting this distribution wind potential of each of the cities has been evaluated. We present a formula that approximates the gamma value of (1+1/k), used to calculate scale parameter (c). The formula is given below.

$$\Gamma\left(1+\frac{1}{k}\right) = 0.088\left(1-\exp\left(-0.32(k-2)\right)\right)^2 + 0.885 \tag{19}$$

The values of $\Gamma\left(1+\frac{1}{k}\right)$ are calculated by gamma function and the function given above, the difference between values calculated by known gamma function, and values calculated by the new formula. Fig. 3 provides a comparison with scale parameters calculated by gamma function and new formula. The difference between them is less than 0.2%, a good agreement is seen between them.

The iteration method is employed to calculate Weibull parameters by MLM, MMLM, and MoM methods. In the case of MLM and MMLM, mostly the Newton-Raphson method is used for the estimation. This study used a different algorithmic approach to estimate the value of the shape parameter 'k'. It is known that the value of 'k' is always greater than 1.5 and less than the maximum wind speed in the distribution. This fact was used in the algorithm, and the value of k using formulae in the methods mentioned above was calculated with a step of 0.001, and absolute values of ordinates are recorded, the value of abscissa corresponds to a minimum value of ordinate e.g.

$$k - \left[\frac{\sum_{i=1}^{n} f_i v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} f_i \ln(v_i)}{\sum_{i=1}^{n} f_i}\right]^{-1} = 0$$

gives the estimation of 'k'. Figs. 4(a)-4(e) shows the histogram drawn by the recorded values of wind distribution and the Weibull curves drawn by the pdf values calculated by the five methods above. In all figs., All Weibull curves almost overlap (except a few) and are true representatives of recorded wind speed distribution. The magnitude of statistical error RMSE, MABE, MAPE, and

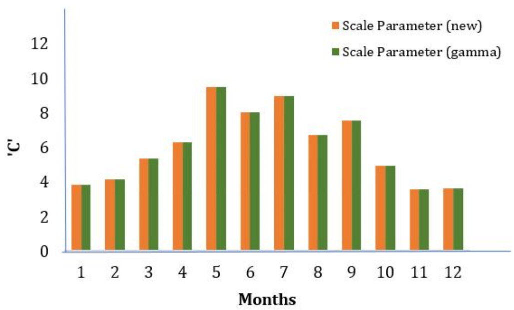


Fig. 3 Comparison of values of $\Gamma\left(1+\frac{1}{k}\right)$ Calculated by gamma function and new formula

$\chi 2$, are very comparable. Five methods calculate the values of Weibull parameters; the calculation error is least for the modified maximum likelihood method; therefore, in Tables 2-6, the values of parameters calculated by MLM are given. Tables 2-6 shows the statistical results of the calculation for five cities. Each table has four columns; the first column shows months of the year, the second column shows values of Weibull parameters, it has two sub-columns, which show the values of 'k'

Table 2 Weibull parameters, statistical errors, and power density for Karachi

Month	Parameters		Statistical errors					Power density
	K	C	RMSE	MABE	MAPE	chi-square	AIC	Watt/m^2
January	2.164	2.791	0.318	0.233	0.559	0.147	13577.215	26.152
February	2.001	3.272	0.309	0.201	0.899	0.137	14431.023	42.231
March	2.766	4.39	0.449	0.358	1.267	0.273	16654.292	103.297
April	3.807	5.135	0.339	0.204	0.895	0.144	14735.691	173.200
May	4.361	5.965	0.324	0.218	0.817	0.152	15800.728	277.815
June	3.239	6.144	0.393	0.298	1.002	0.229	18094.554	289.193
July	4.88	5.702	0.346	0.255	0.818	0.172	14562.032	247.582
August	4.578	5.106	0.271	0.173	0.663	0.107	14413.826	175.774
September	5.132	5.706	0.32	0.21	0.836	0.138	13592.343	250.200
October	2.866	3.892	0.417	0.324	0.899	0.233	15489.822	72.299
November	2.218	2.728	0.282	0.21	1.172	0.126	12888.637	24.428
December	2.127	2.611	0.36	0.278	0.705	0.179	13141.671	21.429

Table 3 Weibull parameters, statistical errors, and power density for Hyderabad

Month	Parameters		Statistical errors					Power density
	K	C	RMSE	MABE	MAPE	chi-square	AIC	Watt/m^2
January	2.578	3.827	0.413	0.311	0.728	0.229	15904.201	67.958
February	2.276	4.158	0.35	0.245	0.714	0.178	15805.492	86.525
March	2.39	5.349	0.397	0.282	0.807	0.224	19040.667	184.623
April	2.746	6.29	0.461	0.348	0.84	0.288	19178.074	303.768
May	4.177	9.491	0.397	0.259	0.806	0.217	20908.904	1110.932
June	3.079	8.029	0.384	0.276	0.808	0.229	20411.488	640.731
July	5.45	8.98	0.371	0.246	0.795	0.189	17810.394	985.395
August	2.566	6.724	0.532	0.447	0.813	0.383	20712.488	368.595
September	4.339	7.57	0.45	0.332	0.779	0.267	17548.332	567.390
October	3.219	4.916	0.343	0.232	0.775	0.168	16423.323	147.964
November	2.535	3.591	0.372	0.274	0.705	0.19	14437.189	56.097
December	2.366	3.615	0.315	0.226	0.671	0.152	15554.698	56.948

Table 4 Weibull parameters, statistical errors, and power density for Lahore

Month	Parameters		Statistical errors					Power density
	K	C	RMSE	MABE	MAPE	chi-square	AIC	Watt/m^2

January	2.296	2.833	0.289	0.163	0.587	0.107	12786.075	27.395
February	2.414	3.223	0.32	0.201	0.46	0.134	12555.078	40.414
March	2.414	3.223	0.32	0.201	0.46	0.134	16102.784	40.414
April	2.216	4.658	0.274	0.143	0.625	0.107	17344.649	121.582
May	2.262	4.791	0.294	0.15	0.719	0.117	18027.5	132.361
June	2.133	4.67	0.313	0.181	0.493	0.139	17396.374	122.537
July	2.104	4.239	0.289	0.147	0.62	0.111	16670.37	91.670
August	2.234	3.937	0.309	0.158	0.741	0.119	16089.248	73.423
September	2.573	3.059	0.23	0.114	0.576	0.07	12356.53	34.722
October	2.574	2.929	0.25	0.127	0.614	0.079	12497.466	30.461
November	2.395	3.151	0.333	0.202	0.473	0.14	12818.808	37.731
December	2.342	2.724	0.265	0.169	0.553	0.101	11018.513	24.363

Table 5 Weibull parameters, statistical errors, and power density for Quetta

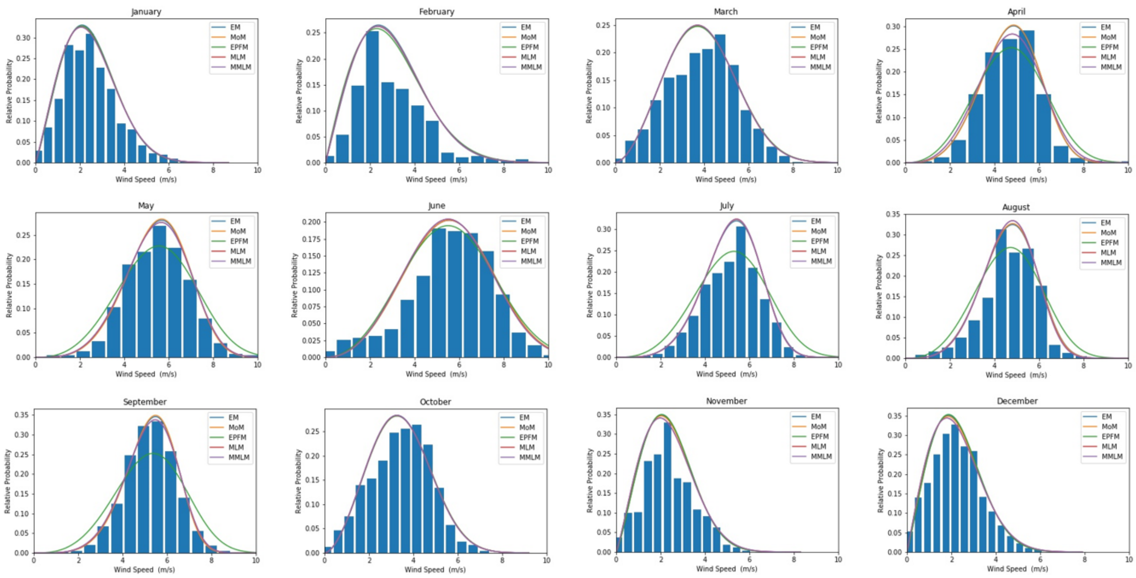
Month	Parameters		Statistical errors					Power density
	K	C	RMSE	MABE	MAPE	chi-square	AIC	Watt/m^2
January	2	3.392	0.282	0.183	0.681	0.121	15785.767	47.074
February	2.012	3.578	0.313	0.199	0.712	0.139	15124.844	55.188
March	2	4.232	0.394	0.29	0.765	0.228	18087.242	91.367
April	2	4.131	0.343	0.246	0.763	0.185	17231.747	84.981
May	2	4.552	0.355	0.252	0.82	0.198	19277.13	113.740
June	2.197	4.84	0.357	0.237	0.778	0.187	17937.656	136.460
July	2.392	5.414	0.487	0.366	0.842	0.305	19398.157	191.474
August	2.398	4.36	0.351	0.232	0.75	0.176	17398.337	100.021
September	2.244	3.648	0.307	0.215	0.683	0.147	15362.409	58.420
October	2	3.435	0.315	0.229	0.707	0.158	15425.207	48.880
November	2	3.109	0.332	0.232	0.694	0.159	13327.791	36.216
December	2	3.025	0.28	0.165	0.743	0.11	14353.419	33.373

and ‘c’. Column three shows statistical errors; this column has five sub-columns, RMSE, MABE, MAPE, and $\chi 2$, and AIC, respectively. The last column shows the values of wind power density available for the months January to December. The recorded data was time series data, but we used frequency distribution format for MLM and MMLM to calculate Weibull parameters.

Table 6 Weibull parameters, statistical errors, and power density for Peshawar

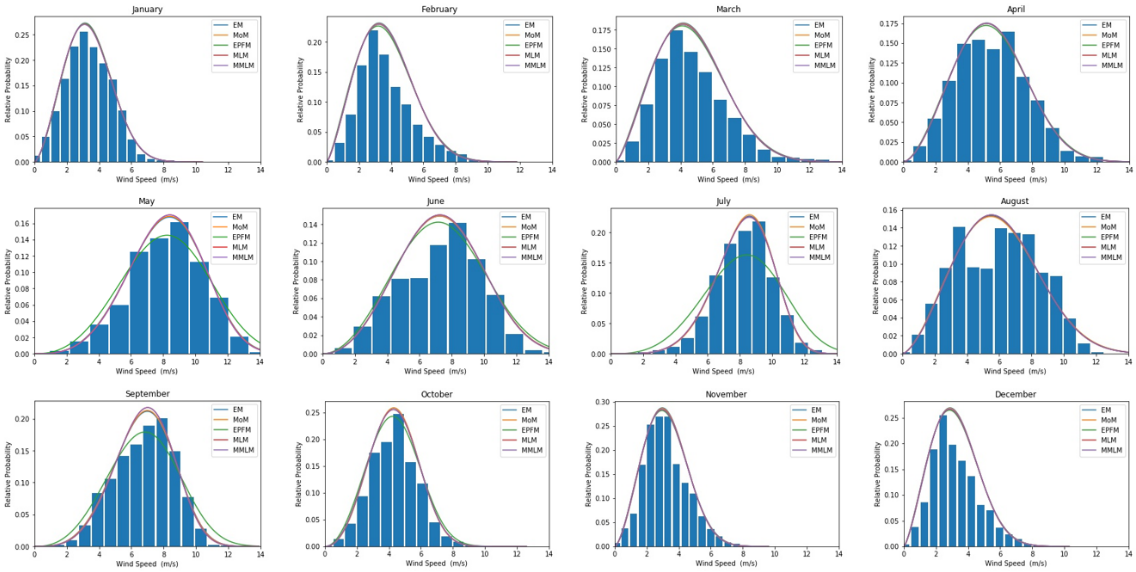
Month	Parameters		Statistical errors					Power density
	K	C	RMSE	MABE	MAPE	chi-square	AIC	Watt/m^2

January	2.258	2.16	0.266	0.156	0.576	0.093	10889.553	12.138
February	2.665	2.239	0.218	0.133	0.499	0.066	9813.352	13.658
March	2	2.286	0.283	0.162	0.689	0.101	11374.029	14.410
April	2	2.476	0.229	0.111	0.618	0.066	12236.306	18.309
May	2.09	2.839	0.229	0.109	0.685	0.068	13786.372	27.533
June	2.456	3.451	0.256	0.108	0.858	0.075	15530.028	49.677
July	2.421	2.851	0.264	0.162	0.682	0.098	13044.336	27.990
August	2.165	2.669	0.22	0.105	0.703	0.063	13142.723	22.882
September	2.477	2.748	0.291	0.151	0.808	0.101	12828.548	25.099
October	2.464	1.883	0.17	0.091	0.531	0.04	9269.716	8.074
November	2.127	1.759	0.259	0.196	0.922	0.107	8731.796	6.554
December	2.251	1.829	0.34	0.288	0.618	0.16	9442.524	7.368

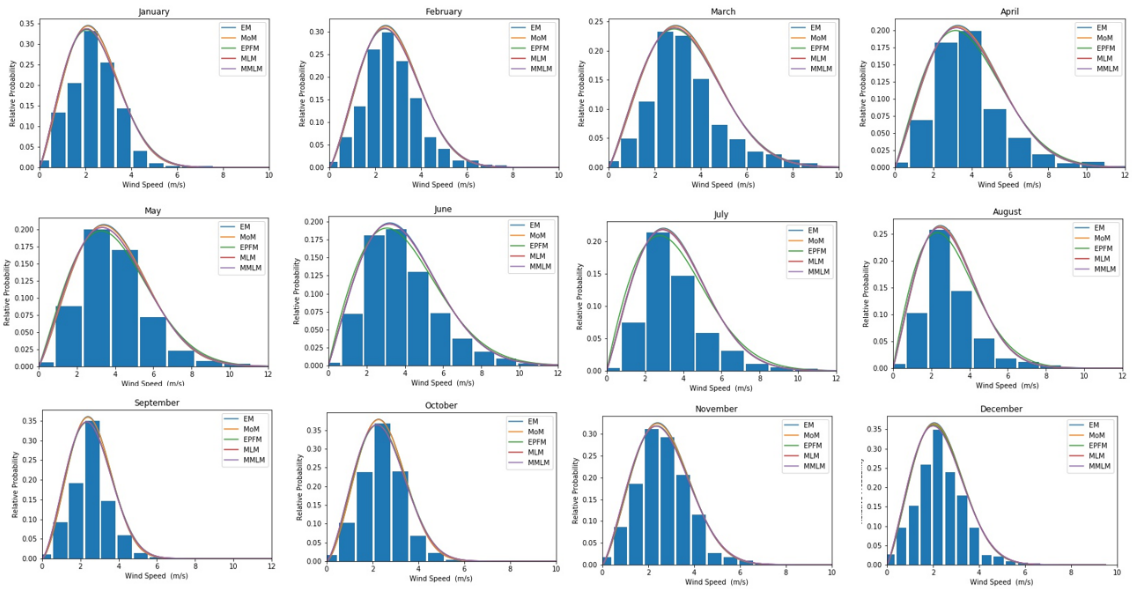


(a) Month-wise wind distribution and fitted Weibull distribution curves for Karachi

Fig. 4 Month-wise wind distribution and fitted Weibull distribution curves

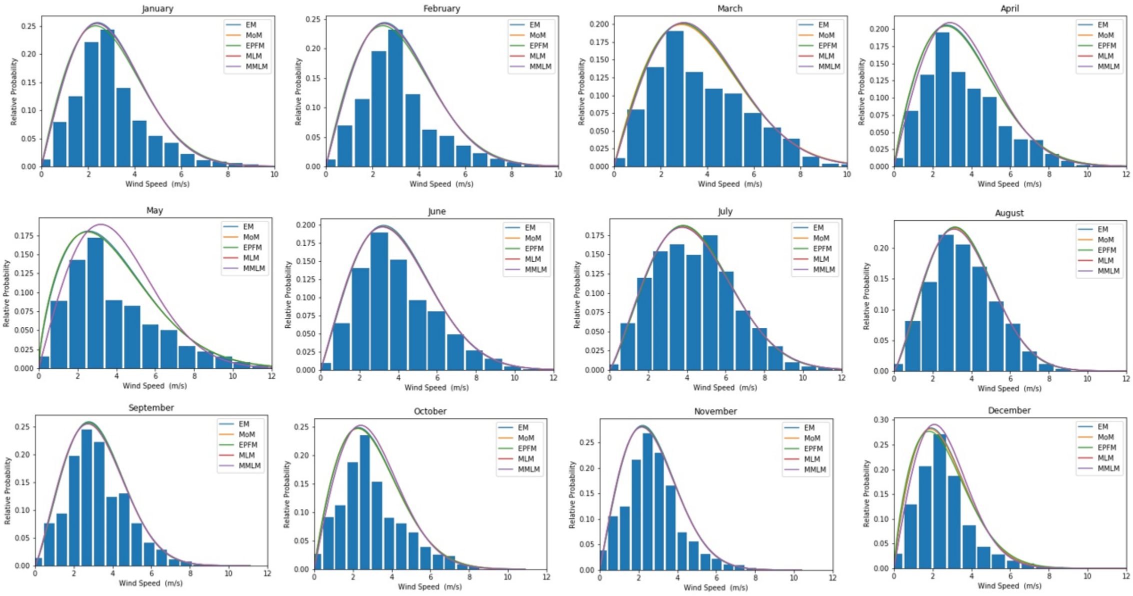


(b) Month-wise wind distribution and fitted Weibull distribution curves for Hyderabad

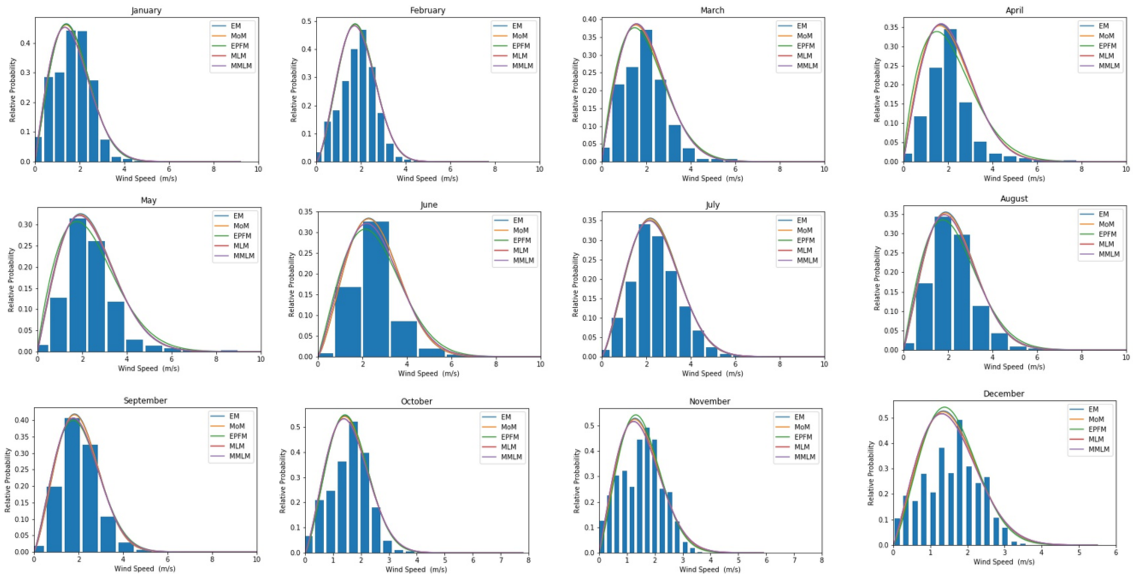


(c) Month-wise wind distribution and fitted Weibull distribution curves for Lahore

Fig. 4 Continued



(d) Month-wise wind distribution and fitted Weibull distribution curves for Quetta



(e) Month-wise wind distribution and fitted Weibull distribution curves for Peshawar

Fig. 4 Continued

6. Conclusions

The wind distribution of five different cities of Pakistan has been modeled using Weibull two parameters distribution. In conclusion, we have the following points to mention:

- Five different statistical methods are employed to calculate the Weibull parameters of the distribution.
- A new formula is suggested for the calculation of gamma function in the expression of the scale parameter. The new formula works well; the percentage error between values of scale parameters calculated from gamma and the new formula is less than 0.2%. Fig. 3 gives a comparison between values calculated by the new formula and gamma function.
- The Weibull distribution calculated by five methods represents the histogram drawn by recorded wind speeds for five cities. The best results are obtained for MLM; table 2-6 shows '*k*' and '*c*' values.
- A new and simple algorithm is used to calculate '*k*' by MLM, MMLM, MoM methods. We avoided the iteration method; instead, absolute values are calculated, and '*k*' is found by finding the lowest value of the function.
- Based on statistical errors given in Tables 2-6. we conclude that the Hyderabad region's wind potential compared to Karachi, Lahore, Quetta, and Peshawar is the greatest throughout the year from January to December. Karachi stands second for wind potential; the potential is highest from March-October. The ranking order according to corresponding wind potential for other cities is Quetta, Lahore, and Peshawar.
- Most wind turbines' cut-off speed is 3 or more (Adaramola *et al*. 2014, Abid *et al*. 2015); it is not feasible to install a wind turbine in Peshawar. Quetta and Lahore Also don't have appreciable wind potential except for April, May, June, and July. Hyderabad and Karachi are two possible sites where wind turbines can produce reasonable electricity. However, the best site among these five cities under study, Hyderabad, has the most significant available wind power.

References

Abid, M., Karimov, K.S., Wajid, H.A., Farooq, F., Ahmed, H. and Khan, O.H. (2015), "Design, development and testing of a combined Savonius and Darrieus vertical axis wind turbine", *Iran. J. Energy Environ*., **6**(1), 1-4.

Adaramola, M.S., Agelin-Chaab, M. and Paul, S.S. (2014), "Assessment of wind power generation along the coast of Ghana", *Energy Convers. Manag*., **77**, 61-69. https://doi.org/10.1016/j.enconman.2013.09.005

Akdağ, S.A. and Dinler, A. (2009), "A new method to estimate Weibull parameters for wind energy applications", *Energy Convers. Manag*., **50**(7), 1761-1766. https://doi.org/10.1016/j.enconman.2009.03.020

Akgül, F.G., Şenoğlu, B. and Arslan, T. (2016), "An alternative distribution to Weibull for modeling the wind speed data: Inverse Weibull distribution", *Energy Convers. Manag*., **114**, 234-240. https://doi.org/10.1016/j.enconman.2016.02.026

Arik, I., Kantar, Y.M. and Usta, I. (2019), "The new odd-burr rayleigh distribution for wind speed characterization", *Wind Struct*., *Int. J*., **28**(6), 369-380. https://doi.org/10.12989/was.2019.28.6.369

Arslan, T., Bulut, Y.M. and Yavuz, A.A. (2014), "Comparative study of numerical methods for determining Weibull parameters for wind energy potential", *Renew. Sustain. Energy Rev*., **40**, 820-825.

https://doi.org/10.1016/j.rser.2014.08.009

Baloch, M.H., Kaloi, G.S. and Memon, Z.A. (2016), "Current scenario of the wind energy in Pakistan challenges and future perspectives: A case study", *Energy Reports*, **2**, 201-210. https://doi.org/10.1016/j.egyr.2016.08.002

Bilir, L., Imir, M., Devrim, Y. and Albostan, A. (2015), "Seasonal and yearly wind speed distribution and wind power density analysis based on Weibull distribution function", *Int. J. Hydrogen Energy*, **40**(44), 15301-15310. https://doi.org/10.1016/j.ijhydene.2015.04.140

Carta, J.A., Ramirez, P. and Velazquez, S. (2009), "A review of wind speed probability distributions used in wind energy analysis: Case studies in the Canary Islands", *Renew. Sustain. Energy Rev.*, **13**(5), 933-955. https://doi.org/10.1016/j.rser.2008.05.005

Chai, T. and Draxler, R.R. (2014), "Root mean square error (RMSE) or mean absolute error (MAE)-Arguments against avoiding RMSE in the literature", *Geosci. Model Develop.*, **7**(3), 1247-1250. https://doi.org/10.5194/gmd-7-1247-2014

Chang, T.P. (2011), "Estimation of wind energy potential using different probability density functions", *Appl. Energy*, **88**(5), 1848-1856. https://doi.org/10.1016/j.apenergy.2010.11.010

Deep, S., Sarkar, A., Ghawat, M. and Rajak, M.K. (2020), "Estimation of the wind energy potential for coastal locations in India using the Weibull model", *Renew. Energy*, **161**, 319-339. https://doi.org/10.1016/j.renene.2020.07.054

Gugliani, G.K., Sarkar, A., Ley, C. and Mandal, S. (2018), "New methods to assess wind resources in terms of wind speed, load, power and direction", *Renew. Energy*, **129**, 168-182. https://doi.org/10.1016/j.renene.2018.05.088

Gugliani, G.K., Sarkar, A., Ley, C. and Matsagar, V. (2021), "Identification of optimum wind turbine parameters for varying wind climates using a novel month-based turbine performance index", *Renew. Energy*, **171**, 902-914. https://doi.org/10.1016/j.renene.2021.02.141

Justus, C.G. and Mikhail, A. (1976), "Height variation of wind speed and wind distributions statistics", *Geophys. Res. Lett.*, **3**(5), 261-264. https://doi.org/10.1029/GL003i005p00261

Kadhem, A.A., Abdul Wahab, N.I., Aris, I., Jasni, J. and Abdalla, A.N. (2017), "Advanced wind speed prediction model based on a combination of Weibull distribution and an artificial neural network", *Energies*, **10**(11), 1744. https://doi.org/10.3390/en10111744

Khahro, S.F., Tabbassum, K., Soomro, A.M., Liao, X., Alvi, M.B., Dong, L. and Manzoor, M.F. (2014), "Techno-economical evaluation of wind energy potential and analysis of power generation from wind at Gharo, Sindh Pakistan", *Renew. Sustain. Energy Rev.*, **35**, 460-474. https://doi.org/10.1016/j.rser.2014.04.027

Khan, J.K., Ahmed, F., Uddin, Z., Iqbal, S.T., Jilani, S.U., Siddiqui, A.A. and Aijaz, A. (2015), "Determination of Weibull parameter by four numerical methods and prediction of wind speed in Jiwani (Balochistan)", *J. Basic Appl. Sci.*, **11**, 62-68. https://doi.org/10.6000/1927-5129.2015.11.30

Khan, M.T.I., Ali, Q. and Ashfaq, M. (2018), "The nexus between greenhouse gas emission, electricity production, renewable energy and agriculture in Pakistan", *Renew. Energy*, **118**, 437-451. https://doi.org/10.1016/j.renene.2017.11.043

Lun, I.Y. and Lam, J.C. (2000), "A study of Weibull parameters using long-term wind observations", *Renew. Energy*, **20**(2), 145-153. https://doi.org/10.1016/S0960-1481(99)00103-2

Mahmood, F.H., Resen, A.K. and Khamees, A.B. (2020), "Wind characteristic analysis based on Weibull distribution of Al-Salman site, Iraq", *Energy Reports*, **6**, 79-87. https://doi.org/10.1016/j.egyr.2019.10.021

Mohammadi, K., Alavi, O., Mostafaeipour, A., Goudarzi, N. and Jalilvand, M. (2016), "Assessing different parameters estimation methods of Weibull distribution to compute wind power density", *Energy Convers. Manag.*, **108**, 323-335. https://doi.org/10.1016/j.enconman.2015.11.015

Mohiuddin, O., Mohiuddin, A., Obaidullah, M., Ahmed, H. and Asumadu-Sarkodie, S. (2016), "Electricity production potential and social benefits from rice husk, a case study in Pakistan", *Cogent Eng.*, **3**(1), 1177156. https://doi.org/10.1080/23311916.2016.1177156

Rehman, A. and Deyuan, Z. (2018), "Investigating the linkage between economic growth, electricity access, energy use, and population growth in Pakistan", *Appl. Sci.*, **8**(12), 2442. https://doi.org/10.3390/app8122442

Rocha, P.A.C., de Sousa, R.C., de Andrade, C.F. and da Silva, M.E.V. (2012), "Comparison of seven numerical methods for determining Weibull parameters for wind energy generation in the northeast region of Brazil", *Appl. Energy*, **89**(1), 395-400. https://doi.org/10.1016/j.apenergy.2011.08.003

Saeed, M.K., Salam, A., Rehman, A.U. and Saeed, M.A. (2019), "Comparison of six different methods of Weibull distribution for wind power assessment: A case study for a site in the Northern region of Pakistan", *Sustain. Energy Technol. Assess.*, **36**, 100541. https://doi.org/10.1016/j.seta.2019.100541

Safari, B. and Gasore, J. (2010), "A statistical investigation of wind characteristics and wind energy potential based on the Weibull and Rayleigh models in Rwanda", *Renew. Energy*, **35**(12), 2574-2880. https://doi.org/10.1016/j.renene.2010.04.032

Salahaddin, A.A. (2013), "Comparative study of four methods for estimating Weibull parameters for Halabja, Iraq", *Int. J. Phys. Sci.*, **8**(5), 186-192. https://doi.org/10.5897/IJPS12.697

Sarkar, A., Gugliani, G. and Deep, S. (2017), "Weibull model for wind speed data analysis of different locations in India", *KSCE J. Civil Eng.*, **21**(7), 2764-2776. https://doi.org/10.1007/s12205-017-0538-5

Seguro, J.V. and Lambert, T.W. (2000), "Modern estimation of the parameters of the Weibull wind speed distribution for wind energy analysis", *J. Wind Eng. Indust. Aerodyn.*, **85**(1), 75-84. https://doi.org/10.1016/S0167-6105(99)00122-1

Sukkiramathi, K., Rajkumar, R. and Seshaiah, C.V. (2020), "Mathematical representation to assess the wind resource by three parameter Weibull distribution", *Wind Struct., Int. J.*, **31**(5), 419-430. https://doi.org/10.12989/was.2020.31.5.419

Sumair, M., Aized, T., Gardezi, S.A.R., Bhutta, M.M.A., Rehman, S.M.S. and Ur Rehman, S.U. (2020), "Comparison of three probability distributions and techno-economic analysis of wind energy production along the coastal belt of Pakistan", *Energy Explorat. Exploitat.*, 0144598720931587. https://doi.org/10.1177/0144598720931587

Teyabeen, A.A., Akkari, F.R. and Jwaid, A.E. (2018), "Mathematical Modelling of Wind Turbine Power Curve", *Int. J. Simul. Syst. Sci. Technol.*, **19**(5), 1-13. https://doi.org/10.5013/IJSSST.a.19.05.15

Tizgui, I., El Guezar, F., Bouzahir, H. and Benaid, B. (2017), "Comparison of methods in estimating Weibull parameters for wind energy applications", *Int. J. Energy Sector Manag.* https://doi.org/10.1108/IJESM-06-2017-0002

Ullah, K. (2013), "Electricity infrastructure in Pakistan: an overview", *Int. J. Energy Inform. Commun.*, 4(3), 11-26.

Usta, I. (2016), "An innovative estimation method regarding Weibull parameters for wind energy applications", *Energy*, **106**, 301-314. https://doi.org/10.1016/j.energy.2016.03.068

Voinov, V., Pya, N., Makarov, R. and Voinov, Y. (2016), "New invariant and consistent chi-squared type goodness-of-fit tests for multivariate normality and a related comparative simulation study", *Commun. Statist. – Theory and Methods*, **45**(11), 3249-3263. https://doi.org/10.1080/03610926.2014.901370

Wadi, M. and Elmasry, W. (2021), "Statistical analysis of wind energy potential using different estimation methods for Weibull parameters: a case study", *Electr. Eng.*, **103**, 2573-2594. https://doi.org/10.1007/s00202-021-01254-0

Wakeel, M., Chen, B. and Jahangir, S. (2016), "Overview of energy portfolio in Pakistan", Energy Procedia, **88**, 71-75. https://doi.org/10.1016/j.egypro.2016.06.024

Zameer, H. and Wang, Y. (2018) "Energy production system optimization: Evidence from Pakistan", *Renew. Sustain. Energy Rev.*, **82**, 886-893. https://doi.org/10.1016/j.rser.2017.09.089

Zhou, J., Erdem, E., Li, G. and Shi, J. (2010), "Comprehensive evaluation of wind speed distribution models: A case study for North Dakota sites", *Energy Convers. Manag.*, **51**(7), 1449-1458. https://doi.org/10.1016/j.enconman.2010.01.020

CC

Nomenclature

k	shape parameter
c	scale parameter
v	wind speed (m/s)
$\bar{v}$	mean value of wind speed
MLM	Maximum Likelihood method
MMLM	Modified Maximum Likelihood Method
EPF	Energy Pattern Factor
EM	Empirical Method
MOM	Method of Moment
MABE	Mean Absolute Bias Error
MAPE	Mean Absolute Percentage Error
RMSE	Root Mean Square Error
X^2	Chi-square

Greek letters

Γ	gamma function
σ	standard deviation of wind speed