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Memory response in elasto-thermoelectric spherical cavity

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Abstract. A mathematical model of electro-thermoelasticity subjected to memory-dependent derivative (MDD) heat conduction law is applied to a one-dimensional problem of a thermoelectric spherical cavity exposed to a warm stun that is an element of time in the presence of a uniform magnetic field. Utilizing Laplace transform as an instrument, the issue has been fathomed logically within the changed space. Numerical inversion of the Laplace transform is carried for the considered distributions and represented graphically. Some comparisons are shown in the figures to estimate the effects of MDD parameters and thermoelectric properties on the behavior of all considered fields.

Keywords: electro-thermoelasticity; memory-dependent derivative; thermoelectric properties; spherical cavity; Laplace transforms; numerical results

1. Introduction

Heat transfer keeps on being a field of real enthusiasm to building and logical scientists, and also designers, developers, and makers. Extensive exertion has been dedicated to inquire about in conventional applications, for example, general power frameworks, heat exchangers.

The set up coupled dynamical hypothesis of thermoelasticity was made by Biot (1956) by expecting that the flexible changes influence the temperature and the different way. Regardless, this hypothesis relied upon Fourier law of warmth conduction. In this way when this hypothesis was joined with the law of preservation of vitality, an illustrative sort heat conduction condition was gotten and hence foreseen an unbounded speed of warmth banner which revoked the physical hypothesis (1956), the subject summed up hypothesis of thermoelasticity has showed up and pulled in a couple of investigators amid the latest couple of decades. The generalized hypothesis was remarkably intended to account theconstrained speed of spread of warm flag which was named as second solid effect and in such manner, we should need to determine here the a standout amongst the most timely progression of the second stable hypothesis for thermoelasticity by Fox (1969) in which he associated the gauges of present-day continuum thermodynamics.

Further, the two settled and all-around thought about summed up thermoelasticity hypothesis was also made by Lord and Shulman (1967) and Green and Lindsay (1972). One warm loosening up

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time parameters were shown within the speculation made by Master and Shulman (1967) and two unwinding times was considered within the speculation proposed by Green and Lindsay, (1972). Afterward on, Chandrashekhariah (1998), Hetnarski and Ignaczak (2000) besides detailed a few survey articles. The book by Ignaczak and Ostoja-Starzewaski (2009) were in like way kept an eye on a point-by-point examination of the summed up thermoelasticity theory. Interior the hypothetical duties to the subject are the affirmations of uniqueness speculations by Sherief (1992), Ezzat (2006). As such, a handful of examinations: Ezzat *et al*. (2001), Othman *et al*. 2002, Mukhopadhyay *et al*. 2009, Alzahrani and Abbas (2016), Lata (2018a, b), Lata *et al*. (2016), Abbas and Kumar (2016), Marin *et al*. (2013, 2017), Marin and Florea (2014), Marin and Craciun (2017), Marin and Nicaise (2016), Sherief *et al*. (2016) and Zenkour (2017), subject to these generalized hypotheses were inquired about.

A prompt alter among power and warm by using thermoelectric materials has pulled in much thought as a result of their potential applications in different coolers too, thermoelectric control generators (Rowe 1995). The thermoelectric figure of authenticity gives a degree of the nature of such materials for applications and characterized by Hiroshige *et al*. (2007), with a particular conclusion objective to achieve a tall figure of authenticity; one requires a tall thermopower. Among the commitments in continuum mechanics of thermoelectric materials are created by Ezzat and Youssef (2010).

Differential conditions of fractional order have been the focal point of numerous examinations because of their continuous appearance in different applications in liquid mechanics, viscoelasticity, science, material science, and building. As of late, Ezzat and El-Bary (2017a) set up another model of partial warmth conduction condition utilizing the Taylor-Riemann arrangement extension of time-fractional order. Sherief *et al*. (2010) have presented a partial recipe of warmth conduction and demonstrated a uniqueness hypothesis and a reciprocity relation and a variational principle. Kothari and Mukhopadhyay (2011) introduced the solution of a problem on fractional-order theory of thermoelasticity for an elastic medium.

One can allude to Yu *et al*. (2013) solved a one-dimensional issue in fractional order generalized electro-magneto-thermoelasticity. Bo *et al*. (2015) displayed a conservative numerical strategy for tackling the two dimensional non-straight fractional reaction-sub diffusion equations, while Zhang *et al*. (2018) presented a period space ghastly technique for the time-space fragmentary Fokker-Planck condition and its contrary issue. Lata (2019) investigated a two-dimensional thermoelastic problem of thick circular plate of finite thickness under fractional order theory of thermoelastic diffusion in frequency domain. Li *et al*. (2019) used ultra-short laser technology in micro-machining due to its high power, precision of operation, low cost, high control, and extremely short duration and investigate the transient response of a bi-layered structure subjected to a non-Gaussian laser beam on its bounding surface in the context of the time-fractional derivative based on generalized thermoelastic theories.

The use of memory subordinate subsidiary (MDD) in warmth conduction law suggests that the warmth transport condition is balanced and therefore the constitutive conditions are changed and the new memory-subordinate model, might be superior to anything partial ones: directly off the bat, the new model is exceptional in the shape, while the partial request hypotheses incorporate assorted pictures inside different makers; besides, physical significance of the past is even more clear watching the encapsulation of MDD's definition; thirdly, the new model is portrayed by number demand differential and crucial, which is progressively profitable in numerical calculation stood out from fragmentary; taking everything into account, the Kernel capacity and time postponement of MDD can be abstractly picked, thusly, gives more approaches to manage delineate material's rational response, as a result, it is more versatile in applications than partial ones, in which the essential variable is the fragmentary request parameter (2004). As of late, Ezzat *et al*. (2017) developed another warmth conduction law with memory-dependent derivative. One can allude to Ezzat and El-Bary (2017b), Tiwari and Mukhopadhyay (2018), Xue *et al*. (2018), Shaw (2019) and He *et al*. (2019), for an overview of utilizations of memory-dependent derivative analytics.

2. Governing equations

Ezzat *et al*. (2016) proposed the generalized Ohm and Fourier laws for elasto-thermoelectric materials subjected to MDD heat transfer when the medium is permeated by an external magnetic field of intensity $\boldsymbol{H}$, as (Shercliff 1979)

$$\boldsymbol{J} = \sigma_o \left(\boldsymbol{E} + \frac{\partial \boldsymbol{u}}{\partial t} \wedge \mu_o \boldsymbol{H} - S\,\nabla T \right), \tag{1}$$

$$\boldsymbol{q} + \omega D_\omega \boldsymbol{q} = -k\,\nabla T + \Pi \boldsymbol{J}, \tag{2}$$

is time delay and the dynamic coupled theory of warmth conduction law seeks after as the limit case when $\omega \to 0$ so that, $\left| D_\omega f(\boldsymbol{x},t) \right| \le \left| \frac{\partial f(\boldsymbol{x},t)}{\partial t} \right| = \left| \lim_{\omega \to 0} \frac{f(\boldsymbol{x},t+\omega) - f(\boldsymbol{x},t)}{\omega} \right|$. This model is more intuitionistic for understanding the physical significance and the comparing memory-dependent derivative condition is progressively expressive. The first Thomson relation see Morelli (1997) is given by

$$\Pi = S\,T \tag{3}$$

in which relates the figure of merit to See beck coefficient S (Tritt 2000) as

$$ZT = \frac{\sigma_o S^2}{k} T \tag{4}$$

Displacement equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + F_i \tag{5}$$

where F_i is the Lorenz force given by (Ezzat 2004)

$$F_i = \mu_o (\boldsymbol{J} \wedge \boldsymbol{H})_i \tag{6}$$

Constitutive equation

$$\sigma_{ij} = \lambda e_{kk}\,\delta_{ij} + 2\mu e_{ij} - \gamma (T - T_o)\,\delta_{ij} \tag{7}$$

The heat equation with MDD heat transfer

$$\begin{aligned} kT_{,ii} - \Pi J_{i,i} &= \rho C_E \frac{\partial T(x,t)}{\partial t} + \gamma T_o \frac{\partial e(x,t)}{\partial t} \\ &+ \int_{t-\omega}^{t} K(t-\xi) \left(\rho C_E \frac{\partial^2 T(x,\xi)}{\partial \xi^2} + \gamma T_o \frac{\partial^2 e(x,\xi)}{\partial \xi^2} \right) d\xi \end{aligned} \tag{8}$$

where $K(t-\omega)$ is the kernel function in which can be chosen freely as

$$K(t-\xi)=1-\frac{2n}{\omega}(t-\xi)+\frac{m^2(t-\xi)^2}{\omega^2}=\begin{cases}1 & if\ \ m=n=0\\ 1-\frac{(t-\xi)}{\omega} & if\ \ ,m=0,n=\frac{1}{2}\\ 1-(t-\xi) & if\ \ m=0\ \ n=\omega/2\\ (1-\frac{t-\xi}{\omega})^2 & if\ \ m=n=1,\end{cases} \tag{9}$$

Strain-displacement relation

$$e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \tag{10}$$

together with the past conditions, establish a total arrangement of generalized electro-thermoelasticity with memory-dependent derivative for a medium with a limited electric conductivity. In the above equations a comma signifies material subsidiaries and the summation tradition are utilized.

3. Physical problem

Let (r,ψ,φ) mean the spiral facilitates, the co-scope, and the longitude of a round directions framework, separately at the focal point of a circular cavity with sweep possessing the district in the nearness a steady attractive field with consistent force. Because of circular symmetry, all the considered capacity will be elements of *r* and *t* just and the dislodging vector has one segment in the redial course. At that point, the parts of the Lorentz compel showing up in Eq. (6) are given by (Ezzat *et al*. 1996)

$$F_{rr}=-\sigma_o\,\mu_o^2\,H_o^2\left(\frac{\partial u}{\partial t},\ 0,0\right). \tag{11}$$

The components of the stress tensor are given by

$$\sigma_{rr}=\lambda e+2\,\mu\frac{\partial u}{\partial r}-\gamma(T-T_o), \tag{12}$$

$$\sigma_{\psi\psi}=\sigma_{\varphi\varphi}=\lambda e+2\,\mu\frac{u}{r}-\gamma(T-T_o), \tag{13}$$

$$\sigma_{r\psi}=\sigma_{r\varphi}=\sigma_{\psi\varphi}=0, \tag{14}$$

where *e* is the cubical dilatation given by

$$e=e_{rr}+e_{\psi\psi}+e_{\varphi\varphi}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2u). \tag{15}$$

The equation of motion

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\frac{\partial e}{\partial r} - \sigma_o \mu_o^2 H_o^2 \frac{\partial u}{\partial t} - \gamma \frac{\partial T}{\partial r}. \tag{16}$$

The figure-of-merit ZT_o at some reference temperature T_o

$$ZT_o = \frac{\sigma_o k_o^2}{k} T_o, \tag{17}$$

where k_o is Seebeck coefficients at T_o.

The first Thomson relation at T_o

$$\pi_o = k_o T_o, \tag{18}$$

where π_o is the Peltier coefficient at T_o.

The MDD energy equation (Ezzat *et al*. 2014)

$$k \nabla^2 T - \pi_o \nabla J = (1 + \omega D_\omega)\left(\rho C_E \frac{\partial T}{\partial t} + \gamma T_o \frac{\partial e}{\partial t}\right), \tag{19}$$

wherein the memory-dependent derivative theory, the first order of function *f* which is essentially characterized in a vital type of a typical subordinate with a part work on a slipping interim (Wang and Li 2011)

$$D_\omega f(t) = \frac{1}{\omega}\int_{t-\omega}^{t} K(t-\xi)\, f'(\xi)\, \mathrm{d}\xi . \tag{20}$$

To solve the problem, we assumed that the initial conditions of the problem are taken to be homogeneous, i.e.

$$u(r,0) = \dot{u}(r,0) = \sigma_{rr}(r,0) = \dot{\sigma}_{rr}(r,0) = T(r,0) = \dot{T}(r,0) = 0, \quad t \le 0. \tag{21}$$

while the boundary conditions are taken as follows:

(i) The thermal boundary condition is that the surface of the cavity subjected to a thermal shock that is a function of time

$$T(r,t) = f(t), \quad r = a. \tag{22}$$

(ii) The surface of the cavity are traction free (zero stress) i.e.

$$\sigma_{rr}(r,t) = 0, \quad r = a. \tag{23}$$

Let us introduce the following non-dimensional variables:

$$r' = c_o \zeta_o r, \quad u' = c_o \zeta_o u, \quad t' = c_o{}^2 \zeta_o t, \quad \sigma' = \frac{\sigma}{\mu}, \quad \theta^* = \frac{\gamma(T - T_o)}{\rho c_o^2}, \quad q^* = \frac{\gamma}{k\rho c_o^3 \zeta_o} q \quad ,$$

$$\zeta_o = \rho C_E / k, \quad c_o^2 = \frac{\lambda + 2\mu}{\rho}, \quad \varepsilon = \frac{\gamma^2 T_o}{\rho c_o^2 C_E}, \quad M = \frac{\sigma_o \mu_o^2 H_o^2}{\rho c_o^2 \zeta_o}, \quad T_o = \frac{\rho c_o^2}{\gamma}.$$

The Eqs. (12), (13), (16) and (19) in non-dimensional form become

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial e}{\partial r} - M \frac{\partial u}{\partial t} - \frac{\partial \theta}{\partial r}, \tag{24}$$

$$(1+ZT_o)\nabla^2\theta = \left(1+\omega D_\omega\right)\left(\frac{\partial \theta}{\partial t} + \varepsilon \frac{\partial e}{\partial t}\right), \tag{25}$$

$$\sigma_{rr} = (\beta^2 - 2)e + 2\frac{\partial u}{\partial r} - \beta^2 \theta, \tag{26}$$

$$\sigma_{\psi\psi} = \sigma_{\varphi\varphi} = (\beta^2 - 2)e + 2\mu\frac{u}{r} - \beta^2\theta, \tag{27}$$

where $\beta^2 = \dfrac{\lambda + 2\mu}{\mu}$.

4. The analytical solutions in the Laplace-transform domain

Performing the Laplace transform defined by the relation

$$\bar{g}(s) = \int_0^\infty e^{-st} g(t)\, dt,$$

of both sides Eqs. (24)-(27), we get the following equations

$$s(s+M)\bar{u} = \frac{\partial \bar{e}}{\partial r} - \frac{\partial \bar{\theta}}{\partial r}, \tag{28}$$

$$\left(\nabla^2 - \varpi\right)\bar{\theta} = \varepsilon\,\varpi\bar{e}, \tag{29}$$

$$\bar{\sigma}_{rr} = (\beta^2 - 2)\bar{e} + 2\frac{\partial \bar{u}}{\partial r} - \beta^2\bar{\theta}, \tag{30}$$

$$\bar{\sigma}_{\psi\psi} = \bar{\sigma}_{\varphi\varphi} = (\beta^2 - 2)\,\bar{e} + 2\frac{\bar{u}}{r} - \beta^2\bar{\theta}, \tag{31}$$

where (see Ref. Ezzat *et al*. 2016).

$$G(s) = (1-e^{-s\omega})(1 - \frac{2n}{\omega s} + \frac{2m^2}{\omega^2 s^2}) - (m^2 - 2n + \frac{2m^2}{\omega s})e^{-s\omega} \text{ and } \varpi = s\left(\frac{1+G(s)}{1+ZT_o}\right). \tag{32}$$

The boundary conditions (22) and (23) have the form in the Laplace transform

$$\bar{\theta}(r,s) = \bar{f}(s), \quad r = a, \tag{33}$$

$$\bar{\sigma}_{rr}(r,s)=0, \qquad r=a. \tag{34}$$

Applying the divergence operator on both sides of Eq. (28) we obtain the equation of motion in the form

$$\left[\nabla^2 - s(s+M)\right]\bar{e} = \nabla^2\bar{\theta}. \tag{35}$$

Eliminating $\bar{\theta}$ between (29) and (35), we get

$$\left\{\nabla^4 - \left[s(s+M)+\varpi(1+\varepsilon)\right]\nabla^2 + \varpi s(s+M)\right\}\bar{e} = 0 \tag{36}$$

The above equation can be made as a factor

$$\left(\nabla^2-\xi_1^2\right)\left(\nabla^2-\xi_2^2\right)\bar{e}=0, \tag{37}$$

where ξ_1, ξ_2 are the roots with positive real parts of the characteristic equation

$$\xi^4 - \left[s(s+M)+\varpi(1+\varepsilon)\right]\xi^2 + \varpi s(s+M) = 0. \tag{38}$$

The roots of the characteristic equation are given by

$$\xi_{1,2}^2 = \frac{1}{2}\left[s(s+M)+\varpi(1+\varepsilon) \pm \sqrt{\left[\varpi(1+\varepsilon)-s(s+M)\right]^2 + 4s(s+M)\varpi\,\varepsilon}\right].$$

Because of linearity, Eq. (37) solution can be written as

$$\bar{e} = \bar{e}_1 + \bar{e}_2,$$

where

$$\left(\nabla^2-\xi_1^2\right)\bar{e}_1 = 0, \quad \left(\nabla^2-\xi_2^2\right)\bar{e}_2 = 0. \tag{39}$$

We look at the equation

$$\left(\nabla^2-\xi^2\right)f = 0,$$

we can write this as

$$\frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} - \xi^2 f = 0.$$

Taking the replacement

$$f = \frac{h}{\sqrt{r}}.$$

The equation above is reduced to

$$\frac{\mathrm{d}^2 h}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}h}{\mathrm{d}r} - \left(\xi^2 + \frac{1}{4}\right)h = 0.$$

The solution of this equation bounded at infinity has the form

$$h = K_{1/2}(\xi r),$$

where $K_{1/2}(\cdot)$ is the modified Bessel function of the second kind of the order of 1/2.

The solution of Eq. (39) can be written on the basis of the above outcomes as

$$\bar{e}(r,s)=\frac{1}{\sqrt{r}}\left[A\xi_1^2K_{1/2}(\xi_1 r)+B\xi_2^2K_{1/2}(\xi_2 r)\right] \tag{40}$$

where A and B are s-dependent parameters.

In the same manner, eliminating $\bar{e}$ between Eqs. (29) and (35), we get

$$\left(\nabla^2-\xi_1^2\right)\left(\nabla^2-\xi_2^2\right)\bar{\theta}=0. \tag{41}$$

The solutions of Eq. (41) that is compatible with Eqs. (29) and (40) is given by

$$\bar{\theta}(r,s)=\frac{1}{\sqrt{r}}\left[A\left(\xi_1^2-s\,[s+M\,]\right)K_{1/2}(\xi_1 r)+B\left(\xi_2^2-s\,[s+M\,]\right)K_{1/2}(\xi_2 r)\right]. \tag{42}$$

Differentiating Eqs. (40) and (42) concerning r and substituting the results into Eq. (28) gives

$$\bar{u}(r,s)=-\frac{1}{\sqrt{r}}\left[A\xi_1K_{3/2}(\xi_1 r)+B\xi_2K_{3/2}(\xi_2 r)\right]. \tag{43}$$

Differentiating Eqs. (43) with respect to r and substituting the result into Eqs. (30) and (31), we have

$$\begin{aligned}\bar{\sigma}_{rr}(r,s)=\frac{1}{\sqrt{r}}\Bigg(&A\left[\beta^2s(s+M\,)K_{1/2}(\xi_1 r)+\frac{4}{r}\xi_1K_{3/2}(\xi_1 r)\right]\\&+B\left[\beta^2s(s+M\,)K_{1/2}(\xi_2 r)+\frac{4}{r}\xi_2K_{3/2}(\xi_2 r)\right]\Bigg),\end{aligned} \tag{44}$$

$$\begin{aligned}\sigma_{\psi\psi}(r,s)=\sigma_{\varphi\varphi}(r,s)=\frac{1}{\sqrt{r}}\Bigg(&A\left[\left(\beta^2s(s+M\,)-2\xi_1^2\right)K_{1/2}(\xi_1 r)-\frac{2}{r}\xi_1K_{3/2}(\xi_1 r)\right]\\&+B\left[\left(\beta^2s(s+M\,)-2\xi_2^2\right)K_{1/2}(\xi_2 r)-\frac{2}{r}\xi_2K_{3/2}(\xi_2 r)\right]\Bigg).\end{aligned} \tag{45}$$

Using the boundary conditions (33) and (34), we get the following system of linear equations where the unknowns are the parameters A and B

$$A\left[\beta^2s(s+M\,)K_{1/2}(\xi_1 a)+\frac{4}{a}\xi_1K_{3/2}(\xi_1 a)\right]+B\left[\beta^2s(s+M\,)K_{1/2}(\xi_2 a)+\frac{4}{a}\xi_2K_{3/2}(\xi_2 a)\right]=0, \tag{46}$$

$$A\left(\xi_1^2-s\,[s+M\,]\right)K_{1/2}(\xi_1 a)+B\left(\xi_2^2-s\,[s+M\,]\right)K_{1/2}(\xi_2 a)=\sqrt{a}\,\bar{f}(s). \tag{47}$$

Starting now and into the foreseeable future, we will utilize the exponential type of the modified Bessel functions of the subsequent kind, in particular

$$K_{1/2}(z)=\sqrt{\frac{\pi}{2z}}e^{-z},\quad K_{3/2}(z)=\sqrt{\frac{\pi}{2z}}\left(1+\frac{1}{z}\right)e^{-z}. \tag{48}$$

By solving the system of two Eqs. (46) and (47) and using Eq. (52), we get the values of the two parameters A and B as

$$A = \frac{a}{\gamma_1}\sqrt{\frac{2}{\pi}}\left(\sqrt{\xi_1}\left[a^2\beta^2 s(s+M) + 4a\xi_2 + 4)\right]e^{\xi_1 a}\right)\bar{f}(s), \tag{49}$$

$$B = -\frac{a}{\gamma_1}\sqrt{\frac{2}{\pi}}\left(\sqrt{\xi_2}\left[a^2\beta^2 s(s+M) + 4a\xi_1 + 4)\right]e^{\xi_2 a}\right)\bar{f}(s), \tag{50}$$

where

$$\gamma_1 = (\xi_1 - \xi_2)\left([a^2\beta^2 s(s+M) + 4](\xi_1 + \xi_2) + 4a[s(s+M) + \xi_1\xi_2]\right).$$

Substituting from Eqs. (49) and (50) into Eqs. (42), (43) and (44), we get upon using Eq. (48), the temperature, displacement and, radial stress distributions, respectively in the form

$$\begin{aligned}\bar{\theta}(r,s) = \frac{a}{\gamma_1}\Big([\xi_1^2 - s(s+M)]\left[a^2\beta^2 s(s+M) + 4a\xi_2 + 4)\right]e^{-\xi_1(r-a)} \\ - [\xi_2^2 - s(s+M)]\left[a^2\beta^2 s(s+M) + 4a\xi_1 + 4)\right]e^{-\xi_2(r-a)}\Big)\bar{f}(s),\end{aligned} \tag{51}$$

$$\begin{aligned}\bar{u}(r,s) = -\frac{a}{\gamma_1 r}\Big((1+\xi_1 r)\left[a^2\beta^2 s(s+M) + 4a\xi_2 + 4)\right]e^{-\xi_1(r-a)} \\ - (1+\xi_2 r)\left[a^2\beta^2 s(s+M) + 4a\xi_1 + 4)\right]e^{-\xi_2(r-a)}\Big)\bar{f}(s),\end{aligned} \tag{52}$$

$$\begin{aligned}\bar{\sigma}_{rr}(r,s) = \frac{a}{\gamma_1 r^2}\Big(\left[r^2\beta^2 s(s+M) + 4r\xi_1 + 4)\right]\left[a^2\beta^2 s(s+M) + 4a\xi_2 + 4)\right]e^{-\xi_1(r-a)} \\ - \left[r^2\beta^2 s(s+M) + 4r\xi_2 + 4)\right]\left[a^2\beta^2 s(s+M) + 4a\xi_1 + 4)\right]e^{-\xi_2(r-a)}\Big)\bar{f}(s).\end{aligned} \tag{53}$$

This completes the solution in the Laplace transform domain.

5. Inversion of the Laplace transforms

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{f}(s)$ be the Laplace transform of a function $f(t)$. The inversion formula for Laplace transforms can be written as (Honig and Hirdes 1984)

$$f(t) = \frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\,\bar{f}(d+iy)\,\mathrm{d}y,$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(s)$.

Expanding the function $h(t) = \exp(-dt)f(t)$ in a Fourier series in the interval [0, 2L], we obtain the approximate formula (2004)

$$f(t) \approx f_N(t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k, \quad \text{for} \quad 0 \le t \le 2L, \tag{54}$$

where

$$c_k = \frac{e^{dt}}{L}\mathrm{Re}\left[e^{ik\pi t/L}\,\bar{f}\left(d + ik\pi/L\right)\right]. \tag{55}$$

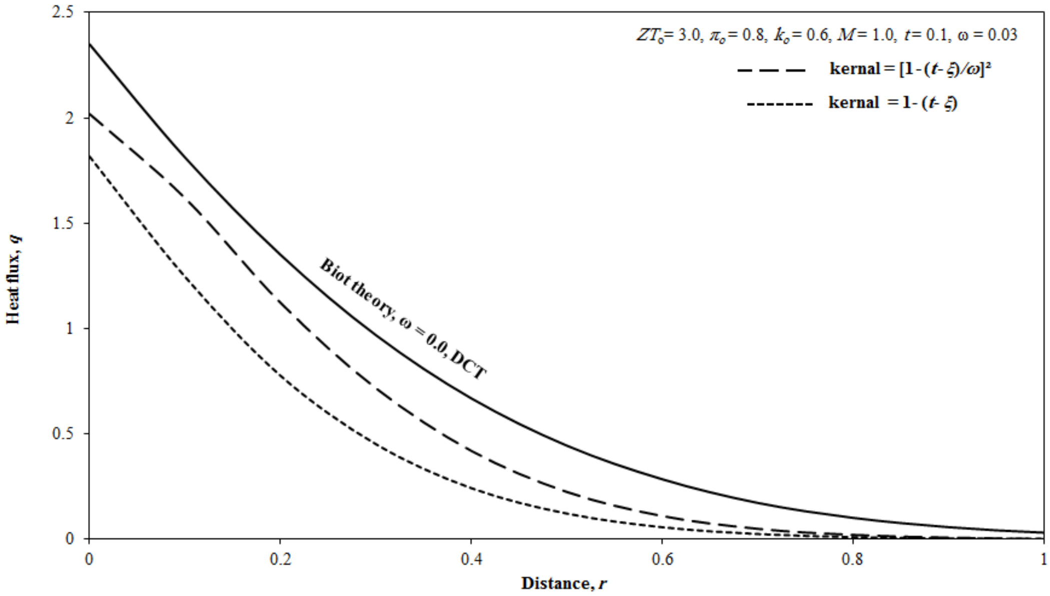

Fig. 1 The variation of heat flux vs. distance r for different forms of Kernel function $K(t, \xi)$

Two methods are used to reduce the total error. First, the 'Korrektur' method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The Korrektur-method uses the following formula to evaluate the function $f(t)$

$$f\ (t) = f_{NK}(t) = f_N\ (t) - e^{-2dL} f_{N'}(2L+t). \tag{56}$$

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in (54). Let N be an odd natural number and let $s_m = \sum_{k=1}^{m} c_k$, be the sequence of partial sums of (54). We define the ε-sequence by

$$\varepsilon_{0,m} = 0,\ \varepsilon_{1,m} = s_m, \quad m = 1, 2, 3, ...$$

and $\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + 1/\left(\varepsilon_{n,m+1} - \varepsilon_{n,m}\right), \quad n, m = 1, 2, 3, ...$

It can be shown that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, ..., \varepsilon_{N,1}, ...$ converges to $f(t) - c_0/2$ faster than the sequence of partial sums (1984).

6. Numerical results and discussion

We have chosen for the motivations behind numerical assessment $f(t)$ in the form

$$f(t) = tH(t) \qquad \text{or} \qquad \bar{f}(s) = \frac{1}{s^2}, \tag{57}$$

where $H(t)$ is the Heaviside unit step function.

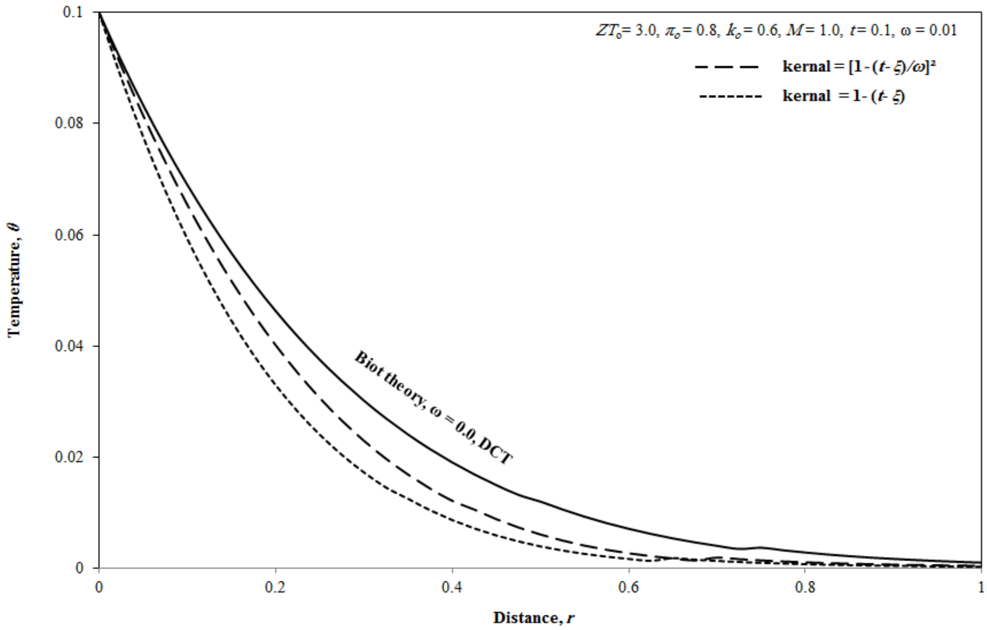

Fig. 2 The vatiation of temperature vs. distance r for different forms of Kernel function $K(t, \xi)$

The copper material was picked for motivations behind numerical assessments and the constants of the issue were taken as pursues (Ezzat *et al*. 2014)

$$T_o = 293K,\ k = 386\ N/Ks,\ \alpha_T = 1.78\times10^{-5}\ K^{-1},\ C_E = 383.1\ m^2/K,\ \beta^2 = 4,$$

$$\zeta_o = 886.73 s/m^2, \mu = 3.86\times10^{10}\ N/m^2, \lambda = 7.76\times10^{10}\ N/m^2,\ \omega \approx 10^{-3}\sec,$$

$$\rho = 8954\ kg/m^3,\ c_o = 4158 m/s,\ \varepsilon = 0.0168,\ B_o = \mu_o H_o = 1T,\ a = 1.$$

Thinking about the above physical information, we have assessed the numerical estimations of the field amounts with the assistance of a PC program created by utilizing MATLAB programming. The precision kept up was 7 digits for the numerical program.

The calculations were completed for certain parameters, where an estimation of time, namely, t=0.1. The investigation of the effect of the figure-of-merit on thermoelectric material within the sight of a consistent attractive field was done in the former areas. Run of the mill numerical outcomes are appeared in Figs. 1-6.

Figs. 1and 2 speak to the dimensionless estimation of warmth motion and temperature for a wide scope of outspread separation r ($0\leq r\leq1$) and different forms of kernel function. In these figures, strong lines speak to the arrangement got in the casing of Biot theory (ω=0) and broken lines represent the solution corresponding to using generalized electro-thermoelasticity (ω>0) with MDD when the kernel function is taken as the form $[1-(t-\xi)/\omega]^2$, while dotted lines when the kernel function is $1-(t-\xi)$. We learned from these figures that vital wonder saw in these assumes that the arrangement of any of the considered capacity in the new model is confined in a limited locale. Past this area, the varieties of these appropriations try not to occur. This implies to the arrangements concurring the new summed up hypothesis show the conduct of limited rates of wave spread.

Fig. 3 indicates the variation of heat flux against the figure-of-merit for Biot theory (ω=0) and for the generalized electro-thermoelasticity theory with MDD (ω>0) when the kernel function has

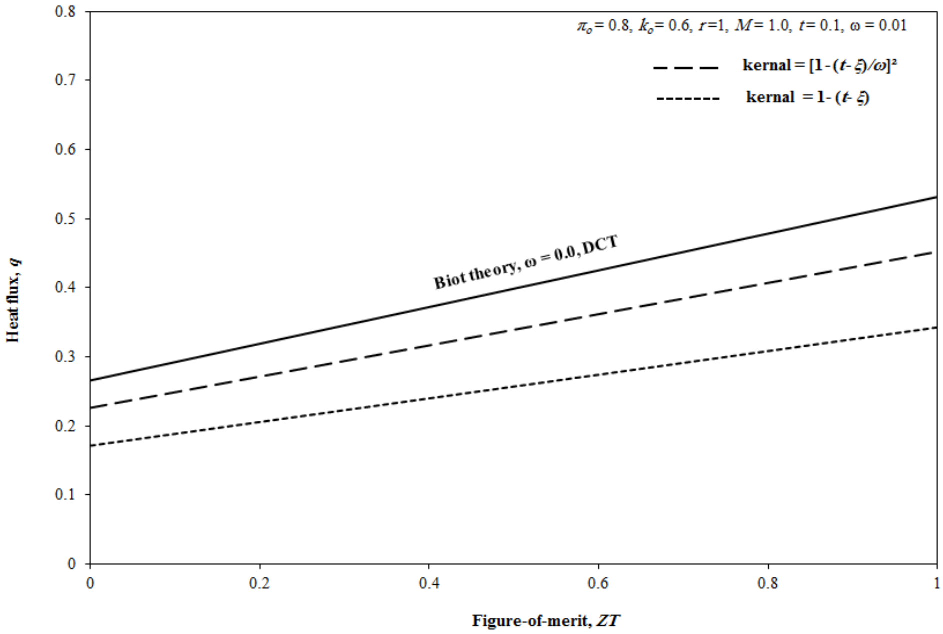

Fig. 3 The vatiation of heat flux vs. figure-of-merit ZT for different forms of Kernel function $K(t, \xi)$

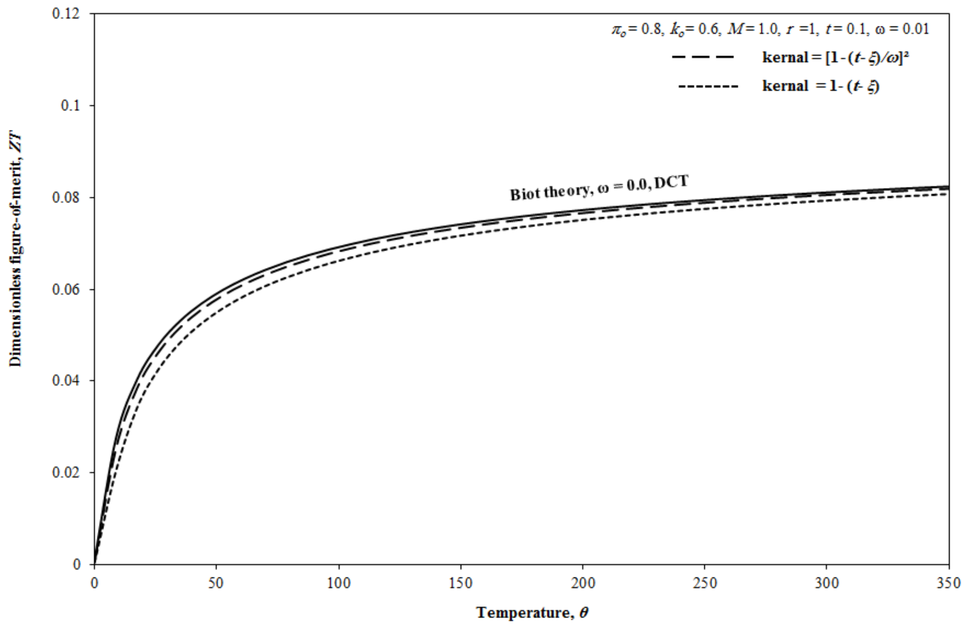

Fig. 4 The dimensionless figure-of-merit ZT is plotted vs. temperature for different forms of Kernel function $K(t, \xi)$

two forms, namely, $[1-(t-\xi)/\omega]^2$ and $1-(t-\xi)$. We learned from this figure that the choice of the kernel function forms has a significant effect on the heat flux field.

Fig. 4 presents some data on the figure-of-merit ZT as a function of temperature for different

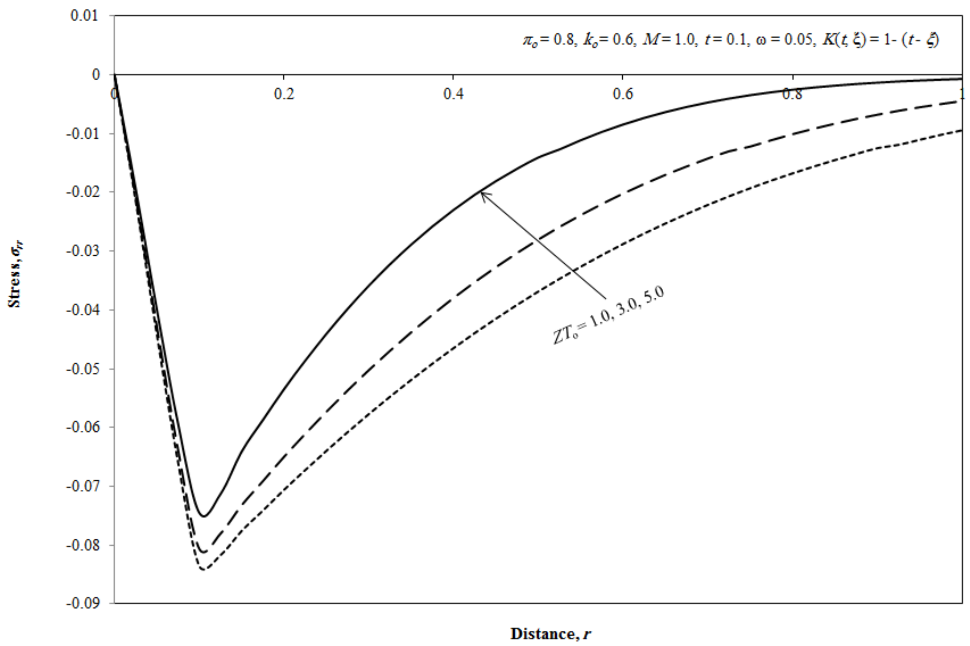

Fig. 5 The variation of stress vs. distance r for different values of figure-of-merit ZT_o

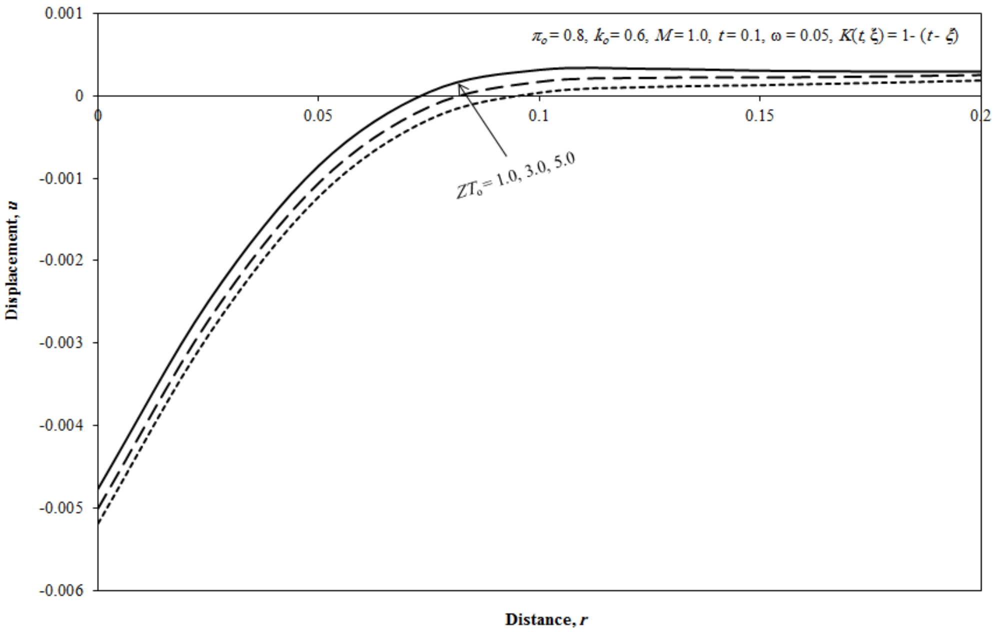

Fig. 6 The variation of displacement vs. distance r for different values of figure-of-merit ZT_o

theories. We noticed that the efficiency of a thermoelectric material figure-of-merit is proportional to the temperature of the solid particles (Ezzat *et al*. 2017, Tiwari *et al*. 2018).

Fig. 5 and 6 show the variety of displacement and stress circulations in thermoelectric circular depression with spiral separation *r* ($0 \le r \le 1$) for three values of figure-of-merit at room temperature ZT_o, namely, ZT_o=1, 3 and 5. We noticed that the stress and displacement field has been affected by the figure-of-merit values, where the expanding of the estimation of figure-of-merit causes decreasing in the magnitude of the stress and displacement field.

8. Conclusions

The primary objective of this work is to take care of certain issues of warm excitations in the hypothesis of coupled fields have a place with the electro-thermoelasticity. The expanding wide use in detecting and activation has pulled in much consideration towards hypotheses about materials displaying couplings between versatile, electric, attractive and warm fields.

The conditions of wave hypothesis of electro-thermoelasticity exposed to MDD based on the change of the Fourier law was built rough phenomenological conditions of thermo-electromagnetic versatility described by a limited speed of engendering of electromagnetic and flexible excitations.

As per the aftereffects of the work, we can see the nearness of MDD's parameters in Fourier law of warmth conduction can assume a crucial job in expanding or diminishing the speed of the wave proliferation of all fields through the thermoelectric medium.

From the considered model we can set up some fundamental hypotheses on the straight coupled and summed up speculations of electro-thermoelasticity; for example, the coupled hypothesis (ω=0) and the generalized case hypothesis (ω>0).

Representative results for the all functions for generalized theory are distinctly different from those obtained for the coupled theory. This because thermal waves in the coupled theory travel with an infinite speed of propagation as opposed to finite speed in the generalized case. It is clear that for small values of time the solution is localized in a finite region. This region grows with increasing time and its edge is the location of the wave front. This region is determined by the values of time *t* and time-delay. The predictions of the new theory are discussed and compared with dynamic classical coupled theory.

The advantage of the considered a new model consists in:

i) The discontinuities in temperature distribution disappeared.

ii) The negative values of temperature that usually appear in the generalized theories of thermoelasticity vanished.

iii) The Kernel functions and time-delay of memory-dependent derivative can be arbitrarily chosen freely according to the necessity of applications (Ezzat 2011).

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DC

Nomenclature

λ, μ	Lame's constants
ρ	density
t	time
C_E	specific heat at constant strain
k	thermal conductivity
T	temperature
T_o	reference temperature
μ_o	magnetic permeability
ε_o	electric permittivity
σ_o	electric conductivity
σ_{ij}	components of stress tensor
u_i	components of displacement vector
C_o	$=[(\lambda+2\mu)/\rho]^{1/2}$, speed of propagation of isothermal elastic waves
η_o	$= \rho C_E/K$
θ	$=T-T_0$, such that $\lvert\theta/T_0\rvert<<1$
q_i	components of heat flux vector
E_i	components of electric field vector
J_i	components electric density vector
H_i	magnetic field intensity
S, k_o	Seebeck coefficient

$\prod$, π_o	Peltier coefficient
e	dilatation
δ_o	non-dimensional constant for adjusting the reference
α_T	coefficient of linear thermal expansion
ε	thermoelastic coupling parameter
γ	$=(3\lambda+2\mu)\alpha_T$
δ_{ij}	Kronecker delta function