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Axisymmetric deformation in transversely isotropic thermoelastic medium using new modified couple stress theory

Parveen Lata[a] and Harpreet Kaur*

Department of Basic and Applied Sciences, Punjabi University, Patiala, Punjab, India

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Abstract. The present study is concerned with the thermoelastic interactions in a two dimensional axisymmetric problem in transversely isotropic thermoelastic solid using new modified couple stress theory without energy dissipation and with two temperatures. The Laplace and Hankel transforms have been employed to find the general solution to the field equations. Concentrated normal force, normal force over the circular region, concentrated thermal source and thermal source over the circular region have been taken to illustrate the application of the approach. The components of displacements, stress, couple stress and conductive temperature distribution are obtained in the transformed domain. The resulting quantities are obtained in the physical domain by using numerical inversion technique. The effect of two temperature varying by taking different values for the two temperature on the components of normal stress, tangential stress, conductive temperature and couple stress are depicted graphically.

Keywords: transversely isotropic; thermoelastic; Laplace transform; Hankel transform; concentrated and distributed sources; new modified couple stress

1. Introduction

Couple stress theory is an extension to continuum theory that includes the effects of couple stresses, in addition to the classical direct and shear forces per unit area. The classical continuum theories are incapable of predicting the size effects in micro and nanoscales. So, higher order continuum theories have been proposed to account for the size effects. Couple stress theory is such a higher order theory. First mathematical model to examine the materials with couple stresses was presented by Cosserat and Cosserat (1909). This theory could not establish the constitutive relationships. Mindlin and Tierstein (1962) and Koiter (1964) developed initial version of couple stress theory, based on the Cosserat continuum theory (1909). Koiter introduced the constitutive relationships for couple stress theory, involving length scale parameters to predict the size effects. It involves four material constants for isotropic elastic materials which are very difficult to determine (1964). So, modified couple stress theory (M-CST) with one length scale parameter was

*Corresponding author, Ph.D. Scholar, E-mail: mehrokpreet93@gmail.com
[a]Associate Professor, E-mail: parveenlata@pbi.ac.in

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presented by Yang *et al.* (2002), in which the couple stress tensor is symmetrical. This theory suffers from some inconsistencies, e.g. M-CST cannot describe the pure bending of plate properly. So, Hadjesfandiari *et al.* (2011) gave consistent couple stress theory (C-CST) with the skew-symmetric couple-stresses, that settles all the discrepancies of modified couple stress theory. Modified couple stress theory was not applicable to anisotropic materials. So, Chen and Li (2014) presented the new modified couple stress theory (NM-CST) for anisotropic materials containing three length scale parameters. Park and Gao (2006) studied the Bernoulli- Euler beam model based on a modified couple stress theory. Sharma and Sharma (2011) studied the damping in micro-scale generalized thermoelastic circular plate resonators under clamped plate and simply-supported plate. Lakes (1982) dynamical studied the effects of couple stress in human compact bone. Fakhrabadi studied the electromechanical behaviors of carbon nanotubes on the basis of modified couple stress theory and Homotopy perturbation method. Darijani and Shahdadi (2015) developed shear deformation based a new non-classical plate model in modified couple stress theory including two unknown functions. Ke and Wang (2011) investigated the size effect on dynamic stability of functionally graded microbeams based on a modified couple stress theory. Chen *et al.* (2011) presented a new modified couple stress model for bending analysis of composite laminated beams with first order shear deformation. Asghari (2012) studied the geometrically nonlinear micro-plate formulation based on the modified couple stress theory. Farokhi *et al.* (2018) formulated the modified couple stress theory in orthogonal curvilinear coordinates. Zozulya (2018) developed higher order couple stress model for plates and shells in orthogonal system of coordinates. Simsek and Reddy (2013) investigated the bending and vibration of functionally graded microbeams using a new higher order beam theory and the modified couple stress theory. Fang *et al.* (2013) examined the problem of thermoelastic damping in the axisymmetric vibration of circular microplate resonators using two dimensional couple stress heat conduction model. Ansari *et al.* (2014) studied the free vibration behavior of post-buckled functionally graded (FG) Mindlin rectangular microplates based on the modified couple stress theory (MCST). Ansari *et al.* (2014) presented an exact solution for the vibration analysis of piezoelectric microbeams on the basis of the modified couple stress theory for both Euler-Bernoulli and Timoshenko beam models using Hamilton's principle. It was shown that when the length of microbeams is decreased, effects of piezoelectricity and size effects are more prominent. Gao and Zhang (2016) constructed a non-classical Kirchhoff plate model by applying modified couple stress theory, surface elasticity theory and two–parameter elastic foundation. Marin *et al.* (2017) discussed the problem of effect of microtemperatures for micropolar thermoelastic bodies. Marin *et al.* (2017c) studied the Saint-Venant's problem in the context of the theory of porous dipolar bodies. Shaat *et al.* (2017) studied the bending analysis of nano-sized Kirchhoff plates using modified couple-stress theory in connection with surface elasticity theory of Gurtin and Murdoch to consider the surface energy effects. effect of nonuniformity and small scale effects were studied on varying the frequency terms. An axisymmetric problem of thick circular plate in modified couple stress theory of thermoelastic diffusion using Laplace and Hankel transforms technique have been investigated by Kumar and Shaloo (2016). Atanasov *et al.* (2017) examined the thermal effect on the free vibration and buckling of the Euler-Bernoulli double microbeam system based on the modified couple stress theory using Bernoulli–Fourier method. Malikan (2017) investigated the buckling of a thick sandwich plate under the biaxial non-uniform compression using the modified couple stress theory with various boundary conditions. Alimirzaei *et al.* (2019) presented the nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using finite element method and modified strain gradient theory. Bourada *et al.* (2019) studied the composite laminated materials using shear deformation theory. Zarga *et al.* (2019) studied the thermomechanical bending for functionally graded sandwich plates using a simple quasi-3D shear deformation theory. Abbas and Youssef (2009) and Abbas and Zenkour (2014) studied different problems under two-temperature generalized thermoelastic theory by finite element method. Abbas (2014c, 2015) studied phase lag models in fiber-reinforced anisotropic materials using generalized thermoelasticity. Abbas (2016) studied the exact solution for free vibration of thermoelastic hollow cylinder with two temperature and using generalized thermoelasticity theory. Lata *et al.* (2016) and Kumar *et al.* (2016,2017,2017a,2016a) studied the deformation in transversely isotropic material using thermoelasticity. Despite of this several researchers worked on different theory of thermoelasticity as Marin (1997,1997a) , Marin and Craciun (2017), Othman and Marin (2017), Hassan *et al.* (2018), Rafiq *et al.* (2019), Arif *et al.* (2018), Othman *et al.* (2015), Lata and Kaur (2019,2019a), Ezzat and AI-Bary (2016), Ezzat *et al.* (2017), Lata (2018,2018a), Karami *et al.* (2019a, b), Medani *et al.* (2019), Chaabane *et al.* (2019), Boulefrakh *et al.* (2019), Boukhlif *et al.* (2019), Boutaleb *et al.*(2019), Othman and Abbas (2012), Zenkour and Abbas (2014) , Abbas (2014a,2014b). Nowaki (1974) developed the theory of thermoelasticity with mass diffusion.

In the present study we deal with the thermoelastic interactions in a two dimensional homogeneous, transversely isotropic thermoelastic solids without energy dissipation and with two temperatures in the context of new modified couple stress model. The Laplace and Hankel transforms have been employed to find the general solution to the field equations. Concentrated normal force, normal force over the circular region and concentrated thermal source and thermal source over the circular region have been taken to illustrate the application of the approach. The components of displacements, stresses and conductive temperature distribution are obtained in the transformed domain. The resulting quantities are obtained in the physical domain by using numerical inversion technique. Numerically simulated results are depicted graphically to show the effect of two temperature on the components of normal stress, tangential stress and conductive temperature.

2. Basic equations

Following Chen and Li (2014), Kumar and Devi (2015), the field equations transversely isotropic thermoelastic medium using new modified couple stress theory in the absence of body forces, body couple and without energy dissipation are given by

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + \frac{1}{2}e_{ijk}m_{lk,l} - \beta_{ij}T, \tag{1}$$

$$c_{ijkl}\varepsilon_{kl,j} + \frac{1}{2}e_{ijk}m_{lk,lj} - \beta_{ij}T_{,j} = \rho\ddot{u}_{i,} \tag{2}$$

$$K_{ij}\varphi_{,ij} - \rho C_E\ddot{T} = \beta_{ij}T_0\ddot{\varepsilon}_{ij,} \tag{3}$$

where

$$\beta_{ij} = c_{ijkl}\alpha_{ij}, \tag{4}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{5}$$

$$m_{ij} = l_i^2 G_i \chi_{ij} + l_j^2 G_j \chi_{ji}, \tag{6}$$

$$\chi_{ij} = \omega_{i,j}, \tag{7}$$

$$\begin{gathered}\omega_i = \frac{1}{2} e_{ijk} u_{k,j}, \\ T = \varphi - a_{ij}\varphi_{,ij}.\end{gathered} \tag{8}$$

Here, $u = (u, v, w)$ is the components of displacement vector, $c_{ijkl}(c_{ijkl} = c_{ijlk} = c_{jikl} = c_{jilk})$ are elastic parameters, a_{ij} are the two temperature parameters, σ_{ij} are the components of stress tensor, ε_{ij} are the components of strain tensor,e_{ijk} is alternate tensor, $m_{ij}$are the components of couple-stress, $\alpha_{ij}$are the coefficients of linear thermal expansion,β_{ij} is thermal tensor,T is the thermodynamical temperature, φ is the conductive temperature, l_i(i = 1,2,3) are material length scale parameters ,$\chi_{ij}$is curvature,$\omega_i$is the rotational vector,$\rho$is the density, K_{ij} is the thermal conductivity, c_E is the specific heat at constant strain, $T_0$is the reference temperature assumed to be such that $T/T_0 \ll 1$, G_i are the elasticity constants and $\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$, $\beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$.

3. Formulation and solution of the problem

We consider a homogeneous transversely isotropic , thermoelastic body initially at uniform temperature T_0. We take a cylindrical polar co-ordinate system (r, θ, z) with symmetry about z – axis. As the problem considered is plane axisymmetric, the field component $v = 0$, and u, w, φ are independent of θ. We have used appropriate transformation following Slaughter (2002) on the set of Eqs. (1)-(3) to derive the equations for transversely isotropic thermoelastic solid without energy dissipation and with two temperature and restrict our analysis to the two dimensional problem with $\vec{u} = (u, 0, w)$, we obtain

Equation of motion

$$\begin{gathered}c_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r\,\partial r} + \frac{u}{r}\right) + c_{44}\frac{\partial^2 u}{\partial z^2} + (c_{13} + c_{44})\frac{\partial^2 w}{\partial r \partial z} + \frac{1}{4}\left(l_2^2 G_2\left(-\frac{\partial^4 u}{\partial r^2 \partial z^2} + \frac{\partial^4 w}{\partial r^3 \partial z} - \frac{\partial^4 u}{\partial z^4} + \frac{\partial^4 w}{\partial r \partial z^3}\right)\right) \\ -\beta_1 \frac{\partial}{\partial r}\left(1 - a_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\right) - a_3\frac{\partial^2}{\partial z^2}\right)\varphi = \rho\ddot{u},\end{gathered} \tag{9}$$

$$\begin{gathered}c_{33}\frac{\partial^2 w}{\partial z^2} + (c_{44} + c_{13})\left(\frac{\partial^2 u}{\partial r \partial z} + \frac{\partial u}{r\,\partial z}\right) + c_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial w}{r\,\partial r}\right) - \frac{1}{4}\Bigg(-l_2^2 G_2\Bigg(-\frac{\partial^4 u}{\partial r^3 \partial z} + \frac{\partial^4 w}{\partial r^4} + \\ \frac{1}{r}\left(-\frac{\partial^3 u}{\partial r^2 \partial z} + \frac{\partial^3 w}{\partial r^3}\right)\Bigg) + l_2^2 G_2\left(\frac{\partial^4 u}{\partial r^3 \partial z} - \frac{\partial^4 w}{\partial r^2 \partial z^2} + \frac{1}{r}\left(\frac{\partial^3 u}{\partial z^3} - \frac{\partial^3 w}{\partial r^2 \partial z}\right)\right)\Bigg) \\ -\beta_3 \frac{\partial}{\partial z}\left(1 - a_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\right) - a_3\frac{\partial^2}{\partial z^2}\right)\varphi = \rho\ddot{w},\end{gathered} \tag{10}$$

Equation of heat conduction without energy dissipation

$$K_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{\varphi}{r}\right) + K_3\frac{\partial^2 \varphi}{\partial z^2} - \rho c_E\frac{\partial^2}{\partial t^2}\left(1 - a_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\right) - a_3\frac{\partial^2}{\partial z^2}\right)\varphi = T_0\frac{\partial^2}{\partial t^2}\left(\beta_1\frac{\partial u}{\partial r} + \beta_3\frac{\partial w}{\partial z}\right). \tag{11}$$

The constitutive relationships are

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - \beta_3 T,$$
$$\sigma_{rz} = 2c_{44}e_{rz} - \frac{1}{4}\left((l_1^2G_1 - l_2^2G_2)\left(-\frac{\partial^3 u}{\partial z\partial r^2} + \frac{\partial^3 w}{\partial r^3}\right) + (l_3^2G_3 - l_2^2G_2)\left(-\frac{\partial^3 u}{\partial z^3} + \frac{\partial^3 w}{\partial r\partial z^2}\right)\right),$$
$$\sigma_{\theta\theta} = c_{21}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T, \tag{12}$$
$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T,$$
$$m_{\theta z} = \frac{1}{2}(l_2^2G_2 - l_3^2G_3)\left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial r\partial z}\right),$$

where $e_{rr} = \frac{\partial u}{\partial r}$, $e_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)$, $e_{\theta\theta} = \frac{u}{r}$, $e_{zz} = \frac{\partial w}{\partial z}$, $T = \left(1 - a_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\right) - a_3\frac{\partial^2}{\partial z^2}\right)\varphi$.

In the above equation we use contracting subscript notation $(1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 31, 6 \to 12)$ to relate c_{ijkl} to c_{mn}. The basis of the symmetries of C_{ijkl} is due to

i. The stress tensor is symmetric, which is only possible if $(C_{ijkl} = C_{jikl})$

ii. If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkl} = C_{klij}$

iii. From stress tensor and elastic stiffness tensor symmetries infer $(C_{ijkl} = C_{ijlk})$ and $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$

To facilitate the solution, we define the dimensionless quantities as

$$\theta' = \frac{\theta}{L},\ r' = \frac{r}{L},\ z' = \frac{z}{L},\ t' = \frac{c_1}{L}t,\ u' = \frac{\rho c_1^2}{L\beta_1 T_0}u,\ w' = \frac{\rho c_1^2}{L\beta_1 T_0}w,\ T' = \frac{T}{T_0},\ \varphi' = \frac{\varphi}{T_0},\ \sigma'_{zr} = \frac{\sigma_{zr}}{\beta_1 T_0}, \sigma'_{rr} = \frac{\sigma_{rr}}{\beta_1 T_0},\ \sigma'_{z\theta} = \frac{\sigma_{z\theta}}{\beta_1 T_0},\ m'_{32} = \frac{m_{32}}{L\beta_1 T_0}, a_1' = \frac{a_1}{L}, a_3' = \frac{a_3}{L}. \tag{13}$$

Defining Laplace and Hankel transformation as

$$\hat{f}(r,z,s) = \int_0^\infty f(r,z,t)e^{-st}dt, \tag{14}$$

$$\tilde{f}(\xi,z,s) = \int_0^\infty \hat{f}(r,z,s)rJ_n(r\xi)d\,r\,. \tag{15}$$

Applying the dimensionless quantities defined by (13) and Laplace Hankel defined by (14)-(15) to the Eqs. (9)-(11) , we obtain

$$(-\epsilon_1 + \delta_2 D^2)\tilde{u} - \delta_1\xi D\tilde{w} + \frac{1}{4L^2c_{11}}l_2^2G_2((\xi^2D^2 - D^4)\tilde{u} - (\xi^3 D + \xi D^3)\tilde{w}) + \xi(1 + \frac{a_1}{L}\xi^2 - \frac{a_3}{L}D^2)\tilde{\varphi} = 0, \tag{16}$$

$$\delta_1\epsilon_2 D\tilde{u} + (\epsilon_8 + \delta_3 D^2)\tilde{w} - \frac{\xi}{4L^2c_{11}}l_2^2G_2(\,(\xi^2 D - D^3)\tilde{u} - (\xi^3 + D^2\xi)\tilde{w}) - \epsilon_9 D(1 + \frac{a_1}{L}\xi^2 - \frac{a_3}{L}D^2)\tilde{\varphi} = 0, \tag{17}$$

$$\epsilon_6\xi s^2\tilde{u} + \epsilon_7 D s^2\tilde{w} + \left(\epsilon_2 + \epsilon_5 D^2 - \epsilon_4 s^2(1 + \frac{a_1}{L}\xi^2 - \frac{a_3}{L}D^2)\right)\tilde{\varphi} = 0, \tag{18}$$

where

$\delta_1=\frac{c_{13}+c_{44}}{c_{11}}$, $\delta_2=\frac{c_{44}}{c_{11}}$, $\delta_3=\frac{c_{33}}{c_{11}}$, $\epsilon_1=s^2+\xi^2$, $\epsilon_2=\frac{-\xi^2+1}{\xi}$, $\epsilon_4=\frac{\rho c_E c_1}{K_1 L}$, $\epsilon_5=\frac{K_3}{K_1}$,

$\epsilon_6=\frac{T_0\beta_1^2}{K_1\rho}$, $\epsilon_7=\frac{T_0\beta_1\beta_3}{K_1\rho}$, $\epsilon_8=-\delta_2\xi^2-s^2$, $\epsilon_9=\frac{\beta_3}{\beta_1}$, $\epsilon_{10}=\delta_2-\frac{l_2^2G_2}{4L^2c_{11}}(-\xi^2)$,

$$\epsilon_{11}=-\delta_1\xi-\frac{l_2^2G_2}{4L^2c_{11}}\xi^3,\ \epsilon_{12}=\epsilon_8+\frac{l_2^2G_2}{4L^2c_{11}}\xi^4,\ \epsilon_{13}=\epsilon_2-\epsilon_4 s,\ \epsilon_{14}=\delta_3+\xi^2\frac{l_2^2G_2}{4L^2c_{11}}.$$

The non trivial solution of the system of Eqs. (16)-(18) yields

$$(PD^8+QD^6+RD^4+SD^2+T)=0, \tag{19}$$

where

$P=-\epsilon_{26}\xi^2{\alpha_1}^2$,

$Q=\epsilon_{10}(\epsilon_{14}\epsilon_{26}-\epsilon_{16}\epsilon_{22})+\alpha_1(\epsilon_{12}\epsilon_{26}+\epsilon_{14}\epsilon_{25}-\epsilon_{16}\epsilon_{21})-\xi\epsilon_{20}\alpha_1\epsilon_{16}-\epsilon_{20}\epsilon_{14}\epsilon_{15}+\xi\epsilon_{11}\alpha_1\epsilon_{26}+\alpha_1\xi(\epsilon_{27}\epsilon_{26}+\epsilon_2\epsilon_{25}\alpha_1-\epsilon_{22}\epsilon_{15})$,

$R=-\epsilon_1(\epsilon_{14}\epsilon_{26}-\epsilon_{16}\epsilon_{22})+\epsilon_{10}(\epsilon_{12}\epsilon_{26}+\epsilon_{14}\epsilon_{25}-\epsilon_{16}\epsilon_{21})+\alpha_1(\epsilon_{12}-\epsilon_{25})+\epsilon_{20}\epsilon_{27}\epsilon_{16}+\xi\epsilon_{19}\alpha_1\epsilon_{16}+\epsilon_{15}\epsilon_{19}\epsilon_{14}-\epsilon_{11}(\epsilon_{27}\epsilon_{26}-\xi\epsilon_{25}\alpha_1-\epsilon_{22}\epsilon_{15})+\alpha_1\xi(\epsilon_{27}\epsilon_{25}+\epsilon_{15}\epsilon_{21})$,

$S=-\epsilon_{11}(\epsilon_{27}\epsilon_{25}+\epsilon_{15}\epsilon_{21})-\epsilon_{19}\epsilon_{27}\epsilon_{16}-\epsilon_{20}\epsilon_{15}\epsilon_{12}-\epsilon_1(\epsilon_{12}\epsilon_{26}+\epsilon_{14}\epsilon_{25}-\epsilon_{16}\epsilon_{21})+\epsilon_{12}\epsilon_{10}\epsilon_{25}$,

$T=-\epsilon_1\epsilon_{12}\epsilon_{25}+\epsilon_{19}\epsilon_{12}$.

The roots of Eq. (19) are$\pm\lambda_i(i=1,2,3,4,5)$,using the radiation condition that $\hat{u},\hat{w},\ \hat{\varphi},\to 0$ as $z\to\infty$ the solution of equation (24) may be written as

$$(\tilde{u},\tilde{w},\tilde{\varphi})=\sum_{i=1}^{4}(1,R_i,S_i)A_ie^{-\lambda_i z}, \tag{20}$$

$$R_i=\frac{-\epsilon_1\epsilon_{25}+\epsilon_{15}\epsilon_{19}+(-\epsilon_1\epsilon_{26}+\epsilon_{10}\epsilon_{25}+\epsilon_{15}\epsilon_{20})\lambda_i^2+(\epsilon_{10}\epsilon_{26}+\alpha_1\epsilon_{13})\lambda_i^4+\alpha_1\epsilon_{26}\lambda_i^6}{\epsilon_1\epsilon_{25}+(\epsilon_{12}\epsilon_{26}+\epsilon_{14}\epsilon_{25}+\epsilon_{16}\epsilon_{21})\lambda_i^2+(\epsilon_{14}\epsilon_{26}-\epsilon_{16}\epsilon_{22})\lambda_i^4}, \tag{21}$$

$$S_i=\frac{-\epsilon_1\epsilon_{12}+(-\epsilon_1\epsilon_{14}+\alpha_1\epsilon_{12}-\epsilon_{27}\epsilon_{11})\lambda_i^2+(\epsilon_{10}\epsilon_{14}+\alpha_1(\epsilon_{12}+\xi\epsilon_{27}+\xi\epsilon_{11}))\lambda_i^4-\alpha_1(-\epsilon_{14}+\xi^2\alpha_1)\lambda_i^6}{\epsilon_1\epsilon_{25}+(\epsilon_{12}\epsilon_{26}+\epsilon_{14}\epsilon_{25}+\epsilon_{16}\epsilon_{21})\lambda_i^2+(\epsilon_{14}\epsilon_{26}-\epsilon_{16}\epsilon_{22})\lambda_i^4}, \tag{22}$$

where

$$\epsilon_{15}=\epsilon_6 s^2\xi,\ \epsilon_{16}=\epsilon_7 s^2,\ \epsilon_{17}=1+\frac{a_1}{L}\xi^2,\ \epsilon_{18}=\frac{a_3}{L},\ \epsilon_{19}=-\xi\epsilon_{17},\ \epsilon_{20}=\xi\epsilon_{18},$$
$$\epsilon_{21}=\epsilon_9\epsilon_{17},\ \epsilon_{22}=\epsilon_9\epsilon_{18},\ \epsilon_{23}=\epsilon_4 s^2\epsilon_{17},\ \epsilon_{24}=\epsilon_4 s^2\epsilon_{18},$$
$$\epsilon_{25}=-\epsilon_2+\epsilon_{23},\ \epsilon_{26}=-\epsilon_5-\epsilon_{24},\ \epsilon_{27}=\epsilon_2\delta_1+\alpha_1\xi^3,\ \alpha_1=-\frac{l_2^2G_2}{4L^2c_{11}},$$

4. Boundary conditions

For Mechanical forces/ Thermal sources acting on the surface
The boundary conditions are

$$\sigma_{zz}(r,z,t)=-P_1(r,t),$$
$$\sigma_{zr}(r,z,t)=0,$$

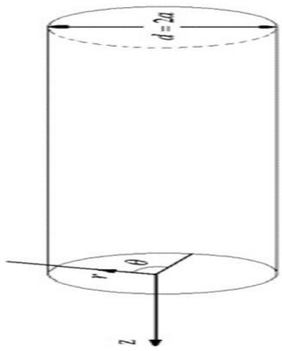

Fig. (I) The coordinate system used for derivation of the equations

$$\frac{\partial \varphi}{\partial r}(r,z,t) = P_2(r,t),$$
$$m_{\theta z} = 0. \tag{23}$$

$P_1(r,t)$ and $P_2(r,t)$ are well behaved functions.

Here $P_2(r,t)$ =0 corresponds to plane boundary subjected to normal force and $P_1(r,t)$ =0 corresponds to plane boundary subjected to thermal point source.

Case 1. Concentrated normal force/ Thermal point source

When plane boundary is subjected to concentrated normal force/ thermal point force, then $P_1(r,t), P_2(r,t)$ take the form

$$(P_1(r,t), P_2(r,t)) = \left(\frac{P_1\delta(r)\delta(t)}{2\pi r}\ ,\frac{P_2\delta(r)\delta(t)}{2\pi r}\right). \tag{24}$$

P_1 is the magnitude of the force applied , P_2 is the magnitude of the constant temperature applied on the boundary and $\delta(r)$ is the Dirac delta function.

Making use of equations (23), (24), (12)-(14) and (20) the components of distance , stress , couple stress and conductive temperature are given by (26)-(31).

Case 2. Normal force over the circular region/ Thermal source over the circular region

Let a uniform pressure of total magnitude / constant temperature applied over a uniform circular region of radius a is obtained by setting

$$(P_1(r,t), P_2(r,t)) = \left(\frac{P_1}{\pi a^2}H(a-r)\delta(t)\ ,\frac{P_2}{\pi a^2}H(a-r)\delta(t)\right), \tag{25}$$

where $H(a-r)$is the Heaviside unit step function.

Making use of dimensionless quantities defined by (11) and then applying Laplace and Hankel transforms defined by (13)-(14) on (25) ,we obtain

$$\left(\widetilde{P_1}(\xi,s), \widetilde{P_2}(\xi,s)\right) = \left(\frac{P_1}{\pi a\xi}J_1(a\xi)\ ,\frac{P_2}{\pi a\xi}J_2(a\xi)\right).$$

The expressions for the components of displacements, stress, couple stress and conductive temperature are obtained by replacing $\frac{P_1}{2\pi}$ with $\frac{P_1 J_1(a\xi)}{\pi a\xi}$ and by replacing $\frac{P_2}{2\pi}$ with $\frac{P_2 J_1(a\xi)}{\pi a\xi}$ in Eqs. (26)-(31) respectively and are given by (32)-(37).

$$\tilde{u} = \frac{1}{\Delta}\sum_{i=1}^{4}(\frac{P_1}{2\pi}B_{1i} + \frac{P_2}{2\pi}B_{3i})e^{\lambda_i z}, \tag{26}$$

$$\tilde{w} = \frac{1}{\Delta}\sum_{i=1}^{4} R_i(\frac{P_1}{2\pi}B_{1i} + \frac{P_2}{2\pi}B_{3i})e^{\lambda_i z}, \tag{27}$$

$$\tilde{\varphi} = \frac{1}{\Delta}\sum_{i=1}^{4} S_i(\frac{P_1}{2\pi}B_{1i} + \frac{P_2}{2\pi}B_{3i})e^{\lambda_i z}, \tag{28}$$

$$\widetilde{\sigma_{zz}} = \frac{1}{\Delta}\sum_{i=1}^{4}\left(\frac{\beta_1 T_0}{\rho c_1^2}(C_{13}\epsilon_2 - C_{33}\lambda_i R_i) - \beta_3 T_0 S_i\right)(\frac{P_1}{2\pi}B_{1i} + \frac{P_2}{2\pi}B_{3i})e^{\lambda_i z}, \tag{29}$$

$$\widetilde{\sigma_{zr}} = \frac{1}{\Delta}\sum_{i=1}^{5}\left(\frac{\beta_1 T_0}{\rho c_1^2}C_{44}(-\lambda_i - \xi R_i) - \beta_1 T_0\left(\alpha_1(-\xi^2\lambda_i - \xi^3 R_i) + \alpha_2(-\lambda_i^3 + \xi\lambda_i^2 R_i)\right)\right)(\frac{P_1}{2\pi}B_{1i} + \frac{P_2}{2\pi}B_{3i})e^{\lambda_i z}, \tag{30}$$

$$\widetilde{m_{\theta z}} = \frac{\beta_1 T_0(l_1^2 G_1 - l_2^2 G_2)}{2\Delta\rho c_1^2 L^2}\sum_{i=1}^{4}(\frac{P_1}{2\pi}B_{1i} + \frac{P_2}{2\pi}B_{3i})\,(\lambda_i^3 + \xi\lambda_i^2 R_i)e^{\lambda_i z}. \tag{31}$$

For circular region

$$\tilde{u} = \frac{1}{\Delta}\sum_{i=1}^{4}(\frac{P_1 J_2(a\xi)}{\pi a\xi}B_{1i} + \frac{P_2 J_1(a\xi)}{\pi a\xi}B_{3i})e^{\lambda_i z}, \tag{32}$$

$$\tilde{w} = \frac{1}{\Delta}\sum_{i=1}^{4} R_i\,(\frac{P_1 J_2(a\xi)}{\pi a\xi}B_{1i} + \frac{P_2 J_1(a\xi)}{\pi a\xi}B_{3i})e^{\lambda_i z}, \tag{33}$$

$$\tilde{\varphi} = \frac{1}{\Delta}\sum_{i=1}^{4} S_i(\frac{P_1 J_2(a\xi)}{\pi a\xi}B_{1i} + \frac{P_2 J_1(a\xi)}{\pi a\xi}B_{3i})e^{\lambda_i z}, \tag{34}$$

$$\widetilde{\sigma_{zz}} = \frac{1}{\Delta}\sum_{i=1}^{4}\left(\frac{\beta_1 T_0}{\rho c_1^2}(C_{13}\epsilon_2 - C_{33}\lambda_i R_i) - \beta_3 T_0 S_i\right)(\frac{P_1 J_2(a\xi)}{\pi a\xi}B_{1i} + \frac{P_2 J_1(a\xi)}{\pi a\xi}B_{3i})e^{\lambda_i z}, \tag{35}$$

$$\widetilde{\sigma_{zr}} = \frac{1}{\Delta}\sum_{i=1}^{5}\left(\frac{\beta_1 T_0}{\rho c_1^2}C_{44}(-\lambda_i - \xi R_i) - \beta_1 T_0\left(\alpha_1(-\xi^2\lambda_i - \xi^3 R_i) + \alpha_2(-\lambda_i^3 + \xi\lambda_i^2 R_i)\right)\right)(\frac{P_1 J_2(a\xi)}{\pi a\xi}B_{1i} + \frac{P_2 J_1(a\xi)}{\pi a\xi}B_{3i})e^{\lambda_i z}, \tag{36}$$

$$\widetilde{m_{\theta z}} = \frac{\beta_1 T_0(l_1^2 G_1 - l_2^2 G_2)}{2\Delta\rho c_1^2 L^2}\sum_{i=1}^{4}(\frac{P_1 J_2(a\xi)}{\pi a\xi}B_{1i} + \frac{P_2 J_1(a\xi)}{\pi a\xi}B_{3i})\,(\lambda_i^3 + \xi\lambda_i^2 R_i)e^{\lambda_i z}, \tag{37}$$

where

$$A_{1i} = \frac{\beta_1 T_0}{\rho c_1^2}(C_{13}\epsilon_2 - C_{33}\lambda_i R_i) - \beta_3 T_0 S_i, \qquad A_{2i} = \frac{\beta_1 T_0}{\rho c_1^2}C_{44}(-\lambda_i - \xi R_i) - \beta_1 T_0\left(\alpha_1(-\xi^2\lambda_i - \xi^3 R_i) + \alpha_2(-\lambda_i^3 + \xi\lambda_i^2 R_i)\right),$$

$$A_{3i} = -\lambda_i S_i,$$

$$A_{4i} = \frac{\beta_1 T_0}{2\rho c_1^2 L^2}(l_1^2 G_1 - l_2^2 G_2)(\lambda_i^2 - \xi\lambda_i R_i),$$

$$\Delta = \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4,$$

$$\Delta_1 = A_{11}A_{22}(A_{33}A_{44} - A_{43}A_{34}) - A_{11}A_{23}(A_{32}A_{44} - A_{42}A_{34}) + A_{11}A_{24}(A_{32}A_{43} - A_{42}A_{33}),$$

$$\Delta_2 = A_{12}A_{21}(A_{33}A_{44} - A_{43}A_{34}) - A_{12}A_{23}(A_{31}A_{44} - A_{41}A_{34}) + A_{24}A_{12}(A_{31}A_{43} - A_{41}A_{33}),$$

$$\Delta_3 = A_{13}A_{21}(A_{32}A_{44} - A_{42}A_{34}) - A_{22}A_{13}(A_{31}A_{44} - A_{41}A_{34}) + A_{13}A_{24}(A_{31}A_{42} - A_{41}A_{32}),$$

$$\Delta_4 = A_{14}A_{21}(A_{32}A_{43} - A_{42}A_{33}) - A_{22}A_{14}(A_{31}A_{43} - A_{41}A_{33}) + A_{14}A_{23}(A_{31}A_{42} - A_{41}A_{32}),$$

$$B11 = \Delta_1/A_{11},$$
$$B12 = -\Delta_2/A_{12},$$
$$B13 = \Delta_3/A_{13},$$
$$B14 = -\Delta_4/A_{14},$$
$$A_i = \frac{1}{\Delta}(\widetilde{P_1}(\xi,s)B_{1i} + (\widetilde{P_2}(\xi,s)B_{3i}),$$
$$B_{31} = A_{12}(A_{23}A_{44} - A_{43}A_{24}) - A_{13}(A_{22}A_{44} - A_{42}A_{24}) + A_{14}(A_{22}A_{43} - A_{42}A_{23}),$$
$$B_{32} = -A_{11}(A_{23}A_{44} - A_{43}A_{24}) + A_{13}(A_{21}A_{44} - A_{41}A_{24}) - A_{14}(A_{21}A_{43} - A_{41}A_{23}),$$
$$B_{33} = A_{11}(A_{22}A_{44} - A_{42}A_{24}) - A_{12}(A_{21}A_{44} - A_{41}A_{24}) + A_{14}(A_{21}A_{42} - A_{41}A_{22}),$$
$$B_{34} = -A_{11}(A_{22}A_{43} - A_{42}A_{23}) + A_{12}(A_{21}A_{43} - A_{41}A_{23}) - A_{13}(A_{21}A_{42} - A_{41}A_{22}).$$

5. Particular cases

1. If $a_1 = a_3 = 0$ from equations (26)-(31) we obtain the corresponding expressions for displacements, stresses, couple stress and conductive temperature in thermoelastic medium without energy dissipation.

2. If we take $a_1 = a_3 = a$, $c_{11} = \lambda + 2\mu = c_{33}$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu, \beta_1 = \beta_3 = \beta, \alpha_1 = \alpha_3 = \alpha, K_1 = K_3 = K$ in equations (26)-(31) , we obtain the corresponding expressions for displacements, stresses, couple stress and conductive temperature for isotropic thermoelastic solid without energy dissipation.

6. Inversion of the transformations

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (30)-(36). Here the distance components, normal and tangential stresses ,conductive temperature and couple stress are functions of z, the parameters of Hankel and laplace transforms are ξ and s respectively and hence are of the form $\tilde{f}\ (\xi\ , z, s)$. To obtain the function $f(r, z, t)$ in the physical domain, we first invert the Hankel transform using

$$\hat{f}(r, z, s) = \int_0^\infty \xi \tilde{f}(\xi\ , z\ , s) J_n(\xi r)\, d\xi. \tag{38}$$

Now for the fixed values of ξ ,r and z the function $\hat{f}(r\ , z\ , s)$ in the expression above can be considered as the Laplace transform $\hat{g}\ (s)$ of $g\ (t)$. Following Honig and Hirdes (1984), the Laplace transform function $\hat{g}\ (s)$ can be inverted. The function $g(t)$ can be obtained by using

$$g(t) = \frac{1}{2\pi i}\int_{C+i\infty}^{C+i\infty} e^{st}\hat{g}(s) ds, \tag{39}$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\hat{g}(s)$. Taking $s\ =\ C\ +\ iy$ we get

$$g(t) = \frac{e^{Ct}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\hat{g}(C + iy) dy, \tag{40}$$

Now, taking $e^{-Ct}g(t)$ as $h(t)$ and expanding it as Fourier series in [0, 2L], we obtain approximately the formula

$$g(t) = g_\infty(t) + E_D,$$

where

$$g_\infty(t) = \frac{C_0}{2} + \sum_{K=1}^{\infty} C_k, \quad 0 \le t \le 2L,$$

and

$$C_k = \frac{e^{Ct}}{L} Re\left[e^{\frac{\pi i k t}{L}} \hat{g}\left(C + \frac{ik\pi t}{L}\right))\right]. \tag{41}$$

E_D is the discretization error and can be made arbitrarily small by choosing C large enough.

The value of C and L are chosen according to the criteria outlined by Honig & Hirdes (1984).

Since the infinite series in (42) can be summed up only to a finite number of N terms, so the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{C_0}{2} + \sum_{K=1}^{N} C_K, \quad 0 \le t \le 2L. \tag{42}$$

Now, we introduce a truncation error E_T, that must be added to the discretization error to produce the total approximate error in evaluating $g(t)$ using the above formula. To accelerate the convergence, the discretization error and then the truncation error is reduced by using the 'Korrecktur method' and the 'ϵ-algorithm', respectively as given by Honig & Hirdes (1984).

The Korrecktur method formula, to evaluate the function $g(t)$is

$$g(t) = g_\infty(t) - e^{-2CL} g_\infty(2L + t) + E_D',$$

where

$$\left|E_D'\right| \ll |E_D|.$$

Thus, the approximate value of $g(t)$ becomes

$$g_{Nk}(t) = g_N(t) - e^{-2CL} g_{N'}(2L + t), \tag{43}$$

where N' is an interger such that $N' < N$.

We shall now describe the ϵ-algorithm, which is used to accelerate the convergence of the series in (42). Let N be an odd natural number and $S_m = \sum_{k=1}^{m} C_k$ be the sequence of partial sums of (42). We define the 'ϵ -sequence' by

$$\epsilon_{0,m} = 0, \epsilon_{1,m} = S_m, \epsilon_{n+1,m} = \epsilon_{n-1,m+1} \frac{1}{\epsilon_{n,m+1} - \epsilon_{n,m}}; n, m = 1,2,3 \ldots \ldots \ldots$$

The sequence $\epsilon_{1,1}, \epsilon_{3,1}, \ldots \ldots \ldots \epsilon_{N,1}$ converges to $g(t) + E_D - \frac{C_0}{2}$ faster than the sequence of partial sums S_m, $m = 1,2,3, \ldots \ldots$ The actual procedure to invert the Laplace transform consists of (43) together with the 'ϵ −algorithm'.

The last step is to calculate the integral in Eq. (38). The method for evaluating this integral is described in Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

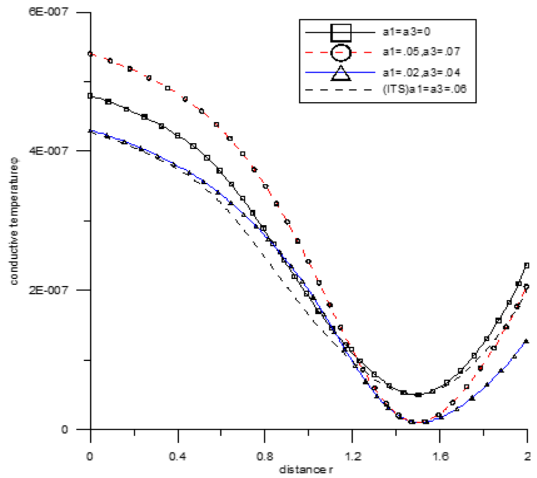

Fig. 1 Variation of conductive temperature φ with the distance r(concentrated normal force)

7. Results and discussions

For numerical computations, we take the copper material which is transversely isotropic. Physical data for a single crystal of copper is given by

$$c_{11} = 18.78 \times 10^{10}\ Kgm^{-1}s^{-2}, \quad c_{12} = 8.76 \times 10^{10}\ Kgm^{-1}s^{-2}, \quad c_{13} = 8.0 \times 10^{10}\ Kgm^{-1}s^{-2},$$

$$c_{33} = 17.2 \times 10^{10}\ Kgm^{-1}s^{-2}, c_{44} = 5.06 \times 10^{10}\ Kgm^{-1}s^{-2}, \quad C_E = 0.6331 \times 10^{3} JKg^{-1}K^{-1},$$

$$\alpha_1 = 2.98 \times 10^{-5}K^{-1}, \quad \alpha_3 = 2.4 \times 10^{-5}K^{-1}, \quad T_0 = 293K, \quad \rho = 8.954 \times 10^{3}Kgm^{-3},$$

$$K_1 = 0.433 \times 10^{3}Wm^{-1}K^{-1}, \quad K_3 = 0.450 \times 10^{3}Wm^{-1}K^{-1}, \quad G_1 = 0.1, \quad G_2 = 0.2,$$

$$G_3 = 0.3, \quad L = 1, \quad l_1 = l_2 = l_3 = .843.$$

Following Dhaliwal and Singh (1980), magnesium crystal is chosen for the purpose of numerical calculation (isotropic solid). In case of magnesium crystal like material for numerical calculations, the physical constants used are

$$\lambda = 2.17 \times 10^{10}Nm^{2}, \quad \mu = 3.278 \times 10^{10}Nm^{2} \quad ,K = 1.7 \times 10^{2}\ Wm^{-1}K^{-1},$$

$$\beta = 2.68\ \times 10^{6}\ Nm^{-2}K^{-1}\ \ ,\ \rho = 8.954 \times 10^{3}Kgm^{-3},\ T_0 = 298\ K,\ C_E = 1.04 \times 10^{3}JKg^{-1}K^{-1}.$$

The values of normal force stress σ_{zz} , tangential stress σ_{zr} , conductive temperature φ and couple stress $m_{z\theta}$for a transversely isotropic thermoelastic solid with two temperature (TITWT) , isotropic thermoelastic solid with two temperature(ITS)and thermoelastic solid without two temperature (TSWT) are presented graphically to show the impact of two temperature.

i). The solid line with central symbol square(− □ −) corresponds to (TSWT) for $a_1 = a_3 = 0$.

ii) small dashed line with central symbol circle $(- - o - -)$ corresponds to(TITWT) for $a_1 = .05, a_2 = .07$.

iii) solid line with centre symbol triangle $(-\ \Delta -)$ corresponds to (TITWT) for $a_1 = .02, a_2 = .04$.

iv) dashed line with no central symbol$(-\ - - -)$ corresponds to (ITS) for $a_1 = a_3 = .06$.

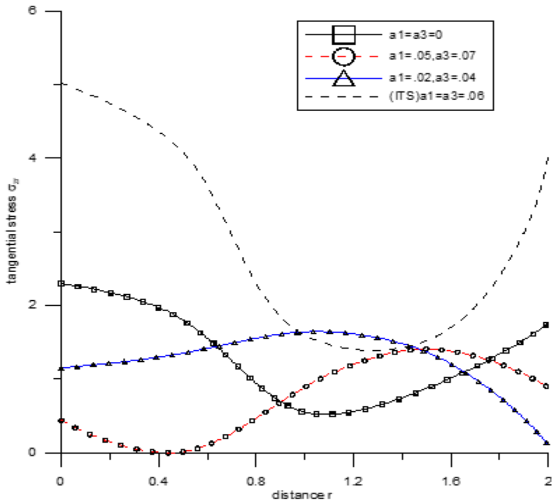

Fig. 2 variation of tangential stress σ_{zr} with the distance r(concentrated normal force)

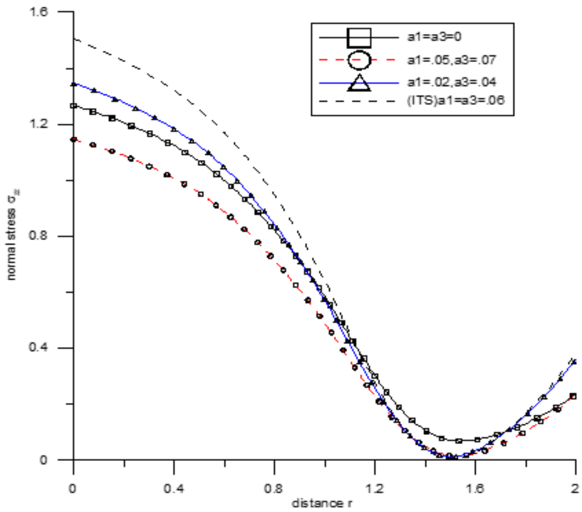

Fig. 3 Variation of normal stress σ_{zz} with the distance r(concentrated normal force)

7.1 Normal force on the boundary of the half-space

Case 1: Concentrated normal force

In Fig. 1, value of conductive temperature φ decreases for $0 \leq r \leq 1.5$ and increases in the remaining range. It is clear from the figure value of φ is small for all the four cases. In Fig. 2 variation of tangential stress σ_{zr} shows oscillatory behavior for $0 \leq r \leq 2$. For $a_1 = a_3 = 0$ and $a_1 = .05, a_3 = .07$ curves are opposite oscillatory. For $a_1 = .02, a_2 = .04$ curves first rises for $0 \leq r \leq 1.2$ and then falls in the remaining range. Amplitude in above mentioned three cases are smaller. But for isotropic case (ITS) curve follow a different trend with large amplitude. Effect of two temperature parameter is clearly observed from the figure. In Fig. 3 variation of normal stress σ_{zz} is similar as that

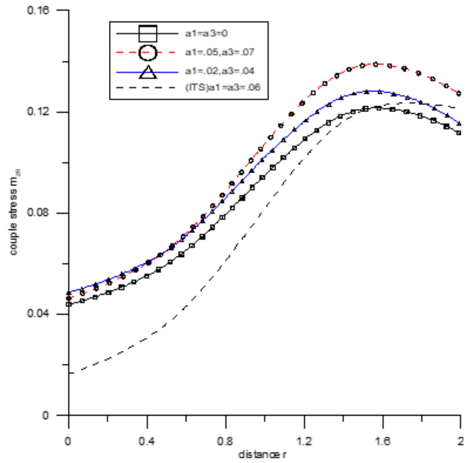

Fig. 4 Variation of couple stress $m_{z\theta}$ with the distance r(concentrated normal force)

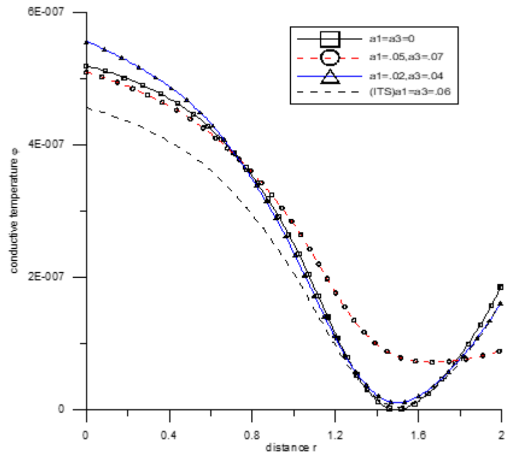

Fig. 5 Variation of conductive temperature φ with the distance r(normal force over the circular region)

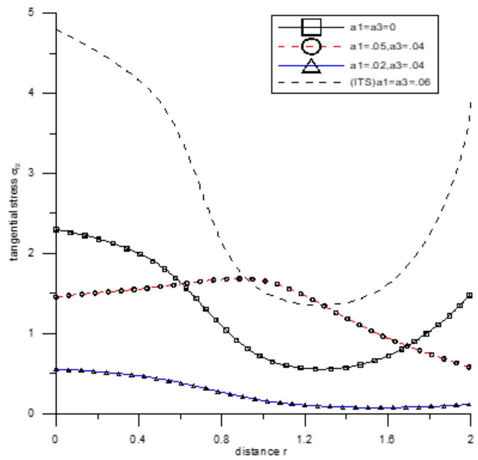

Fig. 6 Variation of tangential stress $\sigma_{zr}$with the distance r(normal force over the circular region)

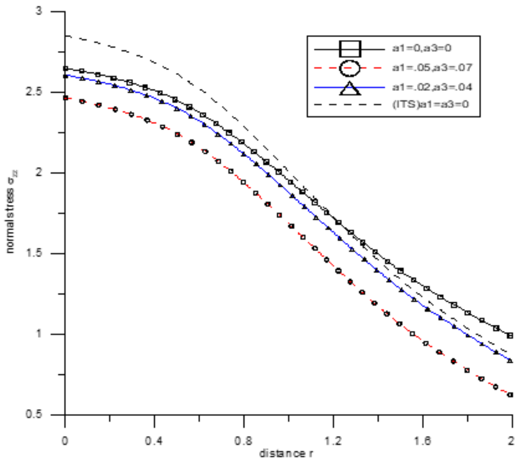

Fig. 7 Variation of normal stress σ_{zz} with the distance r(normal force over the circular region)

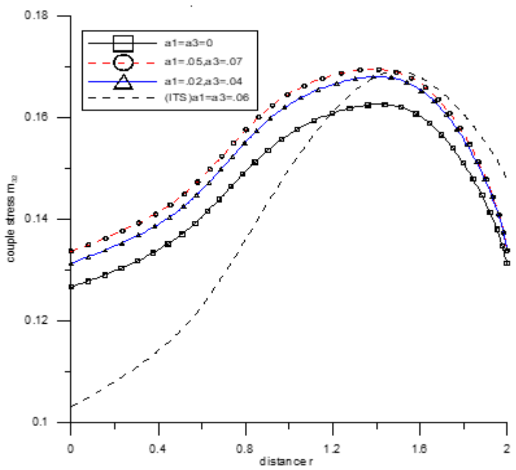

Fig. 8 Variation of couple stress $m_{z\theta}$ with the distance r (normal force over the circular region)

of Fig. 1 except that of the amplitude/value of the curves. The value of σ_{zz} is higher than that of the corresponding value of φ. Very near the loading surface values of φ and $\sigma_{zz}$are high. In fig. 4 couple stress $m_{z\theta}$ first monotonically decreases for $0 \leq r \leq 1.6$ and increases slightly in the rest of distance axes. Near the loading surface value of m_{32} is smallest for (ITS) $a_1 = a_3 = 0$ than the remaining three cases.

Case 2: Normal force over the circular region

In Fig. 5 variation of φ with the distance r is similar to that of Fig. 1. Value of φ are also almost same for the same value of r. In Fig. 6 variation of σ_{zr} is almost similar to that of Fig. 2. For $a_1 = .02, a_3 = .04$ curve is descending oscillatory at the lowest position from the all four curves with very small amplitude. For $a_1 = .05, a_3 = .07$ and For $a_1 = a_3 = 0$ curves are opposite oscillatory with almost same amplitudes. For isotropic solid curve first decreases for $0 \leq r \leq 1.2$ and then increases in the remaining range. Amplitude is greatest in this case. In Fig. 7 all the four curves decrease as

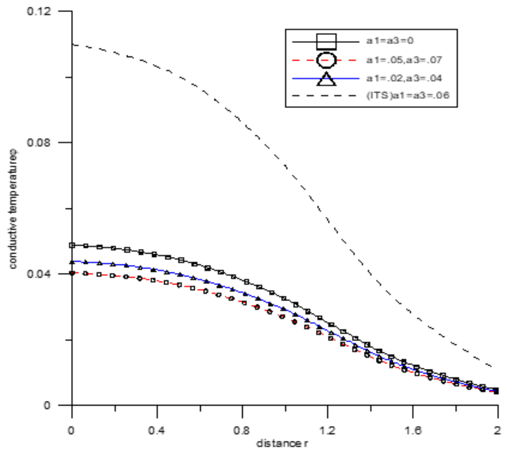

Fig. 9 Variation of conductive temperature φ with the distance r(thermal point source)

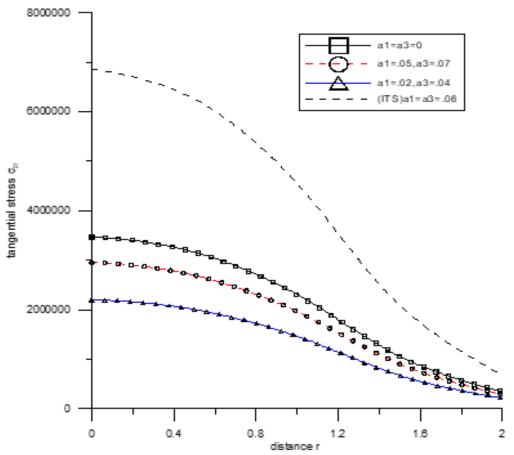

Fig. 10 Variation of tangential stress σ_{zr} with the distance r(thermal point source)

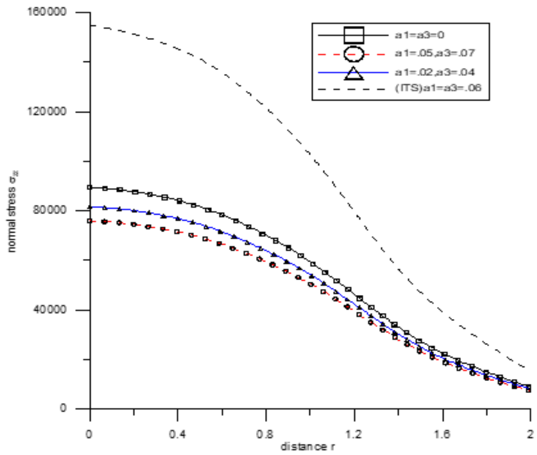

Fig. 11 Variation of normal stress σ_{zz} with the distance r(thermal point source)

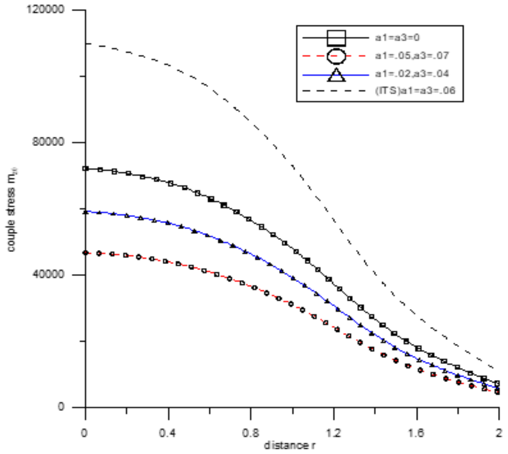

Fig. 12 Variation of couple stress $m_{z\theta}$ with the distance r(thermal point source)

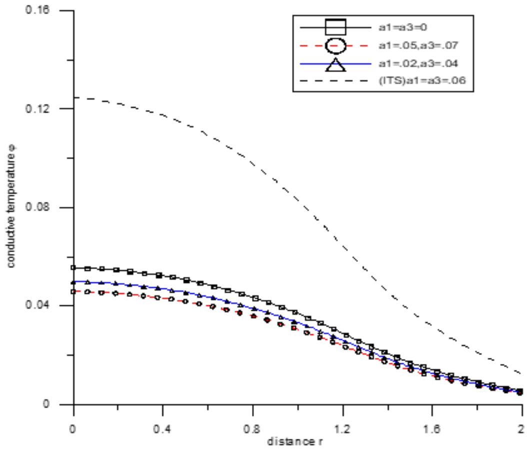

Fig. 13 Variation of conductive temperature φ with the distance r (thermal source over the circular region)

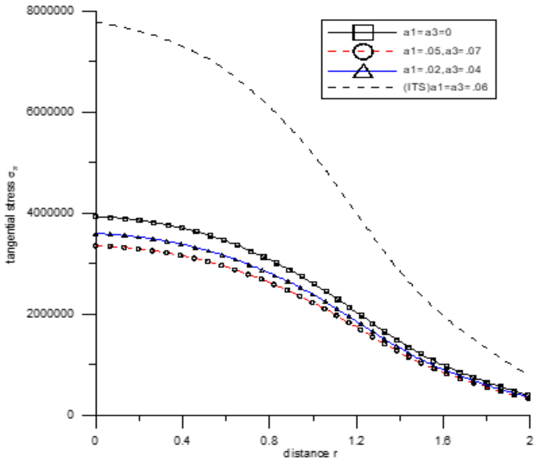

Fig. 14 Variation of tangential stress σ_{zr} with the distance r(thermal source over the circular region)

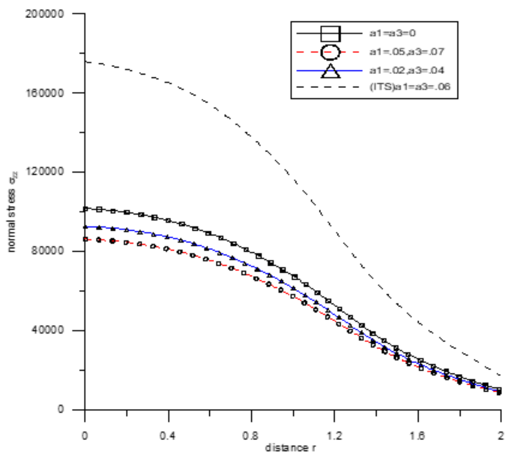

Fig. 15 Variation of normal stress σ_{zz} with the distance r(thermal source over the circular region)

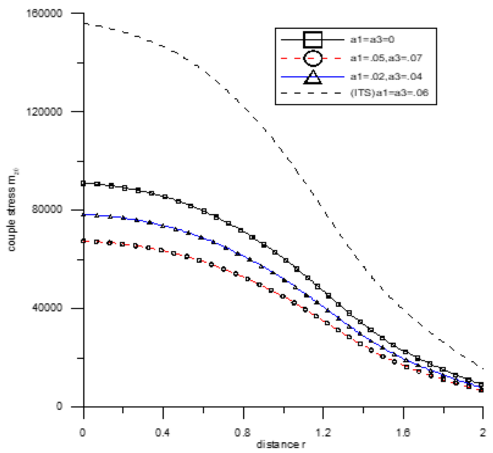

Fig. 16 Variation of couple stress $m_{z\theta}$ with the distance r(thermal source over the circular region)

distance r increases from $\sigma_{zz} = 2.75$ to 00.5 appx. The curve for $a_1 = .05, a_3 = .07$ is at the lowest positon, then curve for $a_1 = .02, a_3 = .04$ is at above of that . Then comes the curve for $a_1 = a_3 = 0$. Curve in case of isotropic thermoelastic solid starts from the uppermost position and cuts the curve for $a_1 = .02, a_3 = .04$ at $r = 1.2$. In Fig. 8 curves for the $m_{z\theta}$ first increase in $0 \le r \le 1.3$ and then decrease with the moderate amplitude in the rest of the range.

7.2 Thermal source on the boundary of half-space

Case-I: Thermal point source and Case-II: Thermal source over the circular region

Figs. 9-12 show the characteristics for thermal source for circular region and Figs. 13-16 show the characteristics for concentrated thermal source. It is depicted from Figs.9-16 that the distribution curves for normal stress σ_{zz}, conductive temperature φ ,tangential stress σ_{zr} and couple stress $m_{z\theta}$ for thermal source for circular region and concentrated thermal source ,decrease with the increase in the distance r with difference in magnitudes/ value in their respective patterns for all the cases of $a_1 =$

$.02, a_3 = .04 \quad a_1 = .05, a_3 = .07$, $a_1 = a_3 = 0$ and isotropic thermoelastic solid($a_1 = a_3 = .06$). Values of physical quantities are higher near the loading surface than the remaining range. Curve for (ITS) $a_1 = a_3 = 0$ is at uppermost position than the remaining three curves , with the largest amplitude in all the Figs. 9-16.

7. Conclusions

From the above investigation, it is clear that effect of two temperature plays an important part in the study of the deformation of the transversely isotropic thermoelastic body using new modified couple stress theory. As r varies from the point of application of the source the components of normal stress, tangential stress, couple stress and conductive temperature for concentrated normal force and normal force over the circular region follow different types of pattern. For thermal point source and thermal source over the circular region, it is observed that the variations of normal stress, tangential stress, couple stress and conductive temperature are monotonically decreasing with the increase of r with difference in magnitude/value. As the disturbances travel through different constituents of the medium, it suffers sudden changes, resulting in a variable/ non- uniform pattern of curves. The trend of curves exhibits the properties of two temperature of the medium and satisfies the required condition of the problem. The results of this problem are very useful in the two dimensional problem of dynamic response of the transversely isotropic thermoelastic solid without energy dissipation and with two temperature which has various geophysical, biological and industrial applications.

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