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Nonlinear thermal displacements of laminated composite beams

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Abstract. In this paper, nonlinear displacements of laminated composite beams are investigated under non-uniform temperature rising with temperature dependent physical properties. Total Lagrangian approach is used in conjunction with the Timoshenko beam theory for nonlinear kinematic model. Material properties of the laminated composite beam are temperature dependent. In the solution of the nonlinear problem, incremental displacement-based finite element method is used with Newton-Raphson iteration method. The distinctive feature of this study is nonlinear thermal analysis of Timoshenko Laminated beams full geometric non-linearity and by using finite element method. In this study, the differences between temperature dependent and independent physical properties are investigated for laminated composite beams for nonlinear case. Effects of fiber orientation angles, the stacking sequence of laminates and temperature on the nonlinear displacements are examined and discussed in detail.

Keywords: composite laminated beams; thermal nonlinear analysis; Timoshenko beam theory; total Lagragian; Finite Element Method; temperature dependent physical properties

1. Introduction

Laminated composite structures have been used many engineering applications, such as aircrafts, space vehicles, automotive industries, defence industries and civil engineering applications because these structures have higher strength-weight ratios, more lightweight and ductile properties than classical materials. With the great advances in technology, the using of the laminated composite structures is growing in applications.

Laminated composite structures have many practical applications in high temperature systems as thermal barrier systems, for example, reactor vessels, aircrafts, space vehicles, defense industries and other engineering structures. As nuclear power plants, aerospace vehicles, thermal power plants etc. are subjected to large thermal loadings, laminated composite structures have found extensive applications in these applications. The design of laminated composite structural elements (beams, plates, shells etc.) in the high thermal environments is very important. In high temperature, large deflection problems can be occurred in the laminated composite structures. As a result of high temperature, stability loss and fracture can be occurring in large deflections. It is

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known that large deflections problems are nonlinear type problems. So, understanding the nonlinear analysis and behavior of laminated composite structure are very important in high temperature values.

In the literature, much more attention has been given to the linear analysis of laminated composite beam structures. However, nonlinear studies of laminated composite beams have not been investigated broadly. In the open literature, studies of the nonlinear behavior of laminated composite beams are as follows; Ghazavi and Gordaninejad (1989) studied geometrically nonlinear static of laminated bimodular composite beams by using mixed finite element model. Singh *et al.* (1992) investigated nonlinear static responses of laminated composite beam based on higher shear deformation theory and von Karman's nonlinear type. Pai and Nayfeh (1992) presented three-dimensional nonlinear dynamics of anisotropic composite beams with von Karman nonlinear type. Di Sciuva and Icardi (1995) investigated large deflection of anisotropic laminated composite beams with Timoshenko beam theory and von Karman nonlinear strain-displacement relations by using Euler method. Donthireddy and Chandrashekhara (1997) investigated thermoelastic nonlinear static and dynamic analysis of laminated beams by using finite element method. Fraternali and Bilotti (1997) analyzed nonlinear stress of laminated composite curved beams. Ganapathi *et al.* (2009) studied nonlinear vibration analysis of laminated composite curved beams. Patel (1999) examined nonlinear post-buckling and vibration of laminated composite orthotropic beams/columns resting on elastic foundation with Von-Karman's strain-displacement relations. Oliveira and Creus (2003) investigated flexure and buckling behaviors of thin-walled composite beams with nonlinear viscoelastic model. Valido and Cardoso (2003) developed a finite element model for optimal desing of laminated composite thin-walled beams with geometrically nonlinear effects. Machado (2007) studied nonlinear buckling and vibration of thin-walled composite beams. Civalek (2008) examined cross-ply laminated plates by using first order shear deformation theory. Cardoso *et al.* (2009) investigated geometrically nonlinear behavior of the laminated composite thin-walled beam structures with finite element solution. Emam and Nayfeh (2009) investigated post-buckling of the laminated composite beams with different boundary conditions. Malekzadeh and Vosoughi (2009) studied large amplitude free vibration of laminated composite beams resting on elastic foundation by using differential quadrature method. Akgöz and Civalek (2011) and Civalek (2013) examined nonlinear vibration laminated plates resting on nonlinear-elastic foundation. Youzera *et al.* (2012) presented nonlinear dynamics of laminated composite beams with damping effect. Gürses *et al.* (2009), Baltacıoglu *et al.* (2010,2011), Civalek (2006) investigated static and vibration of laminated plates by using discrete singular convolution technique. Patel (2014) examined nonlinear static of laminated composite plates with the Green-Lagrange nonlinearity. Akbaş (2013, 2014, 2015a, 2015b, 2015c, 2017), Akbaş and Kocatürk (2013) and Kocatürk and Akbaş (2012, 2013) investigated nonlinear of functionally graded beams. Stoykov and Margenov (2014) studied Nonlinear vibrations of 3D laminated composite Timoshenko beams. Civalek and Demir (2016), Mercan and Civalek (2016), Wan *et al.* (2013) studied composite structures by using nonlocal elasticity theories. Cunedioğlu and Beylergil (2014) examined vibration of laminated composite beams under thermal loading. Akbaş (2018a, 2018b, 2018c, 2018d) studied nonlinear and post-buckling behaviors of laminated and fiber reinforced composite beams. Li and Qiao (2015), Shen *et al.* (2016, 2017), Li and Yang (2016) investigated nonlinear postbuckling analysis of composite laminated beams. Kurtaran (2015), Mororó *et al.* (2015), Pagani and Carrera (2017) analyzed large deflections of laminated composite beams. Benselama *et al.* (2015), Liu and Shu (2015), Topal (2017) investigated buckling behavior of composite laminate beams. Latifi *et al.* (2016), Ebrahimi and Hosseini (2017) presented nonlinear dynamics of laminated composite structures.

As seen from literature study, nonlinear thermal analyses of laminated composite beams have not been studied at large. In the nonlinear studies of laminated beams, the Von-Karman nonlinearity type is used generally. It is known, the Von-Karman nonlinearity type has restriction on the deflections and rotations. However, there are not any restriction on the deflections and rotations in the total Lagrangian nonlinear approach. The objective of presented paper is to fill this blank for laminated composite beams with considering the total Lagrangian nonlinear approach. In this study, temperature dependent nonlinear analysis of laminated Timoshenko beams is investigated under non-uniform temperature rising. The total Lagrangian approach is considered in the nonlinear kinematics model of the problem. The Newton-Raphson approach is utilized in nonlinear solution. The effects of the fiber orientation angles, the stacking sequence and temperature rising of laminates on the nonlinear thermal displacements of the composite laminated beam are investigated. Also, the difference between the temperature dependent and temperature independent physical properties are investigated in detail with different temperature values.

2. Theory and formulation

A simply supported laminated composite beam with three layers of length L, width b and height h, as shown in Fig. 1. The beam is subjected to non-uniform temperature rising with temperature rising values at the bottom surface ΔT_B and top surface ΔT_T as seen from Fig. 1. It is assumed that the layers are located as symmetry according to mid-plane axis. The height of each layer is equal to each other.

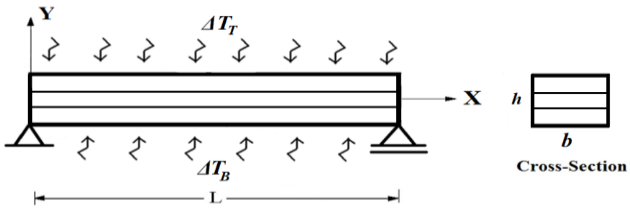


Fig. 1 A simply supported laminated beam subjected to non-uniform temperature rising and cross-section

In this study, the material properties are temperature-dependent. The effective material properties of the laminated beam, Young's modulus, coefficient of thermal expansion and shear modulus are a function of temperature T as follows (Shen 2001 and Li and Qiao 2015);

$$E_{11}(T) = E_{01}(1 - 0.5\ 10^{-3}T)\ \text{GPa} \tag{1a}$$

$$E_{22}(T) = E_{02}(1 - 0.2\ 10^{-3}T)\ \text{GPa} \tag{1b}$$

$$G_{12}(T) = G_{13} = G_{012}(1 - 0.2\ 10^{-3}T)\ \text{GPa} \tag{1c}$$

$$G_{23}(T) = G_{023}(1 - 0.2\ 10^{-3}T)\ \text{GPa} \tag{1d}$$

$$\alpha_{11}(T) = \alpha_{011}(1 + 0.5\ 10^{-3}T)\ \ 1/\ {}^{0}C \tag{1e}$$

$$\alpha_{22}(T) = \alpha_{022}(1 + 0.5\ 10^{-3}T)\ \ 1/\ {}^{0}C \tag{1f}$$

where, E_{11} and E_{22} indicate the Young's modulus in the longitudinal and transverse directions, respectively, G_{12}, G_{13}, G_{23} are the shear modulus, α_{11} and α_{22} are the thermal expansion coefficients in the longitudinal and transverse directions, respectively. $m=cos\,\theta$ and $n=\ sin\,\theta$, θ indicates the fiber orientation angle. E_{01}, E_{02}, G_{012}, G_{023}, α_{011}, α_{022} are the material constants at initial temperature value. T is final temperature; $T= T_0 +\Delta T$. where, T_0 is installation temperature and ΔT is the temperature rising.

The equivalent Young's modulus of kth layer in the X direction (E_x^k) as is used the following formulation as a function of temperature T (Vinson and Sierakowski 2002)

$$\frac{1}{E_x^k(T)} = \frac{cos^4(\theta_k)}{E_{11}(T)} + \left(\frac{1}{G_{12}(T)} - \frac{2\nu_{12}}{E_{11}(T)}\right) cos^2(\theta_k)\ sin^2(\theta_k) + \frac{sin^4(\theta_k)}{E_{22}(T)} \tag{2}$$

The temperature rising ΔT is governed by heat transfer equation of

$$-\frac{d}{d(Y)} = \left[K(Y)\frac{dT(Y)}{d(Y)}\right] = 0 \tag{3}$$

By integrating Eq. (3) using boundary conditions for kth layer $T(h_k/2)=\Delta T_T^k$ and $T(-h_k/2)=\Delta T_B^k$, the following expression can be obtained

$$\Delta T^k(Y) = \frac{\Delta T_B^k + \Delta T_T^k}{2} + \frac{\Delta T_T^k - \Delta T_B^k}{h_k} \quad -0.5h_k \leq Y \leq 0.5h_k \tag{4}$$

where, h_k is the thickness of kth layer, $\Delta T_T^k$is the temperature value of top surface in the kth layer and $\Delta T_B^k$is the temperature value of bottom surface in the kth layer.

As is known post-buckling case is a geometrically nonlinear problem. In the deriving of the geometrically nonlinear kinematic formulations, the total Lagrangian (TL) model is used with the finite element formulations of a two-node beam element for the Timoshenko beam theory which consist of shear deformation effects. The finite beam element with two nodes is shown in Fig. 2. The freedom degrees of each nodes are horizontal displacement u, vertical displacement v and the rotation φ.

In the TL kinematic model, all quantities of the body are expressed as functions of initial coordinate system (X, Y). So, the final coordinates of any point (B) of the beam at the deformed configuration are as follows

$$x = X + u - Ysin\varphi \tag{5}$$

$$y = v + Ycos\varphi \tag{6}$$

In the deformed configuration as seen from Fig. 2, the differential arc length ds-displacement relations are given as follows

$$ds = \sqrt{(1+u')^2 + (1+v')^2}\ dX \tag{7}$$

$$1 + u' = \frac{L\,cos\varphi}{L_i} \quad v' = \frac{L\,sin\varphi}{L_i} \quad s' = \frac{L}{L_i} \tag{8}$$

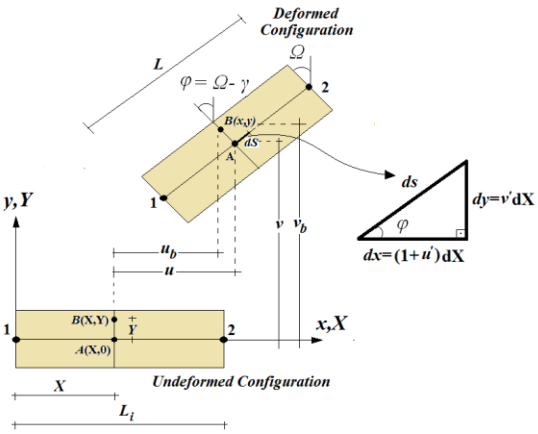


Fig. 2 The kinematics and coordinates of the beam element in the Lagrangian approach

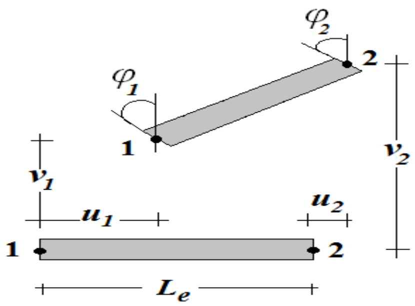


Fig. 3 Finite beam element model

The deformation gradient matrix ($\boldsymbol{F}$) can be obtained as;

$$[\boldsymbol{F}] = \begin{bmatrix} 1 + u' - Y\kappa\, cos\varphi & -sin\varphi \\ v' - Y\kappa\, sin\varphi & cos\varphi \end{bmatrix} \tag{9}$$

The Green-Lagrange strain tensor is as follows;

$$[\boldsymbol{\epsilon}] = \frac{1}{2}\left[\boldsymbol{F}^{\boldsymbol{T}}\boldsymbol{F} - \boldsymbol{I}\right] \tag{10}$$

where $\boldsymbol{I}$ indicates the identity matrix. With using a consistent-linearization technique, the Green-Lagrange strain-displacement relation is given as;

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_{XX} \\ 2\epsilon_{XY} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{XX} \\ \gamma \end{Bmatrix} = \begin{Bmatrix} (1 + u')cos\varphi + v'\, sin\varphi - Y\kappa - 1 \\ (1 + u')sin\varphi + v'\, sin\varphi \end{Bmatrix} = \begin{Bmatrix} e - Y\kappa \\ \gamma \end{Bmatrix} \tag{11}$$

where $\epsilon_{XX}, \gamma, \kappa = \varphi'$ are the axial strain, the shear strain and the curvature, respectively. In Eq. (11), $e = (1 + u')cos\varphi + v'\, sin\varphi - 1$. The finite element model of problem is displayed in Fig. 3. Each node of the finite element has three freedom degrees and the displacement vector $\{\boldsymbol{u}\}^{(e)}$ for a finite element is given as follows;

$$\{u\}^{(e)} = [\, u_1,\, v_1,\, \varphi_1,\, u_2,\, v_2,\, \varphi_2\,]^T \tag{12}$$

The displacement fields ($\{\boldsymbol{u}\}^{(e)}$) of a finite beam element are given in terms of the node displacements

$$u_x^{(e)} = N_1^{(u)} u_1 + N_2^{(u)} u_2 = \left[N^{(u)}\right]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{13}$$

$$\begin{aligned} u_y^{(e)} &= N_1^{(v)} v_1 + N_2^{(v)} \varphi_1 + N_3^{(v)} v_2 + N_4^{(v)} \varphi_2 \\ &= \left[N^{(v)}\right]\begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} \end{aligned} \tag{14}$$

$$\begin{aligned} \varphi^{(e)} &= N_1^{(\varphi)} v_1 + N_2^{(\varphi)} \varphi_1 + N_3^{(\varphi)} v_2 + N_4^{(\varphi)} \varphi_2 \\ &= \left[N^{(\varphi)}\right]\begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} \end{aligned} \tag{15}$$

where $N_{\mathrm{i}}^{(u)}$, $N_{\mathrm{i}}^{(v)}$ and $N_{\mathrm{i}}^{(\varphi)}$ are the interpolation functions for axial displacement, vertical displacement and rotation, respectively. The strain-displacement relation is presented with the nodal displacements in matrix form;

$$\{\epsilon\} = [D]\{u\}^{(e)} \tag{16}$$

where $[D]$ is a differential operator matrix which related between the strain and the displacement vector. With using linear interpolation functions, the differential operator is given as follows

$$[D] = \frac{1}{L_i}\begin{bmatrix} -cos\varphi & -sin\varphi & \gamma\, N_2^{(\varphi)} L_i & cos\varphi & sin\varphi & \gamma\, N_4^{(\varphi)} L_i \\ sin\varphi & -cos\varphi & -N_2^{(\varphi)}(1+e)L_i & -sin\varphi & cos\varphi & -N_4^{(\varphi)}(1+e)L_i \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

The second Piola-Kirchhoff stresses-the Green-Lagrange strain relation is given for linear elastic material as follows;

$$\begin{aligned} \{\boldsymbol{S}\} = \begin{Bmatrix} S_{XX} \\ S_{XY} \end{Bmatrix} &= \begin{bmatrix} E_x^k(T) & 0 \\ 0 & G_x^k(T) \end{bmatrix}\begin{Bmatrix} \epsilon_{XX-} - \alpha 11(Y,T)\Delta \mathrm{T}(Y,T) \\ \gamma \end{Bmatrix}, \\ &= [C]\{\epsilon\} \end{aligned} \tag{18}$$

S_{XX} and S_{XY} the second Piola-Kirchhoff stresses axial and shear components, respectively. where $[C]$ is;

$$[C] = \begin{bmatrix} E_x^k(T) & 0 \\ 0 & G_x^k(T) \end{bmatrix} \tag{19}$$

where G_x^k is the equivalent shear modulus of kth layer. Substituting Eqs. (16) into Eq. (18), the constitutive relation is expressed as follows

$$\{\boldsymbol{S}\} = [C][D]\{u\}^{(e)} \tag{20}$$

The virtual work equation of the beam element based on the TL approach with neglecting the body forces is given as follows;

$$\int_V (s_{XX}\delta\varepsilon_{XX} + s_{XY}\delta\gamma)dV - \int_0 (r_X\delta u + r_Y\delta v)dS = 0 \tag{21a}$$

$$\int_V \{\boldsymbol{S}\}\{\delta\epsilon\}dV - \int_S \left\{\delta u^{(e)}\right\}^T [N]^T \begin{Bmatrix} r_X \\ r_Y \end{Bmatrix} dS = 0 \tag{21b}$$

where r_X and r_Y are the boundary forces in the X and Y directions respectively. V indicates the volume of the body. Substituting Eqs. (20) and (16) into Eq. (21b) and applying the variational process, the virtual work equation is expressed as follows;

$$\int_V \left([C][D]\{u\}^{(e)}\right)\left(\frac{\partial[D]}{\partial u}\{\delta u\}^{(e)} + [D]\{\delta u\}^{(e)}\right)dV - \int_0^{L_e} \left\{\delta u^{(e)}\right\}^T [N]^T \begin{Bmatrix} r_X \\ r_Y \end{Bmatrix} dX = 0 \tag{22a}$$

$$\{\delta u\}^{(e)}\left(\int_V \left(\frac{\partial[D]^T}{\partial u}[C][D] + [D]^T[C][D]\right)\{u\}^{(e)}dV - \int_0^{L_e} [N]^T \begin{Bmatrix} r_X \\ r_Y \end{Bmatrix} dX\right) = 0 \tag{22b}$$

After integration process and the regulation of the Eq. (22b), the equation of motion is expressed as follows

$$\{u\}^{(e)}\left(\int_0^{L_e} \sum_{k=1}^{k}\left(\frac{\partial[D]^T}{\partial u}[C][D] + [D]^T[C][D]\right)A_k dX - \int_0^{L_e} [N]^T \begin{Bmatrix} r_X \\ r_Y \end{Bmatrix} dX\right) = 0 \tag{23}$$

In Eq. (23), $\{u\}^{(e)}$ is the displacement vector. The element tangent stiffness matrix is presented as follows

$$\boldsymbol{K}_T = \boldsymbol{K}_M + \boldsymbol{K}_G \tag{24}$$

where $\boldsymbol{K}_G$ and $\boldsymbol{K}_M$ are the geometric and material stiffness matrixes, respectively, as follows

$$\boldsymbol{K}_M = \int_0^{L_e} \sum_{k=1}^{k} [D]^T[C][D]A_k\, dX \tag{25a}$$

$$\boldsymbol{K}_G = \int_0^{L_e} \sum_{k=1}^{nk} \frac{\partial[D]^T}{\partial u}[C][D]A_k\, dX \tag{25b}$$

The load vector $\boldsymbol{F}$ is given as follows

$$\boldsymbol{F} = \int_0^{L_e} [N]^T \begin{Bmatrix} r_X \\ r_Y \end{Bmatrix} dX \tag{26}$$

In the nonlinear finite element solution, Newton-Raphson iteration procedure is considered. In solution procedure, the load is divided by an appropriate number with the small-step incremental. For n+1 st load increment and i th iteration, increment displacement vector is given as follows;

$$d\boldsymbol{u}_n^i = \left(\boldsymbol{K}_T^i\right)^{-1}\boldsymbol{F}_{n+1}^i \tag{27}$$

where $\boldsymbol{K}_T^i$, $d\boldsymbol{u}_n^i$ and $\boldsymbol{F}_{n+1}^i$ are tangent stiffness matrix, the increment displacement vector and load vector respectively for i th iteration and n+1 st load increment. The iteration tolerance form is selected in the Euclidean norm as follows;

$$\sqrt{\frac{\left[\left(d\boldsymbol{u}_n^{i+1} - d\boldsymbol{u}_n^i\right)^T\left(d\boldsymbol{u}_n^{i+1} - d\boldsymbol{u}_n^i\right)\right]^2}{\left[\left(d\boldsymbol{u}_n^{i+1}\right)^T\left(d\boldsymbol{u}_n^{i+1}\right)\right]^2}} \le \xi_{tol} \tag{28}$$

The updated displacement vector after the end of the i th iteration and n+1 st load increment is given as follows;

$$\boldsymbol{u}_{n+1}^{i+1} = \boldsymbol{u}_{n+1}^i + d\boldsymbol{u}_{n+1}^i = \boldsymbol{u}_n + \Delta\boldsymbol{u}_n^i \tag{29}$$

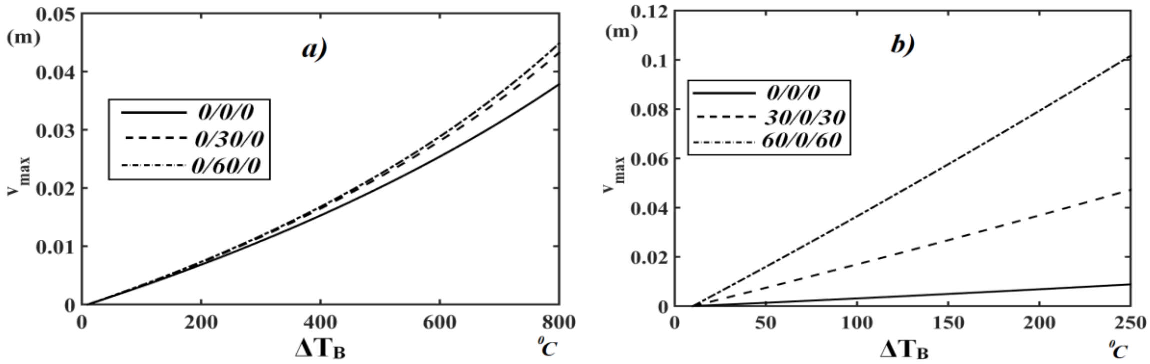
where

$$\Delta\boldsymbol{u}_n^i = \sum_{k=1}^{i} d\boldsymbol{u}_n^k \tag{30}$$

3. Numerical results

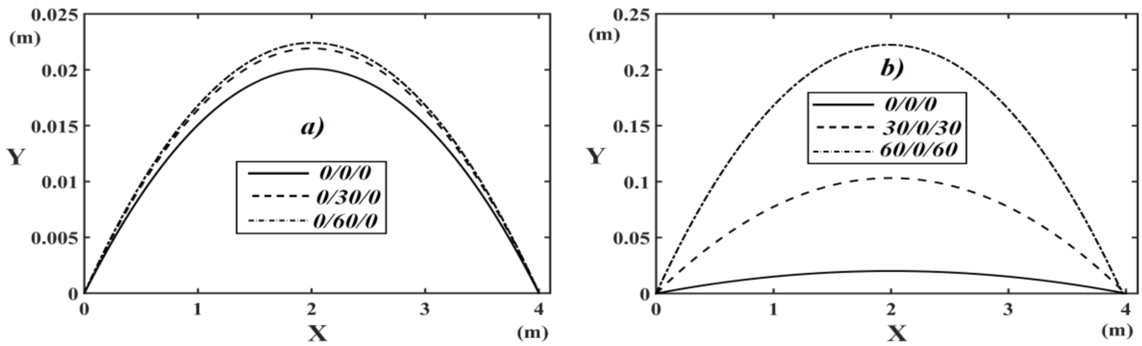
In the numerical study, nonlinear thermal displacements of the simply supported laminated beam are calculated and presented for different fibre orientation angles and the stacking sequence of laminates under non-uniform temperature rising in temperature dependent and independent physical properties. The difference between the temperature dependent and temperature independent physical properties of the laminated beam is examined discussed. In integration process, five-point Gauss rule is used. In the iterations, the temperature rising is divided by a suitable number according to the value of temperature. The temperature rising is divided by large numbers. After completing an iteration process, the temperature is increased by adding temperature increment to the accumulated temperature.

The material of the laminated composite beam is considered as Graphite/Epoxy in numerical examples. The material properties of Graphite/Epoxy are temperature dependent. For Eq. (1), the material constants of Graphite/Epoxy are expressed as follows (from Wang *et al.* 2002, Oh *et al.* 2000); E_{01}=150 GPa, E_{02}=9 GPa, G_{012}=7,1 GPa, G_{023}=2,5 GPa, α_{011}=1,1x10^{-6} $1/\,^0C$, α_{022}= 25,2x10^{-6} $1/\,^0C$ at 30 ^0C. In this study, the Possion's ratio is taken as ν=0.3. The geometry properties of the beam are considered as follows: b=0.2 m, h=0.2 m and L=4 m. It is mentioned before that the thickness of layers is equal to each other. In the numerical results, the installation temperature is T_0=30 ^0C. In the non-uniform temperature rising, the temperature rising at the bottom surface ΔT_B is varied as the temperature rising at the top surface ΔT_T is kept constant as 10 ^0C. The number of finite elements is taken as 100.



(a) for the stacking sequence [*0/θ/0*] (b) for the stacking sequence [*θ/0/θ*]

Fig. 4 The maximum transversal displacements v_{max} versus temperature rising ΔT_B with different fiber orientation angles in the temperature dependent physical properties.

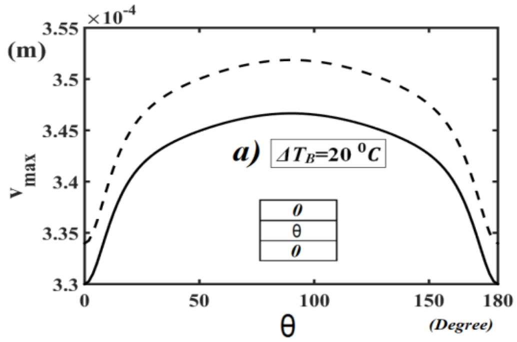


(a) for the stacking sequence [*0/θ/0*] (b) for the stacking sequence [*θ/0/θ*]

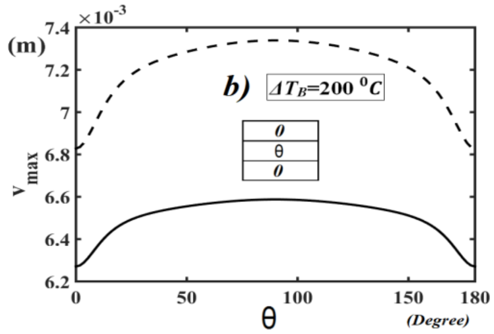
Fig. 5 Thermal nonlinear deflected configuration of the laminated beam for different values of the fiber orientation angles for ΔT_B=500 ^0C

In Fig. 4, the maximum vertical displacements (at the midpoint of the beam) versus temperature rising ΔT_B are presented for different values of fiber orientation angles (θ) for stacking sequences [*0/θ/0*] and [*θ/0/θ*] in nonlinear case and temperature dependent physical properties. In Fig. 5, the effect of stacking sequences on nonlinear thermal configuration of the laminated beam for a constant temperature rising ΔT_B= 500 ^0C for the stacking sequences [*0/θ/0*] and [*θ/0/θ*].

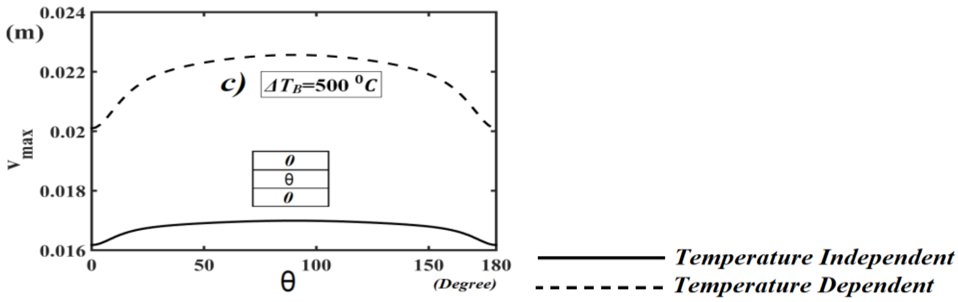
It is seen from Figs. 4 and 5 that increasing the fiber orientation angles (θ) causes an increase in the nonlinear thermal deflections in both [*0/θ/0*] and [*θ/0/θ*]. This is because; the equivalent Young's modulus and bending rigidity decrease according to the Eq. (2) with increasing θ. As a result, the strength of the material decreases and the displacements increases naturally. It is observed from Figs. 4 and 5 that the nonlinear thermal deflections in [*θ/0/θ*] are bigger than [*0/θ/0*]'s. The nonlinear responses in the [*θ/0/θ*] are very sensitive and thermal nonlinear displacements change quickly with increasing the fiber orientation angles in contrast with [*0/θ/0*]. It shows that the stacking sequence plays very important role on the thermal nonlinear behavior of the laminated beams. This situation is seen more specifically in Fig. 5 which shows the displaced



(a) for ΔT_B=20 ^0C



(b) for ΔT_B=200 ^0C



(c) for ΔT_B=500 ^0C

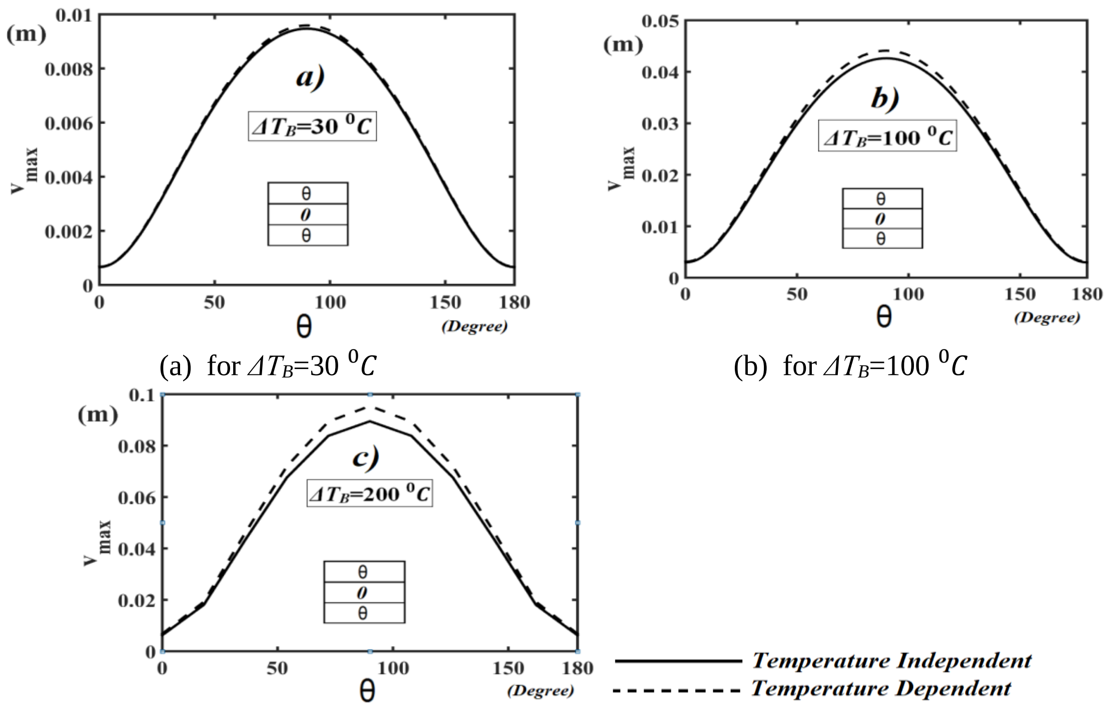
Fig. 6 The relationship between fiber orientation angles and nonlinear maximum thermal deflections with temperature dependent and independent physical properties for [*0/θ/0*]

configuration of the laminated beam. Also, it is observed from Fig. 4 that the difference among of the results of fiber orientation angles increases significantly with increasing temperature.

In Figs. 6 and 7, the effects of the fiber orientation angles on the maximum vertical displacements are plotted for the stacking sequences [*0/θ/0*] and [*θ/0/θ*], respectively, for different values of temperature rising ΔT_B. Also, the difference between the temperature dependent and temperature independent physical properties is investigated in Figs. 6 and 7.

It is observed from Figs. 6 and 7 that increasing the fiber orientation angles to 0° from 90°, the deflections increase significantly. At the fiber orientation angle θ=90 $^{\circ}$, the deflections are the greatest value for each layer arrangements because the equivalent Young's modulus and bending rigidity is the smallest values at θ=90 $^{\circ}$.

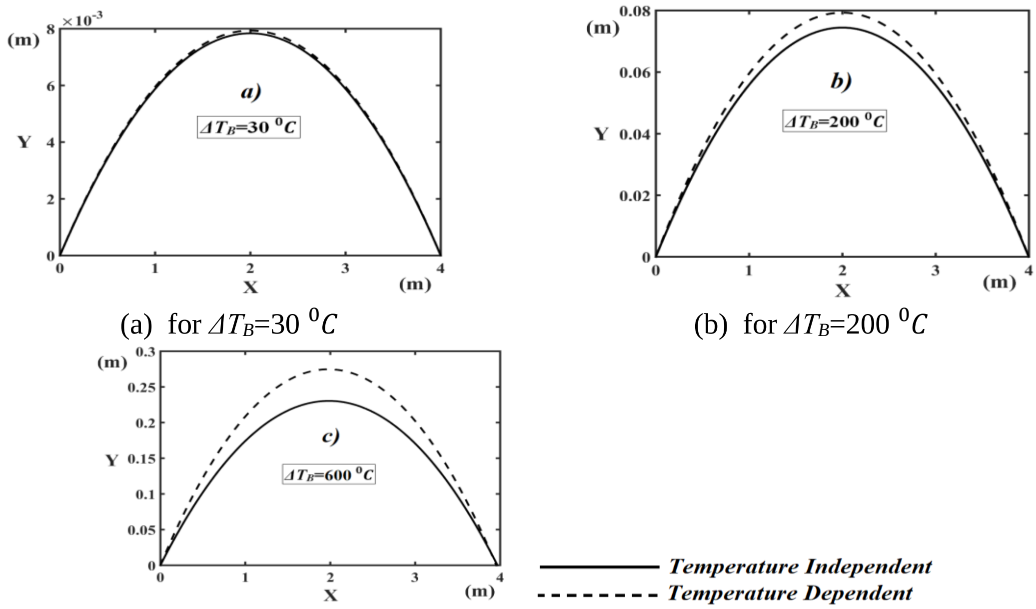
It is seen from Figs. 6 and 7, with increase in temperature, the difference between temperature dependent and independent physical properties increases considerably. Figs. 6 and 7 show that the nonlinear thermal displacements for the temperature-dependent physical properties are greater than those for the temperature-independent physical properties. This situation may be explained as follows: in the temperature-dependent physical properties, with the temperature increase, the intermolecular distances of the material increase and intermolecular forces decrease. As a result, the strength of the material decreases. Hence, the beam becomes more flexible in the case of the temperature-dependent physical properties. Also, as seen from Figs. 6 and 7, the difference between the temperature dependent and independent physical properties in the stacking sequences [*θ*/0/*θ*] is very sensitive in contrast with [*θ*/0/*θ*]. It shows that the stacking sequences is very



(a) for ΔT_B=30 ^0C (b) for ΔT_B=100 ^0C

(c) for ΔT_B=200 ^0C

Fig. 7 The relationship between fiber orientation angles and maximum deflections for linear and nonlinear solution for the stacking sequence [$\theta/0/\theta$]



(a) for ΔT_B=30 ^0C (b) for ΔT_B=200 ^0C

(c) for ΔT_B=600 ^0C

Fig. 8 The difference between temperature dependent and independent physically properties on the nonlinear deflected configuration for the stacking sequence 30/0/30 important role in the thermal and nonlinear behavior of laminated composite beams.

In Fig. 8, the difference between the temperature dependent and independent physical properties on the nonlinear thermal configuration is shown for different temperature rising for the stacking sequence 30/0/30.

As seen from Fig. 8, increasing temperature leads to increase the difference between the temperature dependent and independent physical properties considerably. In the higher temperature values, the temperature dependent physical properties must be taken into account for safe design of composite laminated beams and for obtaining more realistic results.

4. Conclusions

Nonlinear thermal behavior of a simply supported laminated composite beam is analyzed under non-uniform temperature rising with temperature dependent physical properties. In the nonlinear kinematic model, total Lagrangian approach is used by using finite element method with the Timoshenko beam theory. The considered non-linear problem is solved by using incremental displacement-based finite element method and the Newton-Raphson method. The fibber orientation angles and the stacking sequence of laminates on nonlinear thermal deflections of the laminated beam are studied and discussed. The differences between temperature dependent and independent physical properties are examined.

It is observed from the investigations that the fiber orientation angles and the stacking sequences of laminates have a great influence on the nonlinear thermal behavior of the laminated beams. There are significant differences of the analysis results for the temperature dependent and independent physical properties in the nonlinear case. With change the fiber orientation angles and the stacking sequences of laminates, difference between the temperature dependent and independent physical properties results change considerably. The stacking sequences is very important role in the thermal and nonlinear behavior of laminated composite beams. In order to obtain more realistic results and real displacement values for laminated composite structures in higher temperature values, the nonlinear case and the temperature-dependent physical properties must be taken into account for safe design of beams. Also, it is seen from the investigation, with the finite element method, nonlinear analysis of composite structure can be solved without any difficulty.

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