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Dynamic analysis of a laminated composite beam under harmonic load

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Abstract. Dynamic responses of a laminated composite cantilever beam under a harmonic are investigated in this study. The governing equations of problem are derived by using the Lagrange procedure. The Timoshenko beam theory is considered and the Ritz method is implemented in the solution of the problem. The algebraic polynomials are used with the trivial functions for the Ritz method. In the solution of dynamic problem, the Newmark average acceleration method is used in the time history. In the numerical examples, the effects of load parameter, the fiber orientation angles and stacking sequence of laminas on the dynamic responses of the laminated beam are investigated.

Keywords: dynamic analysis; laminated composite beams; Ritz method

1. Introduction

Laminated composite structures have been used in a lot of engineering fields such as civil, mechanical and aerospace engineering projects because of higher strength and low density properties. With the development of production technology, the using of laminated composite has been increasing in engineering applications. In the laminated composite structures, the moving-load dynamic problems are very important topic as such in other structures elements. As is known, the mechanical results of dynamic loads are bigger than those of static loadings. Also, instantaneous failures and local cracks can be occurred affected by dynamically loads in contrast with other load types. So, learn to dynamical behavior of laminated composites are very important for their design in the dynamic load problems.

In the literature, many researchers investigated the static, dynamic and stability analyses of laminated structures (Bozyiğit *et al*. 2020a, 2020b, 2020c, Phung-Van *et al*. 2017, 2019a, 2019b, Gillich *et al*. 2016, Thanh *et al*. 2019, Nguyen 2017, Al-Furjan *et al*. 2020a, 2020b, 2020c, Shariati *et al*. 2020, Alimirzaei *et al*. 2019, Bourada *et al*. 2020, Bousahla *et al*. 2020, Semmah *et al*. 2019, Belbachir *et al*. 2019, 2020, Draoui *et al*. 2019). Some investigations of dynamics of laminated and like composites are presented as follows; Wang *et al*. (2005) studied dynamics of the cracked fiber reinforced composites beams with bending and torsion effects. Palanivel (2006) performed the free vibration analysis of laminated composite beams by using two high-order shear deformation theory and finite elements method. Zenkour *et al*. (2010) investigated bending results of functionally

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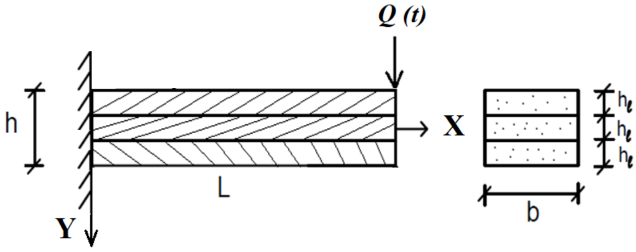

Fig. 1 A cantilever laminated composite beam under a dynamic point load at free end

graded viscoelastic sandwich beams embedded elastic foundation. Eltaher *et al*. (2012) presented free vibration analysis of functionally graded nanobeams based on nonlocal elasticity theory by using finite element method. DeValve and Pitchumani (2014) investigated damping vibration analysis of rotating composite beams with embedded carbon nanotubes. Bahmyari *et al*. (2014) analysed the dynamics of laminated beams under distributed moving masses by using finite element method and first shear deformation theory. Tornabene *et al*. (2014) investigated static and vibration analysis of laminated doubly-curved shells and panels embedded in elastic foundation bycusing the generalized differential quadrature. Mohanty *et al*. (2015) investigated dynamic responses of functionally graded pre-twisted beams by using Timoshenko beam theory and finite element method. Akbaş (2014, 2015a, 2015b, 2017a, 2018c, 2018h, 2018i, 2019a) presented dynamic analysis of functionally graded beams with different mechanical cases. Hadji *et al*. (2017) analyzed wave propagation of functionally graded beams with higher order shear deformation theory. Akbaş (2013, 2017b, 2018a, 2018d, 2018f, 2018g, 2019b, 2019f, 2019g) investigated geometrically nonlinear analysis of composite beams such as functionally graded, laminated composites by using finite element method. Li *et al*. (2018) investigated nonlinear dynamics of laminated beams under both blast and thermal loads. Ghayesh (2018) analyzed forced nonlinear vibration of axially functionally graded micro beams by using coupled stress theory. Draiche *et al*. (2019) presented static analysis of laminated reinforced composite plates based on first-order shear deformation theory by using the Navier method. Akbaş (2017c, 2018b, 2018e, 2019c, 2019d, 2019e) presented post-buckling, stability behavior of composite structures with functionally graded and laminated materials. Yaylı (2019) presented free lateral vibration behavior of a functionally graded nanobeam in an elastic matrix with rotationally restrained ends.

The aim of this paper is to investigate dynamic analysis of a laminated beam under harmonic load by using Ritz method. The governing equations of problem are obtained by using the Lagrange procedure. In the solution of the forced vibration problem, the Newmark average acceleration method is used in the time history. The main purpose of this paper is to investigate the effects of fibre orientation angles and stacking sequence of laminas on the dynamic responses of the laminated composite beam under harmonic load in detail.

2. Theory and formulation

A cantilever laminated composite beam under a dynamic point load $Q(t)$ at free end with the length L, the height h and width b with three layers. Three identical laminas are considered in the laminated beam. The dynamic point load $Q(t)$ is assumed to be sinusoidal harmonic in time domain as following

$$Q(t) = Q_0 \sin(\overline{\omega} t), \qquad 0 \leq t \ll \infty \tag{1}$$

Where, Q_0 and $\overline{\omega}$ indicate the amplitude and the frequency of the dynamic load.

The axial strain (ε_z) and shear strain (γ_{zy}) are given according to the Timoshenko beam theory as follows

$$\varepsilon_z = \frac{\partial u_0}{\partial z} - Y\frac{\partial \emptyset}{\partial z} \tag{2a}$$

$$\gamma_{zy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial z} \tag{2b}$$

where,u_0, v_0 and $\emptyset$ are axial displacement, vertical displacement and rotation, respectively. The equivalent Young's modulus of *i*th layer for *Z* direction (E_z^i) is used considered as following model with temperature function (Vinson and Sierakowski 2008)

$$\frac{1}{E_x^i(T)} = \frac{cos^4(\theta_i)}{E_{11}(T)} + \left(\frac{1}{G_{12}(T)} - \frac{2v_{12}}{E_{11}(T)}\right) cos^2(\theta_i)\, sin^2(\theta_i) + \frac{sin^4(\theta_i)}{E_{22}(T)} \tag{3}$$

where, E_{11} and E_{22} are the Young's modulus in Z and Y directions, respectively. G_{12}, G_{13}, G_{23} indicate the shear modulus. θ is the fiber orientation angle, $m=cos\,\theta$ and $n=sin\,\theta$. The constitutive equations are presented as follows;

$$\sigma_{xx} = E_x^i(T)\left(\frac{\partial \mathrm{u}(X,t)}{\partial X} - Y\frac{\partial \emptyset(x,t)}{\partial X}\right) \tag{4a}$$

$$\tau_{xy} = k_s\, G_{12}{}_x^i(T)\left(\frac{\partial v}{\partial \mathrm{x}} - \emptyset(x,t)\right) \tag{4b}$$

where, σ_{xx} and τ_{xy} are the normal and shear stresses, respectively. k_s is the shear correction factor. The strain energy (U), the kinetic energy (K) and potential energy of the external loads (U_e) are presented as follows

$$U = \frac{1}{2}\int_0^L \left(A_0\left(\frac{\partial u}{\partial X}\right)^2 - 2B_0\left(\frac{\partial u}{\partial X}\right)\left(\frac{\partial \emptyset}{\partial X}\right) + D_0\left(\frac{\partial \emptyset}{\partial X}\right)^2 + A_5\left(\frac{\partial v}{\partial \mathrm{x}} - \emptyset(x,t)\right)^2\right) dX \tag{5a}$$

$$K = \frac{1}{2}\int_0^L \left(I_0\left(\frac{\partial u}{\partial t}\right)^2 - 2I_1\left(\frac{\partial u}{\partial t}\right)\left(\frac{\partial \emptyset}{\partial t}\right) + I_2\left(\frac{\partial \emptyset}{\partial t}\right)^2 + I_0\left(\frac{\partial v}{\partial t}\right)^2\right) dX \tag{5b}$$

$$U_e = -Q(t)\, v(z_p, t) \tag{5c}$$

where, $\mathrm{A_0}$, $\mathrm{B_0}$, $\mathrm{D_0}$, $\mathrm{A_5}$, A_{XT}, A_{YT}, $\mathrm{I_0}$, $\mathrm{I_1}$, $\mathrm{I_2}$ are given as follows;

$$\mathrm{A_0} = \sum_{i=1}^{n} \mathrm{bE_x^i(y_{i+1} - y_i\ )} \tag{6a}$$

$$\mathrm{B_0} = \frac{1}{2}\sum_{i=1}^{n} \mathrm{bE_x^i(y_{i+1}^2 - y_i^2)} \tag{6b}$$

$$\mathrm{D_0} = \frac{1}{3}\sum_{i=1}^{n} \mathrm{bE_x^i(y_{i+1}^3 - y_i^3)} \tag{6c}$$

$$\mathrm{A_{55}} = \frac{5}{4}\sum_{i=1}^{n} \mathrm{bQ_{55}^i(z_{i+1} - z_i\ } - \frac{4}{3\mathrm{h}^2}\left(\mathrm{y_{i+1}^3 - y_i^3}\right)) \tag{6d}$$

$$\mathrm{I_0} = \sum_{i=1}^{n} \mathrm{b}\rho^{\mathrm{i}}(\mathrm{y_{i+1} - y_i\ }) \tag{6e}$$

$$\mathrm{I_1} = \frac{1}{2}\sum_{i=1}^{n} \mathrm{b}\rho^{\mathrm{i}}(\mathrm{y_{i+1}^2 - y_i^2}) \tag{6f}$$

$$\mathrm{I}_2 = \frac{1}{3}\sum_{i=1}^{n} b\rho^{i}(y_{i+1}^3 - y_i^3) \tag{6g}$$

where n is number of layers, ρ^{i} is the mass density of ith layer and Q_{55}^{k} is given below

$$Q_{55}^{i} = G_{13}(T)\cos^2(\theta_i) + G_{23}(T)\sin^2(\theta_i) \tag{7}$$

The Lagrangian functional of the problem is presented as follows

$$I = K - (U_i + U_e) \tag{8}$$

In the solution of the problem in Ritz method, approximate solution is given as a series of i terms of the following form

$$u_0(z,t) = \sum_{i=1}^{\infty} a_i\,(t)\alpha_i(z) \tag{9a}$$

$$v_0(z,t) = \sum_{i=1}^{\infty} b_i\,(t)\beta_i(z) \tag{9b}$$

$$\emptyset(z,t) = \sum_{i=1}^{\infty} c_i\,(t)\gamma_i(z) \tag{9c}$$

where a_i, b_i and c_i are the unknown coefficients, $\alpha_i(z,t)$, $\beta_i(z,t)$, $\gamma_i(z,t)$ are the coordinate functions depend on the boundary conditions over the interval $[0,L]$. The coordinate functions for the cantilever beam are given as algebraic polynomials

$$\alpha_i(z) = z^{i} \tag{10a}$$

$$\beta_i(z) = z^{(i+1)} \tag{10b}$$

$$\gamma_i(z) = z^{i} \tag{10c}$$

where i indicates the number of polynomials involved in the admissible functions.

After substituting Eq. (9) into energy Eq. (5), and then using the Lagrange's equation gives the following equation

$$\frac{\partial I}{\partial q_i} - \frac{\partial}{\partial t}\frac{\partial I}{\partial \dot{q}_i} = 0 \tag{11}$$

where q_i is the unknown coefficients which are a_i, b_i and c_i. After implementing the Lagrange procedure, the motion equation of the problem is obtained as follows;

$$[\mathrm{K}]\{\mathrm{q(t)}\} + [\mathrm{M}]\{\ddot{\mathrm{q}}(\mathrm{t})\} = \{\mathrm{F(t)}\} \tag{12}$$

where $[K]$, $[M]$ and $\{\mathrm{F(t)}\}$ are the stiffness matrix, the mass matrix and load vector, respectively. The detail of these expressions are given as follows

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \tag{13}$$

where

$$\mathrm{K}_{11} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L \mathrm{A}_0 \frac{\partial\alpha_i}{\partial x}\frac{\partial\alpha_j}{\partial x}dx \;,\; \mathrm{K}_{12} = 0,$$

$$\mathrm{K}_{13} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L \mathrm{B}_0 \frac{\partial\alpha_i}{\partial x}\frac{\partial\gamma_j}{\partial x}dx, \quad \mathrm{K}_{21} = 0,$$

$$\mathrm{K}_{22} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L \mathrm{A}_{55} \frac{\partial\beta_i}{\partial x}\frac{\partial\beta_j}{\partial x}dx,$$

$$\mathrm{K}_{23} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L \mathrm{A}_{55} \frac{\partial\beta_i}{\partial x}\gamma_j dx$$

$$K_{31} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L B_0 \frac{\partial\gamma_i}{\partial x}\frac{\partial\alpha_j}{\partial x} dx$$
$$K_{32} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L A_{55}\gamma_i \frac{\partial\beta_j}{\partial x} dx$$
$$K_{33} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L D_0 \frac{\partial\gamma_i}{\partial x}\frac{\partial\gamma_j}{\partial x} dx \tag{14}$$

$$[M] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \tag{15}$$

where

$$M_{11} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_0\, \alpha_i\alpha dx \ , \ M_{12} = 0,$$
$$M_{13} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_1\, \alpha_i\gamma_j dx, \quad M_{21} = 0,$$
$$M_{22} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_0\, \beta_i\beta_j dx,$$
$$M_{23} = M_{32} = 0$$
$$M_{31} == -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_1\, \gamma_i\alpha_j dx$$
$$M_{33} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_2\, \gamma_i\gamma_j dx \tag{16}$$
$$\{F(t)\} = Q\beta_j \tag{17}$$

The governing equation of motions Eq. (12) is solved numerically by using implicit Newmark average acceleration method in the time domain.

3. Numerical results

In this section, dynamical displacements of the laminated composite cantilever beam under the sinusoidal harmonic load are presented and discussed according to material and load parameters. The material constants of Graphite/Epoxy are E_{01}=150 GPa, E_{02}=9 GPa, G_{012}=7,1 GPa, G_{023}=2,5 GPa, v=0.3, ρ=1800 kg/m^3. The geometry values are selected as b=0.1 *m*, h=0.1 *m* and L=1 *m*. The magnitude of load is selected as Q_0=1 kN. In the numerical results, number of the series term is taken as 10.

In order to validate the used formulations, the maximum vertical displacements (at the free end) of the 0/0/0 laminated beam are obtained and compared with ANSYS Workbench 14 structural analysis program $\overline{\omega}$=80 rad/s in Fig. 2. It is seen from Fig. 2, that results of this study agree well with results of ANSYS Workbench14.

In Figs. 3 and 4, effects of the fiber orientation angles (θ) on the lateral dynamical free-end displacements of the laminated beams are presented for different stacking sequence of laminas for $\overline{\omega} = 100\ rd/s$ in time history.

It is seen from Figs. 3 and 4, the dynamical displacements of the laminated composite beams increase with increasing of the fiber orientation angles (θ). This is because that with increasing the fiber orientation angles, the stiffness of the beam decrease according to the Eq. (3). The dynamical displacements of [θ/0/θ] are greater than those of [0/θ/0]. The stacking sequence of [θ/0/θ] is more

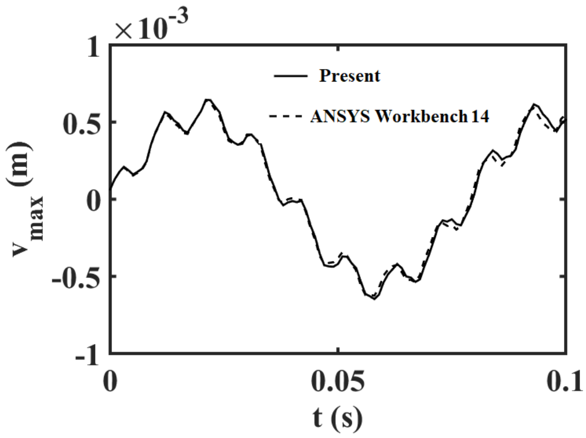

Fig. 2 Comparison study: Time responses of the 0/0/0 laminated beam for $\overline{\omega}$=2 rad/s

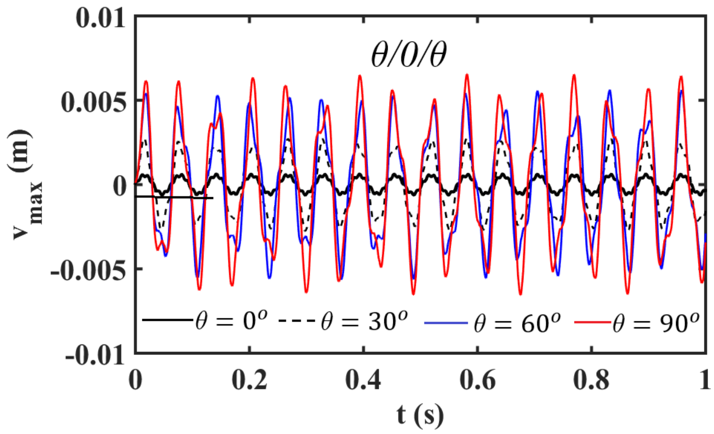

Fig. 3 Time history of dynamic displacements at free end of laminated beam with different values of fiber orientation angles for *θ/0/θ* stacking sequence

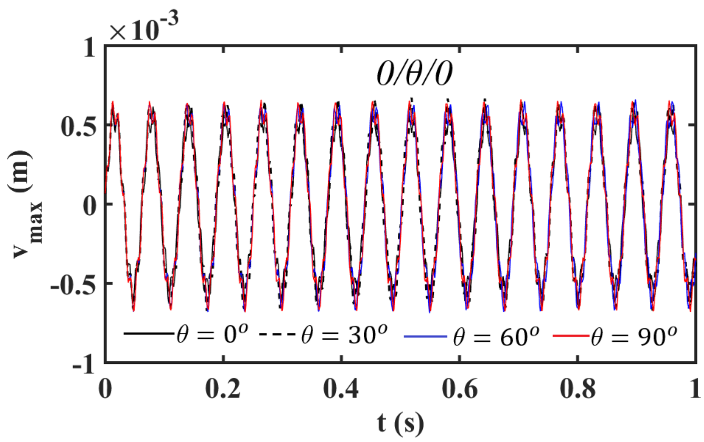

Fig. 4 Time history of dynamic displacements at free end of laminated beam with different values of fiber orientation angles for *0/θ/0* stacking sequence

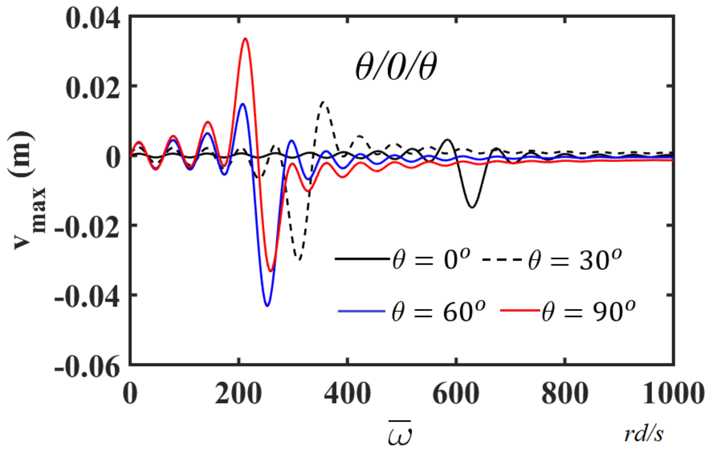

Fig. 5 The relationship between of the maximum displacements and the frequency of the dynamic load ($\overline{w}$) of laminated beam with different values of fiber orientation angles for *θ/0/θ* stacking sequence

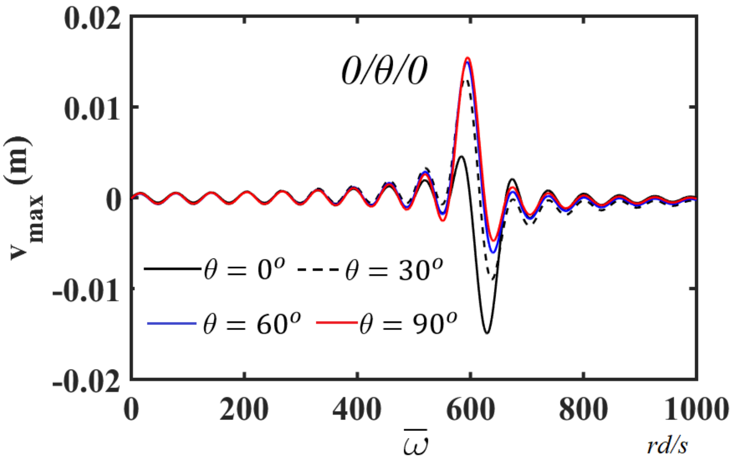

Fig. 6 The relationship between of the maximum displacements and the frequency of the dynamic load ($\overline{w}$) of laminated beam with different values of fiber orientation angles for *0/*θ*/0* stacking sequence

effect on the dynamical responses rather that the stacking sequence of [0/*θ*/0]. The stacking sequence play important role on the dynamic responses of laminated beams.

In Figs. 5 and 6, the relationship between of the maximum displacements and the frequency of the dynamic load ($\overline{w}$) of laminated cantilever beam is presented for different values the fiber orientation angles for different stacking sequence of laminas for *t*=0.1 s.

In Figs. 5 and 6, the resonance frequencies can be observed in the values of amplitude hit. It is seen from Figs. 5 and 6, the resonance frequencies of the laminated beam decrease with increasing of fiber orientation angles. Because of increasing the fiber orientation angles, the laminated beam gets more flexible and so, the resonance frequencies decrease naturally. Also, the resonance responses of the laminated beams are more sensitive in the [*θ*/0/*θ*]. In [0/*θ*/0], the resonance and dynamic responses of the laminated beams change very little.

4. Conclusions

The dynamical displacements of a laminated composite cantilever beam under a sinusoidal dynamic load are studied based on the Timoshenko beam theory by using the Ritz method. The governing equations of problem are derived by using the Lagrange procedure. In the dynamic solution, the Newmark average acceleration method is used in the time history. In the numerical results, the effects of fibre orientation angles, stacking sequence and frequency of dynamic load on dynamically displacements of the laminated composite beam are investigated.

It is observed from presented results that the stacking sequence of laminas have important role on the dynamic responses of laminated composite beams. The stacking sequence of [θ/0/θ] is more influence of the dynamical responses rather that the stacking sequence of [0/θ/0]. Increasing the fiber orientation angles cause to increase dynamic responses of laminated beams significantly. With choosing suitable values of fiber orientation angles and stacking sequence, the dynamic effects can be reduced in the laminated beams.

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