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Mesoscopic analysis of reinforced concrete beams

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Abstract. Reinforced concrete can be considered as a heterogeneous material consisting of coarse aggregate, mortar mix and reinforcing bars. This paper presents a two-dimensional mesoscopic analysis of reinforced concrete beams using a simple two-phase mesoscopic model for concrete. The two phases of concrete, coarse aggregate and mortar mix are bonded together with reinforcement bars so that inter force transfer will occur through the material surfaces. Monte Carlo's method is used to generate the random aggregate structure using the constitutive model at mesoscale. The generated models have meshed such that there is no material discontinuity within the elements. The proposed model simulates the load-deflection behavior, crack pattern and ultimate load of reinforced concrete beams reasonably well.

Keywords: mesoscopic analysis; reinforced concrete beam; deep beam; two-phase

1. Introduction

Concrete is the most widely used material for construction. It is a composite material with complex behavior and a highly heterogeneous microstructure. For obtaining a better understanding of the macroscopic constitutive behavior of concrete, the effects of microstructure and properties of the individual components of concrete on the macroscopic material behavior have to be taken into account. Mesoscopic models have proven to be the most practical and useful approach to study the nonlinear behavior of concrete composition on the macroscopic properties (Nagarajan *et al.* 2010). A numerical approach to investigate the property of concrete at the mesoscopic level is given by Roelfstra (1989). A method of random computer generation of the particle system meeting the prescribed particle size distribution was developed by Bazant *et al.* (1990), using the assumption that the particles are elastic in nature and have only axial interactions, as in a truss. A lattice model is presented by Schlangen and Mier (1992) for the simulation of typical failure mechanism and crack face bridging in concrete. The influence of lattice element type and lattice orientation on the fracture pattern (Schlangen and Garboczi 1997) was investigated by simulating a shear loading experiment on a concrete plate.

A random aggregate generation procedure based on Monte Carlo's simulation principle was

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developed by Wang *et al.* (1999). A method of mesh generation is also developed for studying the nonlinear behavior of concrete. For implementing heterogeneity in the model, a method has been developed using digital images of the real microstructure of the material by Asai *et al.* (2003). A computational study on size effect in tensile fracture of concrete and simulation of fracture process of plain concrete using two-dimensional lattice model is presented by Prasad and Sagar (2006). A single edge notched concrete plate subjected to uniaxial tension is adopted and triangular lattice network is used to simulate the heterogeneous structure of plain concrete. A random structure of aggregates at the mesoscopic level resembling the real concrete proposed by Wriggers and Moftah (2006) was meshed using aligned approach so that the finite element boundaries are coincident with the material interfaces and no material discontinuity within the elements. A numerical method from the mesoscopic point of view was proposed by Ying *et al.* (2007) to describe the fracture process of concrete. The fracture in concrete was developed using 2D lattice modeling (Kozicki and Tejchman, 2007, Sagar *et al.* 2010). The mesoscopic analysis of reinforced concrete beams using a modified micro-truss model (Nagarajan *et al.* 2010) was developed based on the framework method. Kwan *et al*. (2017), proposes a method based on both tensile strength and fracture toughness. Kim and Al-Rub (2011) have the influence of the aggregate shapes on material behavior. Size effect in concrete under tension using Monte Carlo simulations of mesoscale finite element models containing random inclusions were studied by Wang *et al.* (2015). Borges and Pituba (2017) presented a computational homogenization model for concrete using the concept of the representative volume element. In all these studies the concrete is considered as a three-phase composite material consisting of mortar matrix, aggregates, and an interfacial transition zone.

In the modified micro-truss model (Nagarajan *et al*. 2010), the random aggregate structure is developed over the truss model and the constitutive model is analyzed at the meso level. The aggregate was randomly distributed. An alternate approach for studying the mesoscopic model, (Sreedevi 2010) without introducing lattice bar elements, whereas the continuity between the different phases was done by overlapping the aggregate phases into the base material, was developed for studying the nonlinear properties of concrete. Rodrigues *et al.* (2016) proposed a two-dimensional mesoscale model in which the concrete is modeled as a heterogeneous three-phase material. Interface solid finite elements with a high aspect ratio are used to represent the interfacial transition zone. Considering the effects of mesostructure constituents, the proposed two-dimensional model shows the same characteristics as that of real three dimensional concrete. In the present study, an attempt has been made to predict the behavior of reinforced concrete beams using a simple two-phase two-dimensional model (coarse aggregate and mortar) with reinforcement bar for mesoscopic analysis. The method is validated by comparing the results for simply supported shallow beams (under-reinforced and over-reinforced) and a deep beam.

2. Generation of concrete mesostructure model

Concrete is considered as a homogeneous material at a macroscopic scale. At a mesoscopic scale, it is a heterogeneous material consisting of coarse aggregate, mortar mix, and interfacial zones. The shape, size, and distribution of coarse aggregates closely resembling real concrete are randomly generated at the mesoscopic level. The structure to be generated consists of randomly distributed coarse aggregate particles and mortar mix filling the space between the particles (Fig. 1). The generation of the random geometrical configuration of the coarse aggregate must satisfy the basic statistical character of the real concrete. In order to produce the geometrical configuration

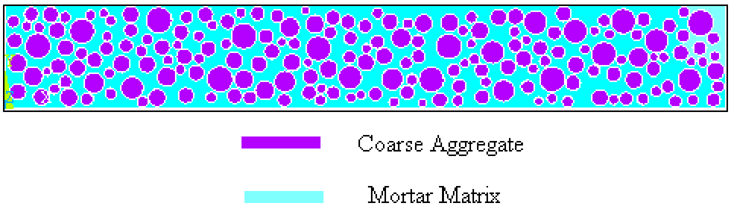


Fig. 1 2D Concrete mesostructure

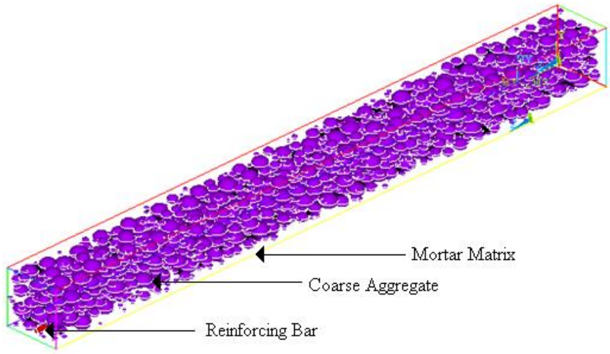


Fig. 2 Concrete mesostructure with reinforcing bar

to meet these requirements, the random sampling principle of Monte Carlo's simulation method is used. It is a method for iteratively evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than just a couple of uncertain parameters. This random principle is applied by taking samples of aggregate particles from a source whose size distribution follows a given grading curve and placing the aggregate particles one by one into the concrete in such a way that there is no overlapping with the particles already placed.

An algorithm for generating a realistic aggregate structure by Wriggers and Moftah (2006) is used for the generation realistic aggregate structure. This is a take and place method of generation. The take process and place process are to be performed concurrently such that a particle generated by the take process is immediately placed into the concrete. These are conducted in a sequence starting with the largest size particles, proceeding until the last particle of the size range has been placed, and then repeating for successively smaller size particles because it is generally easier to pack the particles into the concrete in this way. Hence, it is first necessary to obtain the aggregate grading curve. Using this whenever the placing process of a particle reveals an overlapping, the particle being placed is translated to the least distance allowed between the particles in order to get away from the overlapping particle until all the placing requirements are completely satisfied.

The shape of the particles depends on the aggregate type such as gravel, crushed stone, etc. The rebar elements were placed in the concrete model (Fig. 2) and full compatibility is assumed between the interface of reinforcement and concrete.

2.1 Aggregate size distribution

Aggregate size distribution plays an essential role in the concrete mix design. The size distribution of aggregate particles can be described by means of grading curves obtained from sieve analysis. The particle size distribution is determined by grading and it is expressed by cumulative percentage passing through a series of standard sieve sizes. In two dimensional mesoscopic studies, the border effect is neglected and the area ratio of aggregate is taken equal to the volumetric ratio (Wang *et al.* 1999, 2015) and is given by,

$$A_r = \frac{c}{a+b+c} \tag{1}$$

where, A_r is the area fraction of aggregate, c is the ratio of coarse aggregate, a is the ratio of cement content and, b is the ratio of fine aggregate. If the size distribution of the aggregate particles is given by the particle size distribution curves, then the amount of aggregate within the grading segment$[d_s, d_{s+1}]$, can be calculated as

$$A_p[d_s, d_{s+1}] = \frac{P(d_s)-P(d_{s+1})}{P(d_{max})-P(d_{min})} \times A_r \times A_{con} \tag{2}$$

where, $A_r\ [d_s, d_{s+1}]$ is the area of aggregate within the grading segment, A_{con} is the area of concrete, d_s, d_{s+1}, are the aperture size of the sieve, $d_{\min}$ and $d_{\max}$ are the minimum and maximum sizes of coarse aggregate particles, respectively and P(d) is the cumulative percentage passing a sieve with aperture diameter d.

The random aggregate structure generation consists of taking process and placing process. The take and place process can be initiated if the grading of aggregates in a concrete mix is known. In this process, each aggregate particle is generated according to the given grading curve and is immediately placed into the concrete domain.

Three conditions are considered to place the aggregate at a free position within the concrete domain. The first condition is that the whole particle must be completed within the boundary of the concrete area. The second condition is that there should not be any overlapping with the previously placed aggregate particles. The third condition is that each particle generated should incorporate a minimum coating space of mortar film all-around. This ensures a minimum distance between the edge of a particle and the boundary of the concrete specimen and also a minimum gap width between the adjacent particles.

The random models were developed for a 2D random aggregate structure with the mix ratio 1:2:3(cement: fine aggregate: coarse aggregate), using the above procedure. The input information required is the size of the concrete domain, mix ratio, and the aggregate size distribution. In this study, the shape of the aggregates particles is assumed as circular. Here particle size distribution curve is used for the aggregate size distribution.

2.2 Steps in mesoscopic analysis

The analysis was done using finite element analysis software ANSYS. Programs were written for generating the random aggregate structure. For introducing the heterogeneity in the concrete structure, elastic properties were assigned for both the phases. Coarse aggregate and mortar matrix are modeled as linearly elastic or ideally brittle materials. The Young's modulus (E) of these phases recommended by Prasad and Sagar (2006) and Nagarajan *et al.* (2010) were used in this study (Table 1). Proper bonding between the constituents is made using the "glue" option given in ANSYS.

The basic steps in the mesoscopic numerical analysis of reinforced concrete beams are as follows:

Table 1 Elastic property of the phases

Phase	E (N/mm^2)	σ_t(N/mm^2)	μ
Coarse aggregate	70000	10	0.20
Mortar	25000	5	0.22

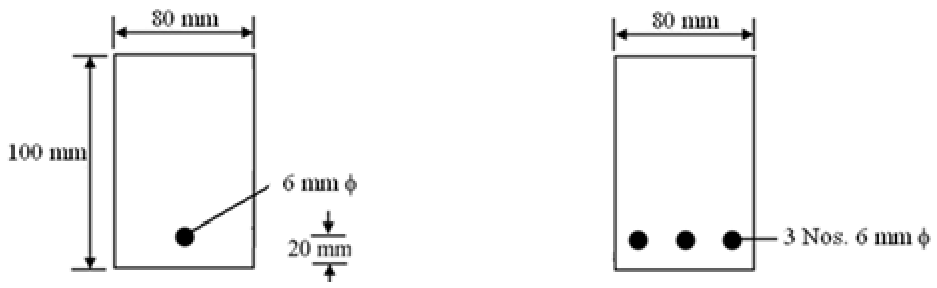
1. The random aggregate model for the structure is developed using the percentage of Coarse aggregates (CA) used.
2. A rectangular domain is defined to represent the area of the beam. The geometric properties for the CA, Mortar mix & Reinforcing bar are assigned within the boundary of the concrete domain and proper bonding between the constituents are made.
3. The degrees of constraints are properly assigned and the area in the rectangular domain meshes such that the finite element boundaries coincide with materials interfaces
4. The maximum load is estimated and is applied as a concentrated load at the mid-span of the beam in a series of load steps.
5. Initially, the load corresponding to the first load step is applied.
6. The structure is analyzed for the applied load and key parameters such as the load value, displacements and axial stress in all the elements are noted.
7. At the completion of each incremental solution corresponding to each load step, the model is to be adjusted to reflect nonlinear changes before proceeding to the next load step.
8. The highly stressed elements are selected in each case and its stiffness is deactivated (if it exceeds the tensile strength,σ_t), for obtaining the crack pattern after each load step.
9. The load corresponding to the next load step is applied and the steps from 6 to 8 are repeated until the analysis shows a divergence in the computed results. The load corresponding to the last converged load step is taken as the ultimate load.
10. By further refining the load steps, much finer results can be obtained, as only the highly stressed elements are deactivated in each load step to obtain the crack pattern.

3. Validation of the modified model

To validate the proposed simplified two-dimensional mesoscopic model, the results obtained from the mesoscopic analysis are compared with the experimental results. The experimental study involved casting and testing of reinforced concrete beams of the identical cross-section. FEM models are made for the beams and mesoscopic analysis is conducted to validate the proposed method.

3.1 Experimental study

To validate the simplified mesoscopic models, experimental studies are conducted using physical specimens. The experimental study involved casting and testing of reinforced concrete beams of identical cross-section. The beams were cast using the mix proportion of 1:2:3 with water cement ratio of 0.6. The under-reinforced and over-reinforced beams were of 900 mm long, 100 mm deep and 80 mm wide. The deep beam was of 900 mm long, 300 mm deep and 150 mm wide. The beam was singly reinforced with 5 numbers of 6 mm diameter bars at the tension zone with a clear cover of 20 mm. The beam was cast using the mix proportion of 1:1.63:3.06 with a



Under reinforced beam Over reinforced beam

Fig. 3 Reinforcement details for the beams

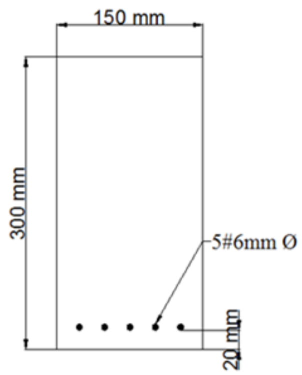


Fig. 4 Reinforcement details for the beams

water-cement ratio of 0.5. The beams were simply supported with an effective span of 700 mm and loaded at the mid-span for three-point bending. The percentage distribution of coarse aggregates used for casting beams is determined using sieve analysis. The yield strength and elastic modulus of steel used were 250 N/mm^2, and 2 x 10^5 N/mm^2 respectively. The reinforcement details of the shallow beams are shown in Fig. 3 and deep beam in Fig. 4. The specimens are tested for the 28^{th} day compressive strength. The load and displacement data were recorded simultaneously before reaching the ultimate load. The results are shown in Table 3. The load-deformation and crack pattern of beams II, III, VI and deep beam are shown in Figs. 5(a) and 5(b), 6(a) and 6(b), 7(a) and 7(b) and 8(a) and 8(b) respectively. The cracks in shallow beams are flexural cracks. In the deep beam initially, a flexural crack is observed at the center and on further loading, failure is due to the cracks connecting the point of application of load and supports (see Fig. 8(b)).

3.2 Mesoscopic model for reinforced concrete beam

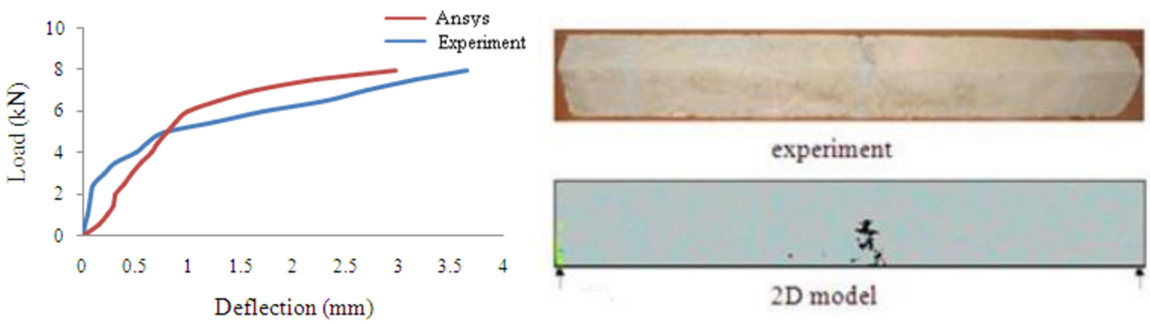
The two-dimensional mesoscopic modeling and analysis were done using finite element analysis software ANSYS 10.0. The types of finite elements used to model different components of the beam are summarized in Table 2. The Young's modulus (*E*) and Poisson's ratio given in Table 1 is used in this study. All six samples of beams used for experimental studies are modeled and mesoscopic analyses were conducted using the procedure explained above.

Table 2 ANSYS Finite elements used for the 2D model

Components	ANSYS Element Type
Mortar mix	PLANE 82
Coarse aggregate	PLANE 82
Reinforcing bar	LINK 1

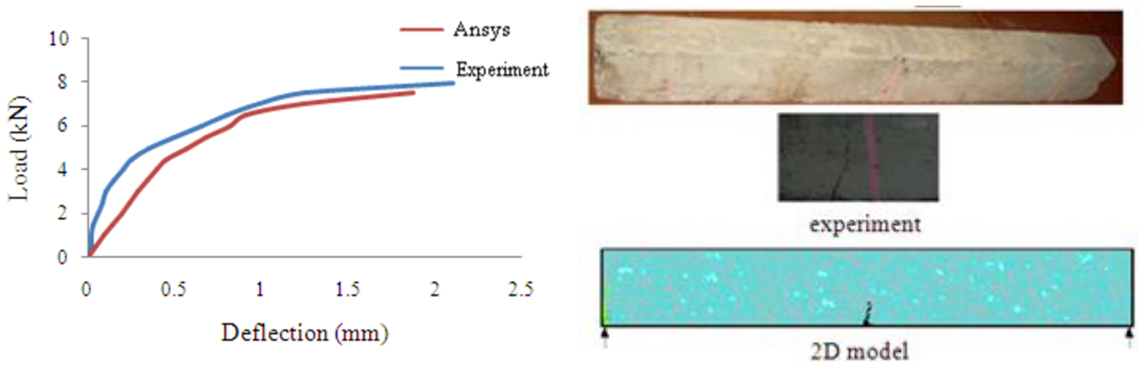
Table 3 Ultimate load of the beams

Type of beam	Sample No	Experiment (kN)	Mesoscopic Analysis, (kN)	Error (%)
	I	6.5	7.2	10.8
Under Reinforced	II	7.0	7.8	11.4
	III	7.8	8.6	10.3
	IV	13.3	12.0	-9.8
Over Reinforced	V	10.9	11.2	2.8
	VI	11.0	10.2	-8.0
Deep Beam	I	133.0	126.7	4.7



(a) (b)

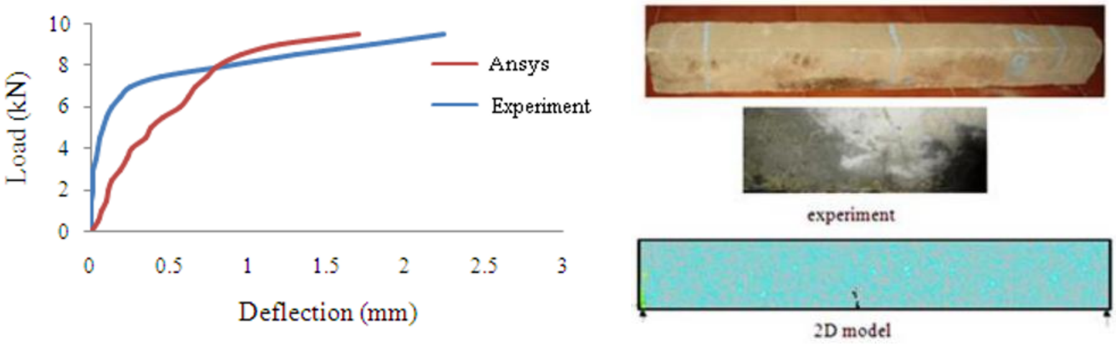
Fig. 5 (a) Load-deflection plot at mid-span and (b) crack Pattern of the beam-II



(a) (b)

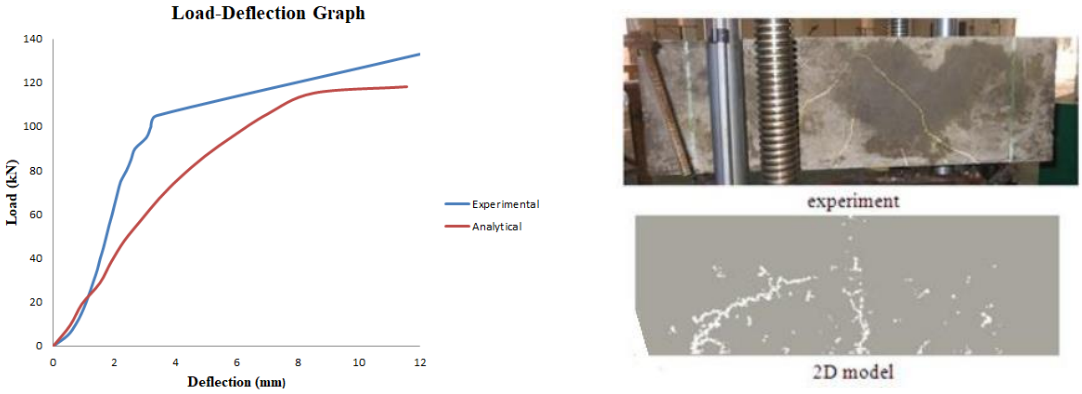
Fig. 6 (a) Load-deflection plot at mid-span and (b) crack Pattern of the beam-III

For validating of the mesoscopic model, the results obtained from the analysis are compared



(a) (b)

Fig. 7 (a) Load-deflection plot at mid-span and (b) crack Pattern of the beam-VI



(a) (b)

Fig. 8 (a) Load-deflection plot at mid-span and (b) crack Pattern of the deep beam

with the experimental study. The load-deflection curves and the crack pattern obtained from the mesoscopic model and experimental studies for beams II, III, VI and deep beam are shown in Figs. 5(a) and 5(b), 6(a) and 6(b), 7(a) and 7(b) and 8(a) and 8(b) respectively. The comparison of ultimate loads of mesoscopic models of under-reinforced, over reinforced and deep beams, is shown in Table 3.

3.3 Comparison of results

The load-deflection plots and crack patterns for the beams are shown in Figs. 5 to 8. The proposed method of mesoscopic analysis shows that the crack pattern and load-deflection curves are closely consistent with the test results. The experimental results exhibit a stiffer response initially when compared to the analytical model. The ultimate loads obtained from the experiment and mesoscopic analyses are given in Table 3. The ultimate load obtained from the analysis matches well with those obtained from the experiment. The average error observed for the under-reinforced section is less than 11% and for the over reinforced sections is less than 7%. For deep beam, the error is less than 5%. The maximum error observed is 11.4% in sample II of the under-reinforced section. The present study on the various samples indicates that the proposed mesoscopic model adequately predicted the behavior of reinforced concrete beams considered in this study.

4. Conclusions

A two dimensional mesoscopic model of the reinforced concrete beam is developed by considering concrete as a two-phase material; coarse aggregate and mortar. A take and place algorithm for generating realistic concrete models in which the aggregates are randomly placed is used. Instead of using micro-truss, proper bonding between the constituents is made so that the forces are transferred through interface bonding. The model has then meshed such that the finite element boundaries coincide with materials interfaces. The mesoscopic model is validated using experimental studies for under reinforced, over reinforced sections and for a deep beam. The results show that the proposed mesoscopic model is closely consistent with the experimental results. The crack pattern shows a very good agreement with the crack patterns of the experiment. The proposed simple model can be used to study the crack pattern of under-reinforced, over reinforced and deep concrete beams, where experimental studies are not feasible.

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