

Spatial interpolation of SPT data and prediction of consolidation of clay by ANN method

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(Received March 8, 2019, Revised November 8, 2019, Accepted November 13, 2019)

Abstract. Artificial Intelligence (AI) is anticipated to be the future of technology. Hence, AI has been applied in various fields over the years and its applications are expected to grow in number with the passage of time. There has been a growing need for accurate, direct, and quick prediction of geotechnical and foundation engineering models especially since the success of each project relies on numerous amounts of data. In this study, two applications of AI in the field of geotechnical and foundation engineering are presented – spatial interpolation of standard penetration test (SPT) data and prediction of consolidation of clay. SPT and soil profile data may be predicted and estimated at any location and depth at a site that has no available borehole test data using artificial intelligence techniques such as artificial neural networks (ANN) based on available geospatial information from nearby boreholes. ANN can also be used to accelerate the calculation of various theoretical methods such as the one-dimensional consolidation theory of clay with high efficiency by using lesser computation resources. The results of the study showed that ANN can be a valuable, powerful, and practical tool in providing various information that is needed in geotechnical and foundation design.

Keywords: geotechnical and foundation design; artificial intelligence; artificial neural networks; SPT; soil profile data; consolidation

1. Introduction

The Standard Penetration Test (SPT) is a well-known field test for subsurface geotechnical investigation because of its ability to extract soil samples for further laboratory testing. In many instances, the standard penetration test number of blows per foot or SPT-N (blows/30cm) is

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directly used with widely established correlations to derive numerous parameters for foundation design such as optimal dimensioning, load capacity, stress analysis, seismic analysis, and settlement of shallow and deep foundations (Kim and Mission 2011, Erzín and Gul 2013, Singh and Sawant 2014, Ganaie and Sawant 2015, Abbas *et al.* 2016, López-Chavarría *et al.* 2017, Velázquez-Santillán *et al.* 2018, López-Chavarría *et al.* 2019, Mandal and Maity 2019). Due to the limited number of SPT borehole investigations at any given particular site, geotechnical engineers may often interpolate, extrapolate, or average from nearby existing available borehole locations to estimate the unknown subsurface parameters at any other point or location. Recently, Artificial Intelligence (AI) has become very popular and has been applied in many engineering problems (Chakraverty and Nayak 2012, Sunny *et al.* 2016, Chore and Magar 2017, Ayat *et al.* 2018) because it allows the modeling of nonlinear processes. In this study, the use of Artificial Neural Networks (ANN) for spatial interpolation of SPT-N and soil profile data (Kim *et al.* 2013) is explored, and its various advantages and benefits compared with the other classification and estimation methods are also compared.

Predicting the soil settlement or consolidation of clay is important in the design of foundations especially in the analysis of negative skin-friction in piles (Liu *et al.* 2012, Cao *et al.* 2014, Kim *et al.* 2018). The typical solution procedure for the prediction of soil settlement and excess pore water pressure profile at any specific time during the consolidation process is performed by numerical analysis of the 1D consolidation equation proposed by Terzaghi (1943) using the finite difference method (FDM) or by manually using tables and design charts. Different methods of predicting the consolidation of clay can also be found in the works of Mikasa (1963), Gibson *et al.* (1976), Fox *et al.* (2014), and Brandenberg (2016). In FDM, solutions are processed incrementally at a small time-step up to the final time of interest. However, having a small time-step interval, a long consolidation time, and a thick clay layer deposit, usually involves a large number of iterations that can consume a lot of computer memory and huge sizes of data file, which can slow down the computational process and may take longer computer processing time to finish (Kim *et al.*, 1995). It is therefore not possible to directly predict the future conditions of the soil without first chronologically solving the current and sequential conditions that lead to the final conditions. Hazzard and Yacoub (2008) presented a hybrid computational scheme for the numerical solution of 1D consolidation, based on the method described by Booker and Small (1975), to speed up the required computational time by increasing the time-step gradually as the solution progresses while maintaining the required stability and accuracy. However the suggested method, still suffers from the chronological or sequential type of solution in which the needed computational resources such as computer memory, output file size, and the processing time accumulates as the solution progresses. To predict the one-dimensional consolidation of a homogeneous clay layer under a uniform surcharge load with high efficiency and accuracy, the program ANN-1DConsol was developed in the MATLAB graphical user interface (GUI). With sufficient training of the ANN model and without resorting to a stepwise progressing and sequential solution procedure, the method provides reliable and direct estimates of the excess pore pressures and settlement in the clay layer at any time during the progress of consolidation using much less computational resources compared to FDM. Numerical examples are presented to validate the prediction performance and to demonstrate the advantages of the ANN in comparison with the conventional finite difference method.

2. Different methods of spatial interpolation of standard penetration test (SPT) data

2.1 Nearest borehole method

The nearest borehole method is a similar adaptation of the nearest neighbor algorithm (Cover and Hart 1967), which is one of the simplest and most classical non-linear classification algorithms. The method classifies objects based on closest training examples in the feature space. Any point within an area on a site has a known distance to existing SPT borehole locations. When a point is closest to a nearby SPT borehole location, then the area bounded by all these points is assumed to have the same SPT- N data and soil profile with the nearest single borehole as shown in Fig. 1.

2.2 Linear interpolation and spatial averaging method

Instead of copying the SPT-N and soil profile data from the nearest single borehole, another method of estimating the soil SPT-N characteristics of any particular point at a site is to linearly interpolate in two (2D) or three (3D) dimensions from nearby boreholes surrounding the point or location in question. When more than two or more nearby borehole locations are available, then a common approximation is to perform spatial averaging of all the SPT-N data at any specific depth, as shown in Fig. 2.

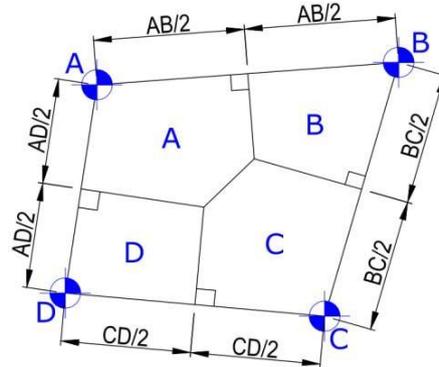


Fig. 1 Nearest borehole method

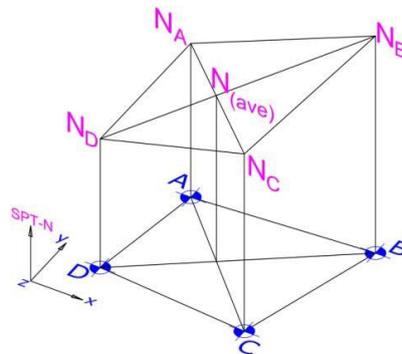


Fig. 2 Linear interpolation and spatial averaging method

2.3 Spatial interpolation by Artificial Neural Network (ANN)

A major drawback of the spatial method using linear interpolation by averaging is its inability to properly characterize or classify the soil profile at any unknown site in question, especially when dealing with non-numeric data (ex. soil type). The spatial interpolation method using Artificial Neural Network (ANN) has all these capabilities to perform soil parameter estimation, pattern recognition, classification, and nonlinear fitting.

ANN is a set of connected input-output units, where each connection has weights and biases associated with it. Training and learning is being performed by a feed forward-back propagation algorithm: (a) the inputs are fed simultaneously into the input layer, (b) weights and biases are initially assigned and the weighted input are fed simultaneously into the hidden layer, (c) the hidden layer's weighted outputs can be input to another hidden layer, (d) the weighted outputs of the last hidden layer are inputs to units making up the output layer, (e) predicted output is compared with the target and the error is propagated backwards by updating the weights and biases to reflect the error of the network classification until a specific performance and termination criteria is achieved.

The structure of the ANN network for spatial interpolation of SPT and soil profile data is shown in Fig. 3. Input layer consists of the spatial coordinates (X, Y, Z) data and depth of groundwater table (W) of the respective existing boreholes. Output layer consists of the SPT-N (blows/30 cm) profile and a general soil classification (cohesive or cohesionless) at the respective depths Z.

Where the ANN input and output data are generally on widely different scales, it is necessary to normalize them to speed up training and obtain better results, such that they are compatible with the range of hidden layer activation functions (ex. -1.0 to 1.0, or 0 to 1.0). The min-max normalization method is then used to rescale the input and output data for training by linear interpolation within the range from minimum of 0 to maximum of 1.0. Let p be the raw input or output data, and given the maximum (p_{max}) and minimum (p_{min}) value of p , a normalized data p_n can be derived as follows,

$$p_n = \frac{(p - p_{min})}{(p_{max} - p_{min})} \quad (1)$$

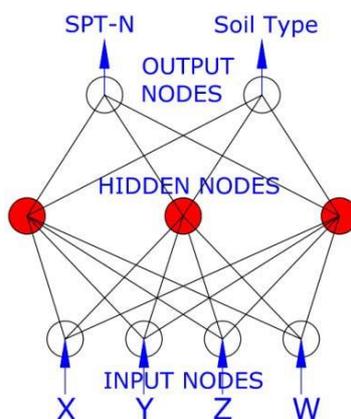


Fig. 3 Artificial Neural Network (ANN) method

2.4 Application example of ANN for predicting SPT-N and soil profile

Shown in Fig. 4 is an area that is being modeled and studied, which is part of the IRPC-UHV project site in Rayong, Thailand (MAA Geotechnics, 2013). The studied site consists of 17 SPT boring holes, in which 13 were used for ANN training and the remaining 4 for validation and testing. Shown in Fig. 5 is the site classification based on nearest borehole method. Linear interpolation and spatial averaging method using nearby boreholes have to be performed at every depth interval in order to completely define the whole SPT-N profile at every location. Shown in Fig. 6 is an example result of SPT-N contours using linear interpolation method of SPT data from nearby boreholes at depth $z = 20.0\text{m}$. As mentioned, the linear interpolation and spatial averaging method, while dealing only with numeric data, lacks the ability to estimate the probable classification of the soil at every depth (since we cannot interpolate or average between say sand and clay). The ANN architecture shown in Fig. 3 is then modeled in MATLAB, in which input and output data were normalized for training and simulation.

Shown in Fig. 7 is a comparison between the measured and predicted SPT-N values and soil profiles. Fig. 7 shows that ANN has the ability to classify and estimate the SPT and soil profile at any location and depth within the area boundary of the trained network, in which good agreement is seen in comparison with measured data and referred results from nearby boreholes. A distinct feature of the predicted results by ANN is the ability to generalize or smoothen the SPT-N profile throughout the depth and thereby removing spatial variations. Results are also compared in terms of the soil classification at the respective depths, in which general predictions by ANN, either cohesive soil (clay=C) or cohesionless soil (sand=S), showed fair agreement with those from field SPT results. When compared with the reliability of soil classification, ANN achieves up to about 70-80% of the prediction, and in which in most cases exceeding those results from nearest borehole method.



Fig. 4 Project study area and borehole location

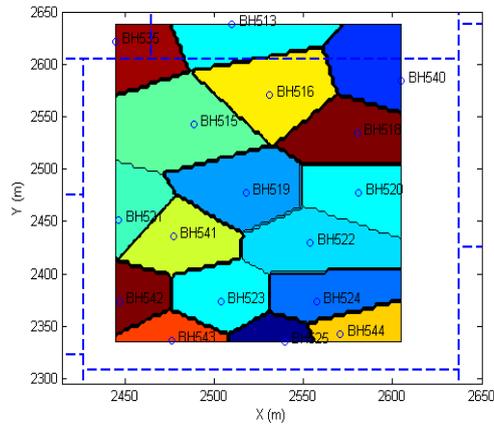


Fig. 5 Nearest borehole classification

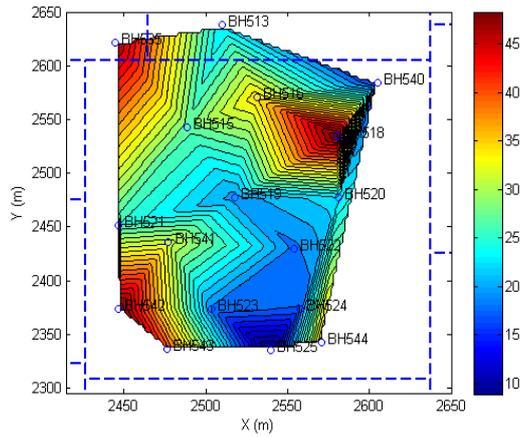


Fig. 6 SPT-N Contour using linear interpolation (at depth=20 m)

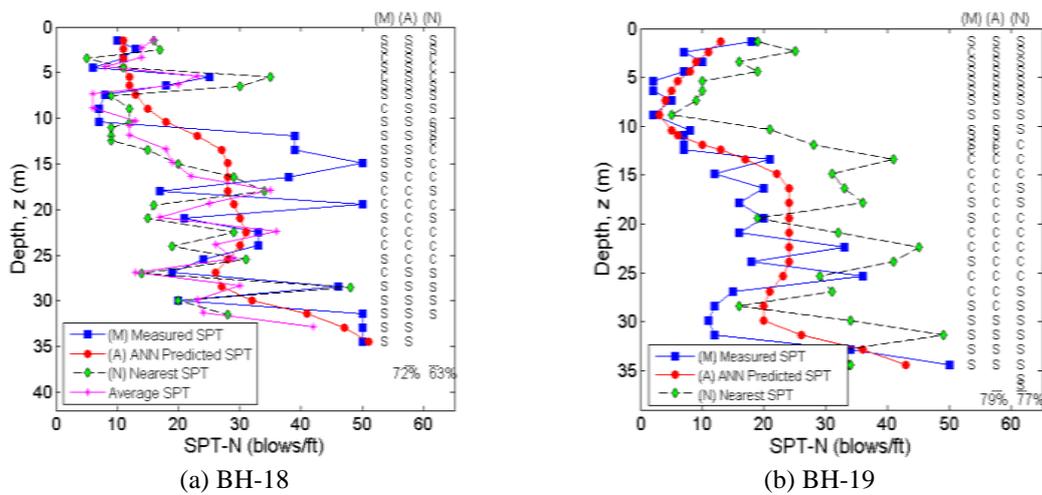


Fig. 7 Comparison of measured and predicted SPT-N and soil profiles

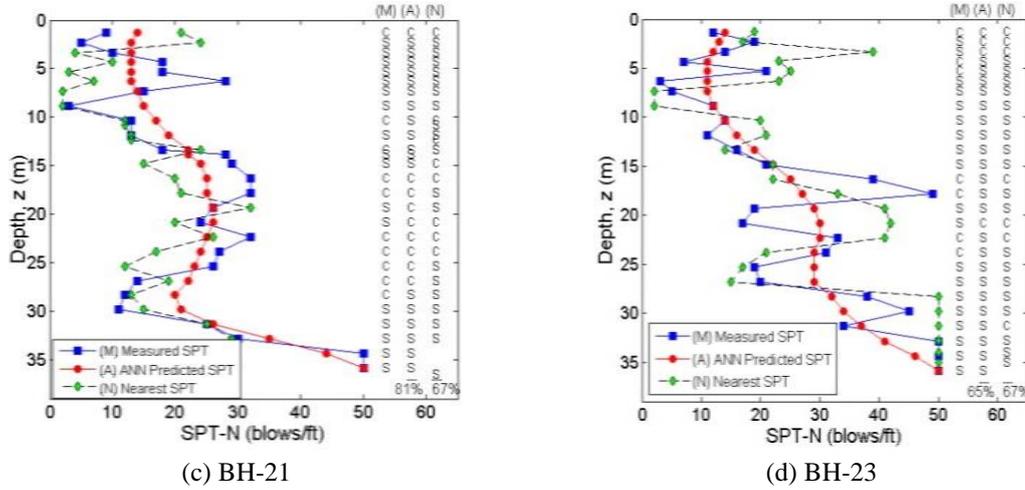


Fig. 7 Continued

3. Prediction of one-dimensional consolidation

3.1 Finite difference solution of the 1D consolidation equation

Terzaghi (1943) derived the one-dimensional consolidation equation for a homogenous layer of clay with thickness H_c under a uniformly distributed surcharge load q that is given as,

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \quad (2)$$

where u is the excess pore water pressure, t is the consolidation time, z is the depth, and C_v is the coefficient of consolidation. Eq. (2) is based on the assumption that the coefficient of consolidation C_v remains constant during the consolidation process, the effect of self-weight consolidation is neglected, the soil profile is fully saturated, and the consolidation settlements are small or infinitesimal. The finite difference form of Eq. (2) for numerical analysis in time $(t+\Delta t)$ is written as,

$$u_{z,(t+\Delta t)} = u_{z,t} + \frac{C_v \Delta t}{\Delta z^2} [u_{(z+\Delta z),t} - 2u_{z,t} + u_{(z-\Delta z),t}] \quad (3)$$

In Eq. (3), Δt = time-step and the depth increment $\Delta z = H_c/n$, where n is the number of sublayer elements in the finite difference grid. Eq. (3) is applied using the following initial and boundary conditions with respect to the excess pore pressure at the depth and time coordinates $u(z,t)$ in which:

$$u(z,0) = q \text{ (initial condition)} \quad (4)$$

$$u(0,t) = 0 \text{ (at a permeable top surface boundary)} \quad (5)$$

$$u(H_c,t) = 0 \text{ (at a permeable bottom surface boundary)} \quad (6)$$

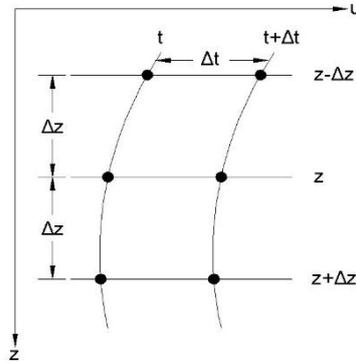


Fig. 8 Finite difference nodes in the numerical solution of the 1D consolidation equation

$$\frac{\partial u}{\partial z}(H_c, t) = 0 \text{ (at an impermeable bottom surface boundary)} \quad (7)$$

The impermeable boundary condition defined by Eq. (7) means that there can be no flow in the perpendicular direction. Eq. (8) is implemented numerically by creating a dummy node in the finite difference grid after the bottom surface, which can be expressed in finite difference form as,

$$\frac{(u_{H_{c+\Delta z}}) - (u_{H_{c-\Delta z}})}{2\Delta z} = 0, \text{ or } (u_{H_{c+\Delta z}}) = (u_{H_{c-\Delta z}}) \quad (8)$$

Eq. (3) implies that if the solution for u has been determined at time t , then the values at time $(t+\Delta t)$ can be calculated by marching the solution downward with depth and forward in time as shown in Fig. 8. To ensure that the approximate solution of Eq. (3) converge to the exact solution as Δt and Δz approaches zero, the following criteria should be satisfied in determining the time and depth increments, Δt and Δz , respectively (Forsythe and Wasow 1960):

$$\frac{C_v \Delta t}{\Delta z^2} = \beta \leq \frac{1}{2} \quad (9)$$

The total settlement S can be calculated using the coefficient of compressibility m_v and excess pore pressure u by numerically integrating along the depth profile as follows:

$$S = m_v \int_0^{H_c} (q - u) dz = m_v q H_c - \frac{m_v \Delta z}{2} \sum_{n=1}^n (u_n + u_{n+1}) \quad (10)$$

3.2 Artificial Neural Network (ANN) model for predicting one-dimensional consolidation

Neural networks are composed of simple elements operating in parallel, which are trained to perform a particular function by adjusting the values of the connections (weights) between elements so that a particular input leads to a specific target output (Demuth *et al.* 2009). For a range of input values for H , C_v , m_v , q , and the consolidation time t , and their corresponding outputs in terms of the excess pore pressure u , input-target pairs were generated from the numerical results using the FDM, which are needed to train the network. To improve training and performance, the

original input-output dataset were preprocessed by normalizing the output excess pore pressure u profile in terms of the applied surcharge load q , that is, u/q , and normalizing the actual consolidation time t from the input in terms of the time factor T_v defined as,

$$T_v = \frac{C_v t}{H^2} \tag{11}$$

where t = actual consolidation time, H = length of the longest drainage path, and in which $H = H_c$ for single drainage and $H = H_c/2$ for double drainage.

The typical architecture of the ANN model for predicting 1D consolidation is shown in Fig. 9. The optimized network architecture of the model has one input layer, one hidden layer with four neurons, and 21 neurons in the output layer, which was trained for a performance goal of 10^{-6} . Two ANN models were being developed and presented in this study: *net1* for consolidation in single drainage boundary conditions, and *net2* for double drainage conditions. From the various combinations of the range of input parameters, a total number of 4,800 samples in each case were used for training, which were obtained from the original number of input-output dataset for

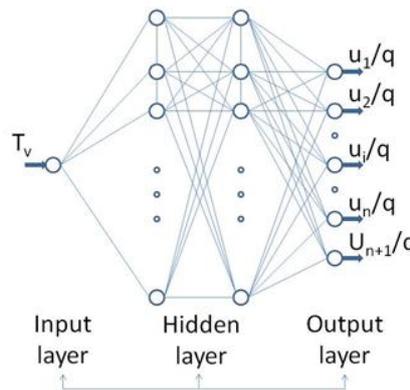


Fig. 9 Typical architecture of the ANN model for 1D consolidation prediction

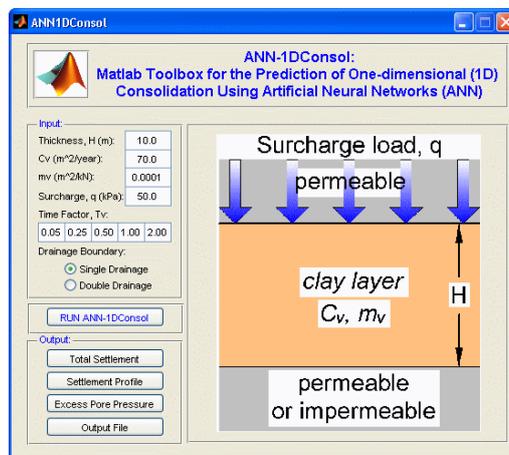


Fig. 10 Problem geometry and program user interface of ANN-1DConsol

ANN corresponding to the total number of required iterations to reach the final time factor $T_v = 2.0$ and using the criteria $\beta = 0.25$. The computer program ANN-1DConsol was then developed by the authors and its graphical user interface (GUI) is also shown in Fig. 10.

3.3 Numerical validation of ANN model in predicting 1D consolidation

A 10.0 m thick single layer of homogeneous clay was subjected to an instantaneous uniform surcharge load $q = 50$ kPa (Fig. 10). The consolidation properties are as follows: $C_v = 70$ m²/year and $m_v = 0.0001$ m²/kN. It is desired to determine the development of excess pore pressures and settlement during the progress of consolidation for single and double drainage conditions. Using finite difference solution, the thickness of the clay layer was subdivided into 20 elements and a time step corresponding to a factor $\beta = 0.25$ was selected. Results were compared between the FDM predictions and that using the ANN models *net1* for single drainage and *net2* for double drainage. Fig. 11 compares the development of the excess pore pressures corresponding to the time factors $T_v = 0.05, 0.25, 0.50,$ and 1.0 in which it can be seen that ANN predictions are in good agreement with FDM results. Shown in Fig. 12 are the development of settlement in the layer in which ANN results are also equivalent to the settlement predictions by FDM.

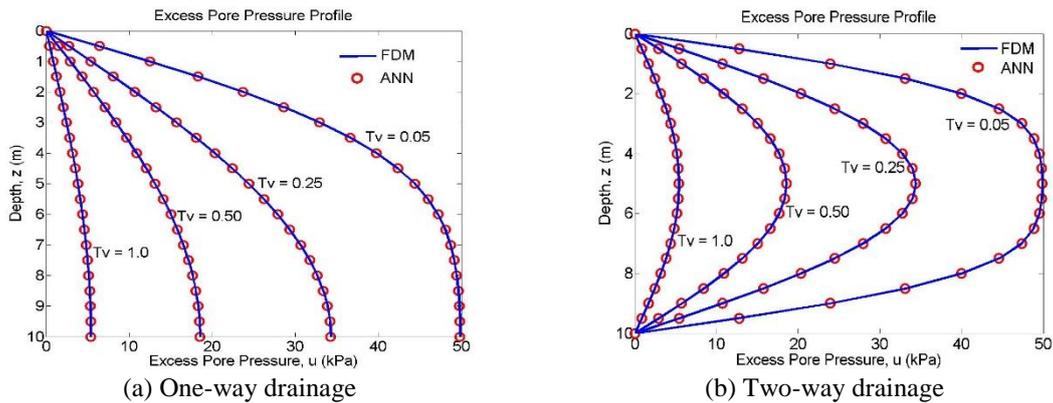


Fig. 11 Comparison of predicted excess pore pressure profiles between FDM and ANN

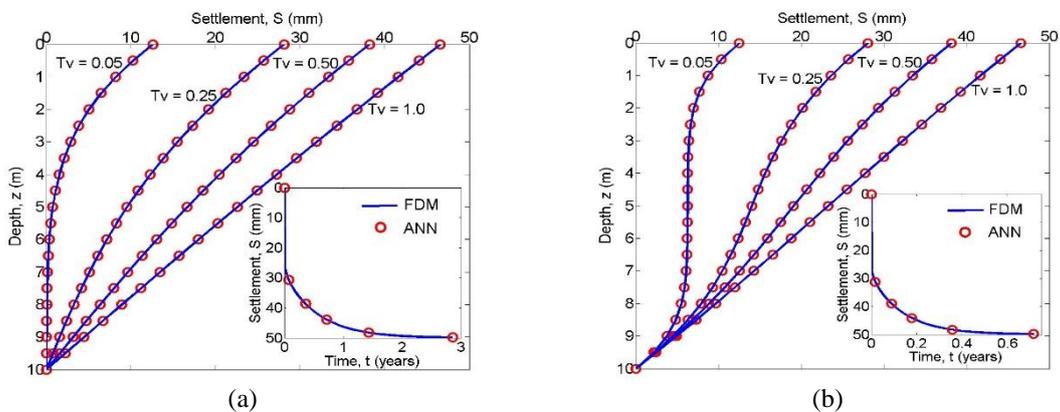


Fig. 12 Comparison of predicted settlements between FDM and ANN

Having validated the accuracy of the ANN for predicting 1D consolidation, whose results are comparable with the FDM, the ANN method is advantageous due to its quick speed and high efficiency. The FDM and ANN predictions were implemented using the MATLAB program in a 2.83 GHz quad-core computer with 6 Gb memory. The central processing unit (CPU) time in seconds (s) as well as the size of the output file in kilobytes (kb) were compared. Due to the sequential nature or the time-marching solution process of the FDM, the CPU time and output file sizes or memory requirements increases especially when consolidation results are needed at longer consolidation times. In contrast, equivalent and accurate predictions are still being provided by ANN in which direct and quick results can be made at any consolidation time of interest. The prediction method of 1D consolidation can therefore be reliably made by ANN and can be more efficient by about 6 % to 278 % compared to the FDM and thus minimizing the needed computational resources.

4. Conclusions

The use of Artificial Neural Network (ANN) as applied in geotechnical and foundation engineering for spatial interpolation of Standard Penetration Test (SPT) data and soil profile classification, and prediction of consolidation of a homogeneous clay, has been demonstrated to be a promising alternative to conventional and classical methods. Predicted results of SPT-N profile by ANN have shown to be in good agreement with measured data, with the ability to generalize the soil profile and remove spatial variations. In addition, predicted performance of soil profile classification method by ANN is shown to generally exceed the results from the approximate nearest borehole classification method. ANN can also provide accurate and direct estimates of the excess pore pressures and settlement at any time during consolidation without resorting to the stepwise progressing solution procedure. The prediction performance of the ANN has been validated by the equivalent results with FDM. Hence, ANN can be used as a direct, accurate, and quick tool for spatial interpolation of SPT and soil profile data, excess pore pressures, and soil settlement without the need to perform any lengthy manual calculations or using tables and charts. In summary, the results of the study showed that ANN can be a valuable, powerful, and practical tool in providing various information that is needed in geotechnical and foundation design.

Acknowledgments

This work was supported by Kunsan National University, and the Korea Institute of Energy Technology Evaluation and Planning (KETEP) and the Ministry of Trade, Industry & Energy (MOTIE) of the Republic of Korea (No. 20183010025200).

References

- Abbas, N.E., Khamlichi, A. and Bezzazi, M. (2016), "Seismic response of foundation-mat structure subjected to local uplift", *Coupled Syst. Mech.*, **5**(4), 285-304. <https://doi.org/10.12989/csm.2016.5.4.285>.
- Ayat, H., Kellouche, Y., Ghrici, M. and Boukhatem, B. (2018), "Compressive strength prediction of limestone filler concrete using artificial neural networks", *Adv. Comput. Des.*, **3**(3), 289-302. <https://doi.org/10.12989/acd.2018.3.3.289>.

- Booker, J.R. and Small, J.C. (1975), "An investigation of the stability of numerical Solutions of Biot's equations of consolidation", *Int. J. Solids Struct.*, **11**(7-8), 907-917. [https://doi.org/10.1016/0020-7683\(75\)90013-X](https://doi.org/10.1016/0020-7683(75)90013-X).
- Brandenberg, S.J. (2016), "iConsol.js: JavaScript implicit finite-difference code for nonlinear consolidation and secondary compression", *Int. J. Geomech.*, **17**(6), 1-11. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0000843](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000843).
- Cao, W., Chen, Y. and Wolfe, W.E. (2014), "New load transfer hyperbolic model for pile-soil interface and negative skin friction on single piles embedded in soft soils", *Int. J. Geomech.*, **14**(1), 92-100. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0000289](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000289).
- Chakraverty, S. and Nayak, S. (2012), "Fuzzy finite element method for solving uncertain heat conduction problems", *Coupled Syst. Mech.*, **1**(4), 345-360. <http://doi.org/10.12989/csm.2012.1.4.345>.
- Chore, H.S. and Magar, R.B. (2017), "Prediction of unconfined compressive and Brazilian tensile strength of fiber reinforced cement stabilized fly ash mixes using multiple linear regression and artificial neural network", *Adv. Comput. Des.*, **2**(3), 225-240. <https://doi.org/10.12989/acd.2017.2.3.225>.
- Cover, T.M. and Hart, P.E. (1967), "Nearest neighbor pattern classification", *IEEE T. Inform. Theor.*, **13**(1), 21-27. <https://doi.org/10.1109/TIT.1967.1053964>.
- Demuth, H., Beale, M., and Hagan, M. (2009), *Neural Networks Toolbox User Guide*, The Math Works Inc., Natick, Massachusetts, U.S.A.
- Erzin, Y. and Gul, T. (2013), "The use of neural networks for the prediction of the settlement of pad footings on cohesionless soils based on standard penetration test", *Geomech. Eng.*, **5**(6), 541-564. <https://doi.org/10.12989/gae.2013.5.6.541>.
- Forsythe, G.E. and Wasow, W.R. (1960), *Finite Difference Methods for Partial Differential Equations*, John Wiley & Sons, Inc., New York, U.S.A.
- Fox, P.J., Pu, H.F. and Berles, J.D. (2014), "CS3: Large strain consolidation model for layered soils", *J. Geotech. Geoenviron. Eng.*, **140**(8), 1-13. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0001128](https://doi.org/10.1061/(ASCE)GT.1943-5606.0001128).
- Ganaie, A.H. and Sawant, V.A. (2015), "Analysis of a strip footing on a homogeneous soil using element free Galerkin method", *Coupled Syst. Mech.*, **4**(4), 365-383. <https://doi.org/10.12989/csm.2015.4.4.365>.
- Gibson, R.E., England, G.L. and Hussey, M.J.L. (1967), "The theory of one-dimensional consolidation of saturated clays", *Géotechnique*, **17**(3), 261-273. <https://doi.org/10.1680/geot.1967.17.3.261>.
- Hazzard, J. and Yacoub, T. (2008), "Consolidation in multi-layered soils: A hybrid computational scheme", *Proceedings of the GeoEdmonton ,08 61st Canadian Geotechnical Conference and 9th Joint CGS/IAH-CNC Groundwater Conference*, Edmonton, Canada, September.
- Kim, H.J. and Mission, J.L. (2011), "Probabilistic evaluation of economical factors of safety for the geotechnical design of pile axial load capacity using Monte Carlo simulation", *KSCE J. Civ. Eng.*, **15**(7), <https://doi.org/10.1007/s12205-011-0948-8>.
- Kim, H.J., Hirokane, S., Yoshikuni, H., Moriwaki, T., and Kusakabe, Q. (1995), "Consolidation behavior of dredged clay ground improved by horizontal drain method," *Proceedings of the International Symposium on Compression and Consolidation Clay Soils*, Hiroshima, Japan, May.
- Kim, H.J., Mission, J.L., Park, T.W. and Dinoy, P.R. (2018), "Analysis of negative skin-friction on single piles by one-dimensional consolidation model test", *Int. J. Civ. Eng.*, **16**(10), 1445-1461. <https://doi.org/10.1007/s40999-018-0299-7>.
- Liu, J., Gao, H. and Liu, H. (2012), "Finite element analyses of negative skin friction on a single pile", *Acta Geotech.*, **7**(3), 239-252. <https://doi.org/10.1007/s11440-012-0163-x>.
- López-Chavarría, S., Luévanos-Rojas, A. and Medina-Elizondo, M. (2017), "Optimal dimensioning for the corner combined footings", *Adv. Comput. Des.*, **2**(2), 107-121. <https://doi.org/10.12989/acd.2017.2.2.169>.
- López-Chavarría, S., Luévanos-Rojas, A., Medina-Elizondo, M., Sandoval-Rivas, R. and Velázquez-Santillán, F. (2019), "Optimal design for the reinforced concrete circular isolated footings", *Adv. Comput. Des.*, **4**(3), 273-294. <https://doi.org/10.12989/acd.2019.4.3.273>.
- MAA Geotechnics Co., Ltd. (2013), *Final Geotechnical Investigation Report for Upstream Project for Hygiene and Value Added Products-IRPC Public Company Limited*, 27 February 2013.
- Mandal, A. and Maity, D. (2019), "Seismic analysis of dam-foundation-reservoir coupled system using

- direct coupling method”, *Coupled Syst. Mech.*, **8**(5), 393-414. <https://doi.org/10.12989/csm.2019.8.5.393>.
- MATLAB (2012), The MathWorks, Inc., www.mathworks.com.
- Mission, J.L., Kim, H.J. and Lee, K.H. (2013), “Artificial neural network (ANN) application for spatial interpolation of standard penetration test (SPT) and soil profile data”, *Proceedings of the 2013 World Congress on Advances in Structural Engineering and Mechanics (ASEM13)*, Jeju, Korea, September.
- Singh, M. and Sawant, V.A. (2014), “Parametric study on flexible footing resting on partially saturated soil”, *Coupled Syst. Mech.*, **3**(2), 233-245. <https://doi.org/10.12989/csm.2014.3.2.233>.
- Sunny, M.R., Mulani, S.B., Sanyal, S. and Kapania, R.K. (2016), “An artificial neural network residual kriging based surrogate model for curvilinearly stiffened panel optimization”, *Adv. Comput. Des.*, **1**(3), 235-251. <https://doi.org/10.12989/acd.2016.1.3.235>.
- Terzaghi, K. (1943), *Theoretical Soil Mechanics*, John Wiley & Sons, Inc. New York, U.S.A.
- Velázquez-Santillán, F., Luévanos-Rojas, A., López-Chavarría, S., Medina-Elizondo, M. and Sandoval-Rivas, R. (2018), “Numerical experimentation for the optimal design for reinforced concrete rectangular combined footings”, *Adv. Comput. Des.*, **3**(1), 49-69. <https://doi.org/10.12989/acd.2018.3.1.049>.