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# Effect of thermal laser pulse in transversely isotropic Magneto-thermoelastic solid due to Time-Harmonic sources

Parveen Lata[1a], Iqbal Kaur*[1] and Kulvinder Singh[2b]

[1]*Department of Basic and Applied Sciences, Punjabi University, Patiala, Punjab, India*
[2]*Kurukshetra University Kurukshetra, Haryana, India*

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**Abstract.** The present research deals with the time-harmonic deformation in transversely isotropic magneto thermoelastic solid with two temperature (2T), rotation due to inclined load and laser pulse. Generalized theory of thermoelasticity has been formulated for this mathematical model. The entire thermo-elastic medium is rotating with uniform angular velocity and subjected to thermally insulated and isothermal boundaries. The inclined load is supposed to be a linear combination of a normal load and a tangential load. The Fourier transform techniques have been used to find the solution to the problem. The displacement components, stress components, and conductive temperature distribution with the horizontal distance are computed in the transformed domain and further calculated in the physical domain using numerical inversion techniques. The effect of angle of inclination of normal and tangential load for Green Lindsay Model and time-harmonic source for Lord Shulman model is depicted graphically on the resulting quantities.

**Keywords:** thermal laser pulse; time-harmonic sources; transversely isotropic thermoelastic; rotation; inclined load; Magneto thermoelastic solid; generalized thermoelasticity

## 1. Introduction

When sudden heat/external force is applied in a solid body, it transmits time-harmonic wave by thermal expansion. Laser technology has dynamic applications in testing and analysis of materials. When a solid is exposed with a laser pulse, it absorbs some energy which results in thermal deformation and generates ultrasonic waves in the material. Waves are generated with two different mechanisms depending on the energy density exposer by the laser pulse. If thin metal is exposed with the high energy density, the surface layer of the solid material melts and particles fly off the surface which give rise to forces that generate ultrasonic waves. However, at low energy density, the surface material does not melt, but it enlarges at a high rate and wave and wave motion is generated due to thermoelastic processes. The change at some point of the medium is beneficial to detect the deformed field near mining shocks, seismic and volcanic sources, thermal power plants, high-energy particle accelerators, and many emerging technologies. The study of a time-harmonic source is one of the broad and dynamic areas of continuum dynamics.

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*Corresponding author, Ph.D., E-mail: bawahanda@gmail.com
[a]Associate Professor, E-mail: parveenlata@pbi.ac.in
[b]Assistant Professor, E-mail: ksingh2015@kuk.ac.in


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Sharma *et al.* (2015) investigated the 2-D deception in a transversely isotropic homogeneous thermoelastic solids in presence of two temperatures due to GN-II theory with an inclined load (linear combination of normal load and tangential load). Kumar *et al.* (2016a) investigated the impact of Hall current in a transversely isotropic magnetothermoelastic in presence and absence of energy dissipation due to the normal force. Kumar *et al.* (2016b) studied the conflicts caused by thermomechanical sources in a transversely isotropic rotating homogeneous thermoelastic medium with magnetic effect as well as two temperature and applied to the thermoelasticity Green-Naghdi theories with and without energy dissipation using thermomechanical sources. Lata *et al.* (2016a) studied two temperature and rotation aspect for GN-II and GN-III theory of thermoelasticity in a homogeneous transversely isotropic magnetothermoelastic medium for the case of the plane wave propagation and reflection. Ezzat *et al.* (2017) proposed a mathematical model of electro-thermoelasticity for heat conduction with a memory-dependent derivative. Lata (2018) studied the impact of energy dissipation on plane waves in sandwiched layered thermoelastic medium of uniform thickness, with two temperature, rotation, and Hall current in the context of GN Type-II and Type-III theory of thermoelasticity. Ezzat and El-Bary (2017) had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space of based on a memory-dependent derivative.

Ezzat and El-Bary (2017) had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space of based on the memory-dependent derivative. Othman *et al.* (2019) discussed the deformation in rotating infinite micro stretch generalized thermoelastic medium. Despite of this several researchers worked on different theory of thermoelasticity as Marin (1994), Abbas and Youssef (2009, 2012), Mohamed *et al.* (2009), Marin (2010), Marin *et al.* (2013), Abd-Alla and Mahmoud (2011), Bouderba *et al.* (2013), Atwa (2014), Marin and Nicaise (2016), Allam *et al.* (2018), Lal *et al*. (2017), Zenkour *et al.* (2018), Mahmoud (2012), Kumar *et al.* (2016c), Marin *et al.* (2016, 2017a), Bhatti *et al*. (2019), Chauthale *et al*. (2017), Farhan and Khder (2019), Hassan *et al.* (2018), Marin *et al*. (2019), Lata and Kaur (2019a, 2019b, 2019c), Kaur and Lata (2019a, 2019b).

Irrespective of these, not much work has been carried out in magneto-thermoelastic transversely isotropic solid with rotation, a time-harmonic source for inclined load with two temperature in generalized thermoelasticity. In this paper, we have attempted to study the deformation in transversely isotropic magneto thermoelastic solid with the combined effects of rotation for inclined load with two temperature by considering the disturbances harmonically time-dependent. The expressions of displacement components, conductive temperature and stress components due to time-harmonic sources are calculated in the transformed domain by using the Fourier transformation. Numerical inversion technique is used to find the resulting quantities in the physical domain and effects of frequency at different values have been represented graphically.

## 2. Basic equations

For a considered transversely isotropic thermoelastic medium, following Kumar *et al.* (2015) the constitutive equation is given by

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)T. \tag{1}$$

and equation of motion as described by Schoenberg and Censor (1973) for a uniformly rotating medium with angular velocity and Lorentz force which governs the dynamic displacement *u* is

$$\boldsymbol{t_{ij,j}} + \boldsymbol{F_i} = \boldsymbol{\rho}\{\ddot{\boldsymbol{u}}_i + (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{u})_i + (2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}})_i\}, \tag{2}$$

where $\Omega = \Omega\mathbf{n}$, $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation, The term $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{u})$ is the additional centripetal acceleration due to the time-varying motion only, and the term $2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}$ is the Coriolis acceleration.

$$F_i = \mu_0(\boldsymbol{j} \times \boldsymbol{H_0}).$$

The heat conduction equation following Kumar *et al.* (2015) is

$$K_{ij}\varphi_{,ij} + \rho\left(Q + \varepsilon\tau_0\dot{Q}\right) = \beta_{ij}T_0\left(\dot{e}_{ij} + \varepsilon\tau_0\ddot{e}_{ij}\right) + \rho C_E\left(\dot{T} + \tau_0\ddot{T}\right), \tag{3}$$

where

$$\beta_{ij} = C_{ijkl}\alpha_{ij}, \tag{4}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i,j = 1,2,3. \qquad T = \varphi - a_{ij}\varphi_{,ij} \tag{5}$$

$\beta_{ij} = \beta_i\delta_{ij}$, $K_{ij} = K_i\delta_{ij}$, i is not summed.

## 3. Formulation and solution of the problem

We consider a homogeneous transversely isotropic magnetothermoelastic medium, permeated by an initial magnetic field $\boldsymbol{H_0} = (0, H_0, 0)$ acting along $y$-axis. The rectangular Cartesian coordinate system $(x, y, z)$ having origin on the surface $(z = 0)$ with $z$-axis pointing vertically into the medium is introduced. The surface of the half-space is subjected to an inclined load acting at $z = 0$.

In addition, we consider that

$$\boldsymbol{\Omega} = (0, \Omega, 0).$$

From the generalized Ohm's law

$$J_2 = 0.$$

The density components $J_1$ and $J_3$ are given as

$$J_1 = -\varepsilon_0\mu_0 H_0\frac{\partial^2 w}{\partial t^2}, \tag{6}$$

$$J_3 = \varepsilon_0\mu_0 H_0\frac{\partial^2 u}{\partial t^2}. \tag{7}$$

In addition, the equations of displacement vector $(u, v, w)$ and conductive temperature $\varphi$ for transversely isotropic thermoelastic solid in presence of two temperature are

$$u \equiv u(x,z,t), v = 0, w \equiv w(x,z,t) \ and \ \varphi \equiv \varphi(x,z,t). \tag{8}$$

Now using the proper transformation on Eqs. (1)-(3) with the aid of (8), following Slaughter (2002) is as under

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{13}\frac{\partial^2 w}{\partial x\partial z} + C_{44}\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x\partial z}\right) - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial x}\left\{\varphi - \left(a_1\frac{\partial^2\varphi}{\partial x^2} + a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\} - \mu_0 J_3 H_0 = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right), \tag{9}$$

$$(C_{13}+C_{44})\frac{\partial^2 u}{\partial x\partial z}+C_{44}\frac{\partial^2 w}{\partial x^2}+C_{33}\frac{\partial^2 w}{\partial z^2}-\beta_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial z}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}$$
$$-\mu_0 J_1 H_0=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}\right), \tag{10}$$

$$K_1\frac{\partial^2\varphi}{\partial x^2}+K_3\frac{\partial^2\varphi}{\partial z^2}+\rho\left(Q+\varepsilon\,\tau_0\dot{Q}\right)=\rho C_E\left(\dot{T}+\tau_0\ddot{T}\right)$$
$$+T_0\frac{\partial}{\partial t}\left\{\beta_1\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x}+\beta_3\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}\right\}, \tag{11}$$

and

$$t_{11}=C_{11}e_{11}+C_{13}e_{13}-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{12}$$

$$t_{33}=C_{13}e_{11}+C_{33}e_{33}-\beta_3\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{13}$$

$$t_{13}=2C_{44}e_{13}, \tag{14}$$

where

$$T=\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right),$$
$$\beta_1=(C_{11}+C_{12})\alpha_1+C_{13}\alpha_3,$$
$$\beta_3=2C_{13}\alpha_1+C_{33}\alpha_3.$$

Where $\tau_0$ and $\tau_1$ are thermal relaxation times with $\tau_0\geq\tau_1\geq 0$

According to Marin *et al.* (2017), the surface of transversely isotropic thermoelastic solid is illuminated by laser pulse given by the heat input:

$$Q=I_0 f(t)g(x)h(z)$$

Where $I_0$ is the energy absorbed i.e., the laser intensity which is defined as the total energy carried by a laser pulse per unit area of the laser beam, $f(t)$ is a temporal profile given as

$$f(t)=\frac{t}{t_0^2}e^{-(t/t_0)}$$

Here $t_0=2ps$ is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in $x$

$$g(x)=\frac{1}{2\pi r^2}e^{-(x^2/r^2)}$$

Where $r$ is the beam radius and as a function of the depth, z, the heat deposition due to the laser pulse is assumed to decay exponentially within the solid:

$$h(z)=\gamma e^{-\gamma z}$$

Therefore

$$Q=\frac{\gamma I_0 t}{2\pi r^2 t_0^2}e^{-\left(\frac{x^2}{r^2}+\frac{t}{t_0}+\gamma z\right)}$$

We consider that the medium is initially at rest. Therefore, the preliminary and symmetry conditions are given by

$$u(x,z,0)=0=\dot{u}(x,z,0),$$
$$w(x,z,0)=0=\dot{w}(x,z,0),$$
$$\varphi(x,z,0)=0=\dot{\varphi}(x,z,0)\text{for } z\geq 0,-\infty<x<\infty,$$
$$u(x,z,t)=w(x,z,t)=\varphi(x,z,t)=0\ for\ t>0 \text{ when } z\to\infty.$$

Assuming the time-harmonic behavior as

$$(u,w,\varphi,Q)(x,z,t)=(u,w,\varphi,Q)(x,z)e^{i\omega t}. \tag{15}$$

To facilitate the solution, following dimensionless quantities are introduced

$$x'=\frac{x}{L},\ \ z'=\frac{z}{L},\ \ t'=\frac{c_1}{L}t,\ \ u'=\frac{\rho c_1^2}{L\beta_1 T_0}u,\ \ w'=\frac{\rho c_1^2}{L\beta_1 T_0}w, T'=\frac{T}{T_0}, t'_{11}=\frac{t_{11}}{\beta_1 T_0},$$
$$t'_{33}=\frac{t_{33}}{\beta_1 T_0}, t'_{31}=\frac{t_{31}}{\beta_1 T_0},\ \ \varphi'=\frac{\varphi}{T_0}, a'_1=\frac{a_1}{L^2}, a'_3=\frac{a_3}{L^2}, h'=\frac{h}{H_0}, \Omega'=\frac{\mathrm{L}}{C_1}\Omega, \tag{16}$$
$$Q'=\frac{1}{\varpi T_0 C_E}Q, \varpi=\frac{\rho C_E C_1^2}{K_1}, \tau'_0=\frac{c_1}{L}\tau_0, \tau'_1=\frac{c_1}{L}\tau_1, \rho C_1^2=C_{11}.$$

Making use of (16) in Eqs. (9)-(11) and after suppressing the primes using equation (15), yield

$$\frac{\partial^2 u}{\partial x^2}+\delta_4\frac{\partial^2 w}{\partial x\partial z}+\delta_2\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\partial z}\right)-(1+\tau_1 i\omega)\frac{\partial}{\partial x}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}$$
$$=\left(\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}+1\right)(-\omega^2 u)-\Omega^2 u+2\Omega i\omega w, \tag{17}$$

$$\delta_1\frac{\partial^2 u}{\partial x\partial z}+\delta_2\frac{\partial^2 w}{\partial x^2}+\delta_3\frac{\partial^2 w}{\partial z^2}-\frac{\beta_3}{\beta_1}(1+\tau_1 i\omega)\frac{\partial}{\partial z}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}$$
$$=\left(\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}+1\right)(-\omega^2 w)-\Omega^2 w+2\Omega i\omega u, \tag{18}$$

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{K_3}{K_1}\frac{\partial^2\varphi}{\partial z^2}+Q_0 f(x,t)e^{-\gamma z}=\delta_5 i\omega(1+\tau_0 i\omega)\left[\varphi-a_1\frac{\partial^2\varphi}{\partial x^2}-a_3\frac{\partial^2\varphi}{\partial z^2}\right]$$
$$+\delta_6 i\omega(1+\varepsilon\tau_0 i\omega)\left[\beta_1\frac{\partial u}{\partial x}+\beta_3\frac{\partial w}{\partial z}\right], \tag{19}$$

where

$$\delta_1=\frac{c_{13}+c_{44}}{c_{11}},\ \ \delta_2=\frac{c_{44}}{c_{11}},\ \ \delta_3=\frac{c_{33}}{c_{11}},\ \ \delta_4=\frac{c_{13}}{c_{11}},$$
$$\delta_5=\frac{\rho C_E C_1 L}{K_1},\ \ \delta_6=-\frac{T_0\beta_1 L}{\rho C_1 K_1}$$
$$Q_0=\frac{L^2\rho\varpi C_E}{K_1}\frac{\gamma I_0}{2\pi r^2 t_0^2},$$
$$f(x,t)=\frac{L}{c_1}t[1+\varepsilon\tau_0 i\omega]e^{-\left(\frac{x^2}{r^2}+\frac{L}{c_1}\frac{t}{t_0}+i\omega t\right)}.$$

Apply Fourier transforms defined by

$$\hat{f}(\xi,z,\omega)=\int_{-\infty}^{\infty} f(x,z,\omega)e^{i\xi x}\,dx \tag{20}$$

On Eqs. (17)-(19), we obtain a system of equations

$$[-\xi^2+\delta_2 D^2+\delta_7\omega^2+\Omega^2]\hat{u}(\xi,z,\omega)+[\delta_4 Di\xi+\delta_2 Di\xi-2\Omega i\omega]\widehat{w}(\xi,z,\omega)$$
$$+(-\mathrm{i}\xi)(1+\tau_1 i\omega)[1+a_1\xi^2-a_3 D^2]\hat{\varphi}(\xi,z,\omega)=0, \tag{21}$$

$$[\delta_1 Di\xi+2\Omega i\omega]\hat{u}(\xi,z,\omega)+[-\delta_2\xi^2+\delta_3 D^2+\delta_7\omega^2+\Omega^2]\widehat{w}(\xi,z,\omega)$$
$$-\frac{\beta_3}{\beta_1}(1+\tau_1 i\omega)D[1+a_1\xi^2-a_3 D^2]\hat{\varphi}(\xi,z,\omega)=0 \tag{22}$$

$$[-\delta_6\omega\delta_8\beta_1\xi]\hat{u}(\xi,z,\omega)+[\delta_6 i\omega\delta_8\beta_3 D]\hat{w}(\xi,z,\omega)$$
$$+\left[\xi^2-\frac{K_3}{K_1}D^2+\delta_5\delta_8 i\omega(1+a_1\xi^2-a_3D^2)\right]\hat{\varphi}(\xi,z,\omega)=Q_0 f(\xi,t)e^{-\gamma z} \quad (23)$$

where

$$\delta_7=\frac{\varepsilon_0\mu_0^2H_0^2}{\rho}+1, \qquad \delta_8=\left(1+\varepsilon\tau_0\frac{c_1}{L}i\omega\right),$$
$$f(\xi,\omega)=\left[\frac{L}{c_1}t[1+\varepsilon\tau_0 i\omega]\right]r\sqrt{\pi}e^{-\left(\frac{r^2\xi^2}{4}+\frac{\frac{L}{c_1}t}{t_0}\right)}.$$

By taking $\hat{Q}(\xi,z,s)=0$, i.e., no external heat is supplied the non-trivial solution of (21)-(23) yields

$$(AD^6+BD^4+CD^2+E)(\hat{u})=f_1(x,\gamma,t)e^{-\gamma z}$$
$$(AD^6+BD^4+CD^2+E)(\hat{w})=f_2(x,\gamma,t)e^{-\gamma z} \quad (24)$$
$$(AD^6+BD^4+CD^2+E)(\hat{\varphi})=f_3(x,\gamma,t)e^{-\gamma z}$$

where

$$\mathrm{A}=\delta_2\delta_3\zeta_7-\zeta_5\delta_2\frac{\beta_3}{\beta_1}a_3,$$
$$\mathrm{B}=\delta_3\zeta_1\zeta_7-a_3(1+\tau_1 i\omega)\zeta_1\zeta_5\frac{\beta_3}{\beta_1}+\delta_2\delta_3\zeta_6+\delta_2\zeta_7\zeta_3-\zeta_5\zeta_9\delta_2-\zeta_8\delta_1 i\xi\zeta_7+$$
$$\zeta_8\zeta_4\frac{\beta_3}{\beta_1}(1+\tau_1 i\omega)a_3-a_3(1+\tau_1 i\omega)\xi^2\zeta_5\delta_1-a_3(1+\tau_1 i\omega)\delta_3\zeta_4 i\xi,$$
$$\mathrm{C}=\delta_3\zeta_1\zeta_6+\zeta_1\zeta_3\zeta_7-\zeta_1\zeta_5\zeta_9+\delta_2\zeta_6\zeta_3+\zeta_4\zeta_8\zeta_9-\zeta_8\delta_1 i\xi\zeta_6-4\Omega^2\omega^2\zeta_7+\zeta_2\delta_1 i\xi\zeta_5-$$
$$\zeta_2\zeta_4\delta_3-a_3(1+\tau_1 i\omega)\zeta_4 i\xi\zeta_3,$$
$$E=\zeta_3\zeta_1\zeta_6-4\Omega^2\omega^2\zeta_6-\zeta_2\zeta_4\zeta_3,$$
$$\zeta_1=-\xi^2+\delta_7\omega^2+\Omega^2,$$
$$\zeta_2=-i\xi(1+\tau_1 i\omega)(1+a_1\xi^2),$$
$$\zeta_3=-\delta_2\xi^2+\delta_7\omega^2+\Omega^2,$$
$$\zeta_4=-\delta_6\delta_8\omega\beta_1\xi,$$
$$\zeta_5=\delta_6\delta_8 i\omega\beta_3,$$
$$\zeta_6=\xi^2+\delta_5\delta_8 i\omega(1+a_1\xi^2),$$
$$\zeta_7=-\frac{K_3}{K_1}-a_3\delta_5\delta_8 i\omega,$$
$$\zeta_8=\delta_1 i\xi,$$
$$\zeta_9=-(1+a_1\xi^2)(1+\tau_1 i\omega)\frac{\beta_{3.}}{\beta_1}$$
$$f_1(\xi,\gamma,t)=Q_0 f(\xi,t)\left[(\gamma\zeta_8+2\Omega i\omega)\left(\zeta_9\gamma+\frac{\beta_{3.}}{\beta_1}a_3\gamma^3\right)-(\zeta_2+a_3 i\xi\gamma^2)(\zeta_3+\delta_3\gamma^2)\right]$$
$$f_2(\xi,\gamma,t)=Q_0 f(\xi,t)\left[(\zeta_1+\delta_2\gamma^2)\left(\zeta_9\gamma+\frac{\beta_{3.}}{\beta_1}a_3\gamma^3\right)+(-\gamma\delta_1 i\xi+2\Omega i\omega)(\zeta_2+a_3 i\xi\gamma^2)\right]$$
$$f_3(\xi,\gamma,t)=Q_0 f(\xi,t)[(\zeta_1+\delta_2\gamma^2)(\zeta_3+\delta_3\gamma^2)+(-\gamma\delta_1 i\xi+2\Omega i\omega)(\gamma\zeta_8+2\Omega i\omega)]$$
$$f_4(\xi,\gamma,t)=A\gamma^6+B\gamma^4+C\gamma^2+E$$

The roots of the Eq. (24) are $\pm\lambda_j$, ($j$=1, 2, 3), the solution of the Eq. (24) is calculated by using the radiation condition of $\hat{u},\hat{v},\hat{w}$ and can be written as

$$\hat{u}(\xi,z,\omega)=\sum_{j=1}^{3}A_j e^{-\lambda_j z}+\frac{f_1}{f_4}e^{-\gamma z}, \quad (25)$$

$$\hat{w}(\xi,z,\omega) = \sum_{j=1}^{3} d_j A_j e^{-\lambda_j z} + \frac{f_2}{f_4} e^{-\gamma z} \tag{26}$$

$$\hat{\varphi}(\xi,z,\omega) = \sum_{j=1}^{3} l_j A_j e^{-\lambda_j z} + \frac{f_3}{f_4} e^{-\gamma z}, \tag{27}$$

where $A_j(\xi,\omega), j = 1,2,3$ being undetermined constants and $d_j$ and $l_j$ are given by

$$d_j = \frac{\delta_2\zeta_7\lambda_j^4 + (\zeta_7\zeta_1 - a_3\zeta_4 i\xi + \delta_2\zeta_6)\lambda_j^2 + \zeta_1\zeta_6 - \zeta_4\zeta_2}{\left(\delta_3\zeta_7 - \frac{\beta_3}{\beta_1}a_3\zeta_5\right)\lambda_j^4 + (\delta_3\zeta_6 + \zeta_3\zeta_7 - \zeta_5\zeta_9)\lambda_j^2 + \zeta_3\zeta_6},$$

$$l_j = \frac{\delta_2\delta_3\lambda_j^4 + (\delta_2\zeta_3 + \zeta_1\delta_3 - \delta_1\zeta_8 i\xi)\lambda_j^2 - 4\Omega^2\omega^2 + \zeta_3\zeta_1}{\left(\delta_3\zeta_7 - \frac{\beta_3}{\beta_1}a_3\zeta_5\right)\lambda_j^4 + (\delta_3\zeta_6 + \zeta_3\zeta_7 - \zeta_5\zeta_9)\lambda_j^2 + \zeta_3\zeta_6}$$

## 4. Boundary conditions

We consider a normal line load $F_1$ per unit length acting in the positive $z$-axis on the plane boundary $z=0$ along the $y$-axis and a tangential load $F_2$ per unit length, acting at the origin in the positive $x$-axis. The appropriate boundary conditions following Kumar and Ailawalia (2010) are

$$t_{33}(x,z,t) = -F_1\psi_1(x)e^{i\omega t}, \tag{28}$$

$$t_{31}(x,z,t) = -F_2\psi_2(x)e^{i\omega t}, \tag{29}$$

$$h_1\frac{\partial\varphi}{\partial z}(x,z,t) + h_2\varphi(x,z,t) = 0, \tag{30}$$

where $h_2 \to 0$ corresponds to insulated boundaries and $h_1 \to 0$ corresponds to isothermal boundaries. $F_1$ and $F_2$ are the magnitude of the forces applied, $\psi_1(x)$ *and* $\psi_2(x)$ specify the vertical and horizontal load distribution function along the x-axis.

Applying Fourier transform defined by (20) on the boundary conditions (28)-(30), (13)-(14) and with the help of Eqs. (25)-(27), we obtain the components of displacement, normal stress, tangential stress, and conductive temperature as

$$\hat{u} = \frac{F_1\hat{\psi}_1(\xi)}{\Lambda}\left[\sum_{j=1}^{3}\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{f_1}{f_4}e^{-\gamma z}, \tag{31}$$

$$\hat{w} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}d_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}d_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{f_2}{f_4}e^{-\gamma z}, \tag{32}$$

$$\hat{\varphi} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}l_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}l_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{f_3}{f_4}e^{-\gamma z}, \tag{33}$$

$$\widehat{t_{11}} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}S_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}S_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t} + g_1e^{-\gamma z}, \tag{34}$$

$$\widehat{t_{13}} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}N_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}N_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t} + g_2e^{-\gamma z}, \tag{35}$$

$$\widehat{t_{33}} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}M_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}M_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t} + g_3e^{-\gamma z}, \tag{36}$$

where

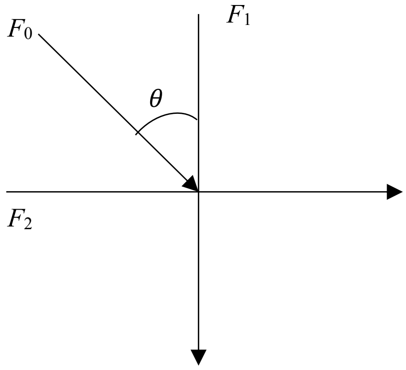


Fig. 1 Inclined load over a transversely isotropic magneto-thermoelastic solid

$$\Gamma_{11} = -N_2R_3 + R_2N_3,$$
$$\Gamma_{12} = N_1R_3 - R_1N_3,$$
$$\Gamma_{13} = -N_1R_2 + R_1N_2,$$
$$\Gamma_{21} = M_2R_3 - R_2M_3,$$
$$\Gamma_{22} = -M_1R_3 + R_1M_3,$$
$$\Gamma_{23} = M_1R_2 - R_1M_2,$$
$$\Gamma = -M_1\Gamma_{11} - M_2\Gamma_{12} - M_3\Gamma_{13},$$
$$N_j = -\delta_2\lambda_j + i\xi d_j,$$
$$M_j = i\xi - \delta_3 d_j\lambda_j - \frac{\beta_3}{\beta_1}(1+\tau_1 i\omega)l_j\left[(1+a_1\xi^2) - a_3\lambda_j^2\right],$$
$$R_j = -h_1\lambda_j l_j + h_2 l_j,$$
$$S_j = -i\xi - \delta_4 d_j\lambda_j - l_j(1+\tau_1 i\omega)\left[(1+a_1\xi^2) - a_3\lambda_j^2\right]$$
$$g_1 = i\xi\frac{f_1}{f_4} - \delta_4\gamma\frac{f_2}{f_4} - (1+a_1\xi^2 - a_3\gamma^2)\frac{f_3}{f_4}$$
$$g_2 = \delta_2\left(\gamma\frac{f_1}{f_4} + i\xi\frac{f_2}{f_4}\right)$$
$$g_3 = i\xi\frac{f_1}{f_4} - \delta_3\gamma\frac{f_2}{f_4} - \frac{\beta_3}{\beta_1}(1+a_1\xi^2 - a_3\gamma^2)\frac{f_3}{f_4}.$$

## 5. Special cases

### *5.1 Concentrated force*

The solution due to the concentrated normal force on the half-space is obtained by setting

$$\psi_1(x) = \delta(x), \psi_2(x) = \delta(x), \tag{37}$$

where $\delta(x)$ is Dirac delta function.

Applying Fourier transform defined by (20) on (37), we obtain

$$\hat{\psi}_1(\xi) = 1, \hat{\psi}_2(\xi) = 1. \tag{38}$$

Using (38) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

### *5.2 Inclined load*

Suppose an inclined load, $F_0$ per unit length is acting on the *y*-axis and its inclination with *z*-axis is $\theta$, we have (see Fig. 1)

$$F_1 = F_0 cos\theta \ and \ F_2 = F_0 sin\theta \tag{39}$$

Using Eqs. (38) and (39) in Eqs. (31)-(36), we obtain the expressions for displacements, and stresses and conductive temperature for concentrated force, on the surface of transversely isotropic magneto-thermoelastic body due to inclined load and Laser pulse.

## 6. Inversion of the transformation

For obtaining the result in the physical domain, invert the transforms in Eqs. (31)-(36) using

$$\tilde{f}(x,z,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\hat{f}(\xi,z,\omega)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |cos(\xi x)f_e - isin(\xi x)f_o|\, d\xi,$$

Where $f_o$ is odd and $f_e$ is the even parts of $\hat{f}(\xi,z,\omega)$ respectively (Honig 1984).

## 7. Particular cases

1. If $Q_0 = 0$ we obtain relations for displacement temperature distribution and stress for without laser pulse from Eqs. (31)-(36) in transversely isotropic magneto thermoelastic solid with two temperature (2*T*), rotation with time harmonic source due to inclined load and Laser pulse.
2. If $\varepsilon = 0$ we obtain relations for displacement temperature distribution and stress for two relaxation times (for Green Lindsay GL model) from Eqs. (31)-(36) in transversely isotropic magneto thermoelastic solid with two temperature (2*T*), rotation with time harmonic source due to inclined load and Laser pulse.
3. If $\tau_1 = 0$ and $\varepsilon = 1$ we obtain relations for displacement temperature distribution and stress for one relaxation times (for Lord Shulman model) from Eqs. (31)-(36) in transversely isotropic magneto thermoelastic solid with two temperature (2*T*), rotation with time harmonic source due to inclined load and Laser pulse.
4. If $\tau_1 = \tau_0 = 0$ we obtain relations for displacement temperature distribution and stress for coupled thermoelastic (CT) half space from Eqs. (31)-(36) in transversely isotropic magneto thermoelastic solid with two temperature (2*T*), rotation with time harmonic source due to inclined load and Laser pulse.
5. If $\theta = \frac{\pi}{2}$ we obtain relations for displacement temperature distribution and stress for tangential load thermoelastic half space from Eqs. (31)-(36) in transversely isotropic magneto thermoelastic solid with two temperature (2*T*), rotation with time harmonic source due to inclined load and Laser pulse.
6. If $\Omega = 0$ we obtain relations for displacement temperature distribution and stress from Eqs. (31)-(36) in transversely isotropic magneto thermoelastic solid with two temperature (2*T*), without rotation with time harmonic source due to inclined load and Laser pulse.
7. If $a_1 = a_3 = 0$ we obtain relations for displacement temperature distribution and stress from

Eqs. (31)-(36) in transversely isotropic magneto thermoelastic solid without two temperature (2*T*), with rotation with time harmonic source due to inclined load and Laser pulse.

## 8. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of inclined load and laser pulse with time harmonic source, we now present some numerical results. Following Dhaliwal and Singh (1980), cobalt material has been taken for thermoelastic material as

$$c_{11} = 3.07 \times 10^{11} Nm^{-2}, c_{33} = 3.581 \times 10^{11} Nm^{-2}, c_{13} = 1.027 \times 10^{10} Nm^{-2},$$
$$c_{44} = 1.510 \times 10^{11} Nm^{-2}, \beta_1 = 7.04 \times 10^{6} Nm^{-2} deg^{-1},\ \beta_3 = 6.90 \times 10^{6} Nm^{-2} deg^{-1},$$
$$\rho = 8.836 \times 10^{3} Kgm^{-3}, C_E = 4.27 \times 10^{2} jKg^{-1} deg^{-1},\ K_1 = 0.690 \times 10^{2} Wm^{-1} Kdeg^{-1},$$
$$K_3 = 0.690 \times 10^{2} Wm^{-1} K^{-1}, \mathrm{T}_0 = 298\,\mathrm{K}, \mathrm{H}_0 = 1\mathrm{Jm}^{-1}\mathrm{nb}^{-1},\ \varepsilon_0 = 8.838 \times 10^{-12}\mathrm{Fm}^{-1},$$
$$L = 1, \gamma = 0.01, \tau_0 = 0.02, \tau_1 = 0.01, r = 0.01, I_0 = 1 \times 10^{11} Jm^{-2}, a_1 = 0.02 \text{ and } a_3 = 0.04$$

Using the above values, the graphical representations of displacement component $u$, normal displacement $w$, conductive temperature $\varphi$, stress components $t_{11}$, $t_{13}$ and $t_{33}$ for transversely isotropic thermoelastic medium have been investigated and the effect of inclination with two temperature has been depicted.

i. The black solid line with square symbols corresponds to transversely isotropic magneto-thermoelastic medium with $\Omega = 0.5, \omega = 0.25$

ii. The red solid line with circle symbols corresponds to transversely isotropic magneto-thermoelastic medium with $\Omega = 0.5, \omega = 0.50$

iii. The green solid line with circle symbols corresponds to transversely isotropic magneto-thermoelastic medium with $\Omega = 0.5,\ \omega = 0.75$

iv. The blue solid line with diamond symbols corresponds to transversely isotropic magneto-thermoelastic medium with $\Omega = 0.5,\ \omega = 1.0$

*Case 1: Concentrated force due to inclined load and with insulated and isothermal boundaries (Green Lindesay Model)*

Fig. 1 to Fig. 6 shows the variations of the displacement components ($u$ and $w$), Conductive

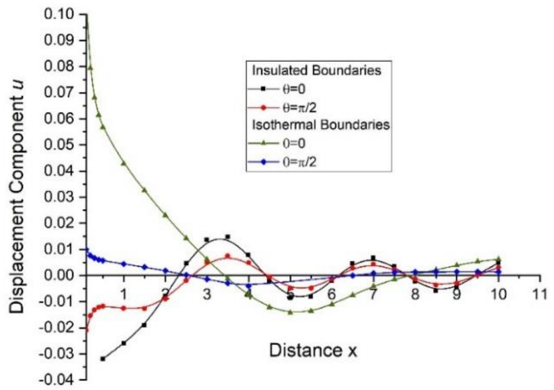


Fig. 2 Variations of displacement component *u* with distance *x*

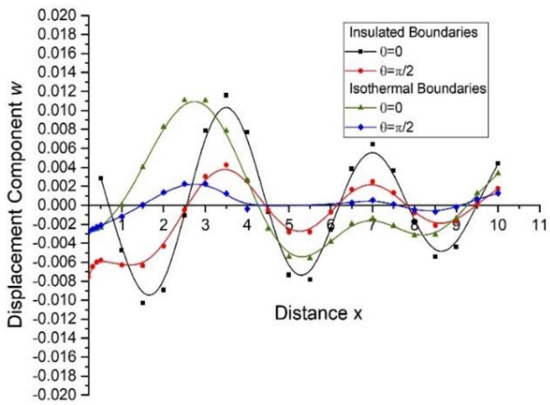


Fig. 3 Variations of displacement component *w* with distance *x*

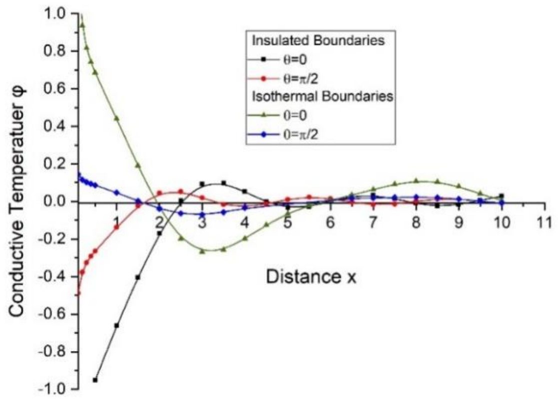


Fig. 4 Variations of conductive temperature $\varphi$ with distance $x$

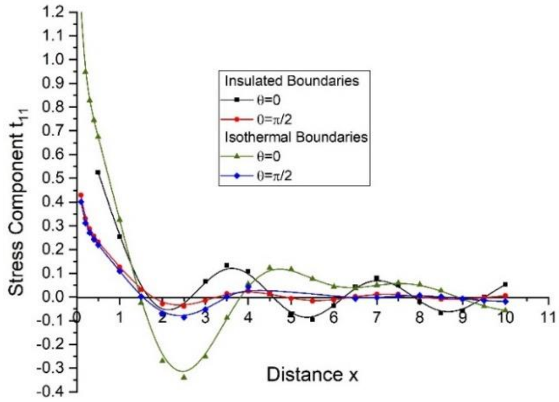


Fig. 5 Variations of stress component $t_{11}$ with distance $x$

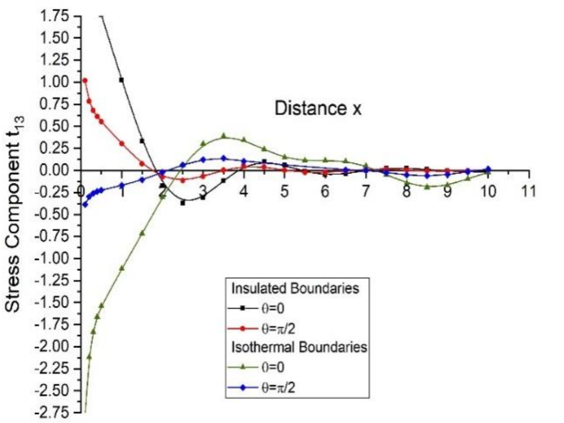


Fig. 6 Variations of the stress component $t_{13}$ with distance $x$

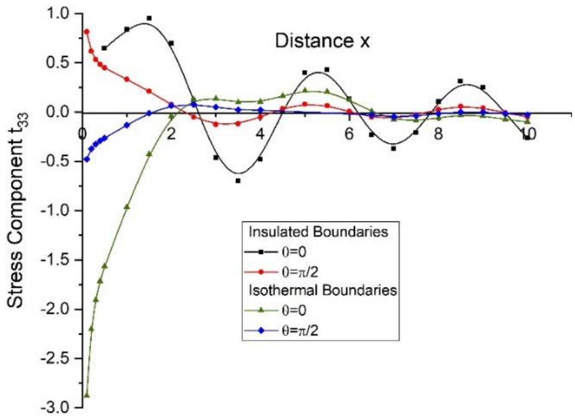


Fig. 7 Variations of the stress component $t_{33}$ with distance $x$

temperature$\varphi$and stress components ( $t_{11}$, $t_{13}$ and $t_{33}$) for a transversely isotropic magneto-thermoelastic medium with concentrated force and with combined effects of rotation, a time-harmonic source for inclined load with two temperature in generalized thermoelasticity respectively. The displacement components ($u$ and $w$), Conductive temperature $\varphi$ and stress components ( $t_{11}$, $t_{13}$ and $t_{33}$) illustrate the same pattern but having different magnitudes for different value of frequency. These components vary (increases or decreases) during the initial range of distance near the loading surface of the time-harmonic source and follow a small oscillatory pattern for the rest of the range of distance. A low value of time-harmonic source frequency shows more stress near the loading surface.

*Case 2: Concentrated force due to inclined load and with time-harmonic frequency (Lord Shulman model)*

Fig. 8 to Fig. 13 shows the variations of the displacement components($u$ and $w$), Conductive temperature$\varphi$and stress components ( $t_{11}$, $t_{13}$ and $t_{33}$) for a transversely isotropic magneto-

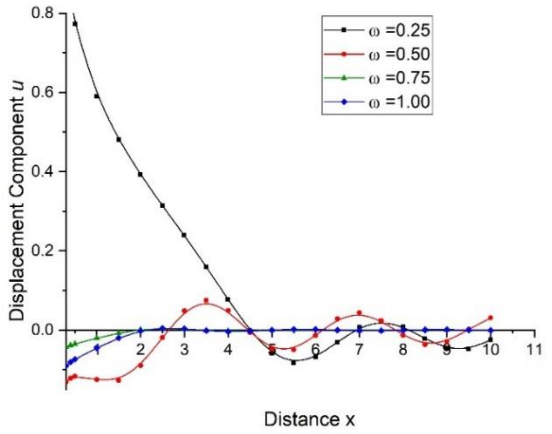


Fig. 8 Variations of displacement component *u* with distance *x*

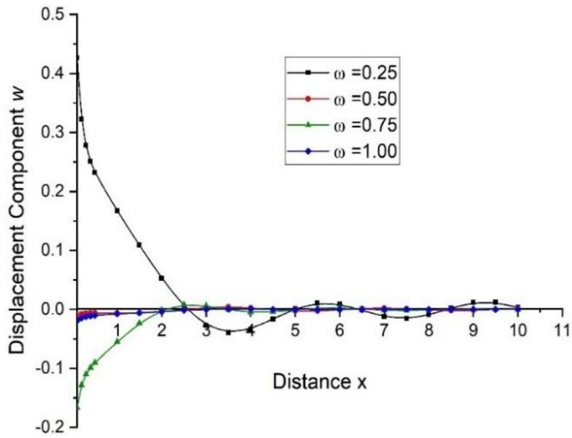


Fig. 9 Variations of displacement component *w* with distance *x*

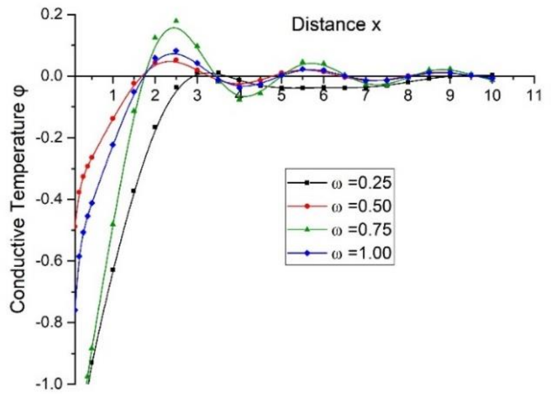


Fig. 10 Variations of conductive temperature $\varphi$ with distance *x*

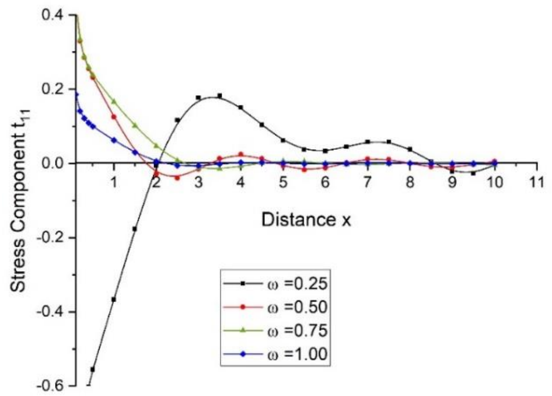


Fig. 11 Variations of stress component $t_{11}$ with distance *x*

thermoelastic medium with linearly distributed force and with combined effects of rotation, a time-harmonic source for inclined load with two temperature in generalized thermoelasticity respectively. As the value of frequency increase displacement component, $u$ increase, normal displacement $w$ decrease, conductive temperature $\varphi$ increase near the loading surface rest remain same for transversely isotropic magneto-thermoelastic medium. However, as the value of frequency increase stress components $t_{11}$ increase for $\omega = 0.25$ $t_{13}$ and $t_{33}$ decrease for transversely isotropic magneto-thermoelastic medium also decrease near the loading surface rest remain the same. The low value of time-harmonic source frequency shows more stress near the loading surface.

## 9. Conclusions

From the above study, it is observed that

• Time-harmonic source plays a key role in the oscillation of physical quantities both close to the

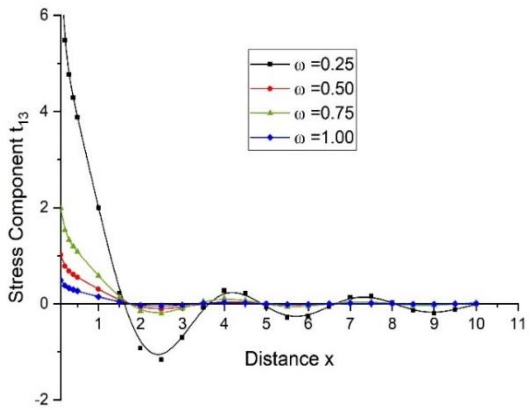


Fig. 12 Variations of the stress component $t_{13}$ with distance *x*

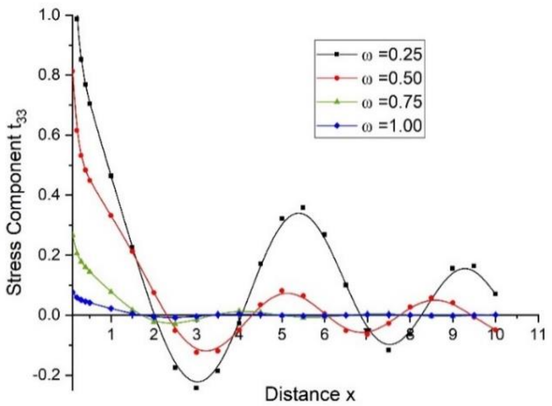


Fig. 13 Variations of the stress component $t_{33}$ with distance *x*

point of use of source as well as just as far from the source. The physical quantities amplitude differ (i.e., either rise or fall) with a change in frequency of the time-harmonic source.

• Moreover, laser pulse, the magnetic effect of two temperature, rotation as well as the angle of inclination of the applied load plays a key part in the deformation of all the physical quantities.

• In the presence of two temperature and inclined load, the displacement components and stress components show an oscillatory nature with respect to x.

• The result gives the inspiration to study magneto-thermoelastic materials as an innovative domain of applicable thermoelastic solids.

• The shape of curves shows the impact of frequency ω on the body and fulfills the purpose of the study.

• The outcomes of this research are extremely helpful in the 2-D problem with dynamic response of time-harmonic sources in transversely isotropic magneto-thermoelastic medium with rotation and two temperature which is beneficial for the analysis of the deformation field such as geothermal engineering; advanced aircraft structure design, thermal power plants, composite engineering, geology, high-energy particle accelerators and in real life as in geophysics, auditory range, geomagnetism etc.

• The proposed model in this research is relevant to different problems in thermoelasticity and thermodynamics.

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## Nomenclature

| | |
|---|---|
| $\delta_{ij}$ | Kronecker delta, |
| $C_{ijkl}$ | Elastic parameters, |
| $\beta_{ij}$ | Thermal elastic coupling tensor, |
| $T$ | Absolute temperature, |
| $T_0$ | Reference temperature, |
| $\varphi$ | conductive temperature, |
| $t_{ij}$ | Stress tensors, |
| $e_{ij}$ | Strain tensors, |
| $u_i$ | Components of displacement, |
| $\rho$ | Medium density, |
| $C_E$ | Specific heat, |
| $a_{ij}$ | Two temperature parameters, |
| $\alpha_{ij}$ | Linear thermal expansion coefficient, |
| $K_{ij}$ | Thermal conductivity, |
| $\omega$ | Frequency , |
| $\tau_0, \tau_1$ | Relaxation Time, |
| $\Omega$ | Angular Velocity of the Solid, |
| $F_i$ | Components of Lorentz force, |
| $\vec{H}_0$ | Magnetic field intensity vector, |
| $\vec{J}$ | Current Density Vector, |
| $\vec{u}$ | Displacement Vector, |
| $\mu_0$ | Magnetic permeability, |
| $\varepsilon_0$ | Electric permeability, |
| $\delta(x)$ | Dirac delta function. |