Coupled Systems Mechanics, *Vol. 3, No. 4 (2014) 345-365*
DOI: http://dx.doi.org/10.12989/csm.2014.3.4.345

Dynamic response of cable-stayed bridges subjected to sudden failure of stays – the 2D problem

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(Received June 7, 2013, Revised September 1, 2013, Accepted September 14, 2013)

Abstract. A significant problem met in engineering practice when designing cable-stayed bridges is the failure of cables. Many different factors can lead to sudden failure of cables, such as corrosion, continuous friction or abrasion, progressive and extended crevice created by fatigue and finally an explosion caused by sabotage or accident, are some of the causes that can lead to the sudden failure of one or more cables. This paper deals with the sudden failure of cables in a special form of cable-stayed bridges with a single line of cables anchored at the central axis of the deck's cross-section. The analysis is carried out by the modal superposition technique where an analytical method developed by the authors in a previous work has been employed.

Keywords: cable-stayed bridges; dynamic response; cable failure; sudden failure

1. Introduction

Cable stayed bridges have been known since the beginning of the 18^{th} century, but they have been of great interest only in the last fifty years, particularly due to their special shape and also because they can serve as an alternative solution to suspension bridges for long spans. The main reason for this delay in their use was the difficulties in their static and dynamic analysis, the involvement of various types of nonlinearities, the absence of computational capabilities, and the lack of high strength materials and construction techniques. Numerous studies exist concerning the static behavior, such as the works of Bruno and Grimaldi (1985), Fleming (1979), Khalil (1999), Kollbruner *et al*. (1980), Gimsing (1997), Michaltsos *et al*. (2003), Virgoreux (1999), the dynamic analysis, such as the works of Freire *et al*. (2006), Chatterjee *et al*. (1994), Nazmy and Abdel-Ghaffar (1990), Abdel-Ghaffar and Khalifa (1991a,b), Fleming and Egeseli (1980), Bruno and Golotti (1994), Achkire and Preumont (1996), Michaltsos (2001), Konstantakopoulos *et al*. (2002), Wang *et al*. (2010), and the stability of cable-stayed bridges, such as the works of Ermopoulos *et al*. (1992), Bosdogianni and Olivari (1997), Michaltsos (2005), Michaltsos *et al*. (2008).

A significant problem arising from the engineering practice is the failure of cables. There are many factors that can lead to sudden failure of stay cables. Corrosion, continuous friction or abrasion, progressive and extended crevice created by fatigue, and finally an explosion caused by

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http://www.techno-press.org/?journal=csm&subpage=7 ISSN: 2234-2184 (Print), 2234-2192 (Online)

Photo 1 The tallest bridge of the world: Bridge of Millau over the Tarn valley

Photo 2 The Cantenario Bridge in Panama

sabotage or an accident are some of the causes that can lead to the sudden failure of one or more cables.

The failure of one or more cables causes a redistribution of forces and stresses not only on the remaining stay cables, but also on the bridge-deck and onto the pylons. Existing codes and recommendations such as SETRA (2001) and PTI (2007) confront this accidental stress state by applying a dynamic amplification factor (DAF) to the forces and stresses, which can be obtained by the static analysis of the bridge. Nevertheless, these regulations after experimental testing (Mozos 2007) and FEM analyses (Mozos and Aparicio 2010a, b) have been proved inadequate (Del Olmo and Bengoechea 2006).

Both SETRA and PTI recommendations propose a dynamic amplification factor equal to 2.0 (maximum value), that is also recommended in many classical references (as for example in Mozos and Aparicio 2009). These values are valid for a single degree-of-freedom system subjected to a rectangular pulse. Mozos and Aparicio (2009) proved that under certain conditions the DAF factor can reach values larger than 2.0 for a multiple degree-of-freedom system under a pulse of infinite duration. The above recent publications along with the ones by Ruiz-Teran and Aparicio (2007), Wolff and Starossek (2009), Starossek (2009) have proven that the proposed DAF factors are unsafe.

This paper deals with the sudden failure of cables in a special form of cable-stayed bridges like the one shown in Photos 1 and 2, where a single line of cables is anchored at the central axis of the deck's cross-section. The analysis is carried out by the modal superposition method using the analytical process presented in Konstantakopoulos (2004), Petalas and Konstantakopoulos (2005), Konstantakopoulos *et al*. (2002). Characteristic examples are solved and useful diagrams and plots are drawn, while interesting results are obtained.

2. Analysis

The following analysis concerns a cable-stayed bridge with a single line of cables anchored at the central axis of the deck's cross-section.

2.1 Stress state of the pylon

The deformation $f(z)$ at the random point $A(z)$ of a pylon with height h (see Fig. 1) is given by

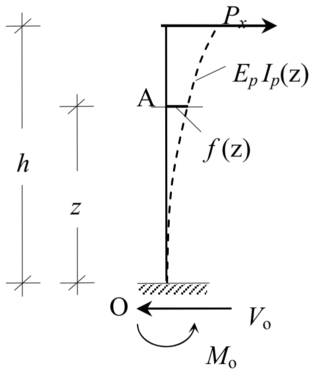

Fig. 1 The deformed pylon

the relation: $E_p I_p(z) f'' = -P_x(z-h)$ or

$$\left.\begin{aligned} f'(z) &= -\int \frac{P_x(z-h)}{E_p I_p} dz + c_1 \\ f(z) &= -\int dz \int \frac{P_x(z-h)}{E_p I_p} dz + c_1 z + c_2 \end{aligned}\right\} \tag{1a}$$

where E_p and I_p are the elasticity modulus and the moment of inertia of the pylon, respectively. The boundary conditions are: $f(0) = f'(0)$ and Eq. (1a) give

$$f(x) = f_o(z) P_x \tag{1b}$$

where

$$f_o(z) = -\int dz \int \frac{(z-h)}{E_p I_p} dz + \left[\int \frac{(z-h)}{E_p I_p} dz\right]_{z=0} + \left[\int dz \int \frac{(z-h)}{E_p I_p} dz\right]_{z=0}$$

For $I_p(z) = I_p$ = constant, the preceding equation becomes $f_o(z) = -\dfrac{z^2(z-3h)}{6E_p I_p}$, which for $z = h$ gives

$$f_o(h) = \frac{h^3}{3E_p I_p} \tag{1c}$$

2.2 The isolated cable

Let us consider the bridge shown in Fig. 2, with one line of cables anchored at the center of the deck's cross-section.

The bridge is stayed by ρ-cables at the left, and κ-cables at the right of pylon a, and by κ-cables at the left and ρ-cables at the right of pylon b.

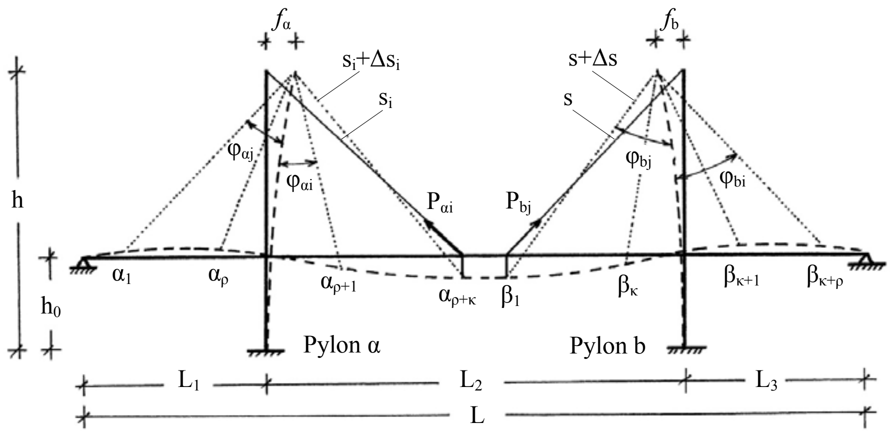

Fig. 2 A three-span cable-stayed bridge

For the cable "*i*" of Fig. 2, we get: $s_i + \Delta s_i = w_i \cos\varphi_i + s_i - f \sin\varphi_i$ or

$$\Delta s_i + f \sin\varphi_i = w_i \cos\varphi_i \tag{2a}$$

And because of $\Delta s_i = s_i P_i / E_c A_i$, the above Eq. (2a) becomes

$$\frac{s_i P_i}{E_c A_i} + f \sin\varphi_i = w_i \cos\varphi_i \tag{2b}$$

where E_c is the modulus of elasticity of the cables and A_i is the cross-sectional area of the cable "*i*".

2.2.1 Thin arrangement of cables

The total deformation at the top of the pylon due to the acting horizontal forces is

$$f(h) = f_o(h)\left[\sum_i P_i \sin\varphi_{ai} - \sum_j P_j \sin\varphi_{aj}\right] = f_o \Phi_a$$

where

$$\Phi_a = \sum_i P_i \sin\varphi_{ai} - \sum_j P_j \sin\varphi_{aj} \tag{3a}$$

Applying Eq. (2b) on both sides of the pylon, we obtain

$$\begin{aligned} &\textit{left side} \quad b_{aj} P_{aj} - f_o \Phi_a \sin\varphi_{aj} = w_a \cos\varphi_{aj} \\ &\textit{right side} \quad b_{ai} P_{ai} + f_o \Phi_a \sin\varphi_{aj} = w_b \cos\varphi_{ai} \end{aligned} \tag{3b}$$

where

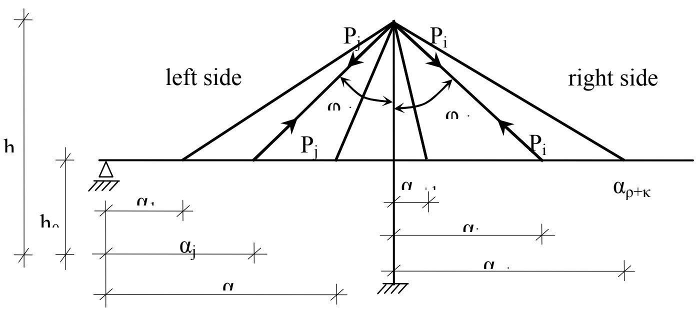

Fig. 3 Notations and symbols right and left of the pylon

$b_{aj} = \dfrac{s_j}{E_c A_j}$ *and* $b_{ai} = \dfrac{s_i}{E_c A_i}$

Multiplying the first of Eq. (3b) by $\sin\varphi_{aj}$, adding the ρ - equations, and then multiplying the second of Eq. (3b) by $\sin\varphi_{ai}$ and adding the κ-equations, we obtain the following relations

$$\begin{aligned} &\textit{left side} \quad \sum_{j=1}^{\rho} P_{aj}\sin\varphi_{aj} - f_o\Phi_a \sum_{j=1}^{\rho}\frac{\sin^2\varphi_{aj}}{b_{aj}} = \sum_{j=1}^{\rho}\frac{\sin 2\varphi_{aj}}{2b_{aj}} w_1 \\ &\textit{right side} \quad \sum_{i=1}^{\kappa} P_{ai}\sin\varphi_{ai} + f_o\Phi_a \sum_{i=1}^{\kappa}\frac{\sin^2\varphi_{ai}}{b_{ai}} = \sum_{i=1}^{\kappa}\frac{\sin 2\varphi_{ai}}{2b_{ai}} w_2 \end{aligned} \tag{3c}$$

Subtracting the above equations from each other, we finally obtain

$$\Phi_a = \frac{1}{1+f_o(A_{aj}+A_{ai})}\left\{\sum_{i=1}^{\kappa}\frac{\sin 2\varphi_{ai}}{2b_{ai}} w_2 - \sum_{j=1}^{\rho}\frac{\sin 2\varphi_{aj}}{2b_{aj}} w_1\right\} \tag{3d}$$

where

$A_{aj} = \sum_{j=1}^{\rho}\dfrac{\sin^2\varphi_{aj}}{b_j}$, $A_{ai} = \sum_{i=1}^{\kappa}\dfrac{\sin^2\varphi_{ai}}{b_i}$

From Eq. (3b) one can easily obtain the cables' stresses, which are

$$\begin{aligned} P_{aj} &= \frac{\cos\varphi_{aj}}{b_{aj}} w_1 + f_o\frac{\sin\varphi_{aj}}{b_{aj}}\Phi_a \\ P_{ai} &= \frac{\cos\varphi_{ai}}{b_{ai}} w_2 - f_o\frac{\sin\varphi_{ai}}{b_{ai}}\Phi_a \end{aligned} \tag{3e}$$

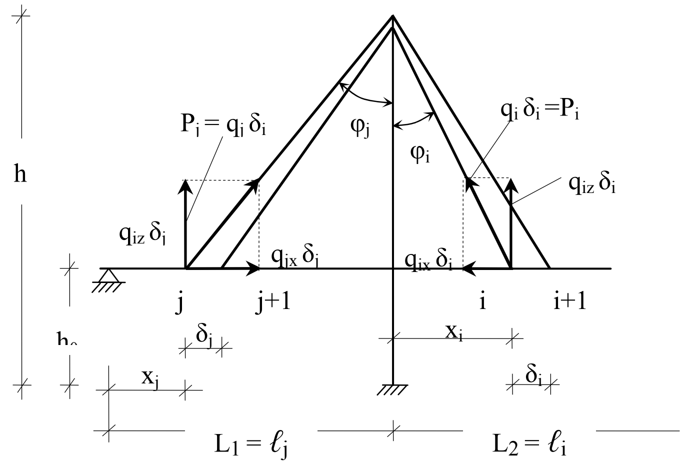

Fig. 3 Notations and symbols right and left of the pylon

2.2.2 Dense arrangement of cables

Let us consider next that the cables are in a dense arrangement and that the distances δ_j and δ_i between two neighboring cables satisfy the conditions

$$\delta_j << \alpha_\rho - \alpha_1 \quad and \quad \delta_i << \alpha_{\rho+\kappa} - \alpha_{\rho+1} \tag{4a}$$

In this case, we may consider an equivalent distributed load $q_z(x)$, applied from position α_1 to α_ρ and from $\alpha_{\rho+1}$ to $\alpha_{\rho+\kappa}$, which for instance at position "i" will be

$$q_i(x) = \frac{1}{\delta_i} \cdot P_i \tag{4b}$$

Following the notations of Fig. 4, we have

$$\begin{aligned} s_i &= \frac{h-h_o}{\cos\varphi_i} \quad , \sin\varphi_i = \frac{x_i}{\sqrt{(h-h_o)^2 + x_i^2}} \quad , \quad \cos\varphi_i = \frac{h-h_o}{\sqrt{(h-h_o)^2 + x_i^2}} \\ s_j &= \frac{h-h_o}{\cos\varphi_j} \quad , \sin\varphi_j = \frac{\ell_j - x_j}{\sqrt{(h-h_o)^2 + (\ell_j - x_j)^2}} \quad , \cos\varphi_j = \frac{h-h_o}{\sqrt{(h-h_o)^2 + (\ell_j - x_j)^2}} \end{aligned} \tag{4c}$$

and through a similar process like the one of §2.3.1, we get for pylon a

$$\begin{aligned} q_{aj}(x) &= \frac{\cos\varphi_{aj}}{b_{aj}} w_1 + f_o \frac{\sin\varphi_{aj}}{b_{aj}} \Phi_a \\ q_{ai}(x) &= \frac{\cos\varphi_{ai}}{b_{ai}} w_2 - f_o \frac{\sin\varphi_{ai}}{b_{ai}} \Phi_a \end{aligned} \tag{4d}$$

where

$$\Phi_a = \frac{1}{1+f_o(I_{aj}+I_{ai})}\left[\int_{\alpha(\rho+1)}^{\alpha(\rho+\kappa)} \frac{\sin 2\varphi_{ai}}{2b_{ai}} w_2 dx_2 - \int_{\alpha 1}^{\alpha\rho} \frac{\sin 2\varphi_{aj}}{2b_{aj}} w_1 dx_1\right]$$

$$I_{aj} = \int_{\alpha 1}^{\alpha\rho} \frac{\sin^2\varphi_{aj}}{b_{aj}} dx_1 \quad , \quad I_{ai} = \int_{\alpha(\rho+1)}^{\alpha(\rho+\kappa)} \frac{\sin^2\varphi_{ai}}{b_{ai}} dx_2$$

Similarly, for pylon b we get

$$q_{bj}(x) = \frac{\cos\varphi_{bj}}{b_{bj}} w_2 - f_o \frac{\sin\varphi_{bj}}{b_{bj}} \Phi_b$$
$$q_{bi}(x) = \frac{\cos\varphi_{bi}}{b_{bi}} w_2 + f_o \frac{\sin\varphi_{bi}}{b_{bi}} \Phi_b \quad (4e)$$

where

$$\Phi_b = \frac{1}{1+f_o(I_{bj}+I_{bi})}\left[\int_{b1}^{b\kappa} \frac{\sin 2\varphi_{bj}}{2b_{bj}} w_2 dx_2 - \int_{b(\kappa+1)}^{b(\kappa+\rho)} \frac{\sin 2\varphi_{bi}}{2b_{bi}} w_3 dx_3\right]$$

$$I_{bj} = \int_{b1}^{b\kappa} \frac{\sin^2\varphi_{bj}}{b_{bj}} dx_2 \quad , \quad I_{bi} = \int_{b(\kappa+1)}^{b(\kappa+\rho)} \frac{\sin^2\varphi_{bi}}{b_{bi}} dx_3$$

2.3 The static problem

The equilibrium equation of the deck of a cable-stayed bridge loaded symmetrically is the following

$$E_b I_b w_o''''(x) = p_{tot}(x) \quad (5a)$$

where
E_b is the modulus of elasticity of the bridge deck,
I_b is the moment of inertia of the cross-section of the bridge deck,
$w_o(x)$ is the total vertical displacement of the deck under the static loads g and p.

$$p_{tot} = g(x) + p(x) - q(x,w)/\cos\varphi \quad (5b)$$

In the last equation
$g(x)$ is the dead load of the bridge,
$p(x)$ is the live load
$q(x,w)$ are the forces due to the cables
Therefore Eq. (5a) becomes

$$E_b I_b w_o''''(x) = g(x) + p(x) - \frac{q(x,w)}{\cos\varphi} \quad (5c)$$

We are searching for a solution of the form

$$w_o(x) = \sum_{i=1}^{n} c_i Z_i(x) \tag{5d}$$

where c_i are unknown coefficients under determination and $Z_i(x)$ are arbitrarily chosen functions of x, which must satisfy the boundary conditions of the deck. In this case, the shape functions of the corresponding continuous beam were chosen (which has the same characteristics with the bridge deck but without cables).

Introducing the above into Eq. (5c), and taking into account Eqs. (4d) and (4e), we get

$$\begin{aligned} EI\sum_n c_n Z_n'''' &= g + p(x) - \frac{1}{b_{aj}}\sum_n c_n Z_{1n} + f_o \frac{\tan\varphi_{aj}}{b_{aj}}\Phi_a \quad for \quad 0 \le x_1 \le L_1 \\ &= g + p(x) - \frac{1}{b_{ai}}\sum_n c_n Z_{2n} - f_o \frac{\tan\varphi_{ai}}{b_{ai}}\Phi_a \quad for \quad 0 \le x_2 \le L_2/2 \\ &= g + p(x) - \frac{1}{b_{bj}}\sum_n c_n Z_{2n} - f_o \frac{\tan\varphi_{bj}}{b_{bj}}\Phi_b \quad for \quad L_2/2 \le x_2 \le L_2 \\ &= g + p(x) - \frac{1}{b_{bi}}\sum_n c_n Z_{3n} + f_o \frac{\tan\varphi_{bi}}{b_{bi}}\Phi_b \quad for \quad 0 \le x_3 \le L_3 \end{aligned} \tag{6a}$$

where Z_{1n}, Z_{2n}, Z_{3n} are the nth shape functions of the first, second and third span.

Multiplying the above equation by Z_ρ, integrating, and taking into account the orthogonality condition, we get

$$\begin{aligned} EIc_n\int_0^L Z_n'''' Z_\rho dx &= \int_0^L (g+p) Z_\rho dx \\ &- \int_0^{L_1} \frac{1}{b_{aj}}\sum_n c_n Z_{n1} Z_{\rho 1} dx_1 + f_o \int_0^{L_1} \frac{\tan\varphi_{aj}}{b_{aj}}\Phi_a Z_{\rho 1} dx_1 \\ &- \int_0^{L_2/2} \frac{1}{b_{ai}}\sum_n c_n Z_{n2} Z_{\rho 2} dx_2 - f_o \int_0^{L_2/2} \frac{\tan\varphi_{ai}}{b_{ai}}\Phi_a Z_{\rho 2} dx_2 \\ &- \int_{L_2/2}^{L_2} \frac{1}{b_{bj}}\sum_n c_n Z_{n2} Z_{\rho 2} dx_2 - f_o \int_{L_2/2}^{L_2} \frac{\tan\varphi_{bj}}{b_{bj}}\Phi_b Z_{\rho 2} dx_2 \\ &- \int_0^{L_3} \frac{1}{b_{bi}}\sum_n c_n Z_{n3} Z_{\rho 3} dx_3 + f_o \int_0^{L_3} \frac{\tan\varphi_{bi}}{b_{bi}}\Phi_b Z_{\rho 3} dx_3 \end{aligned} \tag{6b}$$

where

$$\Phi_a = \frac{1}{1+f_o(I_{aj}+I_{ai})}\left[\int_{a_{\rho+1}}^{a_{\rho+\kappa}} \frac{\sin 2\varphi_{ai}}{2b_{ai}} \sum_n c_n Z_{n2} dx_2 - \int_{a_1}^{a_\rho} \frac{\sin 2\varphi_{aj}}{2b_{aj}} \sum_n c_n Z_{n1} dx_1 \right]$$

$$\Phi_b = \frac{1}{1+f_o(I_{bj}+I_{bi})}\left[\int_{b_1}^{b_\rho} \frac{\sin 2\varphi_{bj}}{2b_{bj}} \sum_n c_n Z_{n2} dx_2 - \int_{b_{\kappa+1}}^{b_{\rho+\kappa}} \frac{\sin 2\varphi_{bi}}{2b_{bi}} \sum_n c_n Z_{n3} dx_3 \right]$$

and n = 1,2,3,… , n

Applying the first of Eqs. (6.b) for n = 1, 2, …, n we get a linear homogeneous system, from which the solution for the unknown $c_1, c_2, \ldots, c_n$ are obtained.

2.4 The dynamic characteristics of the bridge

The equation of motion of a free vibrating bridge is

$$EJ_y w''''(x,t) + c\,\dot{w}(x,t) + m\,\ddot{w}(x,t) = -q_s \tag{7a}$$

We are searching for a solution in the form of separate variables such as

$$w(x,t) = Z(x) \cdot T(t) \tag{7b}$$

and through the well-known process, we get the following equations

$$\begin{aligned} & Z'''' + \frac{1}{EJ_y}\left[q_c(x) + q_s(x)\right] - \lambda^4 Z = 0 \\ & \ddot{T} + \frac{c}{m}\dot{T} + \omega^2 T = 0 \end{aligned} \tag{7c}$$

where

$$\lambda^4 = \frac{m\,\omega^2}{EJ_y}$$

In order for us to apply Galerkin's procedure, we set

$$Z(x) = c_1\Psi_1(x) + c_2\Psi_2(x) + \cdots\cdots + c_n\Psi_n(x) \tag{7d}$$

where c_i are unknown coefficients, under determination, and $\Psi_i(x)$ are functions of x arbitrarily chosen, that satisfy the boundary conditions, of the static system of bridge-deck. As such, the functions we choose for the shape functions of the corresponding static system of beam-deck (a continuous beam or a set of three single-span beams) have the same characteristics with the bridge-deck without cables.

Introducing Eq. (7d) into (7a), multiplying the outcome successively by $\Psi_1, \Psi_2, \ldots, \Psi_n$, and integrating the results from 0 to L, we obtain the following homogeneous linear system of n-equations, with unknowns $c_1, c_2, \ldots, c_n$

$$c_1(A_{i1} - \lambda^4 B_{i1}) + c_2(A_{i2} - \lambda^4 B_{i2}) + \cdots\cdots + c_n(A_{in} - \lambda^4 B_{in}) = 0, \quad with \quad i = 1,2,\ldots,n \tag{7e}$$

where

$$\left. A_{ij} = \int_0^L \left[\Psi_j'''' + \frac{1}{E\,J_y} q_s(\Psi_j) \right] \Psi_i dx \quad , \quad B_{ij} = \int_0^L \Psi_i \Psi_j dx \quad \right\} \tag{7f}$$

In order for the above system to have non-trivial solutions, the determinant of its coefficients must be zero, i.e.

$$\left. \left| \Gamma_{ij} \right| = 0 \quad with \quad i, j = 1, 2, \cdots, n \quad and \quad \Gamma_{ij} = A_{ij} - \lambda^4 B_{ij} \quad \right\} \tag{7g}$$

From Eq. (7g), we determine the values of λ and from Eq. (7c) the spectrum of the flexural eigenfrequencies ω_i. From the first (n-1) equations of the system (7e), we can find

$$\frac{c_j}{c_1} = \frac{\begin{vmatrix} \Gamma_{12} \cdots\cdots\cdots \Gamma_{1(j-1)} \; \Gamma_{11} \; \Gamma_{1(j+1)} \cdots\cdots\cdots \Gamma_{1n} \\ \Gamma_{22} \cdots\cdots\cdots \Gamma_{2(j-1)} \; \Gamma_{21} \; \Gamma_{2(j+1)} \cdots\cdots\cdots \Gamma_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \Gamma_{(n-1)2} \cdots \Gamma_{(n-1)(j-1)} \; \Gamma_{(n-1)1} \; \Gamma_{(n-1)(j+1)} \cdots \Gamma_{(n-1)n} \end{vmatrix}}{\left| \Gamma_{ij} \right|} \tag{7h}$$

$$with \quad i = 1, 2, \cdots, (n-1) \qquad j = 1, 2, \cdots, n$$

$$and\ therefore: \quad Z_n(x) = c_1 \sum_{j=2}^{n} (\Psi_1 + \frac{c_j}{c_1} \cdot \Psi_j)$$

where $X_n(x)$ are the shape functions of the bridge with combined cable system.

2.5 Failure of cables

The following analysis concerns a cable-stayed bridge with a single line of cables anchored at the center of the deck's cross-section. The problem of the failure of cables can be studied as follows.

Let us consider the bridge of Fig. 5(a), which is at rest under the loads g (dead load) and p (live load). Thus, one can determine the deformations of the deck $w_o(x)$, applying § 2.3.

Suddenly, at time $t = 0$ the hatched cables of Fig. 5(b) failed. The static system of the bridge changes to another that is like the initial one but without the failed cables. One can determine the dynamic characteristics of this new static system (i.e., eigenfrequencies and shape functions) by applying § 2.4.

The equation of motion of the bridge after the failure of "s" cables is

$$E\,I_y w'''' + c\dot{w} + m\ddot{w} = g + p(x) \cdot P(t) - q_s(x, w) \tag{8a}$$

We are searching for a solution under the form

$$w(x,t) = \sum_n Z_n(x) T_n(t) \tag{8b}$$

where $T_n(t)$ are unknown time functions (under determination) and $Z_n(x)$ are arbitrarily chosen functions of x, which must satisfy the boundary conditions of the deck. In the present case, the

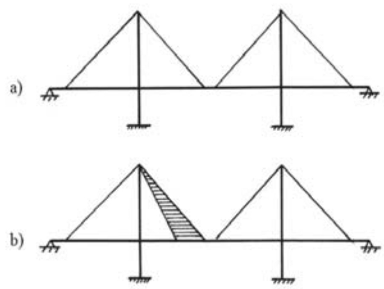

Fig. 5 Initial and damaged bridges

shape functions of the damaged cable-stayed bridge are chosen as those determined by applying § 2.4.

Introducing Eqs. (8b) into (8a), we get

$$E\,I_y \sum_n Z_n'''' T_n + c\sum_n Z_n \dot{T}_n + m\sum_n Z_n \ddot{T}_n + q_s(x, \sum_n Z_n T_n) = g + p(x)\cdot P(t) \quad (8c)$$

Remembering that Z_n satisfies the equation of the free motion of the wounded bridge

$$E\,I_y \sum_n Z_n'''' T_n - m\sum_n \omega_n^2 Z_n T_n + q_s(x, \sum_n Z_n T_n) = 0 \quad (8d)$$

the above (8c) becomes

$$m\sum_n Z_n \ddot{T}_n + c\sum_n Z_n \dot{T}_n + m\sum_n \omega_n^2 Z_n T_n = g + p(x)\cdot P(t) \quad (9a)$$

Multiplying the above by Z_k, integrating from 0 to L, and remembering the orthogonality condition, we finally obtain

$$\ddot{T}_k + \frac{c}{m}\dot{T}_k + \omega_k^2 T_k = \frac{1}{m\int_0^L Z_k^2 dx} G(t) \quad (9b)$$

where

$$G(t) = \left(\int_0^L g\, Z_k dx + P(t)\int_0^L p(x) Z_k dx \right)$$

The solution of the above is given by the Duhamel integral

$$T_k(t) = \frac{1}{m\overline{\omega}_k \int_0^L Z_k^2 dx} \int_0^t e^{-\beta(t-\tau)} G(\tau)\cdot \sin[\overline{\omega}_k (t-\tau)]\, d\tau \quad (9c)$$

with

$$\beta = \frac{c}{2m}, \quad \overline{\omega}_k = \sqrt{\omega_k^2 - \beta^2}$$

Therefore the general solution of Eq. (7b) is given by

$$w(x,t) = \sum_n Z_n \left\{ e^{-\beta t} (An \sin \overline{\omega}_n t + B_n \cos \overline{\omega}_n t) + T_n(t) \right\} \tag{9d}$$

The constants A_n and B_n are determined from the time conditions

$w(x,t_o) = w_o(x)$ *and* $\dot{w}(x,t_o) = 0$

as follows

$$A_n = \frac{\beta \cdot B_n - \dot{T}_n(0)}{\overline{\omega}_n} \quad , \qquad B_n = \frac{\int_0^L w_o(x) Z_n dx}{e^{-\beta t_o} \int_0^L Z_n^2 dx} \tag{9e}$$

3. Numerical results and discussion

In order to study the influence of the sudden failure of cables on the bridge's behavior and static adequacy, we consider a bridge with the following data: L_1=150 m, L_2=350 m, L_3=150m, g = 7000dN/m, (m=700 gr/m), I_b = 0.50 m^4, I_P=1000 I_b, α_1 = 20 m, α_ρ =130 m, $\alpha_{\rho+1}$ = 30 m, $\alpha_{\rho+\kappa}$=170 m, β_1 = 180 m, β_κ = 320 m, $\beta_{\kappa+1}$ = 20 m, $\beta_{\kappa+\rho}$ = 130 m, and live load p = 7000dN/m. The distance between two neighboring cables is 5 m.

Five cases are studied: the non-damaged bridge, a bridge under the sudden failure of 15 cables, a bridge under the sudden failure of 10 cables, a bridge under the sudden failure of 5 cables, a bridge under the sudden failure of 1 cable.

3.1 The non-damaged bridge

Considering the complete bridge (without failed cables) and applying the formulae of §2.3, we get the plots of Fig. 6, where the deformations of the bridge are shown, and also the plots of Fig. 7, where the cables' stresses are shown. The dimensions in these as well as the following figures given are in meters and kN.

3.2 The damaged bridge

Applying the formulae of §2.5, we get the following plots concerning the three first time functions T_{g1}, T_{g2}, T_{g3} and T_{gp1}, T_{gp2}, T_{gp3} for action of the dead loads and for simultaneously action of dead and live loads, respectively.

One can easily see, that after 10 seconds the bridge becomes at rest, while the maximum excitation happens at t = 0.9 sec after the sudden failure of cables.

3.2.1 Failure of 15 cables

For the case of a sudden failure of 15 cables, we get the following plots of Figs. 9 to 12, at time 0.9 sec and 20 sec. We see that both deformations and cables' stresses at both instants are unacceptable.

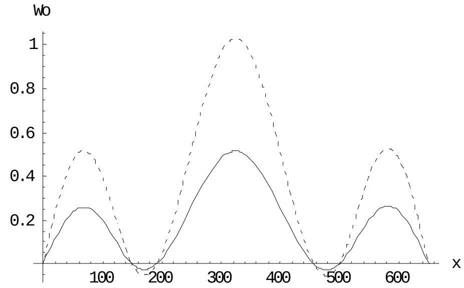

Fig. 6 ___ dead load, _ _ _ dead and live loads

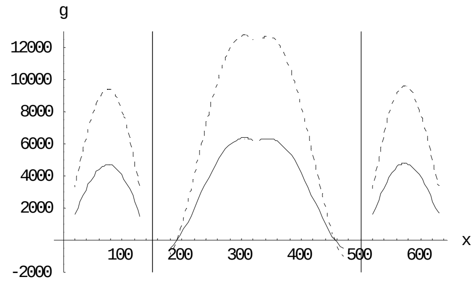

Fig. 7 ____ dead load _ _ _ dead and live loads

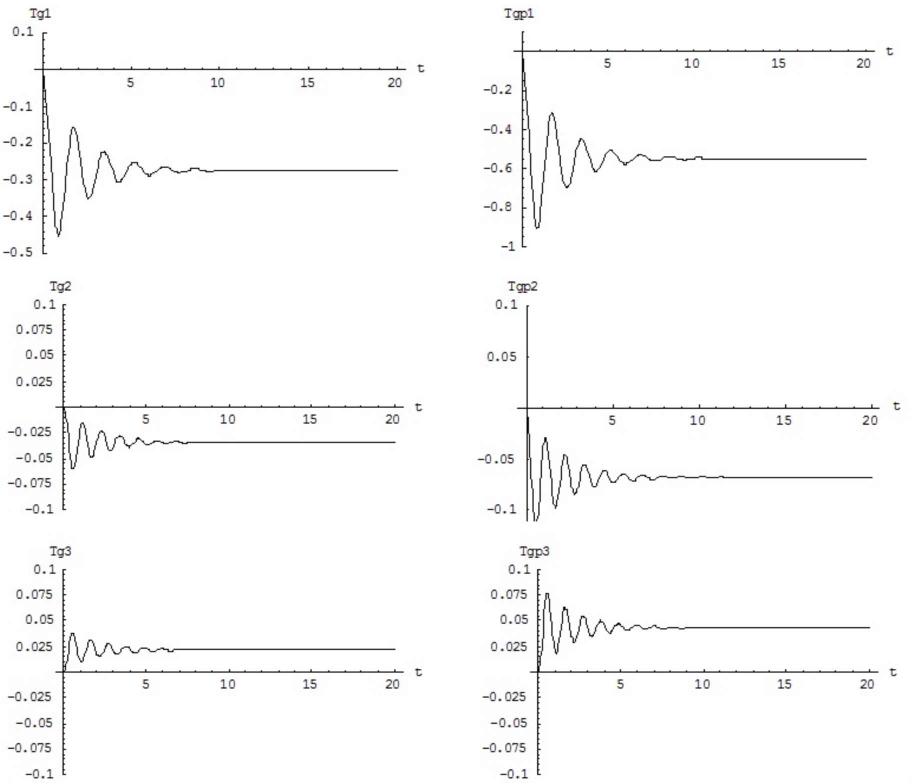

Fig. 8 The time functions T_i (i = 1, 2, 3), for dead and dead+live loads

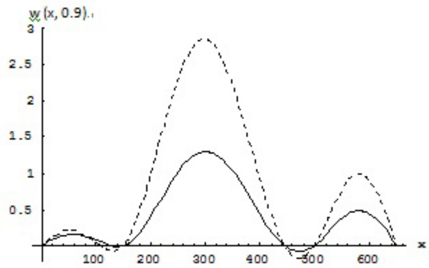

Fig. 9 Deck’s deformations at t = 0.9____ dead load _ _ _ dead and live loads

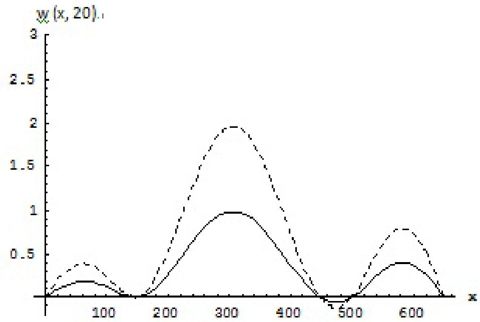

Fig. 10 Deck’s deformations at t = 20____ dead load _ _ _ dead and live loads

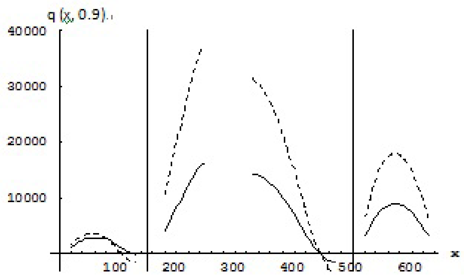

Fig. 11 Stresses of cables at t = 0.9____ dead load _ _ _ dead and live loads

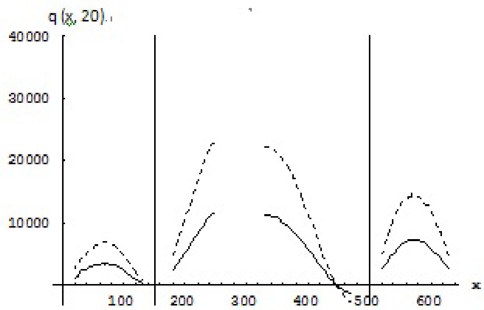

Fig. 12 Stresses of cables at t = 20____ dead load _ _ _ dead and live loads

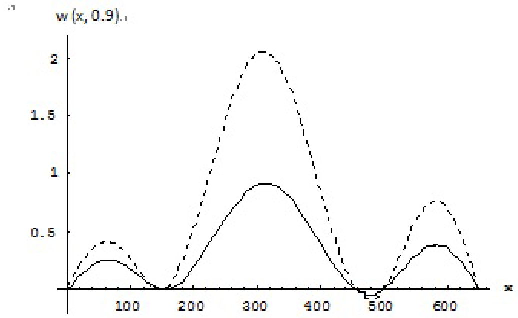

Fig. 13 Deck's deformations at *t* = 0.9____ dead load _ _ _ dead and live loads

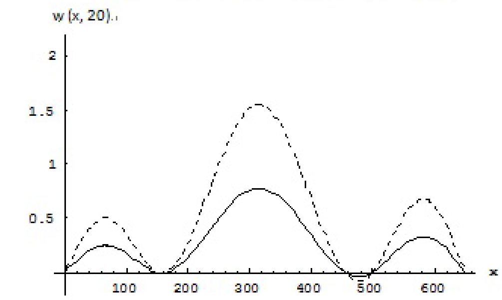

Fig. 14 Deck's deformations at *t* = 20____ dead load _ _ _ dead and live loads

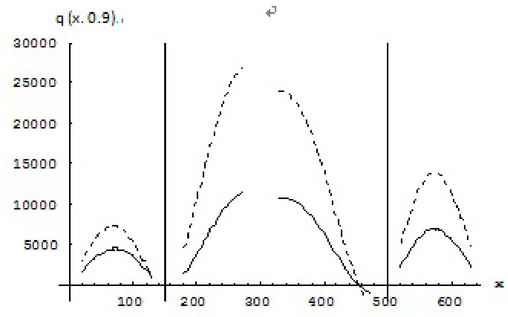

Fig. 15 Stresses of cables at *t* = 0.9____ dead load_ _ _ dead and live loads

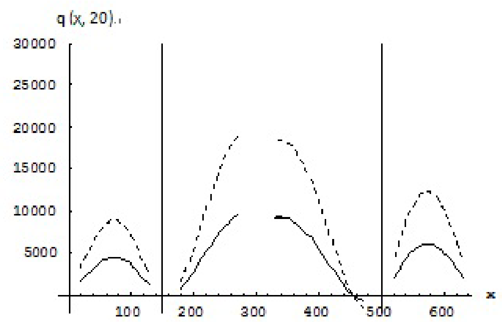

Fig. 16 Stresses of cables at *t* = 20____ dead load _ _ _ dead and live loads

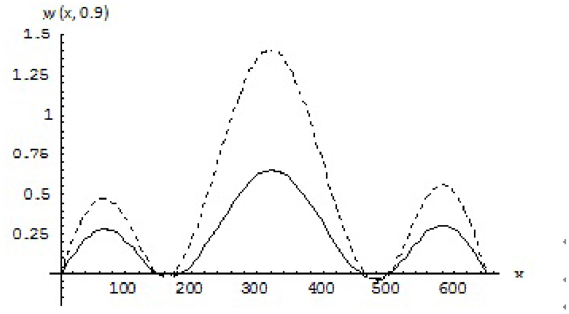

Fig. 17 Deck's deformations at t = 0.9____ dead load_ _ _ dead and live loads

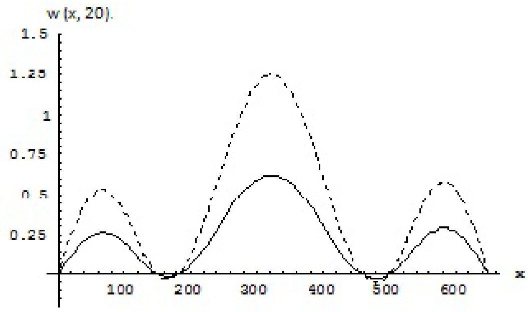

Fig. 18 Deck's deformations at t = 20____ dead load _ _ _ dead and live loads

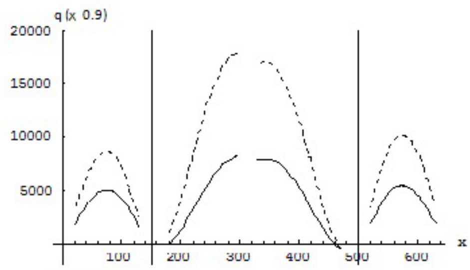

Fig. 19 Stresses of cables at t = 0.9____ dead load _ _ _ dead and live loads

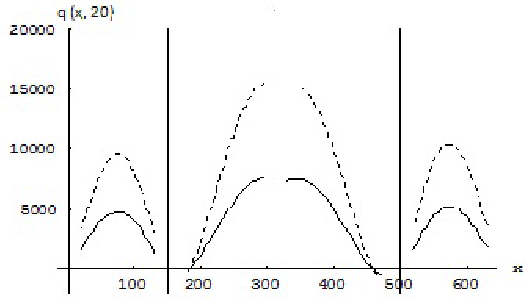

Fig. 20 Stresses of cables at t = 20____ dead load _ _ _ dead and live loads

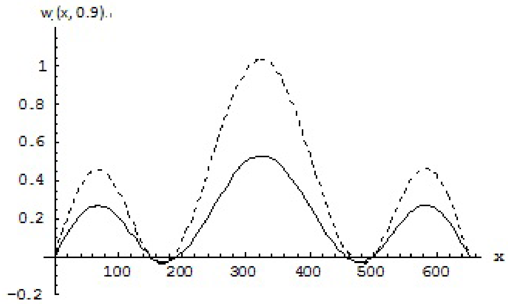

Fig. 21 Deck's deformations at t = 0.9____ dead load _ _ _ dead and live loads

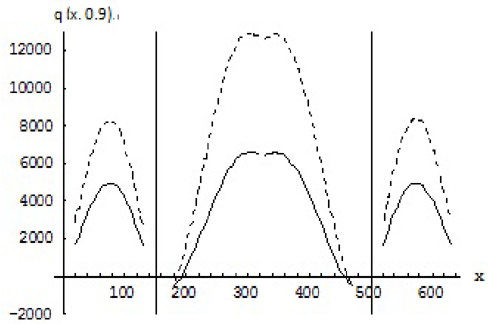

Fig. 22 Deck's deformations at t = 20____ dead load _ _ _ dead and live loads

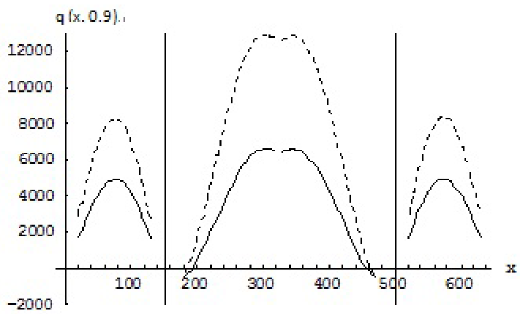

Fig. 23 Stresses of cables at t = 0.9____ dead load _ _ _ dead and live loads

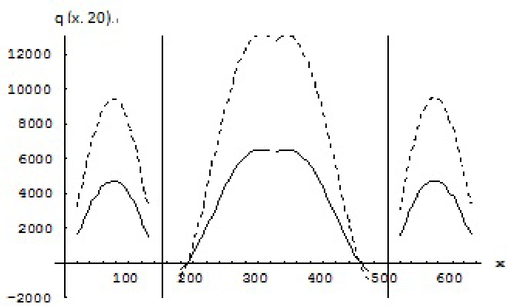

Fig. 24 Stresses of cables at t = 20____ dead load _ _ _ dead and live loads

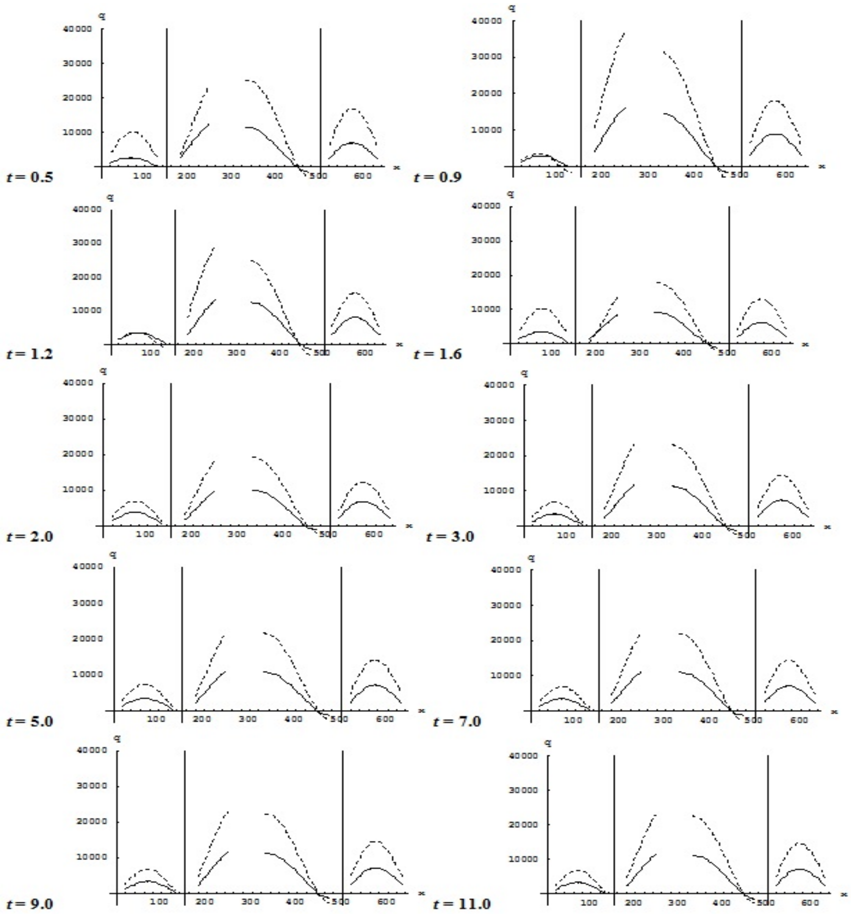

Fig. 25 Cable stresses developed due to failure of one cable

3.2.2 Failure of 10 cables

For the case of a sudden failure of 10 cables, we get the following plots of Figs. 13 to 16, at time 0.9 sec and 20 sec. We see that both deformations and cables' stresses are unacceptable (except the deck's deformations of Fig. 14 at t = 20 sec, that however are very great).

3.2.3 Failure of 5 cables

For the case of a sudden failure of 5 cables, we get the following plots of Figs. 17 to 20, at time 0.9 sec and 20 sec. We see that at t = 0.9 sec we have very large deformations and unacceptable cables' stresses, while when the bridge becomes at rest both deformations and cables' stresses are acceptable.

3.2.4 Failure of 1 cable

For the case of a sudden failure of 1 cable, we get the following plots of Figs. 21 to 24, at time 0.9 sec and 20 sec. We see that both deformations and cables' stresses are acceptable although the cables' stresses at t = 0.9 sec are greater than the designed ones.

3.2.5 A more detailed observation of the cables' stresses

Let us see now the cables' stresses at different instants. Studying the behavior of the bridge in connection with time, we get the following Fig. 25, showing the cables' stresses time history.

We see the passage of the stresses of some cables from their maximum (at t = 0.9 sec, immediately after fracture) to negative stresses (i.e., unstressed cables).

4. Conclusions

A simple approach for studying the response of cable-stayed bridges due to sudden failure of one or more cables is presented. The numerical results have been obtained by closed-form analytical solutions. On the basis of the representative cable-stayed bridge models analyzed herein, the following conclusions can be drawn:

- One must distinguish between the case of a sudden failure of one or more cables caused by an accident or an unexpected incident, and the case of a scheduled (or planned) replacement of some cables. In the first case, even if only one cable has failed, the developed stresses are significant, especially for a traffic-loaded bridge. In the second case where the bridge is at rest, one can replace a big number of cables (preferably unloaded bridge).
- The sudden failure of cables induces unexpected, violent and unforeseen oscillations of high amplitude to the bridge. Therefore, it is obvious that SETRA and PTI codes cannot confront or predict such an incident by simply recommending a maximum dynamic amplification factor equal to 2.
- The consequences of a violent cable failure are so intense, that unexpected phenomena appear as for example instantaneously relaxed (unstressed) cables.
- The deformations of the deck can be amplified from 1.05 to 2.5 times relative to the ones of the same bridge at rest, while the cable stresses can be higher from 1.1 to 3.3 times, respectively.
- The proposed dynamic amplification factor by SETRA and PTI recommendations (with maximum value equal to 2.0) is sufficient for a sudden failure of up to 5 cables. Given thought that such a failure of more than one or two cables is due to an accident or explosion and that additional distress is produced by the accident or explosion itself, it is obvious that the above factor must take significantly higher values than 2.0.

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