

The effects of thermo-mechanical behavior of living tissues under thermal loading without energy dissipation

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Abstract. This study seeks to develop analytical solutions for the biothermoelastic model without accounting for energy dissipation. These solutions are then applied to estimate the temperature changes induced by external heating sources by integrating relevant empirical data characterizing the biological tissue of interest. The distributions of temperature, displacement, and strain were obtained by utilizing the eigenvalues approach with the Laplace transforms and numerical inverse transforms method. The impacts of the rate of blood perfusion and the metabolic activity parameter on thermoelastic behaviors were discussed specifically. The temperature, displacement, and thermal strain results are visually represented through graphical representations.

Keywords: eigenvalues approach; Laplace transformations; thermal and mechanical interactions; tissue; without energy dissipation

1. Introduction

There are several methods available to measure the thermal properties of living organisms, but the outcomes can differ. Achieving accurate measurements of the temperature characteristics of tissues within a living organism is still challenging. The complete understanding of temperature properties in living tissues remains incomplete due to the complexities involved in measuring them *in vivo*. This complexity arises from the potential alterations to tissue temperature caused by postmortem conditions, as well as the absence of the perfusion effect when examining tissue outside of the body. In recent research, the complexity of heat transfer in skin tissues has been recognized as a significant challenge. Therapeutic procedures such as laser tissue welding (Gabay

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et al. 2011), hyperthermia (Mahjoob and Vafai 2009), and laser surgeries (Zhou *et al.* 2009) have become commonplace in modern medicine.

The impact of diseases is influenced by the blood flow reaching the affected region. Accurate monitoring of thermal properties in damaged tissue can aid in the timely and effective application of appropriate therapies. Analyzing heat transfer within living tissues is a complex and demanding task due to the diverse internal structures they possess. To describe heat transfer in living tissues, Pennes' bioheat model (Pennes 1948) is employed, which is based on Fourier's law of thermal conduction. Phase change phenomena are observed across a wide range of biological tissues. To address this, researchers have developed adjusted versions of Penne's bioheat models using various numerical techniques found in the literature. Three models, namely GN-III, GN-II, and GN-I, were introduced by Green and Naghdi (Green and Naghdi 1991, Green and Naghdi 1992, Green and Naghdi 1993). The constitutive formulations of the G-N theories have been linearized. GN-I closely resembles the classical coupled thermoelastic theory, while GN-II exhibits the propagation of thermal signals at a finite velocity without energy dissipations. On the other hand, GN-III suggests finite speed propagation with energy dissipations.

The temperature distribution within skin tissues is subject to intricate phenomena, such as blood circulation and metabolic heating generation. As a result, researchers have extended fundamental relationships to incorporate these complexities. These relationships encompass diverse phenomenological mechanisms, including metabolic heat production, thermal conduction, blood perfusion, radiation, and phase changes. The transformation stages of biological tissue can manifest in various forms. The relevant literature offers modified versions of Penne's bioheat models that employ a variety of numerical approaches. Multiple numerical methods are utilized to solve these models, which include the homotopy perturbation technique (Gupta *et al.* 2010), Legendre wavelets Galerkin approaches (Yadav *et al.* 2014, Kumar *et al.* 2015), and the finite element methods (Gupta *et al.* 2013). In their study, Esneault and Dillenseger (2010) utilized finite difference methods to examine the progression of temperature improvement over time, specifically focusing on cases with abnormally low body temperatures. Ghanmi and Abbas (Ghanmi and Abbas 2019) conducted an analytical investigation into the fractional time derivative within skin tissues through thermal therapy. Marin *et al.* (2021) employed the finite element method to analyze the non-linear bio-heat model in skin tissues induced by an external heating source. Hobiny and Abbas (2021) performed an analytical study on the fractional bioheat model in spherical tissues.

To understand the interactions between heat and mechanical effects in anisotropic laser-induced tissue hyperthermia, Fahmy (2019) proposed a novel boundary element model. Diaz *et al.* (2002) utilized the finite element approach to address the thermo-diffusion problem in biological tissues, aiming to model thermal damage. The thermo-elastic behaviors of tissues are governed by generalized thermo-elastic models, such as the G-N model, G-NII model, DPL, and fractional model. Li *et al.* (2018, 2018, 2019) further studied the impact of heat-induced mechanical responses in skin tissues, taking into account temperature-dependent properties. Youssef and Alghamdi (2020) focused on modeling the thermoelastic dual-phase-lag behavior of living tissues exposed to various thermal loads in a one-dimensional setting. Shen *et al.* (2005) employed a thermo-mechanical model to investigate the interactions between skin tissues and high temperatures, examining both thermal and mechanical effects. Kim *et al.* (2016) focused on studying the mechanical and thermal consequences resulting from the absorption of pulsed laser in human skin. In a recent study, (Lata 2019) examined thermomechanical interactions in a transversely isotropic magneto-thermoelastic solid where two different temperatures were considered without factoring in energy dissipation. (Lata and Kaur 2022) examined the effects of

two temperatures and energy dissipation within an axisymmetric isotropic thermoelastic solid modeled using modified couple stress theory. In one study, (Singh and Lata 2023) examined the impact of two temperatures and nonlocal effects on an isotropic thermoelastic thick circular plate, without considering energy dissipation. Xu *et al.* (2008a, 2008b, 2008c, Marin 2010, Othman 2020, Marin 2021, Marin *et al.* 2022) developed a theoretical framework for understanding the interconnected thermomechanical behaviors of the skin, considering it as a layered material. They emphasized that heat-induced stress can contribute to thermal discomfort and adopted a sequentially coupled approach for ease of solution. Zhu *et al.* (2002) explored rate process models of thermal damage and light energy deposition in tissue using diffusion theory. Numerous researchers have attempted to find numerical or analytical solutions to address the challenges posed by linear and nonlinear models of heat transfer when investigating thermal phenomena in finite media (Zenkour and Abbas 2014, Abbas and Kumar 2016, Li *et al.* 2019, Naik and Sayyad 2020, Mohammed and Ismael 2022, Sobhy and Zenkour 2022).

The objective of this study is to develop an analytical methodology to investigate the thermo-mechanical interactions in living tissue without energy dissipation when subjected to rapid heating and exhibiting varying thermal and mechanical properties. By employing the eigenvalues approach with Laplace transform, precise solutions can be obtained for each physical field, enabling the calculation of thermo-elastic responses in living tissues that experience instantaneous heating. The temperature, displacement, and strain changes are depicted through graphical representations.

2. Statement of the problem

In this study, we assume that the skin tissues are uniform, exhibiting linear, isotropic and homogeneous thermoelastic properties. Consequently, the thermoelastic equations governing the behavior of the skin tissue, considering varying thermal properties according to the bioheat conduction model, can be represented as follows (Li *et al.* 2018), assuming the absence of anybody force

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \gamma T_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (1)$$

$$k^* \nabla^2 T = \frac{\partial}{\partial t} \left(\rho c_e \frac{\partial T}{\partial t} + \rho_b \omega_b c_b (T - T_b) + \gamma T_o \frac{\partial u_{i,i}}{\partial x} - Q_m \right), \quad (2)$$

$$\sigma_{ij} = \left(\lambda u_{k,k} - \gamma (T - T_o) \right) \delta_{ij} + \mu (u_{i,j} + u_{j,i}), \quad (3)$$

where T_o the initial temperature of the local tissues, T is the tissues temperature, ρ is the tissue mass density, λ, μ refer to the Lamé's constants, T_b is the blood temperature, ω_b is the blood perfusion rate, ρ_b is the blood mass density, c_b is the blood specific heat, c_e refer to the specific heat at constant strain, u_i are the displacement components, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t refer to the linear thermal expansion coefficient, k^* is the rate of thermal conductivity in these models, e_{ij} are the strain components, t is the time, σ_{ij} are the components of the stress, δ_{ij} is the Kronecker symbol and Q_m is the metabolic heat generation in skin tissue. According to Mitchell *et al.* (1970), it was observed that the production of metabolic heat relies on the temperature of nearby tissues and can be represented as follows

$$Q_m = Q_{m0} \times 2^{\alpha \left(\frac{T - T_o}{10} \right)}, \quad (4)$$

where Q_{mo} denotes the reference metabolic heat source and α is a constant that pertains to metabolic activity. In practical situations, it is commonly acceptable to approximate the generation of metabolic heat as a linear function of the temperature of the surrounding tissues. This can be expressed as

$$Q_m = Q_{mo} \left(1 + \alpha \left(\frac{T - T_o}{10} \right) \right). \quad (5)$$

Within this context, we assume that the skin tissue in a confined domain, with a thickness of h , possesses surface and bottom boundaries. As a result, the displacement components and strain can be given by

$$u_x = u(x, t), u_y = 0, u_z = 0, e = \frac{\partial u}{\partial x}. \quad (6)$$

Hence, the model can be presented as follows

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (7)$$

$$k^* \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial^2 T}{\partial t^2} + \rho_b \omega_b c_b \frac{\partial T}{\partial t} + \gamma T_o \frac{\partial^3 u}{\partial t^2 \partial x} - \frac{\alpha}{10} Q_{mo} \frac{\partial T}{\partial t}, \quad (8)$$

$$\sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma(T - T_o). \quad (9)$$

3. Initial and boundary conditions

To obtain solutions to the equations, it is essential to establish two sets of initial and boundary conditions that correspond to the specifications of the physical model

$$\sigma(x, 0) = 0, \left. \frac{\partial u(x, t)}{\partial t} \right|_{t=0} = 0, T(x, 0) = T_b, \left. \frac{\partial T(x, t)}{\partial t} \right|_{t=0} = 0, \quad (10)$$

$$\sigma(0, t) = 0, k^* \left. \frac{\partial T(x, t)}{\partial x} \right|_{x=0} = -q_o \frac{t(2t_p - t)e^{-\frac{t}{t_p}}}{16t_p^3}, u(h, t) = 0, T(h, t) = 0, \quad (11)$$

where q_o is constant and the parameter t_p represents the characteristic time associated with the pulsating heat flux. To streamline the governing equations, we will employ the subsequent non-dimensional variables to facilitate the computations

$$(x', u') = \frac{(x, u)}{L}, T' = \frac{T - T_o}{T_o}, \sigma' = \frac{\sigma}{\lambda + 2\mu}, (t', t'_p) = \frac{v}{L}(t, t_p), \quad (12)$$

where $v = \sqrt{\frac{k^*}{\rho c}}$. The governing equations can be expressed using non-dimensional parameters (12) in their dimensionless form after removing the dashes

$$\frac{\partial^2 u}{\partial x'^2} = \epsilon_1 \frac{\partial^2 u}{\partial t'^2} + \epsilon_2 \frac{\partial T}{\partial x'}, \quad (13)$$

$$\frac{\partial^2 T}{\partial x'^2} = \frac{\partial^2 T}{\partial t'^2} + \epsilon_3 \frac{\partial T}{\partial t'} + \epsilon_4 \frac{\partial^3 u}{\partial t'^2 \partial x'}, \quad (14)$$

$$\sigma = \frac{\partial u}{\partial x'} - b_2 T, \quad (15)$$

$$u(x, 0) = 0, \left. \frac{\partial u(x,t)}{\partial t} \right|_{t=0} = 0, T(x, 0) = 0, \left. \frac{\partial T(x,t)}{\partial t} \right|_{t=0} = 0, \quad (16)$$

$$\sigma(0, t) = 0, \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=0} = -\frac{q_0 v}{T_0 k^*} \frac{t(2t_p - t)e^{-\frac{t}{t_p}}}{16t_p^3}, u(h, t) = 0, T(h, t) = 0, \quad (17)$$

where $\epsilon_1 = \frac{\rho v^2}{\lambda + 2\mu}$, $\epsilon_2 = \frac{T_0 \gamma_e}{\lambda + 2\mu}$, $\epsilon_3 = \frac{Lv}{k^*} \left(\rho_b \omega_b c_b - \frac{\alpha}{10} Q_{m0} \right)$, $\epsilon_4 = \frac{\gamma_e}{\rho c_e}$.

Through the utilization of Laplace transforms, Eqs. (13) to (17) can be transformed

$$\bar{g}(x, s) = L[g(x, t)] = \int_0^\infty g(x, t) e^{-st} dt. \quad (18)$$

Hence, we can obtain the following equations

$$\frac{d^2 \bar{u}}{dx^2} = s^2 \epsilon_1 \bar{u} + \epsilon_2 \frac{d\bar{T}}{dx}, \quad (19)$$

$$\frac{d^2 \bar{T}}{dx^2} = s(s + \epsilon_3) \bar{T} + s^2 \epsilon_4 \frac{d\bar{u}}{dx}, \quad (20)$$

$$\bar{\sigma} = \frac{d\bar{u}}{dx} - \epsilon_2 \bar{T}, \quad (21)$$

$$\bar{\sigma}(0, s) = 0, \left. \frac{d\bar{T}(x,s)}{dx} \right|_{x=0} = -\frac{q_0 v s t_p}{8T_0 k^* (1 + s t_p)^3}, \bar{u}(L, s) = 0, \bar{T}(L, s) = 0. \quad (22)$$

Using Eqs. (19) and (20), we can express the vector-matrix differential equation by

$$\frac{dV}{dx} = BV, \quad (23)$$

$$\text{where } V = \begin{pmatrix} \bar{u} \\ \bar{T} \\ \frac{d\bar{u}}{dx} \\ \frac{d\bar{T}}{dx} \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ s^2 \epsilon_1 & 0 & 0 & \epsilon_2 \\ 0 & s(s + \epsilon_3) & s^2 \epsilon_4 & 0 \end{pmatrix}.$$

To solve Eq. (23), we can utilize the eigenvalue techniques outlined in (Das *et al.* 1997, Baksi *et al.* 2006, Santra *et al.* 2014, Abbas *et al.* 2016, Gupta and Das 2016, Kumar *et al.* 2016, Kumar *et al.* 2017, Hobiny and Abbas 2019, Abbas *et al.* 2020) to obtain the characteristic relation of matrix B .

$$\omega^4 - (s^2 \epsilon_1 + s(s + \epsilon_3) + s^2 \epsilon_2 \epsilon_4) \omega^2 + s^3 \epsilon_1 (s + \epsilon_3) = 0, \quad (24)$$

The roots of Eq. (24), denoted as $\pm \omega_1$ and $\pm \omega_2$, correspond to the eigenvalues of matrix B . It is worth noting that the terms in Eq. (24) consist of function of the Laplace parameters s . Consequently, the general solution can be written as follows

$$V(x, s) = A_1 X_1 e^{-\omega_1 x} + A_2 X_2 e^{\omega_1 x} + A_3 X_3 e^{-\omega_2 x} + A_4 X_4 e^{\omega_2 x}. \quad (25)$$

Therefore, within the Laplace domain, the overall solutions for displacement, temperature, and strain can be described as follows

$$\bar{u}(x, s) = B_1 U_1 e^{-\omega_1 x} + B_2 U_2 e^{\omega_1 x} + B_3 U_3 e^{-\omega_2 x} + B_4 U_4 e^{\omega_2 x}. \quad (26)$$

$$\bar{T}(x, s) = B_1 T_1 e^{-\omega_1 x} + B_2 T_2 e^{\omega_1 x} + B_3 T_3 e^{-\omega_2 x} + B_4 T_4 e^{\omega_2 x}. \quad (27)$$

$$\bar{e} = -\omega_1 U_1 B_1 e^{-\omega_1 x} + \omega_1 U_2 B_2 e^{\omega_1 x} - \omega_2 U_3 B_3 e^{-\omega_2 x} + \omega_2 U_4 B_4 e^{\omega_2 x}, \quad (28)$$

here, T_i and U_i represent the eigenvector of temperature and displacement, respectively. The values

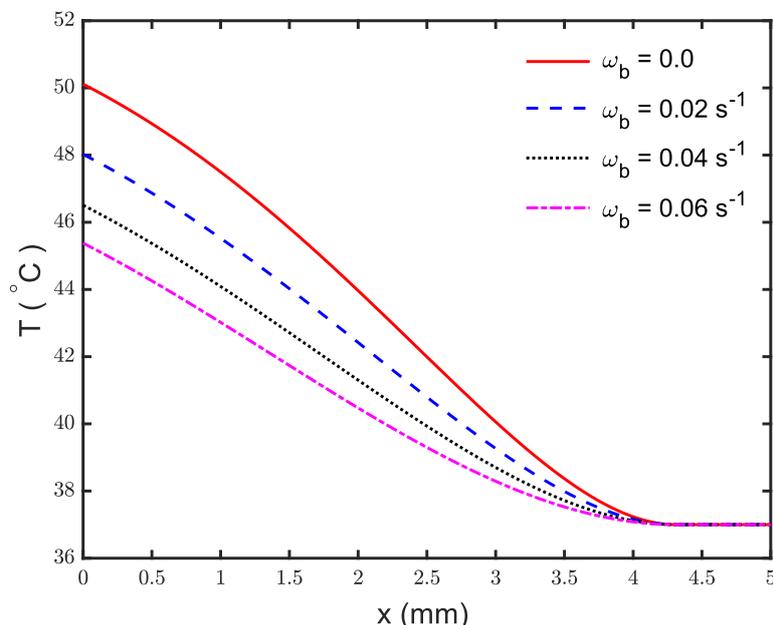


Fig. 1 The effect of blood perfusion rate ω_b in the temperature variations T when $t_p = 32$ s

of B_1, B_2, B_3 and B_4 can be determined by utilizing the boundary conditions of the problem. To obtain the final solutions for temperature, displacement and strain distribution, the Stehfest method (Stehfest 1970) is employed as a numerical inversion strategy.

4. Numerical results and discussions

In order to illustrate the theoretical outcomes discussed in the preceding parts, we present the calculated numerical values of the physical constants. The material constants for skin tissue at the reference temperature are computed as demonstrated below (Li *et al.* 2018)

$$\begin{aligned} \rho_b &= 1060 \text{ (kg)(m}^{-3}\text{)}, c_b = 3770 \text{ (J)(kg}^{-1}\text{)(k}^{-1}\text{)}, \mu = 3.446 \times 10^7 \text{ (N)(m}^{-2}\text{)}, \\ K &= 0.235 \text{ (W)(m}^{-1}\text{)(k}^{-1}\text{)}, \rho = 1190 \text{ (kg)(m}^{-3}\text{)}, c_e = 3600 \text{ (J)(kg}^{-1}\text{)(k}^{-1}\text{)}, \\ \lambda &= 8.27 \times 10^8 \text{ (N)(m}^{-2}\text{)}, Q_m = 1.19 \times 10^3 \text{ (W)(m}^{-3}\text{)}, \alpha_t = 1 \times 10^{-4} \text{ (k}^{-1}\text{)}, T_o = 310 \text{ (k)}. \end{aligned}$$

Using the same set of parameters as before, the computed numerical values for physical quantities under the generalized biothermoelastic model without energy dissipation are presented below. These results are illustrated in Fig. 1-6. At time $t = 40$ s, numerical computations were conducted to determine the variations in displacement, temperature and strain along the distance x . These quantities were evaluated for several values of the studied parameters, as illustrated in Figs. 1-6. The temperature variation in relation to the distance x are depicted in Figs. 1 and 4. The results demonstrate that the temperature initially peaks at the skin surface ($x = 0$) as a result of the exponentially diminishing pulse boundary heating flux. As the distance x continues to increase, the temperature gradually diminishes until it reaches nearly zero. Figs. 2 and 5 show the displacement variations in relation to the distance x . The data clearly shows that the displacement magnitudes begin at their maximum values and progressively diminish as the distance x increases, eventually

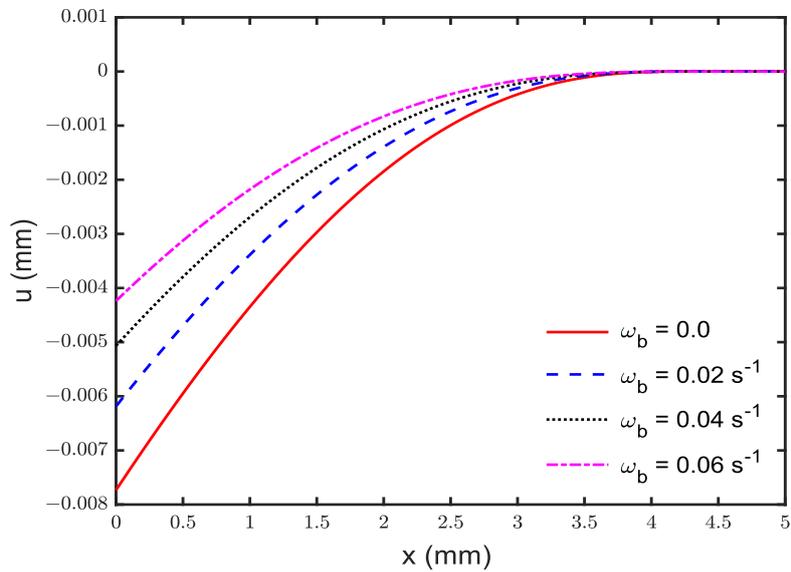


Fig. 2 The effect of blood perfusion rate ω_b in the displacement variation u when $t_p = 32$ s

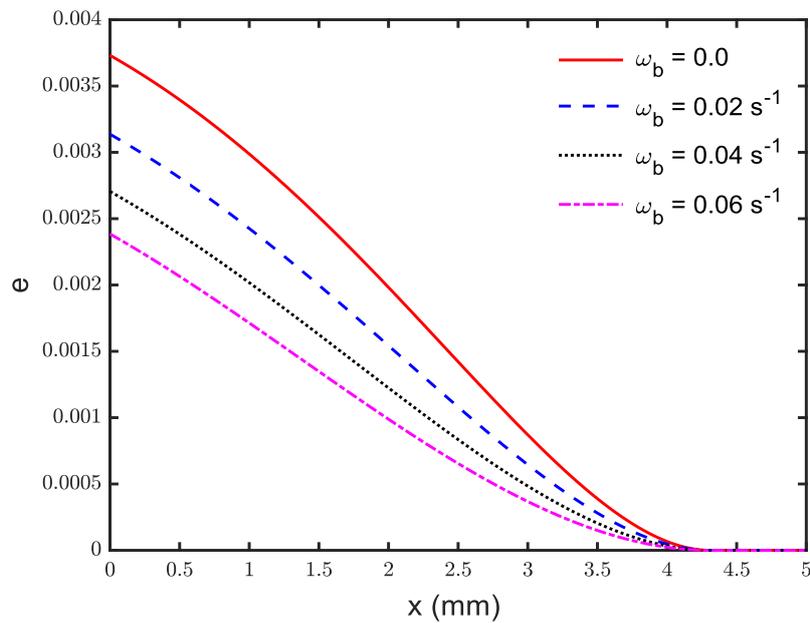


Fig. 3 The effect of blood perfusion rate ω_b in the strain variations e when $t_p = 32$ s

approaching zero. The variations of strain with respect to the distance x are illustrated in Figs. 3 and 6. It is clear from the data that the strain initially reaches its maximum values and then gradually decreases until it approaches zero. The first group of Figures, namely Figs. 1, 2, and 3, portray the variations of temperature, displacement, and strain, respectively, under different blood perfusion rates. The data clearly demonstrates that the blood perfusion rate has a notable impact on

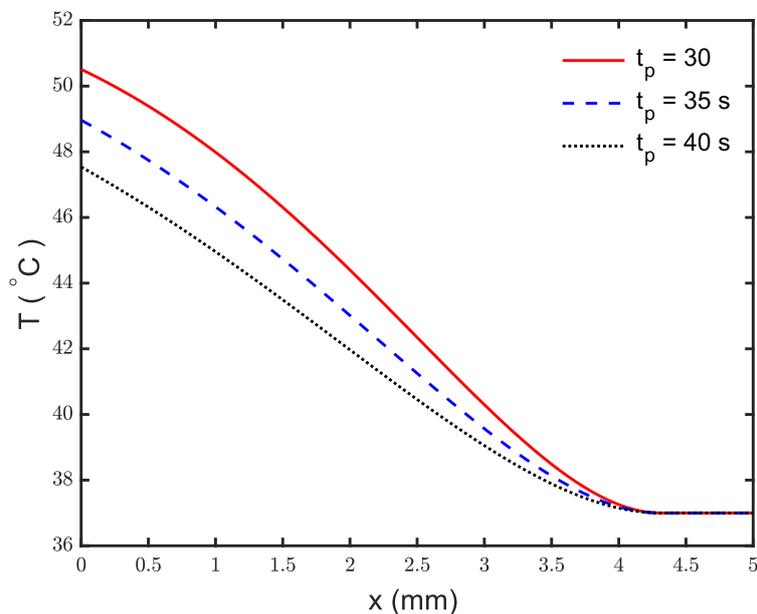


Fig. 4 The temperature variations T for different values of characteristic time of pulsing heat flux t_p

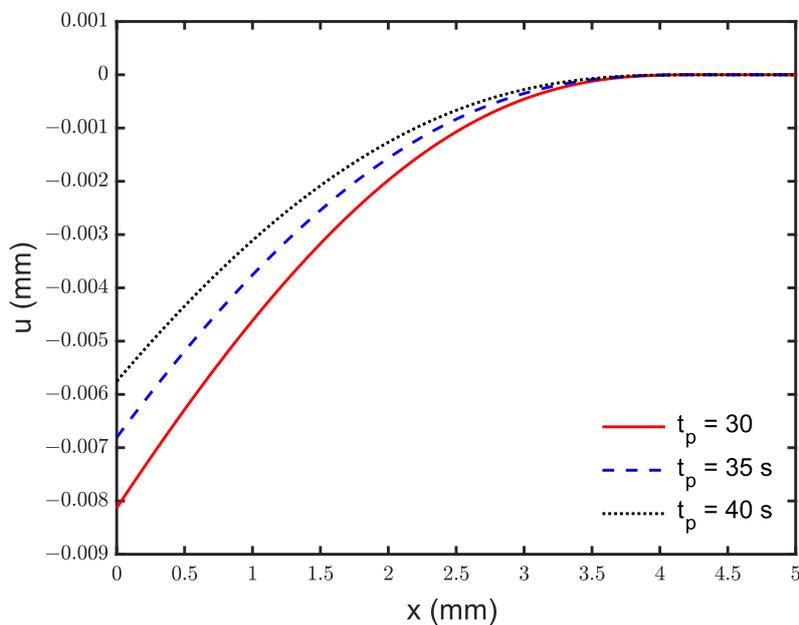


Fig. 5 The displacement variation u for various values of characteristic time of pulsing heat flux t_p

the studied variables. With an increase in the blood perfusion rate, the maximum amplitudes of temperature, displacement, and strain decrease. This suggests that higher blood perfusion rates facilitate greater convective heat loss due to faster blood flow, leading to reduced magnitudes of temperature, displacement, and strain. The second group of Figures, namely Figs. 4, 5, and 6,

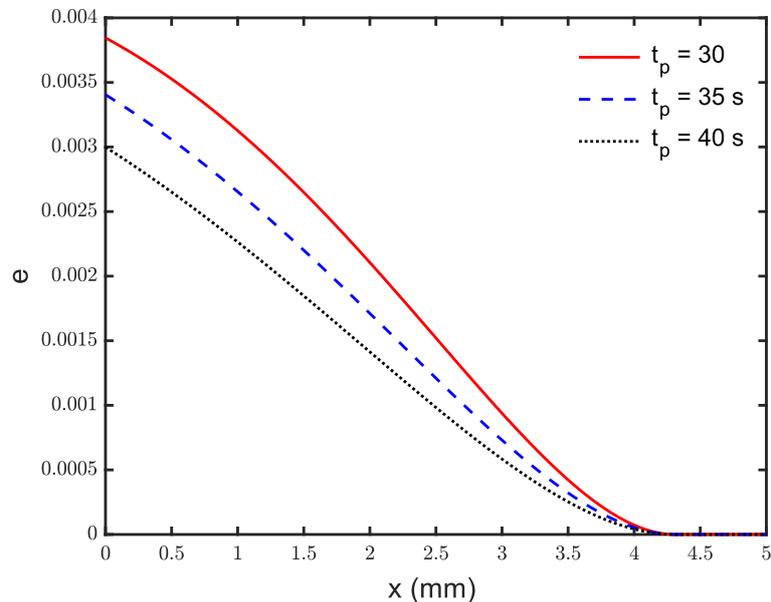


Fig. 6 The strain variation e for various values of characteristic time of pulsing heat flux t_p

depict the variations of temperature, displacement, and strain, respectively, under different characteristic times of pulsing heat flux. The data clearly shows that the characteristic time of the pulsing heat flux has a significant influence on the studied variables. As the characteristic time increases, the maximum amplitudes of temperature, displacement, and strain decrease. This suggests that the characteristic time of the pulsing heat flux tends to attenuate the effects of thermomechanical propagation.

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References

- Abbas, I., Hobiny, A. and Marin, M. (2020), "Photo-thermal interactions in a semi-conductor material with cylindrical cavities and variable thermal conductivity", *J. Taibah Univ. Sci.*, **14**(1), 1369-1376. <https://doi.org/10.1080/16583655.2020.1824465>.
- Abbas, I.A. and Kumar, R. (2016), "2D deformation in initially stressed thermoelastic half-space with voids", *Steel Compos. Struct.*, **20**(5), 1103-1117. <https://doi.org/10.12989/scs.2016.20.5.1103>.
- Abbas, I.A., Abdalla, A.E.N.N., Alzahrani, F.S. and Spagnuolo, M. (2016), "Wave propagation in a generalized thermoelastic plate using eigenvalue approach", *J. Therm. Stress.*, **39**(11), 1367-1377. <https://doi.org/10.1080/01495739.2016.1218229>.
- Baksi, A., Roy, B.K. and Bera, R.K. (2006), "Eigenvalue approach to study the effect of rotation and relaxation time in generalized magneto-thermo-viscoelastic medium in one dimension", *Math. Comput.*

- Model.*, **44**(11-12), 1069-1079. <https://doi.org/10.1016/j.mcm.2006.03.010>.
- Das, N.C., Lahiri, A. and Giri, R.R. (1997), "Eigenvalue approach to generalized thermoelasticity", *Ind. J. Pure Appl. Math.*, **28**(12), 1573-1594.
- Díaz, S.H., Nelson, J.S. and Wong, B.J. (2002), "Rate process analysis of thermal damage in cartilage", *Phys. Medic. Biol.*, **48**(1), 19. <https://doi.org/10.1088/0031-9155/48/1/302>.
- Dillenseger, J.L. and Esneault, S. (2010), "Fast FFT-based bioheat transfer equation computation", *Comput. Biol. Medic.*, **40**(2), 119-123. <https://doi.org/10.1016/j.combiomed.2009.11.008>.
- Fahmy, M.A. (2019), "Boundary element modeling and simulation of biotermomechanical behavior in anisotropic laser-induced tissue hyperthermia", *Eng. Anal. Bound. Elem.*, **101**, 156-164. <https://doi.org/10.1016/j.enganabound.2019.01.006>.
- Gabay, I., Abergel, A., Vasilyev, T., Rabi, Y., Fliss, D.M. and Katzir, A. (2011), "Temperature-controlled two-wavelength laser soldering of tissues", *Laser. Surgery Medic.*, **43**(9), 907-913. <https://doi.org/10.1002/lsm.21123>
- Ghanmi, A. and Abbas, I.A. (2019), "An analytical study on the fractional transient heating within the skin tissue during the thermal therapy", *J. Therm. Biol.*, **82**, 229-233. <https://doi.org/10.1016/j.jtherbio.2019.04.003>.
- Green, A. and Naghdi, P. (1992), "On undamped heat waves in an elastic solid", *J. Therm. Stress.*, **15**(2), 253-264. <https://doi.org/10.1080/01495739208946136>.
- Green, A. and Naghdi, P. (1993), "Thermoelasticity without energy dissipation", *J. Elastic.*, **31**(3), 189-208. <https://doi.org/10.1007/BF00044969>.
- Green, A.E. and Naghdi, P.M. (1991), "A re-examination of the basic postulates of thermomechanics", *Proc. Roy. Soc. London. Ser. A: Math. Phys. Sci.*, **432**(1885), 171-194. <https://doi.org/10.1098/rspa.1991.0012>.
- Gupta, N.D. and Das, N.C. (2016), "Eigenvalue approach to fractional order generalized thermoelasticity with line heat source in an infinite medium", *J. Therm. Stress.*, **39**(8), 977-990. <https://doi.org/10.1080/01495739.2016.1187987>.
- Gupta, P.K., Singh, J. and Rai, K. (2010), "Numerical simulation for heat transfer in tissues during thermal therapy", *J. Therm. Biol.*, **35**(6), 295-301. <https://doi.org/10.1016/j.jtherbio.2010.06.007>.
- Gupta, P.K., Singh, J., Rai, K. and Rai, S. (2013), "Solution of the heat transfer problem in tissues during hyperthermia by finite difference–decomposition method", *Appl. Math. Comput.*, **219**(12), 6882-6892. <https://doi.org/10.1016/j.amc.2013.01.020>.
- Hobiny, A. and Abbas, I. (2019), "A GN model on photothermal interactions in a two-dimensions semiconductor half space", *Result. Phys.*, **15**, 102588. <https://doi.org/10.1016/j.rinp.2019.102588>.
- Hobiny, A. and Abbas, I. (2021), "Analytical solutions of fractional bioheat model in a spherical tissue", *Mech. Bas. Des. Struct. Mach.*, **49**(3), 430-439. <https://doi.org/10.1080/15397734.2019.1702055>.
- Kim, J.Y., Jang, K., Yang, S.J., Baek, J.H., Park, J.R., Yeom, D.I., ... & Chung, S.C. (2016), "Simulation study of the thermal and the thermoelastic effects induced by pulsed laser absorption in human skin", *J. Korean Phys. Soc.*, **68**, 979-988. <https://doi.org/10.3938/jkps.68.979>.
- Kumar, P., Kumar, D. and Rai, K. (2015), "A numerical study on dual-phase-lag model of bio-heat transfer during hyperthermia treatment", *J. Therm. Biol.*, **49**, 98-105. <https://doi.org/10.1016/j.jtherbio.2015.02.008>.
- Kumar, R., Miglani, A. and Rani, R. (2016), "Analysis of micropolar porous thermoelastic circular plate by eigenvalue approach", *Arch. Mech.*, **68**(6), 423-439.
- Kumar, R., Miglani, A. and Rani, R. (2017), "Eigenvalue formulation to micropolar porous thermoelastic circular plate using dual phase lag model", *Multidisc. Model. Mater. Struct.*, **13**(2), 347-362. <https://doi.org/10.1108/mmms-08-2016-0038>.
- Lata, P. (2019), "Thermomechanical interactions in transversely isotropic magneto thermoelastic solid with two temperatures and without energy dissipation", *Steel Compos. Struct.*, **32**(6), 779-793. <https://doi.org/10.12989/scs.2019.32.6.779>.
- Lata, P. and Kaur, H. (2022), "Effect of two temperature and energy dissipation in an axisymmetric modified couple stress isotropic thermoelastic solid", *Couple. Syst. Mech.*, **11**(3), 199-215. <https://doi.org/10.12989/csm.2022.11.3.199>.

- Li, X., Li, C., Xue, Z. and Tian, X. (2018), "Analytical study of transient thermo-mechanical responses of dual-layer skin tissue with variable thermal material properties", *Int. J. Therm. Sci.*, **124**, 459-466. <https://doi.org/10.1016/j.ijthermalsci.2017.11.002>.
- Li, X., Li, C., Xue, Z. and Tian, X. (2019), "Investigation of transient thermo-mechanical responses on the triple-layered skin tissue with temperature dependent blood perfusion rate", *Int. J. Therm. Sci.*, **139**, 339-349. <https://doi.org/10.1016/j.ijthermalsci.2019.02.022>.
- Li, X., Qin, Q.H. and Tian, X. (2019), "Thermomechanical response of porous biological tissue based on local thermal non-equilibrium", *J. Therm. Stress.*, **42**(12), 1481-1498. <https://doi.org/10.1080/01495739.2019.1660599>.
- Li, X., Xue, Z. and Tian, X. (2018), "A modified fractional order generalized bio-thermoelastic theory with temperature-dependent thermal material properties", *Int. J. Therm. Sci.*, **132**, 249-256. <https://doi.org/10.1016/j.ijthermalsci.2018.06.007>.
- Mahjoob, S. and Vafai, K. (2009), "Analytical characterization of heat transport through biological media incorporating hyperthermia treatment", *Int. J. Heat Mass Transf.*, **52**(5-6), 1608-1618. <https://doi.org/10.1016/j.ijheatmasstransfer.2008.07.038>.
- Marin, M. (2010), "Some estimates on vibrations in thermoelasticity of dipolar bodies", *J. Vib. Control*, **16**(1), 33-47. <https://doi.org/10.1177/1077546309103419>.
- Marin, M., Hobiny, A. and Abbas, I. (2021), "Finite element analysis of nonlinear bioheat model in skin tissue due to external thermal sources", *Math.*, **9**(13), 1459. <https://doi.org/10.3390/math9131459>.
- Marin, M., Hobiny, A. and Abbas, I. (2021), "The effects of fractional time derivatives in prothermoelastic materials using finite element method", *Math.*, **9**(14), 1606. <https://doi.org/10.3390/math9141606>.
- Marin, M., Seadawy, A., Vlase, S. and Chirila, A. (2022), "On mixed problem in thermoelasticity of type III for Cosserat media", *J. Taibah Univ. Sci.*, **16**(1), 1264-1274. <https://doi.org/10.1080/16583655.2022.2160290>.
- Mitchell, J.W., Galvez, T.L., Hengle, J., Myers, G.E. and Siebecker, K.L. (1970), "Thermal response of human legs during cooling", *J. Appl. Physiol.*, **29**(6), 859-865. <https://doi.org/10.1152/jappl.1970.29.6.859>.
- Mohammed, B.N. and Ismael, D.S. (2022), "A computational model for temperature monitoring during human liver treatment by Nd: Yag Laser Interstitial Thermal Therapy (LITT)", *Aro-Scientif. J. Koya Univ.*, **10**(2), 38-44. <http://doi.org/10.14500/aro.10949>.
- Naik, N.S. and Sayyad, A.S. (2020), "1D thermal analysis of laminated composite and sandwich plates using a new fifth order shear and normal deformation theory", *Mater. Today: Proc.*, **21**, 1084-1088. <https://doi.org/10.1016/j.matpr.2020.01.009>.
- Othman, M.I., Fekry, M. and Marin, M. (2020), "Plane waves in generalized magneto-thermo-viscoelastic medium with voids under the effect of initial stress and laser pulse heating", *Struct. Eng. Mech.*, **73**(6), 621-629. <https://doi.org/10.12989/sem.2020.73.6.621>.
- Pennes, H.H. (1948), "Analysis of tissue and arterial blood temperatures in the resting human forearm", *J. Appl. Physiol.*, **1**(2), 93-122. <https://doi.org/10.1152/jappl.1948.1.2.93>.
- Santra, S., Lahiri, A. and Das, N.C. (2014), "Eigenvalue approach on thermoelastic interactions in an infinite elastic solid with voids", *J. Therm. Stress.*, **37**(4), 440-454. <https://doi.org/10.1080/01495739.2013.870854>.
- Shen, W., Zhang, J. and Yang, F. (2005), "Modeling and numerical simulation of bioheat transfer and biomechanics in soft tissue", *Math. Comput. Model.*, **41**(11-12), 1251-1265. <https://doi.org/10.1016/j.mcm.2004.09.006>.
- Singh, S. and Lata, P. (2023), "Effect of two temperature and nonlocality in an isotropic thermoelastic thick circular plate without energy dissipation", *Part. Diff. Equ. Appl. Math.*, **7**, 100512. <https://doi.org/10.1016/j.padiff.2023.100512>.
- Sobhy, M. and Zenkour, A.M. (2022), "Refined lord-shulman theory for 1D response of skin tissue under ramp-type heat", *Mater. (Basel)*, **15**(18), 6292. <https://doi.org/10.3390/ma15186292>.
- Stehfest, H. (1970), "Algorithm 368: Numerical inversion of Laplace transforms [D5]", *Commun. ACM*, **13**(1), 47-49. <https://doi.org/10.1145/361953.361969>.

- Xu, F., Lu, T.J. and Seffen, K.A. (2008a), "Biothermomechanics of skin tissues", *J. Mech. Phys. Solid.*, **56**(5), 1852-1884. <https://doi.org/10.1016/j.jmps.2007.11.011>.
- Xu, F., Seffen, K.A. and Lu, T.J. (2008b), "Non-Fourier analysis of skin biothermomechanics", *Int. J. Heat Mass Transf.*, **51**(9-10), 2237-2259. <https://doi.org/10.1016/j.ijheatmasstransfer.2007.10.024>.
- Xu, F., Wen, T., Lu, T.J. and Seffen, K.A. (2008c), "Skin biothermomechanics for medical treatments", *J. Mech. Behav. Biomed. Mater.*, **1**(2), 172-187. <https://doi.org/10.1016/j.jmbbm.2007.09.001>.
- Yadav, S., Kumar, D. and Rai, K.N. (2014), "Finite element legendre wavelet Galerkin approach to inward solidification in simple body under most generalized boundary condition", *Zeitschrift für Naturforschung A*, **69**(10-11), 501-510. <https://doi.org/10.5560/zna.2014-0052>.
- Youssef, H.M. and Alghamdi, N.A. (2020), "Modeling of one-dimensional thermoelastic dual-phase-lag skin tissue subjected to different types of thermal loading", *Sci. Rep.*, **10**(1), 3399. <https://doi.org/10.1038/s41598-020-60342-6>.
- Zenkour, A.M. and Abbas, I.A. (2014), "Nonlinear transient thermal stress analysis of temperature-dependent hollow cylinders using a finite element model", *Int. J. Struct. Stab. Dyn.*, **14**(7), <https://doi.org/10.1142/S0219455414500254>.
- Zhou, J., Chen, J. and Zhang, Y. (2009), "Dual-phase lag effects on thermal damage to biological tissues caused by laser irradiation", *Comput. Biol. Medic.*, **39**(3), 286-293. <https://doi.org/10.1016/j.combiomed.2009.01.002>.
- Zhu, D., Luo, Q., Zhu, G. and Liu, W. (2002), "Kinetic thermal response and damage in laser coagulation of tissue", *Laser. Surgery Medic.*, **31**(5), 313-321. <https://doi.org/10.1002/lsm.10108>.