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The influence of the initial strains of the highly elastic plate on the forced vibration of the hydro-elastic system consisting of this plate, compressible viscous fluid, and rigid wall

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Abstract. The hydro-elastic system consisting of a pre-stretched highly elastic plate, compressible Newtonian viscous fluid, and the rigid wall is considered and it is assumed that on the plate a lineal-located time-harmonic force acts. It is required to investigate the dynamic behavior of this system and determine how the problem parameters and especially the pre-straining of the plate acts on this behavior. The elasticity relations of the plate are described through the harmonic potential and linearized (with respect to perturbations caused by external time-harmonic force) form of these relations is used in the present investigation. The plane-strain state in the plate is considered and the motion of that is described within the scope of the three-dimensional linearized equations of elastic waves in elastic bodies with initial stresses. The motion of the fluid is described by the linearized Navier-Stokes equations and it is considered the plane-parallel flow of this fluid. The Fourier transform with respect to the space coordinate is applied for a solution to the corresponding boundary-value problem. Numerical results on the frequency response of the interface normal stress and normal velocity and the influence of the initial stretching of the plate on this response are presented and discussed. In particular, it is established that the initial stretching of the plate can decrease significantly the absolute values of the aforementioned quantities.

Keywords: compressible viscous fluid; highly elastic plate; initial strain; frequency response; forced vibration; rigid wall

1. Introduction

It is known that the first attempt to investigate of the vibration of the plate + fluid hydro-elastic system was started by Lamb (1921) with the study of the natural vibration of a circular elastic “baffled” plate in contact with still water. In this work it is employed the “non-dimensional added

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virtual mass incremental" (NAVMI) method which assumes the invariability of the modes of vibration of the plate as a result of the contact of that with still water. Namely this assumption allows to the use the Rayleigh quotient for determination of the natural frequencies of the noted hydro-elastic system. The NAVMI method was also used in the further in related investigations carried out in works by Kwak and Kim (1991), Fu and Price (1987), Zhao and Yu (2012), Askari *et al.* (2013) and in many others listed therein. At the same time, there are also sufficient number of related investigations, such as carried out in the papers by Tubaldi and Armabili (2013), Charman and Sorokin (2005), Sun *et al.* (2015), Liao and Ma (2016) and others listed therein which were carried out without employing the NAVMI method.

Up to now without employing the NAVMI method it is also has been made several investigations (see, for instance, Chiba (1994), Shafiee *et al.* (2014), Hasheminejad and Mohammadi (2017), Kutlu *et al.* (2012), Askari and Daneshmand (2010) and others listed therein) which relate to the study of the vibrations of the system consisting of cylindrical tank the lateral wall of which is absolute rigid one, the circular flexible plate which is in bottom of this tank, the incompressible inviscid fluid filled this cylindrical tank and the elastic foundation on which lies the plate and consequently, the tank. The response of the elastic foundation to the plate vibration is described through the Winkler or Pasternak models, however the vibration of the plate is described within the scope of the approximate plate theories such as Kirkchhoff, Mindlin theories. It is assumed that the upper surface of the fluid is free.

The free vibration of two identical circular plates coupled with bounded fluid is considered in the paper by Jeong (2003) in which the motion of the plate is described through the Kirkchhoff theory and fluid is assumed to be inviscid and incompressible.

The influence of the viscosity of the incompressible fluid on the vibration of the membrane or plate which covers this fluid which is in the cylindrical tank the walls of which are absolutely rigid is investigated by Bauer and Chiba (2007).

The vibration of the cylindrical shells which are in contact with the incompressible inviscid fluids is considered in the papers by Moshkelgosha *et al.* (2017), Askari and Jeong (2010), Askari *et al.* (2011), Askari and Daneshmand (2010) and others listed therein.

The other field of the investigations regarding the dynamics of the plate-fluid systems is the corresponding wave propagation problems investigated, for instance, in the paper by Sorokin and Chubinskij (2008) and others listed and reviewed therein. It should be noted that in this paper the role of fluid viscosity in the wave propagation in the plate-fluid system was also considered. However, the investigations carried out in the paper by Sorokin and Chubinskij (2008) and others noted above were made within the scope of the approximate plate theories and, as a result of which, the range of the described wave modes and their dispersion curves, decreases significantly. It is evident that in cases where the wavelength is significantly less than the thickness of the plate more accurate results in the qualitative and quantitative sense, can be obtained by employing the exact equations for describing the plate motion. Moreover, in the foregoing papers, except the paper by Zhao and Yu (2012), the initial stretching of the plates, which can be one of their reference characteristics, are not taken into consideration.

The influence of the initial stress in the plate on the dispersion of the waves propagated in the plate-compressible fluid system is studied within the scope of the corresponding exact linearized equations of motion in the papers by Bagno (2015), Bagno *et al.* (1994) and others, a review of which is given in the survey paper by Bagno and Guz (1997). Note that in these investigations the motion of the viscous fluid is written within the scope of the linearized Navier-Stokes equations and detailed consideration of the results is made in the monograph by Guz (2009). Moreover, note that detailed review of the investigations related to the dynamics of the plate+fluid systems was also made in the paper by Akbarov and Ismailov (2017).

From the authors' point of view, one of the significant investigation fields of the plate-fluid systems is also investigations related to their forced vibration and carried out within the scope of the corresponding exact equations of motion. The first attempt in this field is made in the paper by Akbarov and Ismailov (2014) in which the two-dimensional (plane-strain state) problem on the forced vibration of the pre-strained highly elastic plate+compressible viscous fluid system is studied. Note that the results obtained in the paper by Akbarov and Ismailov (2014) were also detailed in the monograph by Akbarov (2015). Continuation of these studies is made in other papers by these authors the brief review of which is given below.

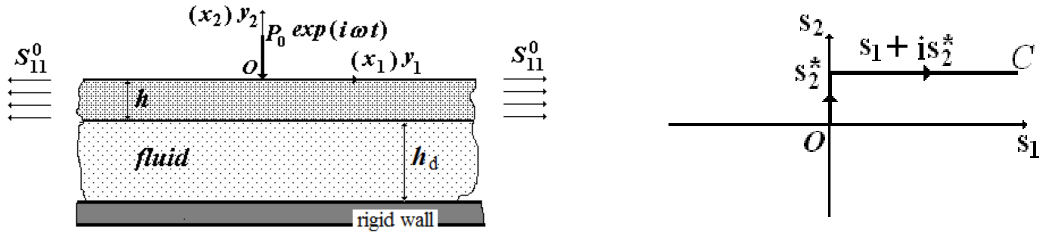
The paper by Akbarov and Ismailov (2017) deals with the study of the forced vibration of the system consisting of the elastic plate, compressible viscous fluid, and rigid wall. The dynamics of the moving load acting on the mentioned hydro-elastic system is considered in the paper by Akbarov and Ismailov (2015). The dynamics of the oscillating moving load acting on the hydro-elastic system consisting of the elastic plate, compressible viscous fluid and rigid wall is considered in the paper by Akbarov and Ismailov (2016a) and it is established that the action of the oscillating moving load on the motion of the mentioned system significantly depend not only on the fluid viscosity but also on the vibration phase of the external load.

The paper by Akbarov and Ismailov (2016b) studies the forced vibration of the initially stretched metal elastic plate loaded with the compressible viscous fluid and it is concluded that the initial stretching causes a decrease in the absolute values of the pressure on the interface plane between the plate and fluid.

The paper by Akbarov and Panakhli (2015, 2017) develops the discrete-analytical solution method for the solution to problems related to the dynamics of the hydro-elastic system consisting of an axially-moving pre-stressed plate, compressible viscous fluid and rigid wall. Concrete numerical results on the influence of the plate moving velocity on the frequency response of the interface fluid pressure and velocity are presented and discussed. It is also studied the influence of the fluid viscosity on this response.

Note that in all the foregoing investigations related to the forced vibration of the plate-fluid systems it is assumed that (except the paper by Akbarov and Ismailov 2014) the plate material is metal (in particular, is steel). Consequently, the foregoing results cannot be applied to the cases where the plate material is the highly elastic one which can be taken place in many modern engineering branches such as bioengineering, chemical engineering, mechanical engineering and etc. Note that one of the characteristic particularities of the mechanical behavior of the highly elastic plates is its very high sensitivity to the initial stretching. Therefore, the study of the forced vibration related to the hydro-elastic system consisting of the highly elastic plate and fluid requires taking into consideration the initial stretching of the plate.

In connection with the foregoing discussions, in the present paper the forced vibration of the hydro-elastic system consisting of the initially strained highly elastic plate, compressible viscous fluid and rigid wall is considered, and the aim of the present investigation is the determination the character of the influence of the initial stretching of the highly elastic plate on the frequency response of the hydro-elastic system under consideration. We recall that, in the paper by Akbarov and Ismailov (2014) the forced vibration of the initially strained highly elastic plate loaded with the compressible viscous fluid which filled the half-plane, was considered. Consequently, the investigations carried out in the present paper can be also estimated as the development of the



(a) (b)

Fig. 1 The sketch of the hydro-elastic system under consideration (a) and Sommerfeld contour (b)

investigations presented in the paper by Akbarov and Ismailov (2014) for the case where the motion of the fluid is bounded not only by the plate but also with the rigid wall.

2. Formulation of the problem

We introduce into consideration the hydro-elastic system consisting of the initially stretched highly elastic plate-layer, compressible viscous fluid and rigid wall the sketch of which is shown in Fig. 1. Assume that the thickness of the plate and the depth of the fluid in the natural state (i.e., before the initial stretching of the plate) are h and h_d, respectively. Assume that after initial stretching of the plate the time-harmonic lineal-located dynamical force acts on that and as a result of this action the forced vibration of the foregoing hydro-elastic system appears. It is required to study the frequency response of the considered hydro-elastic system and the influence of the initial stretching of the highly elastic plate on this response. For mathematical formulation of the problem, first we consider the field equations related to the motion of the initially strained highly elastic plate and to the flow of the fluid.

2.1 Governing field equations for the plate-layer

We distinguish three states of the plate and these states are: the natural state in which the plate has not any deformation and external loading; the initial state in which the plate is stretched along its length by the static forces acting at infinity and the action of these forces continues all further dynamic process; and the perturbed state which causes by the additional lineal-located time harmonic dynamic force acting on the plate. We assume that the plate is in contact with the fluid after initial stretching of that and before the action of the mentioned dynamic force.

We determine positions of the points of the layer in the natural state by the Lagrangian coordinates in the Cartesian system of coordinates $Ox_1x_2x_3$ and suppose that the layer has infinite length in the directions of the Ox_1 and Ox_3 axes. As the Ox_3 axis extends along a direction which is perpendicular to the plane Ox_1x_2 in Fig. 1 and therefore this axis is not shown in this figure.

At the same time, with the initial state of the layer we associate the Cartesian system of coordinates $Oy_1y_2y_3$ and suppose that the origin of this system coincides with the origin of the system $Ox_1x_2x_3$, and the coordinate axes Oy_1, Oy_2 and Oy_3 coincide with the coordinate axes Ox_1, Ox_2 and Ox_3, respectively. We suppose that the material of the layer is compressible and the elastic relations of that are given through the harmonic potential.

Denoting the values related to the initial state by upper index 0, we assume that the displacements in the initial state in the plate can be presented as follows.

$$u_1^0 = (\lambda_1 - 1)x_1 \ , \ u_2^0 = (\lambda_2 - 1)x_2 \, , \ u_3^0 = 0 \ \lambda_1 = const_1 \neq 1 \ , \ \lambda_2 = const_2 \neq 1 , \ \lambda_3 = 1 \, , \\ \lambda_2 = const_2 \neq 1 , \ y_1 = \lambda_1 x_1 \, , y_2 = \lambda_2 x_2 \, , \ y_3 = x_3 \, , \tag{1}$$

where u_k^0 $(k = 1,2,3)$ is a component of the displacement vector in the layer in the initial strain state and λ_k is an elongation factor which characterizes the change in the length of the line element in the Ox_k axis direction. This parameter is determined by the expression $\lambda_k = \sqrt{1 + 2\varepsilon_k}$, where ε_k is the $k - th$ principal value of the Green's strain tensor. The expression of the components of this tensor through the components of the displacement vector will be given below.
Thus, within this initial strains we consider a motion of the layer by the use of coordinates associated with the initial state, i.e. by the use of coordinates y_k $(k = 1,2,3)$. We describe this motion in the framework of the three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB). The started point for construction of the equations and relations of the TLTEWISB is the geometric nonlinear equations and relations written for the sums of the values related to the initial state and of the values related to the perturbed state the meaning of which is explained in the beginning of the present subsection. As a result of the linearization of the mentioned nonlinear equations and relations with respect to the perturbations it is obtained the equations and relations of the TLTEWISB. Note that under this linearization it is assumed that the nonlinear terms with respect to the perturbed state quantities are very small than the corresponding linear terms and these nonlinear terms can be neglected under investigation of a certain class of problems related to dynamics, statics and stability loss of the elements of constructions. The general problems of the TLTEWISB have been elaborated in many investigations such as Biot (1965), Guz (2004), Truestell and Noll (1965) and others.

Thus, as a result of the aforementioned linearization the following basic relations of the TLTEWISB for the compressible body under the plane-strain state in the Oy_1y_2 plane are obtained.

The equation of motion is

$$\frac{\partial Q_{ij}}{\partial y_i} = \rho \frac{\partial^2 u_j}{\partial t^2} \ , \tag{2}$$

and mechanical relations are

$$Q_{ij} = \omega_{ij\alpha\beta} \frac{\partial u_\alpha}{\partial y_\beta} \, , \tag{3}$$

where $i; j; \alpha; \beta = 1,2$ and Einstein summation rule is employed with respect to the repeated indices in Eqs. (2) and (3). At the same time, in equation (2) and (3) the following notation is used: Q_{ij} are the components of the perturbations of the Kirchhoff non-symmetric stress tensor related to the areas of the initial state, u_j are the components of the perturbations of the displacement vector, and ρ is the density also related to the volume of the initial state.

Below we will also consider the determination of the components $\omega_{ij\alpha\beta}$ which are found through the initial strain state (1) and through the corresponding elastic potential. The harmonic potential, as has noted above, is selected in the present investigation for the layer's material and this potential is presented through the following expression

$$\Phi = \frac{1}{2}\lambda(s_1)^2 + \mu s_2 \tag{4}$$

where λ and μ are the mechanical constants of the material and

$$\begin{gathered} s_1 = (\sqrt{1+2\varepsilon_1}-1) + (\sqrt{1+2\varepsilon_2}-1) + (\sqrt{1+2\varepsilon_3}-1), \\ s_2 = (\sqrt{1+2\varepsilon_1}-1)^2 + (\sqrt{1+2\varepsilon_2}-1)^2 + (\sqrt{1+2\varepsilon_3}-1)^2 . \end{gathered} \tag{5}$$

In Eq. (5), ε_i $(i=1,2,3)$ are the principal values of the Green's strain tensor.

For clearer describing the explanation of the linearized equations and relations let us consider briefly the definition of the stress and strain tensors in the large elastic deformation theory through the linearization from which the used equations in the present investigation are obtained. Under this consideration we use the Lagrange coordinates x_i $(i=1,2,3)$ in the Cartesian system of coordinates $Ox_1x_2x_3$ and the position of the points after and before deformations we determine by the vectors $\mathbf{r}^*$ and $\mathbf{r}$ respectively, where $\mathbf{r}^* = \mathbf{r} + \mathbf{u}$. Here $\mathbf{u} = u_i\mathbf{g}_i$ is a displacement vector expressed by the unit basic vectors $\mathbf{g}_i$. Taking the relations $\mathrm{d}\mathbf{r}^* \cdot \mathrm{d}\mathbf{r}^* = \mathrm{d}\mathbf{r}\cdot\mathrm{d}\mathbf{r} + 2\mathrm{d}\mathbf{r}\cdot\mathrm{d}\mathbf{u} + \mathrm{d}\mathbf{u}\cdot\mathrm{d}\mathbf{u}$ (here the symbol " $\cdot$ " means the scalar product of the vectors), $\mathrm{d}\mathbf{u}\cdot\mathrm{d}\mathbf{u} = (\partial u_k/\partial x_i)(\partial u_k/\partial x_j)\,\mathrm{d}x_i\,\mathrm{d}x_j$ and $2\mathrm{d}\mathbf{r}\cdot\mathrm{d}\mathbf{u} = 2(\partial u_k/\partial x_i)\,\mathrm{d}x_k\,\mathrm{d}x_j$ into account, it is obtained that $\mathrm{d}\mathbf{r}^* \cdot \mathrm{d}\mathbf{r}^* - \mathrm{d}\mathbf{r}\cdot\mathrm{d}\mathbf{r} = 2\varepsilon_{ij}\mathrm{d}x_i\mathrm{d}x_j$, where

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_n}{\partial x_i}\frac{\partial u_n}{\partial x_j}\right) \tag{6}$$

Here, ε_{ij} is a component of the symmetric Green's strain tensor $\tilde{\varepsilon}$ and Einstein summation rule takes place in the equation (6) with respect to the repetitive index n.

Let us also consider the definition of the Kirchhoff stress tensor. The use of various types of stress tensors in the large (finite) elastic deformation theory is connected with the reference of the components of these tensors to the unit area of the relevant surface elements in the deformed or un-deformed state, because, in contrast to the linear theory of elasticity, in the finite elastic deformation theory the difference between the areas of the surface elements taken before and after deformation must be accounted for in the derivation of the equation of motion and under satisfaction of the boundary conditions with respect to the forces.

For convenience of the investigation carried out in the present paper, we here consider two types of stress tensors denoted by $\tilde{\boldsymbol{q}}$ and $\tilde{\boldsymbol{S}}$ the components of which refer to the unit area of the relevant surface elements in the un-deformed state, but act on the surface elements in the deformed state. The components S_{ij} of the stress tensor $\tilde{\boldsymbol{S}}$ are determined through the strain energy potential $\Phi = \Phi(\varepsilon_{11}, \varepsilon_{22}, ..., \varepsilon_{33})$, where ε_{ij} is a component of the Green's strain tensor (6), by the use of the following expression

$$S_{ij} = \frac{1}{2}\left(\frac{\partial}{\partial \varepsilon_{ij}} + \frac{\partial}{\partial \varepsilon_{ji}} \right) \Phi(\varepsilon_{11}, \varepsilon_{22}, ..., \varepsilon_{33}) \tag{7}$$

The components q_{ij} of the stress tensor $\tilde{\boldsymbol{q}}$ are determined by the expression

$$q_{ij} = \left(\delta_k^j + \frac{\partial u_j}{\partial x_k} \right) S_{ik} \tag{8}$$

Here, δ_k^j is the Kronecker symbol and Einstein summation rule takes place with respect to the index k . The stress tensor $\tilde{\boldsymbol{q}}$ with components determined by expressions (7) and (8) is called the Kirchhoff stress tensor. According to expressions (6)-(8), the stress tensor $\tilde{\boldsymbol{S}}$ is symmetric, but the Kirchhoff stress tensor $\tilde{\boldsymbol{q}}$ is non-symmetric. Thus, with this we restrict ourselves to the consideration of the definition of the stress and strain tensors in the finite elastic deformation theory. These definitions are given without any restriction related to the association of the selected coordinate systems to the natural or initial state. However, in using the coordinate system associated with the initial deformed state, the initial strain state can be taken as an “un-deformed” state in the foregoing definitions.

Thus, using the foregoing preparation we consider the obtaining the Eq. (3) and the expressions for the components $\omega_{ij\alpha\beta}$ by employing the linearization procedure. First, we found from the Eqs. (1), (4)-(8) that

$$S_{11}^0 = [\lambda(\lambda_1 + \lambda_2 - 2) + 2\mu(\lambda_1 - 1)](\lambda_1), \; S_{22}^0 = [\lambda(\lambda_1 + \lambda_2 - 2) + 2\mu(\lambda_2 - 1)](\lambda_2) = 0, \; S_{12}^0 = 0 . \tag{9}$$

from which follows that

$$\lambda_2 = [2\mu - \lambda(\lambda_1 - 2)] / (\lambda + 2\mu)^{-1} . \tag{10}$$

According to expressions in (1), (9) and (10), we can conclude that for a selected material the magnitude of the initial strains and the initial stresses in the layer can be determined through λ_1 only.

By linearization the expression (8) we obtain the relation

$$q'_{ij} = \left(\delta_k^j + \frac{\partial u_j^0}{\partial x_k} \right) S'_{ik} + S_{ik}^0 \frac{\partial u_j}{\partial x_k} , \tag{11}$$

where q'_{ij} is the perturbation of the components of the non-symmetric Kirchhoff stress tensor q_{ij} and

$$S'_{in} = \left\{ \frac{1}{4} \left(\delta_k^\beta + \frac{\partial u_\beta^0}{\partial x_k} \right) \left(\frac{\partial}{\partial \varepsilon_{k\beta}^0} + \frac{\partial}{\partial \varepsilon_{\beta k}^0} \right) \left(\frac{\partial}{\partial \varepsilon_{in}^0} + \frac{\partial}{\partial \varepsilon_{ni}^0} \right) \Phi^0 \right\} \frac{\partial u_\alpha}{\partial x_\beta} , \tag{12}$$

which is the perturbation of the components of the stress tensor $\tilde{\boldsymbol{S}}$ and the expression in Eq. (12) is obtained from the Eq. (7) by usual linearization procedure and here Einstein summation rule takes place with respect to the indices k and β.

Thus, finally using the relations

$$Q_{11}dy_2dy_3 = q'_{11}\,dx_2dx_3\,,\ \ Q_{22}dy_1dy_3 = q'_{22}\,dx_1dx_3\,,\ Q_{12}dy_2dy_3 = q'_{12}\,dx_2dx_3\,,$$
$$Q_{21}dy_1dy_3 = q'_{21}\,dx_1dx_3\,,\ dy_1 = \lambda_1 dx_1\,,\ dy_2 = \lambda_2 dx_2 \Rightarrow\ Q_{11} = q'_{11}/\lambda_2\ ,\ Q_{22} = q'_{22}/\lambda_1\,, \tag{13}$$
$$Q_{12} = q'_{11}/\lambda_2\,,\ Q_{21} = q'_{21}/\lambda_1$$

and changing $\partial u_j/\partial x_k$ and $\partial u_\alpha/\partial x_\beta$ in Eqs. (11) and (12) with $\lambda_k\partial u_j/\partial y_k$ and $\lambda_\beta\partial u_\alpha/\partial y_\beta$, respectively, Eq. (3) and expressions for components $\omega_{ij\alpha\beta}$ are obtained from Eqs. (11) and (12) after some mathematical calculations.

As an example, we consider the obtaining of the expressions for Q_{11}, ω_{1111} and ω_{1122} given in Eq. (3) and for this purpose we write the following relations obtained from Eqs. (1), (11) and (12).

$$q'_{11} = \lambda_1 S'_{11} + S_{11}^0\frac{\partial u_1}{\partial x_1}\,,\ S'_{11} = \lambda_1\frac{\partial}{\partial \varepsilon_{11}^0}S_{11}^0\frac{\partial u_1}{\partial x_1} + \lambda_2\frac{\partial}{\partial \varepsilon_{22}^0}S_{11}^0\frac{\partial u_2}{\partial x_2} \tag{14}$$

Taking the relations

$$\lambda_1\frac{\partial}{\partial \varepsilon_{11}^0}S_{11}^0 = \frac{\lambda_1}{\lambda_1}\frac{\partial S_{11}^0}{\partial \lambda_1} = \frac{1}{\lambda_1}(\lambda+2\mu) - \frac{1}{(\lambda_1)^2}S_{11}^0\,,\ \lambda_2\frac{\partial}{\partial \varepsilon_{22}^0}S_{11}^0 = \frac{\lambda_2}{\lambda_2}\frac{\partial S_{11}^0}{\partial \lambda_2} = \frac{\lambda}{\lambda_2}\,, \tag{15}$$

which are obtained from the definition of the parameter λ_i and the expression for S_{11}^0 in Eq. (9), and the relations (14) into account, the following mathematical transformations can be made

$$q'_{11} = (\lambda+2\mu)\frac{\partial u_1}{\partial x_1} + \lambda\frac{\partial u_2}{\partial x_2} =$$
$$\lambda_1(\lambda+2\mu)\frac{\partial u_1}{\partial y_1} + \lambda\lambda_2\frac{\partial u_2}{\partial y_2}\,,\ \ Q_{11} = q'_{11}/\lambda_2 = \frac{\lambda_1}{\lambda_2}(\lambda+2\mu)\frac{\partial u_1}{\partial y_1} + \lambda\frac{\partial u_2}{\partial y_2} = \tag{16}$$
$$\omega_{1111}\frac{\partial u_1}{\partial y_1} + \omega_{1122}\frac{\partial u_2}{\partial y_2}\,.\Rightarrow\ \omega_{1111} = \frac{\lambda_1}{\lambda_2}(\lambda+2\mu)\,,\ \omega_{1122} = \lambda\,.$$

Thus, we obtain the foregoing expressions for components ω_{1111} and ω_{1122}. In this way we obtain the expressions for remain components $\omega_{ij\alpha\beta}$ in Eq. (3) which are differ from zero. These expressions are

$$\omega_{2211} = \lambda\,,\ \omega_{1212} = \omega_{2121} = \frac{2\lambda_2\mu}{\lambda_1+\lambda_2}\,,\ \omega_{1221} = \omega_{2112} = \frac{2(\lambda_2)^2\mu}{\lambda_2(\lambda_1+\lambda_2)}\,. \tag{17}$$

This completes the consideration of the basic equations and relations of the TLTEWISB within the scope of which the motion of the pre-strained plate-layer is described. In this case the boundary conditions on the upper face plane of the layer can be written as follows.

$$Q_{21}\big|_{y_2=0} = 0, \; Q_{22}\big|_{y_2=0} = -P_0 e^{i\omega t}\delta(y_1). \tag{18}$$

In Eq. (18), ω is a frequency of the lineal-located external load with amplitude P_0, $\delta(y_1)$ is a delta Dirac function.

2.2 Governing field equations for the compressible Newtonian viscous fluid

Consider the field equations of motion of the Newtonian compressible viscous fluid and under this consideration the density, viscosity constants and pressure related to that will be denoted by upper index (1). We use the Euler coordinates in the coordinate system $Oy_1y_2y_3$ which is associated with the initial state of the plate to write these equations. Taking the smallness of the perturbations in the perturbed state in the system under consideration the Euler and Lagrange coordinates in the coordinate system $Oy_1y_2y_3$ will be identified. Thus, within these assumptions, according to Guz (2009), we write field equations for the fluid flow.

The linearized Navier-Stokes equations

$$\rho_0^{(1)}\frac{\partial v_i}{\partial t} - \mu^{(1)}\frac{\partial v_i}{\partial y_j \partial y_j} + \frac{\partial p^{(1)}}{\partial y_i} - (\lambda^{(1)} + \mu^{(1)})\frac{\partial^2 v_j}{\partial y_j \partial y_i} = 0. \tag{19}$$

The equation of continuity

$$\frac{\partial \rho^{(1)}}{\partial t} + \rho_0^{(1)}\frac{\partial v_j}{\partial y_j} = 0 \tag{20}$$

Rheological relations

$$T_{ij} = \left(-p^{(1)} + \lambda^{(1)}\frac{\partial v_k}{\partial y_k}\right)\delta_{ij} + 2\mu^{(1)}e_{ij} \tag{21}$$

where

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial y_j} + \frac{\partial v_j}{\partial y_i}\right) \tag{22}$$

The equation of state

$$a_0^2 = \frac{\partial p^{(1)}}{\partial \rho^{(1)}} \tag{23}$$

In Eqs. (19)-(23) $i; j; k = 1,2,3$ and the following notation is used: v_i is a component of a perturbation of the velocity vector, $p^{(1)}$ is a perturbation of the pressure, $\mu^{(1)}$ is a coefficient of viscosity, $\lambda^{(1)}$ is the second coefficient of the viscosity, a_0 is a sound speed in the fluid, e_{ij} is a component of a perturbation of the strain rate tensor, T_{ij} is a component of a perturbation of the stress tensor in the fluid, $\rho^{(1)}$ is a perturbation of the density of the fluid, $\rho_0^{(1)}$ is a density of the fluid in the initial state, i.e.,before the perturbation of the fluid and δ_{ij} is a Kronecker symbol. Note that in Eqs. (19)-(21) Einstein summation rule is employed with respect to the repeated indices.

In the present paper we consider the case where

$$v_1 = v_1(y_1, y_2, t)\ ,\ v_2 = v_2(y_1, y_2, t)\ ,\ v_3 = 0\ . \tag{24}$$

According to Guz (2009), the solution of the system Eqs. (19)-(23) in the case given in (24) is reduced to the finding of two potentials φ and ψ which are determined from the following equations.

$$\left[\left(1 + \frac{\lambda^{(1)} + 2\mu^{(1)}}{a_0^2 \rho_0^{(1)}} \frac{\partial}{\partial t}\right)\Delta - \frac{1}{a_0^2} \frac{\partial^2}{\partial t}\right]\varphi = 0\ ,\ \left(\nu^{(1)}\Delta - \frac{\partial}{\partial t}\right)\psi = 0\ ,\ \ \Delta = \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\ , \tag{25}$$

where $\nu^{(1)}$ is a kinematic viscosity, i.e. $\nu^{(1)} = \mu^{(1)} / \rho_0^{(1)}$.

The velocities v_1, v_2 and the pressure $p^{(1)}$ are expressed via the potentials φ and ψ by the following expressions

$$v_1 = \frac{\partial \varphi}{\partial y_1} + \frac{\partial \psi}{\partial y_2}\ ,\ v_2 = \frac{\partial \varphi}{\partial y_2} - \frac{\partial \psi}{\partial y_1}\ ,\ p^{(1)} = \rho_0^{(1)}\left(\frac{\lambda^{(1)} + 2\mu^{(1)}}{\rho_0^{(1)}}\Delta - \frac{\partial}{\partial t}\right)\varphi\ . \tag{26}$$

Assuming that $p^{(1)} = -(T_{11} + T_{22} + T_{33})/3$, we obtain from the relation (21) that

$$\lambda^{(1)} = -\frac{2}{3}\mu^{(1)}\ . \tag{27}$$

This completes the field equation of the fluid flow which is considered in the present paper.

2.3 The compatibility conditions on the interface planes

We assume that the velocities and forces acting on the interface between the fluid and layer are continuous, i.e., we assume that

$$\begin{gathered}\left.\frac{\partial u_1}{\partial t}\right|_{y_2=-\lambda_2 h} = \left.v_1\right|_{y_2=-\lambda_2 h}\ ,\ \left.\frac{\partial u_2}{\partial t}\right|_{y_2=-\lambda_2 h} = \left.v_2\right|_{y_2=-\lambda_2 h}\ , \\ \left.Q_{21}\right|_{y_2=-\lambda_2 h} = \left.T_{21}\right|_{y_2=-\lambda_2 h}\ ,\ \ \left.Q_{22}\right|_{y_2=-\lambda_2 h} = \left.T_{22}\right|_{y_2=-\lambda_2 h}\ .\end{gathered} \tag{28}$$

Moreover, we assume that on the rigid wall the following impermeability conditions satisfy.

$$v_1\Big|_{y_2=-\lambda_2 h-h_d}=0\,,\; v_2\Big|_{y_2=-\lambda_2 h-h_d}=0\,. \tag{29}$$

This completes the formulation of the problem. It should be noted that, with corresponding obvious changes, the foregoing problem formulation can be remake for the case where the fluid is inviscid. Moreover, in the case where $\lambda_1=\lambda_2=1.0$ the foregoing formulation relates to the corresponding classical problem of hydro-elastodynamics which was considered in the paper by Akbarov and Ismailov (2017).

3. Method of solution

Below, we use the dimensionless coordinates $\bar{y}_k = y_k / h$ and omit the over bar on the coordinates. For solution to the problem formulated above, according to the well-known procedure, we represent the sought values as $g(y_1, y_2, t) = g(y_1, y_2)e^{i\omega t}$ and substituting these expressions into the foregoing equations and relations, and replacing the derivatives $\partial(\bullet)/\partial t$ and $\partial^2(\bullet)/\partial t^2$ with $i\omega(\bar{\bullet})$ and $-\omega^2(\bar{\bullet})$ respectively, we obtain corresponding equations, boundary and compatibility conditions for the amplitudes of the sought values. For solution to these equations we use the exponential Fourier transformation with respect to the y_1 coordinate and taking the problem symmetry with respect to $y_1=0$ into account, we can represent the originals of the sought values as follows.

$$\begin{gathered}
u_1=\frac{1}{\pi}\int_0^\infty u_{1F}(s,y_2)\sin(s\,y_1)ds\,,\; u_2=\frac{1}{\pi}\int_0^\infty u_{2F}(s,y_2)\cos(s\,y_1)ds\,,\\
Q_{11}=\frac{1}{\pi}\int_0^\infty Q_{11F}(s,y_2)\cos(s\,y_1)ds\,, Q_{22}=\frac{1}{\pi}\int_0^\infty Q_{22F}(s,y_2)\cos(s\,y_1)ds\,,\\
Q_{12}=\frac{1}{\pi}\int_0^\infty Q_{12F}(s,y_2)\sin(s\,y_1)ds\,,\; Q_{21}=\frac{1}{\pi}\int_0^\infty Q_{21F}(s,y_2)\sin(s\,y_1)ds\,,\\
\varphi=\frac{1}{\pi}\int_0^\infty \varphi_F(s,y_2)\cos(s\,y_1)ds\,,\; \psi=\frac{1}{\pi}\int_0^\infty \psi_F(s,y_2)\sin(s\,y_1)ds\,,\\
v_1=\frac{1}{\pi}\int_0^\infty v_{1F}(s,y_2)\sin(s\,y_1)ds\,,\; v_2=\frac{1}{\pi}\int_0^\infty v_{2F}(s,y_2)\cos(s\,y_1)ds\,,\\
T_{11}=\frac{1}{\pi}\int_0^\infty T_{11F}(s,y_2)\cos(s\,y_1)ds\,, T_{22}=\frac{1}{\pi}\int_0^\infty T_{22F}(s,y_2)\cos(s\,y_1)ds\,,\\
T_{12}=\frac{1}{\pi}\int_0^\infty T_{12F}(s,y_2)\sin(s\,y_1)ds\,,\; T_{21}=\frac{1}{\pi}\int_0^\infty T_{21F}(s,y_2)\sin(s\,y_1)ds\,.
\end{gathered} \tag{30}$$

In Eq. (30), the lower index F indicates the exponential Fourier transformation of the corresponding quantity.

At first, we consider the solution to the following equations obtained from the Eqs. (2), (3), (16) and (17) by substituting the corresponding expressions in Eq. (30) into these equations.

$$Au_{1F}-B\frac{du_{2F}}{dy_2}+C\frac{d^2u_{1F}}{dy_2^2}=0\ ,\ \ Du_{2F}+B\frac{du_{1F}}{dy_2}+G\frac{d^2u_{2F}}{dy_2^2}=0\ , \tag{31}$$

where

$$A=X^2-s^2\omega_{1111}\,,\ B=s(\omega_{1122}+\omega_{2121})\,,\ C=\omega_{2112}\,,\ D=X^2-s^2\omega_{1221}\,,$$
$$G=\omega_{2222}\ ,\ X^2=\omega^2h^2/c_2^2\ ,\ c_2=\sqrt{\mu/\rho}\ . \tag{32}$$

Using the notation

$$A_0=\frac{AG+B^2+CD}{CG}\,,\ B_0=\frac{BD}{CG}\,,\ k_1=\sqrt{-\frac{A_0}{2}+\sqrt{\frac{A_0^2}{4}-B_0}}\ ,\ k_2=\sqrt{-\frac{A_0}{2}-\sqrt{\frac{A_0^2}{4}-B_0}}\ , \tag{33}$$

we can write the solution to the Eq. (31) as follows.

$$u_{2F}=Z_1e^{k_1y_2}+Z_2e^{-k_1y_2}+Z_3e^{k_2y_2}+Z_4e^{-k_2y_2}\ ,$$
$$u_{1F}=Z_1a_1e^{k_1y_2}+Z_2a_2e^{-k_1y_2}+Z_3a_3e^{k_2y_2}+Z_4a_4e^{-k_2y_2}\ , \tag{34}$$

where

$$a_1=\frac{-D-Gk_1^2}{Bk_1^2}\ ,\ a_2=-a_1,\ a_3=\frac{-D-Gk_2^2}{Bk_2^2},\ \ a_4=-a_3\,. \tag{35}$$

Substituting the solutions in Eq. (34) into the expression in Eq. (3) we also obtain expressions for the Fourier transformations Q_{21F} and Q_{22F} of the corresponding stresses which enter into the boundary condition (18) and compatibility condition (28).

$$Q_{21F}=Z_1\left(\omega_{2112}k_1a_1-s\omega_{2121}\right)e^{k_1y_2}+Z_2\left(-\omega_{2112}k_1a_2-s\omega_{2121}\right)e^{-k_1y_2}+$$
$$Z_3\left(\omega_{2112}k_2a_3-s\omega_{2121}\right)e^{k_2y_2}+Z_4\left(-\omega_{2112}k_2a_3-s\omega_{2121}\right)e^{-k_2y_2}\,,$$
$$Q_{22F}=Z_1\left(s\omega_{2211}a_1+k_1\omega_{2222}\right)e^{k_1y_2}+Z_2\left(s\omega_{2211}a_2-k_1\omega_{2222}\right)e^{-k_1y_2}+$$
$$Z_3\left(s\omega_{2211}a_3+k_2\omega_{2222}\right)e^{k_2y_2}+Z_2\left(s\omega_{2211}a_4-k_2\omega_{2222}\right)e^{-k_2y_2}\,. \tag{36}$$

In this way, we determine completely the Fourier transform of the values related to the plate-layer.

Now we consider the determination of the Fourier transformations of the quantities related to the fluid flow for which we begin with the determination of the φ_F and ψ_F from the Fourier transform of the equations in Eq. (25), which taking the relations (27) and

$$\varphi_F=\omega h^2\tilde{\varphi}_F\ ,\ \psi_F=\omega h^2\tilde{\psi}_F \tag{37}$$

into account can be written as follows

$$\frac{d^2\tilde{\varphi}_F}{dy_2^2}+\left(\frac{\Omega_1^2}{1+i4\Omega_1^2/(3N_w^2)}-s^2\right)\tilde{\varphi}_F=0\,,\ \frac{d^2\tilde{\psi}_F}{dy_2^2}-\left(s^2+iN_w^2\right)\tilde{\psi}_F=0\,, \tag{38}$$

where

$$\Omega_1=\frac{\omega h}{a_0}\ ,\ \ N_w^2=\frac{\omega h^2}{\nu^{(1)}}\,. \tag{39}$$

The dimensionless number N_w in Eq. (39) can be taken as Womersley number and characterizes the influence of the fluid viscosity on the mechanical behavior of the system under consideration. When the Womersley number is large (around 10 or greater), it shows that the flow is dominated by oscillatory inertial forces. When the Womersley number is low, viscous forces tend to dominate the flow. However, for hydro-elastodynamic problems the mentioned “large” and “low” limits for the Womersley number can change significantly.

The dimensionless frequency Ω_1 in Eq. (39) can be taken as the parameter which characterizes the compressibility of the fluid on the mechanical behavior of the system under consideration. Thus, the solutions to the equations in Eq. (38) are found as follows

$$\tilde{\varphi}_F=Z_5e^{\delta_1 y_2}+Z_7e^{-\delta_1 y_2}\,,\ \tilde{\psi}_F=Z_6e^{\gamma_1 y_2}+Z_8e^{-\gamma_1 y_2}\,, \tag{40}$$

where

$$\delta_1=\sqrt{s^2-\frac{\Omega_1^2}{1+i4\Omega_1^2/(3N_w^2)}}\ ,\ \gamma_1=\sqrt{s^2+iN_w^2}\ . \tag{41}$$

Using Eqs. (40) and (37) we obtain from the Fourier transformations of the Eqs. (21), (22) and (26) the following expressions for the velocities, pressure and stresses of the fluid.

$$\begin{gathered}
v_{1F}=\omega h\left[-Z_5se^{\delta_1 y_2}-Z_7se^{-\delta_1 y_2}+Z_6e^{\gamma_1 y_2}+Z_8e^{-\gamma_1 y_2}\right],\\
v_{2F}=\omega h\left[Z_5\delta_1e^{\delta_1 y_2}-Z_7\delta_1e^{-\delta_1 y_2}-Z_6se^{\gamma_1 y_2}-Z_8se^{-\gamma_1 y_2}\right],\\
T_{22F}=\mu^{(1)}\omega\left[Z_5\left(\frac{4}{3}\delta_1^2+\frac{2}{3}s^2-R_0\right)e^{\delta_1 y_2}+Z_7\left(\frac{4}{3}\delta_1^2+\frac{2}{3}s^2-R_0\right)e^{-\delta_1 y_2}+\right.\\
\left.Z_6\left(-s\gamma_1-\frac{2}{3}s\gamma_1\right)e^{\gamma_1 y_2}+Z_8\left(s\gamma_1+\frac{2}{3}s\gamma_1\right)e^{-\gamma_1 y_2}\right],\\
T_{21F}=-\mu^{(1)}\omega\left[2s\delta_1Z_5e^{\delta_1 y_2}-2s\delta_1Z_7e^{-\delta_1 y_2}+(s^2+\gamma_1^2)Z_6e^{\gamma_1 y_2}+(s^2+\gamma_1^2)Z_8e^{-\gamma_1 y_2}\right],\\
p_F^{(1)}=\mu^{(1)}\omega R_0\left(Z_5e^{\delta_1 y_2}+Z_7e^{-\delta_1 y_2}\right),
\end{gathered} \tag{42}$$

where

$$R_0 = -\frac{4}{3}\frac{\Omega_1^2}{1+i4\Omega_1^2/(3N_w^2)} - N_w^2 . \tag{43}$$

Substituting expressions (34), (36) and (42) into the boundary condition (18), compatibility condition (28) and impermeability condition (29) we obtain the following system of equations with respect to the unknowns Z_1, $Z_2,\ldots,Z_6$ through which the sought values are determined.

$$\begin{gathered}
\left(Q_{21}/\mu\right)\Big|_{y_2=0} = Z_1\alpha_{11} + Z_2\alpha_{12} + Z_3\alpha_{13} + Z_4\alpha_{14} = 0\,, \left(Q_{22}/\mu\right)\Big|_{y_2=0} = Z_1\alpha_{21} + Z_2\alpha_{22} + \\
Z_3\alpha_{23} + Z_4\alpha_{24} = -P_0/\mu\,, \left.\frac{\partial u_{1F}}{\partial t}\right|_{y_2=-\lambda_2 h} - v_{1F}\Big|_{y_2=-\lambda_2 h} = i\omega(Z_1\alpha_{31} + \\
Z_2\alpha_{32} + Z_3\alpha_{33} + Z_4\alpha_{34}) - \omega h(Z_5\alpha_{35} + Z_7\alpha_{37} + Z_6\alpha_{36} + Z_8\alpha_{38}) = 0\,, \\
\left.\frac{\partial u_{2F}}{\partial t}\right|_{y_2=-\lambda_2 h} - v_{2F}\Big|_{y_2=-\lambda_2 h} = i\omega(Z_1\alpha_{41} + Z_2\alpha_{42} + Z_3\alpha_{43} + Z_4\alpha_{44}) - \\
\omega h(Z_5\alpha_{45} + Z_7\alpha_{47} + Z_6\alpha_{46} + Z_8\alpha_{48}) = 0\,, \\
\left(Q_{21}/\mu\right)\Big|_{y_2=-\lambda_2 h} - \left(T_{21}/\mu\right)\Big|_{y_2=-\lambda_2 h} = Z_1\alpha_{51} + Z_2\alpha_{52} + Z_3\alpha_{53} + Z_4\alpha_{54} - \\
M(Z_5\alpha_{55} + Z_7\alpha_{57} + Z_6\alpha_{56} + Z_8\alpha_{58}) = 0\,, \\
\left(Q_{22}/\mu\right)\Big|_{y_2=-\lambda_2 h} - \left(T_{22}/\mu\right)\Big|_{y_2=-\lambda_2 h} = Z_1\alpha_{61} + Z_2\alpha_{62} + Z_3\alpha_{63} + Z_4\alpha_{64} - \\
M(Z_5\alpha_{65} + Z_7\alpha_{67} + Z_6\alpha_{66} + Z_8\alpha_{68}) = 0\,, \\
v_{1F}\Big|_{y_2=-\lambda_2 h - h_d} = \omega h(Z_5\alpha_{75} + Z_6\alpha_{76} + Z_7\alpha_{77} + Z_8\alpha_{78}) = 0\,, \\
v_{2F}\Big|_{y_2=-\lambda_2 h - h_d} = \omega h(Z_5\alpha_{85} + Z_6\alpha_{86} + Z_7\alpha_{87} + Z_8\alpha_{88}) = 0\,,
\end{gathered} \tag{44}$$

where

$$M = \frac{\mu^{(1)}\omega}{\mu} . \tag{45}$$

It can be easily determined the expressions of the coefficients $\alpha_{nm}(n;m=1,2,...,8)$ in (44) from the expressions (34), (36) and (42), and therefore these expressions are not given here. Thus, unknowns $Z_1, Z_2,..., Z_8$ in the Eq. (44) can be determined via the formulae.

$$Z_k = \frac{\det\left\|\beta_{nm}^k\right\|}{\det\left\|\alpha_{nm}\right\|} , \tag{46}$$

where the matrix $\left(\beta_{nm}^k\right)$ is obtained from the matrix $\left(\alpha_{nm}\right)$ by the replacing of the $k-th$ column of the $\left(\alpha_{nm}\right)$ by the column $(0,-P_0/\mu,0,0,0,0,0,0)^T$.

Thus, after determination the unknowns $Z_1, Z_2,..., Z_8$ we can consider the calculation of the integrals in Eq. (30) for which it is necessary to take into consideration the following reasoning. If we take the Fourier transformation parameter s as the wavenumber, then the equation

$$\det\|\alpha_{nm}\| = 0,\ n;m = 1,2,...,8, \tag{47}$$

coincides with the dispersion equation of the waves propagated in the direction of the Oy_1 axis in the system under consideration. According to the well-known physical-mechanical considerations, the Eq. (47) must have complex roots only because the system under consideration contains the compressible viscous fluid. However, as usual, the viscosity of the Newtonian fluids is insignificant in the qualitative sense and therefore in many cases within the scope of the PC calculation accuracy of the Eq. (47) has real roots. Consequently, these roots become singular points of the integrated expressions in the integrals (30). Therefore, according to works by Tsang (1978), Jensen *et al.* (2011) and many others listed in these references, we will evaluate the wavenumber integrals (30) along the Sommerfeld contour (Fig. 1(b)) in the complex plane $s = s_1 + is_2$ and in this way the real roots of Eq. (47) are avoided.

Thus, using the presentation $g(y_1, y_2, t) = g(y_1, y_2)e^{i\omega t}$ the sought values can be determined through the following two type relations.

$$\{Q_{22}, Q_{11}, u_2, T_{22}, T_{11}, \mathrm{v}_2\} = \frac{1}{\pi}\mathrm{Re}\left\{e^{i\omega t}\int_C [Q_{22F}, Q_{11F}, u_{2F}, T_{22F}, T_{11F}, \mathrm{v}_{2F}] \times \cos(sy_1)ds\right\},$$
$$\{Q_{21}, Q_{12}, u_1, T_{21}, \mathrm{v}_1\} = \frac{1}{\pi}\mathrm{Re}\left\{e^{i\omega t}\int_C [\mathrm{Q}_{21}, Q_{12F}, u_{1F}, T_{21F}, \mathrm{v}_{1F}] \sin(sy_1)ds\right\}. \tag{48}$$

According to Fig. 1(b), we can write the following relation.

$$\int_C f(s)\cos(sy_1)ds = i\int_0^{s_2^*} f(is_2)\cos(is_2\,\mathrm{y}_1)ds_2 + \int_0^{\infty} f(s_1 + is_2^*)\cos((s_1 + is_2^*)\,\mathrm{y}_1)ds_1,$$
$$\int_C f(s)\sin(sy_1)ds = i\int_0^{s_2^*} f(is_2)\sin(is_2\,\mathrm{y}_1)ds_2 + \int_0^{\infty} f(s_1 + is_2^*)\sin((s_1 + is_2^*)\,\mathrm{y}_1)ds_1 \tag{49}$$

Taking the fact that the values of the integrals $\int_C f(s)\cos(sy_1)ds$ and $\int_C f(s)\sin(sy_1)ds$ are independent on the values of the parameter s_2^* into account, as usual (see, for example Jensen *et al.* (2011) and Tsang (1978), to simplify the calculation of these integrals, the parameter s_2^* is assumed as a small parameter.

According to this assumption and to the relation

$$\left|\int_0^{s_2^*} f(is_2)\cos(is_2\,\mathrm{y}_1)ds_2\right| = O(s_2^*), \left|\int_0^{s_2^*} f(is_2)\sin(is_2\,\mathrm{y}_1)ds_2\right| = O(s_2^*)$$

we use the following approximate expressions for calculating of the foregoing integrals

$$\int_C f(s)\cos(sy_1)ds \approx \int_0^{\infty} f(s_1 + \mathrm{i}s_2^*)\cos((s_1 + \mathrm{i}s_2^*)y_1)\,\mathrm{d}s_1,$$
$$\int_C f(s)\sin(sy_1)ds \approx \int_0^{\infty} f(s_1 + \mathrm{i}s_2^*)\sin((s_1 + \mathrm{i}s_2^*)y_1)\,\mathrm{d}s_1 \tag{50}$$

The accuracy of the expressions in (50) with respect to values of the parameter s_2^* was discussed in the monograph by Akbarov (2015) and in the other references listed therein.

At the same time, under calculation procedure, the improper integrals $\int_0^{+\infty} f(s_1)\cos(s_1 y_1)ds_1$ and $\int_0^{+\infty} f(s_1)\sin(s_1 y_1)ds_1$ in (50) are replaced by the corresponding definite integrals $\int_0^{+S_1^*} f(s_1)\cos(s_1 y_1)ds_1$ and $\int_0^{+S_1^*} f(s_1)\sin(s_1 y_1)ds_1$ respectively. The values of S_1^* are determined from the convergence requirement of the numerical results. Note that under calculation of the latter integrals, the integration intervals are further divided into a certain number of shorter intervals, which are used in the Gauss integration algorithm. In this integration procedure it is assumed that in each of the shorter intervals the sampling intervals of the numerical integration Δs_1 must satisfy the relation $|\Delta s_1| << \min\{s_2^*, 1/y_1\}$. All these procedures are performed automatically with the PC by use of the corresponding programs constructed by the authors in MATLAB.

With this we restrict ourselves to consideration of the solution method for the investigation of the problem under consideration. Note that after some obvious changing the foregoing solution method can be applied also for the case where the fluid is inviscid.

4. Numerical results and discussions

4.1 The selection of the problem parameters and on the calculation algorithm

According to the foregoing discussions, it can be concluded that the problem under consideration is characterized with the following parameters: the dimensionless parameters Ω_1 and N_w which are determined by the expressions in (39), M which is determined with the expression (45), λ/μ where λ and μ are the mechanical constants which enter into the expression of the elastic potential (7), and λ_1 through which the initial strains in the layer are characterized. Under numerical investigation we assume that the values of the mechanical constants and the density of the plate material are $\mu = 1.86\times10^9 Pa$, $\lambda = 3.96\times10^9 Pa$ and $\rho = 1160 kg/m^3$, but the material of the fluid is Glycerin with viscosity coefficient $\mu^{(1)} = 1{,}393 kg/(m\cdot s)$, density $\rho = 1260 kg/m^3$ and sound speed $a_0 = 1459.5 m/s$ (Guz 2009). We introduce also the notation $c_2 = \sqrt{\mu/\rho}$ which is the shear wave propagation velocity in the layer material in the case where the initial strains are absent in that. Note that the values selected above for the constants λ, μ and ρ, and related to the plate material under absent of the initial strains corresponds to the Plexiglass (or Lucite) (see Guz 2004, Lai-Yu *et al.* 2006).

So that, after selection of the noted above materials, the foregoing dimensionless parameters can be determined through the following four quantities: h (the thickness of the plate-layer in the natural state), h_d (the thickness of the fluid strip), ω (the frequency of the time-harmonic external forces) and λ_1 (the elongation factor which characterizes the change in the length of the line element in the Ox_1 axis direction). Note that one of the main parameters for the problem under

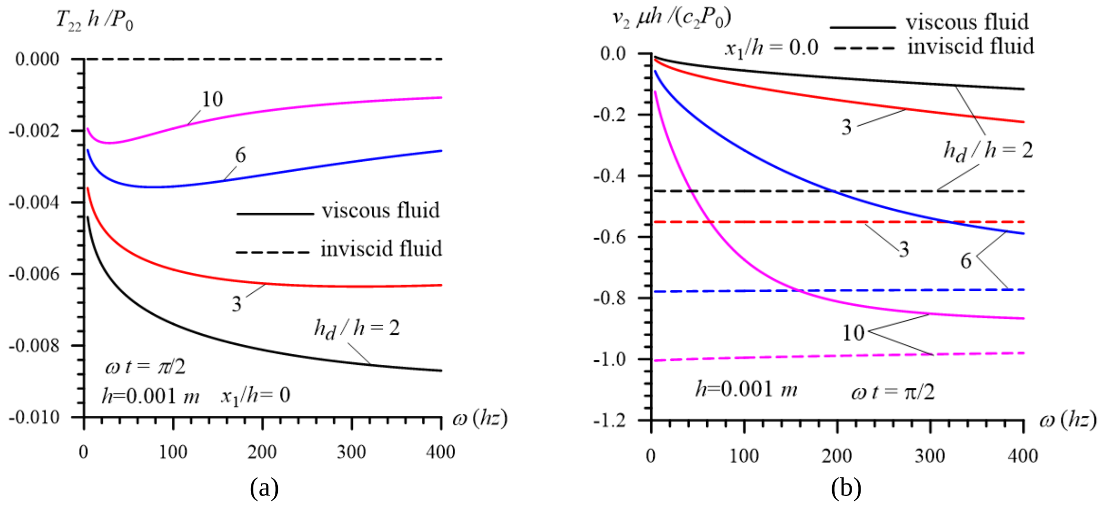


Fig. 2 Frequency response of the stress (a) and velocity (b) in the case where $\omega t = \pi/2$ under various values of ratio h_d/h

consideration is λ_1. Namely, through this parameter the influence of the initial stretching of the plate-layer on the dynamic behavior of the hydro-elastic system will be estimated.

Numerical results, which will be discussed below, relate to the normal stress acting on the interface plane between the fluid and plate-layer and to the velocity of the fluid on the interface plane in the direction of the Oy_2 axis. Note that under obtaining the numerical results the integration interval $\left[0, S_1^*\right]$ is divided into a certain N number of shorter intervals. In each of these shorter intervals with length S_1^*/N the integration is made by the use of the Gauss integration algorithm with ten sample points. The convergence of the mentioned integration algorithm with respect to the S_1^* and N was examined in the papers by Akbarov and Ismailov (2017, 2016a) and therefore here we do not consider again this question. However, according to the convergence requirement of the numerical results discussed in these papers, the all numerical results presented in the present paper are obtained in the case where $S_1^* = 5$ and $N = 2000$ for the case where $h = 0.001m$ and $4hz \le \omega \le 400hz$.

4.2 The case where initial strains in the plate-layer are absent

For estimation the influence of the initial stretching of the plate-layer on the frequency response of the hydro-elastic system under consideration, first we consider some basic particularities of the mentioned response which takes place in the case where the initial stretching in the plate-layer is absent, i.e. in the case where $\lambda_1 = 1.0$. As an examples for these responses numerical results illustrated the change character of the dimensionless interface normal stress $T_{22}h/P_0$ and velocity $v_2\mu h/(c_2P_0)$ with respect to the problem parameters, such as the vibration frequency ω, the vibration phase ωt and the dimensionless distance x_1/h $(=y_1/h)$ from the point at which the

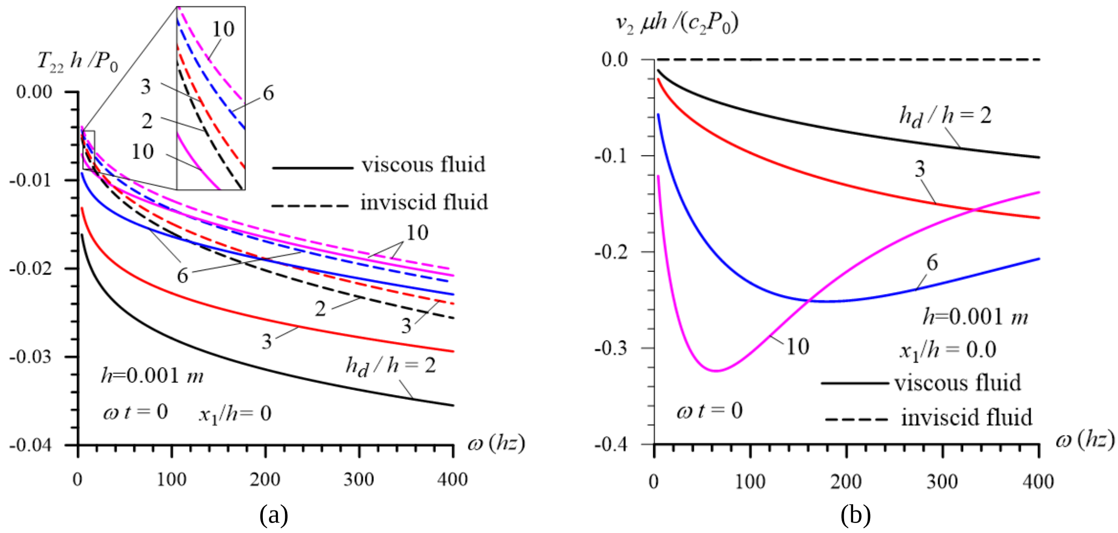


Fig. 3 Frequency response of the stress (a) and velocity (b) in the case where $\omega t = 0$ under various values of ratio h_d/h

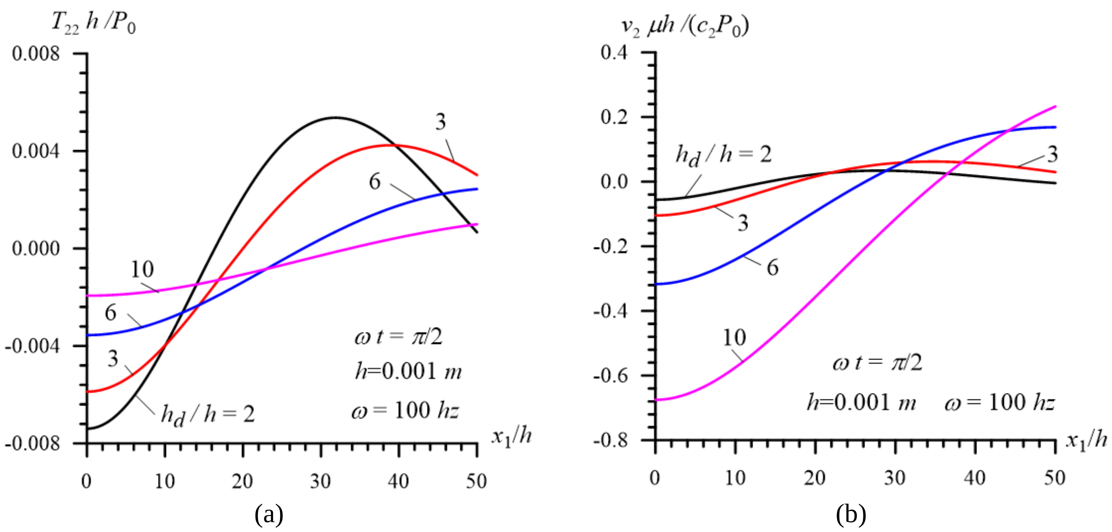
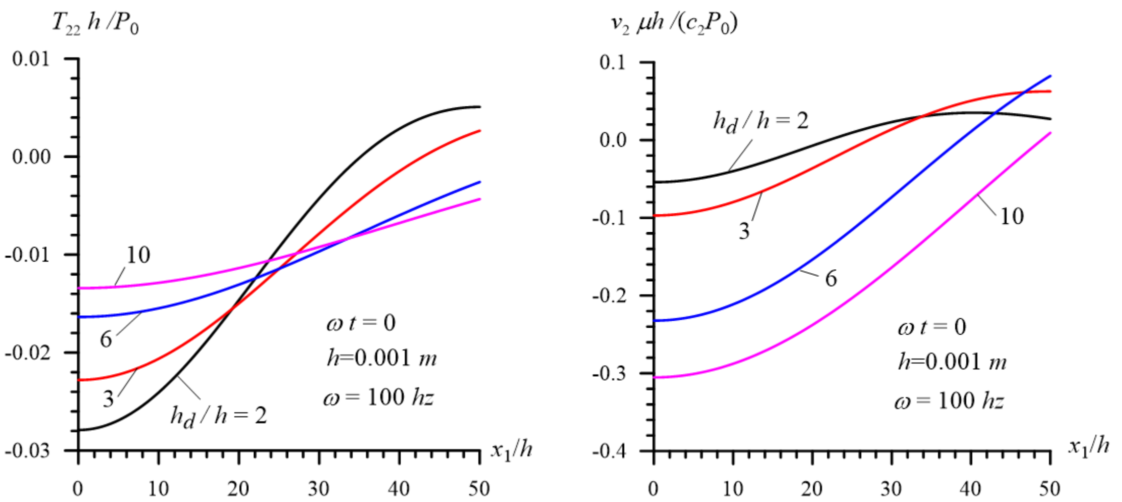


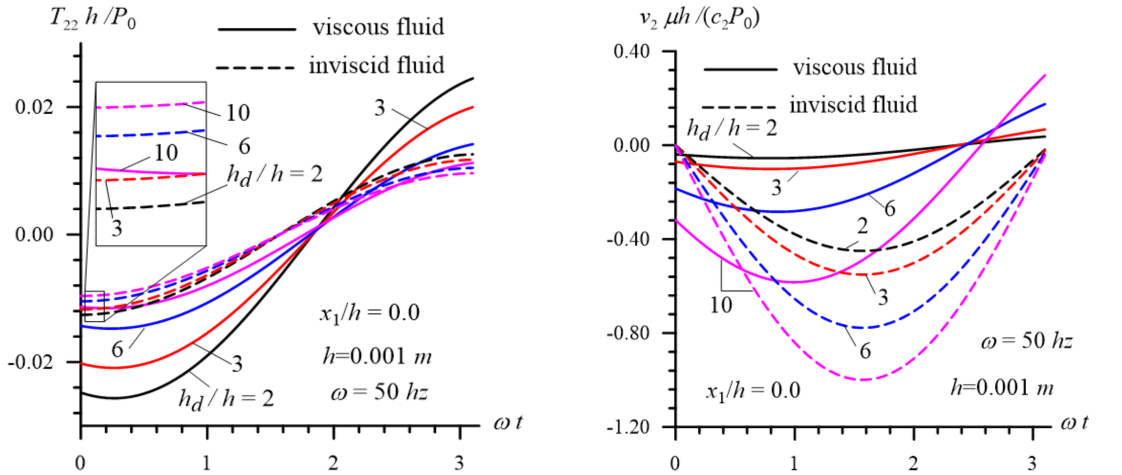
Fig. 4 Distribution of the stress (a) and velocity in the case where $\omega t = \pi / 2$ under various values of ratio h_d/h

external force acts, are given in Figs. 2-6. In these figures the graphs grouped by letter a (b) relate to the dimensionless stress $T_{22}h / P_0$ (velocity $v_2 \mu h / (c_2 P_0)$) and the results shown in Figs. 2 and 4 (in Figs. 3 and 5) are obtained in the case where $\omega t = \pi / 2$ (in the case where $\omega t = 0$). The graphs given in Figs. 2 and 3 illustrate the frequency response of the studied quantities, the graphs given in Figs. 4 and 5 illustrate the distribution of these quantities with respect to the dimensionless coordinate x_1 / h . However, the change of the quantities under consideration with respect to the vibration phase ωt is illustrated with the graphs given in Fig. 6.



(a) (b)

Fig. 5 Distribution of the stress (a) and velocity in the case where $\omega t = 0$ under various values of ratio h_d/h



(a) (b)

Fig. 6 The influence of the vibration phase ωt on the values of stress (a) and velocity (b) under various values of ratio h_d/h

Note that the results given in Figs. 2-6 are obtained for various values of the ratio h_d/h (=2, 3, 6 and 10) which shows the fluid depth. Moreover, note that in Figs. 2-6 the results related to the corresponding inviscid flued case are also given in order to estimate the influence of the fluid viscosity on the frequency response of the considered system. Under inviscid fluid case it is understood that the Glycerin is modeled as inviscid fluid, i.e., it is assumed that $\mu^{(1)} = 0$.

The foregoing results are similar in the qualitative sense with corresponding ones obtained and analyzed in the paper by Akbarov and Ismailov (2017). However, in the paper by Akbarov and Ismailov (2017) it is assumed that the material of the plate is steel with the mechanical constants

$\mu = 79 \times 10^9 Pa$, $\lambda = 94.4 \times 10^9 Pa$ and density $\rho = 7790 kg / m^3$ (see, Guz 2004, Guz and Makhort 2000), and the material of the fluid is also Glycerin as here.

Thus, according to the results analyzed in the paper by Akbarov and Ismailov (2017) and to the results obtained in the present investigation, it can be made the following conclusions with respect to the influence of the vibration phase and fluid viscosity on the values of stress $T_{22}h/P_0$ and velocity $v_2 \mu h / (c_2 P_0)$:

a) in the inviscid fluid case the absolute maximum value of the stress arises under $\omega t = 0 + n\pi$ but under $\omega t = \pi / 2 + n\pi$ the stress is equal to zero. However, the absolute maximum values of the fluid velocities arise under $\omega t = \pi / 2 + n\pi$ and these velocities are equal to zero under $\omega t = 0 + n\pi$;

b) in the viscous fluid case the absolute maximum value of the stress appears under $\omega t = (\omega t)' + n\pi$ and the stress is equal to zero under $\omega t = (\omega t)'' + n\pi$, where $0 < (\omega t)' < \pi / 2$ and $\pi / 2 < (\omega t)'' < \pi$, however, absolute maximum values of the velocity appear under $\omega t = (\omega t)^* + n\pi$ and the velocity becomes zero under $\omega t = (\omega t)^{**} + n\pi$, where $0 < (\omega t)^* < \pi / 2$ and $\pi / 2 < (\omega t)^{**} < \pi$;

c) the values of $(\omega t)'$ and $(\omega t)''$ increase, however the values of $(\omega t)^*$ and $(\omega t)^{**}$ decrease with a decrease in the ratio h_d / h, i.e., with a decrease of the fluid depth;

d) as usual, the values of the $(\omega t)'$ and $(\omega t)^{**}$ are near to zero, however the values of the $(\omega t)''$ and $(\omega t)^*$ are near to $\pi / 2$, therefore the frequency response graphs constructed in the case where $\omega t = 0$ (in the case where $\omega t = \pi / 2$) can be related with a certain accuracy as those obtained in the cases where $\omega t = (\omega t)'$ or $\omega t = (\omega t)^{**}$ (in the case where $\omega t = (\omega t)''$ or $\omega t = (\omega t)^*$);

e) in the case where $\omega t = 0$ (in the case where $\omega t = \pi / 2$) for the considered range change of the vibration frequency ω the absolute values of the stress (velocity) increase monotonically with this frequency;

f) however, in the case where $\omega t = \pi / 2$ (in the case where $\omega t = 0$) the character of the frequency response of the stress (of the velocity) depends on the fluid depth, i.e., on the ratio h_d/h;

g) the absolute values of the stress increase, but the absolute values of the velocity decrease with decreasing in the ratio h_d/h;

h) the fluid viscosity causes an increase under $\omega t = 0$ (decrease under $\omega t = \pi / 2$) in the absolute values of the stress (of the velocity);

i) absolute maximum value of the stress and velocity appear at the point $x_1 / h = 0$;

j) the comparison the results obtained in the present paper with the corresponding ones obtained in the paper by Akbarov and Ismailov (2017) shows that the absolute values of the stress obtained in the present investigation are greater than corresponding ones obtained in the paper by Akbarov and Ismailov (2017), however the absolute values of the velocity obtained under $\omega t = \pi / 2$ in the paper by Akbarov and Ismailov (2017) are greater than the corresponding ones obtained in the present paper. This means that the change of the plate material can influence significantly in the quantitative sense on the frequency response of the studied quantities.

This completes the consideration of the results related to the case where the initial stretching of the plate layer is absent and basing on these results we consider the results illustrated the influence of the initial stretching of the plate-layer on the frequency response of the stress and velocity.

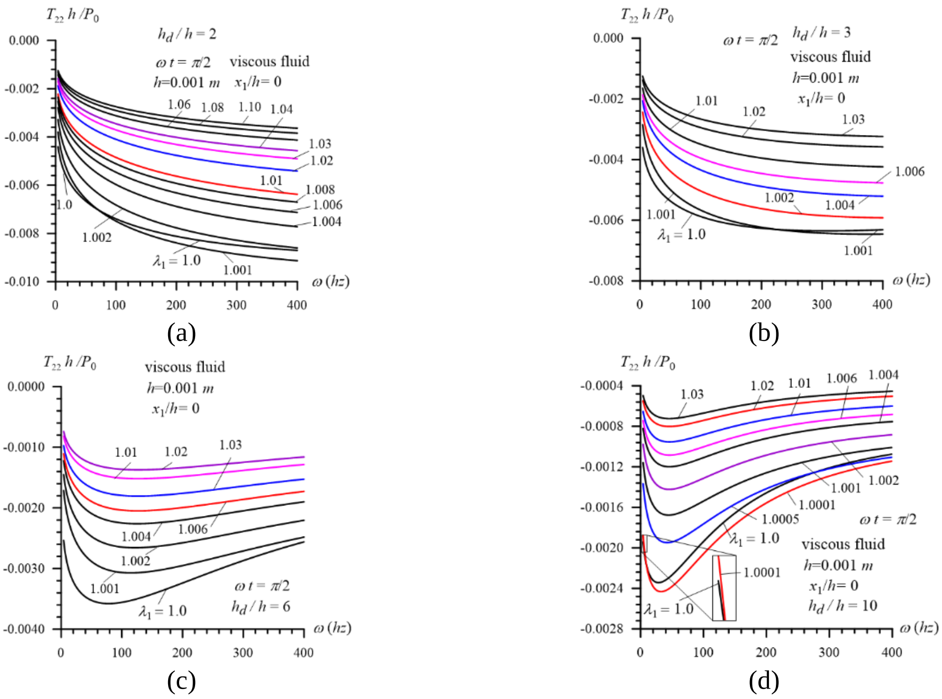


Fig. 7 The influence of the initial strains of the plate-layer on the frequency response of the stress in the case where $\omega t = \pi / 2$ under h_d / h =2 (a), 3 (b), 6 (c) and 10 (d)

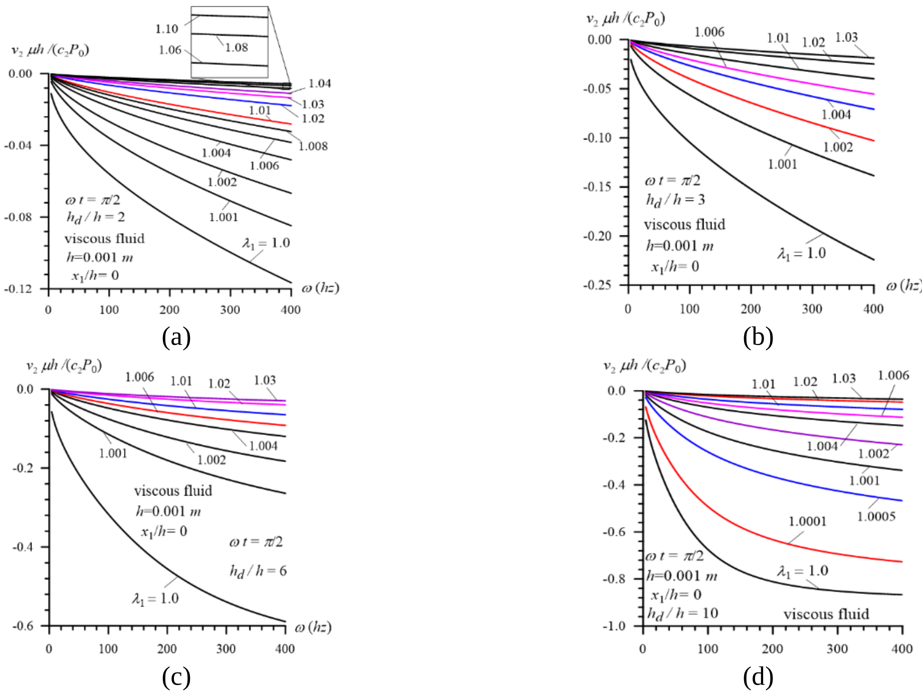


Fig. 8 The influence of the initial strains of the plate-layer on the frequency response of the velocity in the case where $\omega t = \pi / 2$ under h_d / h =2 (a), 3 (b), 6 (c) and 10 (d)

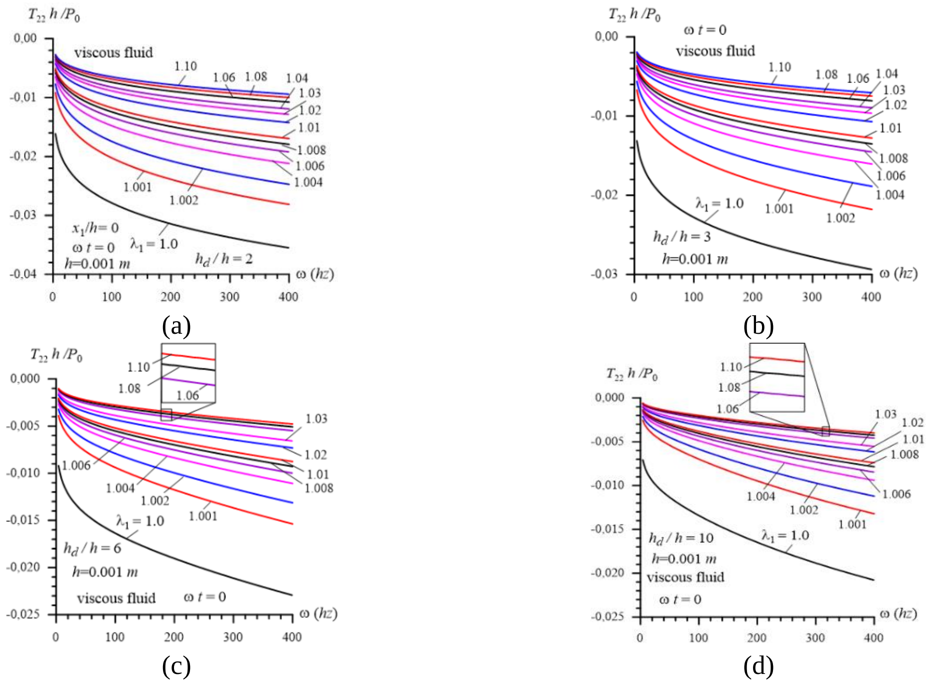


Fig. 9 The influence of the initial strains of the plate-layer on the frequency response of the stress in the case where $\omega t=0$ under h_d/h =2 (a), 3 (b), 6 (c) and 10 (d)

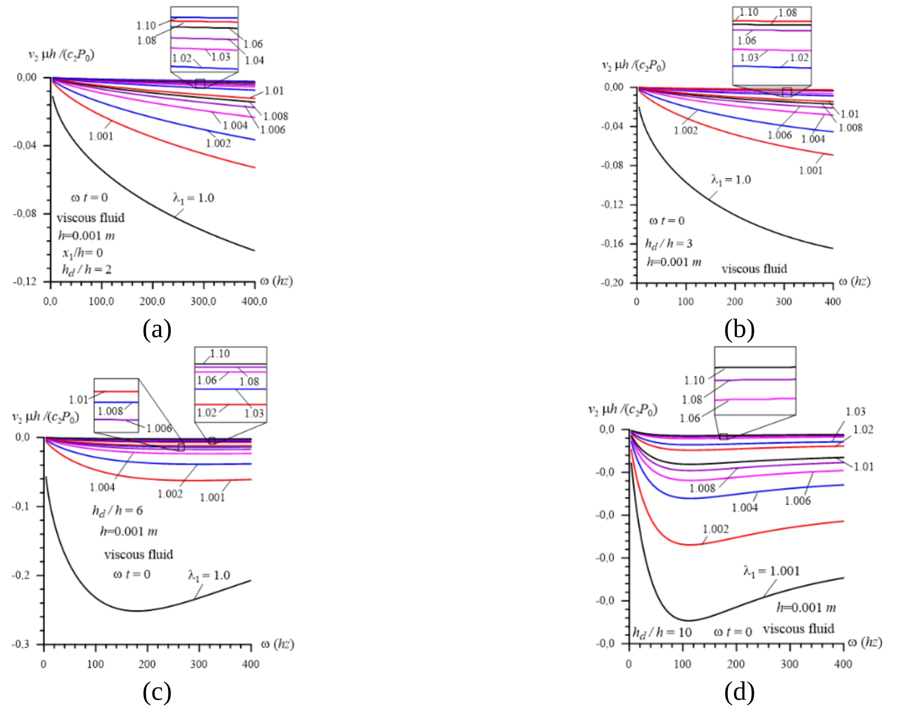


Fig. 10 The influence of the initial strains of the plate-layer on the frequency response of the velocity in the case where $\omega t=0$ under h_d/h =2 (a), 3 (b), 6 (c) and 10 (d)

4.3 The influence of the initial strains in the plate-layer on the frequency response

Thus, taking the foregoing results and conclusions into consideration we consider the influence of the initial stretching of the plate-layer on the frequency response of the studied quantities calculated at $x_1/h=0$ in the cases where $\omega t=0$ and $\omega t=\pi/2$ for various values of the ratio h_d/h(=2, 3, 6 and 10). We analyze only the results related to the viscous fluid case.

Consider the graphs shown in Figs. 7-10 illustrated the influence of the initial stretching of the layer, i.e., the influence of the parameter λ_1 on the frequency response of the stress $T_{22}h/P_0$ (Figs. 7 and 9) and velocity $v_2\mu h/(c_2P_0)$ (Figs. 8 and 10) in various values of the ratio h_d/h. Note that in these figures the graphs grouped by letters a, b, c and d relate to the cases where h_d/h=2, 3, 6 and 10, respectively. Moreover, note the graphs given in Figs. 7 and 8 are constructed in the case where $\omega t=\pi/2$, however the graphs given in Figs. 9 and 10 are constructed in the case where $\omega t=0$.

The analysis of the graphs given in Fig. 7 shows that in the cases where $\lambda_1\geq 1.002$ the absolute values of the stress $T_{22}h/P_0$ calculated under $\omega t=\pi/2$ decrease monotonically with the parameter λ_1. However, in the case where $\lambda_1=1.001$ under h_d/h=2 and 3 the character of the influence of the initial stretching on the values of the stress depends on the vibration frequency, i.e., before (after) a certain value of the vibration frequency the initial stretching of the plate-layer causes a decrease (an increase) in the absolute values of the $T_{22}h/P_0$. Note that this complicated character of the mentioned influence disappears in the cases where $h_d/h=6$ and 10. We recall that the values of the stress illustrated in Fig. 7 appear as a result of the fluid viscosity. Consequently, in the inviscid fluid case the stress $T_{22}h/P_0$ becomes zero under $\omega t=\pi/2$ and all the foregoing discussions losses its meaning. Apart from all this the graphs given in Fig. 7 show that in the cases h_d/h=2 and 5 under $\lambda_1\geq 1.002$ the magnitude of the influence of the initial stretching on the values of the stress under $\omega t=0$ increase monotonically with the vibration frequency, however in the cases where h_d/h=6 and 10 the dependence between the mentioned magnitude and vibration frequency has non-monotonic character.

The graphs given in Fig. 8 show that in all the considered cases under $\omega t=\pi/2$ the absolute values of the velocity $v_2\mu h/(c_2P_0)$ decrease monotonically with the parameter λ_1. The influence of the parameter λ_1 on the absolute values of the $v_2\mu h/(c_2P_0)$ increase monotonically with the vibration frequency ω. Moreover, the magnitude of the influence of the initial stretching of the plate layer on the velocity $v_2\mu h/(c_2P_0)$ becomes more significantly with increasing of the fluid depth, i.e., with the ratio h_d/h. Especially, the difference between the velocities obtained in the cases $\lambda_1=1.0$ and $\lambda_1=1.001$ increases sharply under h_d/h=6 and 10. However the further decreasing of the velocity with increasing λ_1 becomes smoother.

Now we consider the graphs given in Fig. 9 which, as noted above, illustrate the frequency response of the stress $T_{22}h/P_0$ in the case where $\omega t=0$. It follows from these graphs that the initial stretching of the plate causes a decrease of the absolute values of this stress. These graphs also show that the magnitude of the influence of the parameter λ_1 on the values of the stress becomes more considerable with increasing of the fluid depth, i.e., with the ratio h_d/h. Moreover, according to these graphs it can be concluded that the magnitude of the influence of the initial stretching of the plate on the values of the stress increase monotonically with the vibration frequency ω.

Finally, we consider the graphs given in Fig. 10 which, as noted above, show the influence of the parameter λ_1 on the frequency response of the velocity $v_2\mu h/(c_2P_0)$ in the case where $\omega t=0$. We recall that the values of the velocity shown in Fig. 10 appear namely as a result of the fluid viscosity, i.e., in the inviscid fluid case the values of the velocity becomes zero in the case where ωt=0. Thus, it follows from these graphs that, as in the previous cases, the initial stretching of the plate causes to decrease the absolute values of the velocity and this influence becomes more considerable with the fluid depth. Moreover, these graphs show that in the cases where h_d/h=2 and 3 the magnitude of the mentioned influence increase monotonically with the vibration frequency ω, however in the cases where h_d/h=6 and 10 the dependence between the magnitude of this influence and vibration frequency has non-monotonic character.

This completes the consideration of the numerical results illustrated the influence of the initial stretching of the plate-layer on the frequency response of the hydro-elastic system consisting of this plate, compressible viscous fluid and rigid wall.

5. Conclusions

Thus, in the present paper the influence of the initial stretching of the highly elastic plate on the frequency response of the hydro-elastic system consisting of this plate, compressible viscous fluid and rigid wall is investigated. Under this investigation the forced vibration of this system is considered and it is assumed that on the plate the lineal-located time-harmonic forces act. The motion of the plate is described within the scope of the three-dimensional linearized theory of elastic waves in initially stressed bodies and the motion of the fluid is described by employing the linearized Navier-Stokes equations. The elasticity relations of the plate material are determined by the use of the harmonic potential. Under concrete numerical investigations the values of the elastic constant entering into the expression of this potential are taken as the values of the Lame constants of the Plexiglas and Glycerin is taken as a fluid. Numerical results on the aforementioned influence are presented and discussed. According to these numerical results the following concrete conclusions related to the influence of the initial stretching, i.e. of the parameter λ_1, on the values of the dimensionless normal stress $T_{22}h/P_0$ and dimensionless normal velocity $v_2\mu h/(c_2P_0)$ can be made:

-in all the cases (except the case where $\omega t = \pi/2$ and $\lambda_1 = 1.001$ under consideration the stress) considered in the present investigation an increase of the initial stretching of the plate, i.e. an increase in the values of the parameter λ_1 causes a decrease in the absolute values of the stress and velocity;

- the magnitude of the influence of the initial stretching of the plate on the values of the stress in the case where $\omega t = 0$ and on the values of the velocity in the case where $\omega t = \pi/2$ increase monotonically with vibration frequency ω and with the fluid depth, i.e. with the ratio h_d/h ;

- the character of the influence of the initial stretching on the frequency response of the stress in the case where $\omega t = \pi/2$ and on the values of the velocity in the case where $\omega t = 0$ depends on the ratio h_d/h , i.e., in the cases where h_d/h =2 and 3 the magnitude of the mentioned influence increase monotonically with vibration frequency, however in the case where h_d/h = 6 and 10 the dependence between the mentioned magnitude and vibration frequency has non-monotonic character;

- the obtained numerical results agree with the well-known mechanical consideration and allows to develop the approach for controlling of the frequency response of the similar type hydro-elastic system with initial stretching of the highly elastic plate;

- in the present paper the corresponding results related to the case where the initial stretching in the plate is absent, are also given and it is shown that these results agree in the qualitative sense with corresponding ones obtained in the paper by Akbarov and Ismailov (2017);
- the foregoing last two conclusions can also be taken as the validation of the PC programs and calculation algorithm used in the present investigation.

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