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A nonlocal strain gradient theory for scale-dependent wave dispersion analysis of rotating nanobeams considering physical field effects

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Abstract. This paper is concerned with the wave propagation behavior of rotating functionally graded temperature-dependent nanoscale beams subjected to thermal loading based on nonlocal strain gradient stress field. Uniform, linear and nonlinear temperature distributions across the thickness are investigated. Thermo-elastic properties of FG beam change gradually according to the Mori-Tanaka distribution model in the spatial coordinate. The nanobeam is modeled via a higher-order shear deformable refined beam theory which has a trigonometric shear stress function. The governing equations are derived by Hamilton's principle as a function of axial force due to centrifugal stiffening and displacement. By applying an analytical solution and solving an eigenvalue problem, the dispersion relations of rotating FG nanobeam are obtained. Numerical results illustrate that various parameters including temperature change, angular velocity, nonlocality parameter, wave number and gradient index have significant effect on the wave dispersion characteristics of the understudy nanobeam. The outcome of this study can provide beneficial information for the next generation researches and exact design of nano-machines including nanoscale molecular bearings and nanogears, etc.

Keywords: wave propagation; FGMs; nonlocal strain gradient theory; rotating nanobeam; refined beam theory

1. Introduction

Functionally graded materials (FGMs) have been created from a mixture of ceramic and metal with a continuous variation in one or more dimensions which are designed to reach the high structural performance. (Ebrahimi *et al.* 2016a, Ebrahimi and Zia 2015, Ebrahimi and Mokhtari 2015). Thus, it is important to investigate the mechanical specifications of FGM structures. Recently, many papers have been published concerning with analysis of FG nanostructures (Ebrahimi and Salari 2015a, b, 2016, Ebrahimi *et al.* 2015a, 2016c, Ebrahimi and Nasirzadeh 2015, Ebrahimi and Hosseini 2016a, b, c). The physical and mechanical characteristics of structures in the nanobeam, render evident size effects that makes them to demonstrate significant

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mechanical and thermal behavior which are superior to the conventional structural materials. Therefore, nanomaterials have the potential to revolutionize critical technologies. Hence, for investigation on the mechanical behavior of nanostructures in which the interatomic bonds possess a vital role on their regime, the classical continuum theory which disregards such a notable fact is not appropriate for this situation. Accordingly, this issue has been examined in the context of nonlocal continuum theories such as nonlocal elasticity theory (NET) of Eringen (1972, 1983). According to this theory, strain/stress state at any reference point is a function of corresponding states of other points of the continuum body. Reddy (2007) performed Nonlocal theories for bending, buckling and vibration of beams. Narendar and Gopalakrishnan (2009) performed small-scale influences on wave propagation of multi-walled carbon nanotubes. Yang *et al.* (2011) researched wave dispersion of double-walled carbon nanotubes on the basis of size-dependent Timoshenko beam model. Also, Fotouhi *et al*. (2013), Lei *et al*. (2013) and Naderi *et al*. (2014) reported research based on NET. Thermal loading effects on buckling and vibrational behavior of FG nanobeam based on NET is explored by Ebrahimi *et al.* (2015, 2016). Hygro-thermal effects on vibration behavior of FG nano-beams based on nonlocal elasticity theory and unsing Power-Law distribution model are explored by Ebrahimi and Barati (2016, 2016). Thermo-mechanical buckling analysis of curved functionally graded (FG) nanobeams based on nonlocal elasticity model performed by Ebrahimi and Barati (2016). A review on nonlocal elastic models for bending, buckling, vibrations, and wave dispersion of nanoscale beams is explored by Eltaher *et al.* (2016).

Recently, it has been shown that nonlocal differential elasticity based model maybe ill-posed. Of course, due to the simplification of the nonlocal differential elasticity, many works have been focused on the size-dependent behaviors based on the nonlocal differential models. More recently, it is shown that the nonlocal differential and integral elasticity based models may be not equivalent to each other (Zhu and Li 2017a). So nonlocal differential model is an approximate model. (Zhu and Li (2017b, c) have studied the tension and vibration problem for CNTs and Graphene based on nonlocal integral and strain gradient elasticity theory. Most recently Ebrahimi and Barati (2016g, h, i, j, 2017a, b) and Ebrahimi *et al.* (2017) explored thermal and hygro-thermal effects on nonlocal behavior of FG nanobeams and nanoplates.

Moreover, the stiffness enhancement observed in experimental works and strain gradient elasticity (2010) cannot be forecasted well by using Eringen's nonlocal elasticity theory. Nonlocal strain gradient theory (NSGT) accounts the stress for both nonlocal elastic stress field and the strain gradient stress field. It is worth mentioning that the nonlocal strain gradient theory catches the true effect of the two length scale parameters on the physical and mechanical characteristics of small-scale structures. Li *et al.* (2016) reported vibration analysis of nonlocal strain gradient FG nano-beams. In these works, both stiffness-hardening and stiffness-softening influences on vibration behavior of FG nanobeams are presented. Yang *et al*. (2002), Lam *et al*. (2003), Akgöz *et al*. (2013), Li *et al*. (2015) and Farajpour *et al*. (2016) reported researches in the field of nonlocal strain gradient theory. Although the nonlocal strain gradient models are being used more and more extensively in examining the size-dependent effects on the statical and dynamical behaviors of micro/nano-structures. However, for the fake of simplification, the size-dependent effects are often assumed to be neglected in the thickness direction of beams and plates. Recently, a nonlocal strain gradient beam model incorporating the thickness effect is developed for the size-dependent buckling analysis of nanobeams (Li *et al*. 2018). Devices in the nanometer realm with the moving parts called nano-machines. Rotating nanostructures containing molecular bearings, nanogears, nanoturbines and multiple gear systems have received notable consideration from the research community (1997, 2004). Hence, investigation of vibration and wave propagation of nanomachines is significant for their accurate design. Pradhan and Murmu (2010) used a nonlocal beam model to investigate the flap-wise bending-vibration characteristics of a rotating nanocantilever. Narendar and Gopalakrishnan (2011) explored the wave dispersion behavior of a rotating nanotube using the nonlocal elasticity theory. Aranda-Ruiz *et al.* (2012) reported free vibration of rotating non-uniform nano-cantilevers according to the Eringen nonlocal elasticity theory. Recently, Mohammadi *et al.* (2016) investigated vibration analysis of a rotating viscoelastic nano-beam embedded in a visco-Pasternak elastic medium and in a nonlinear thermal environment. Also, Ebrahimi and Shafei (2016) investigated the application of Eringen's nonlocal elasticity theory for vibration analysis of rotating FG nano-beams.

Recently, mechanical (static and dynamic) analysis of FG nano-realm structures attracted great deal of attention of researchers. Wang (2010) studied wave propagation analysis of fluid-conveying single-walled carbon nanotubes applying strain gradient theory. Wave propagation analysis of single-walled carbon nanotubes exposed to an axial magnetic field in the framework of nonlocal Euler-Bernoulli beam model investigated by Narendar *et al.* (2012). Aydogdu (2014) studied longitudinal wave dispersion of carbon nanotubes. Also, Filiz and Aydogdu (2015) performed wave propagation analysis of functionally graded (FG) nanotubes conveying fluid embedded in elastic medium.

Nonlocal thermo-elastic wave propagation in embedded nonhomogeneous FG nanobeams using nonlocal elasticity theory presented by Ebrahimi *et al*. (2016). Thermal environment effects on wave dispersion behavior of inhomogeneous strain gradient nanobeams based on higher order refined beam theory studied by Ebrahimi *et al*. (2016). Wave propagation analysis of quasi-3D FG nanobeams in thermal environment based on nonlocal strain gradient theory investigated by Ebrahimi and Barati (2016). Karami *et al*. (2017) reported the effects of triaxial magnetic field on the anisotropic nanoplates. An efficient shear deformation theory for wave propagation in FGM beams with porosities is presented by Benadouda *et al*. (2017). Flexural wave propagation in size-dependent functionally graded beams based on nonlocal strain gradient theory is performed by Li *et al.* (2015). In another work, Ebrahimi and Barati (2016) explored flexural wave propagation analysis of embedded S-FGM nano-beams under longitudinal magnetic field. Narendar (2016) investigated wave dispersion in functionally graded magneto-electro-elastic nonlocal rod. Recently, Ebrahimi *et al.* (2016) reported wave propagation analysis of rotating strain gradient temperature dependent FG nanobeam in thermal environment based on Euler-Bernoulli beam theory.

It is observable that, most of the researches are dedicated to buckling, static and vibration of FG nano-beams, and just a few number of them are working in the field of wave propagation of FG small-scale beams. Also, the studies in the field of temperature dependent FG nano-scale structures have not done a beneficial comparison between the effective parameter to make their effect clearer to investigate. According to the history, it is clear that wave dispersion analysis of rotating FG thermo-elastic nanobeam based on higher order shear deformable refined beam theory under different temperature distributions is a novel and beneficial topic to study.

This research deals with the wave dispersion characteristics of a rotating FG thermo-elastic nano-beam studying based on refined beam theory by using nonlocal strain gradient theory under different temperature distributions. Material properties are supposed to change gradually across the thickness of nanobeam based on Mori-Tanaka distribution model. The governing partial differential equations are derived by applying the Hamilton's principle in the framework of higher order shear deformable refined beam model. An analytical solution is applied to capture required parameters. It is clear that, wave dispersion characteristics of rotating FG nanobeams are extremely affected by temperature changes, angular velocity, wave number, nonlocal parameter, length scale parameter, and material graduation.

2. Theory and formulation

2.1 Mori-Tanaka FGM nanobeam model

Material properties of the FG nanobeam are assumed to distribute according to Mori-Tanaka model about the spatial coordinate. Mori-Tanaka homogenization technique represents the local effective material properties including effective local bulk modules K_e and shear modules μ_e in the form (Barati *et al*. 2016)

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m (K_c - K_m)/(K_m + 4\mu_m/3)} \tag{1}$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m (\mu_c - \mu_m)/[(\mu_m + \mu_m (9K_m + 8\mu_m)/(6(K_m + 2\mu_m))]} \tag{2}$$

where, subscripts m and c define metal and ceramic, respectively and their volume fractions are related by the following for

$$V_c + V_m = 1 \tag{3}$$

While, the volume fraction of the ceramic phase is given by

$$V_c = (\frac{z}{h} + \frac{1}{2})^P \tag{4}$$

$$V_m = 1 - (\frac{z}{h} + \frac{1}{2})^P \tag{5}$$

Here p indicates the gradient index which determines gradual alteration of material properties through the thickness of the nanobeam. Finally, the effective Young's modulus (E), poison ratio (v) and mass density (ρ) can be expressed by

$$E(z) = \frac{9K_e \mu_e}{3K_e + \mu_e} \tag{6}$$

$$v(z) = \frac{3K_e - 2\mu_e}{6K_e + 2\mu_e} \tag{7}$$

$$\rho(z) = \rho_c V_c + \rho_m V_m \tag{8}$$

And thermal expansion coefficient (α) and thermal conductivity ($\boldsymbol{\kappa}$) may be expressed as

$$\frac{\alpha_e - \alpha_m}{\alpha_c - \alpha_m} = \frac{\frac{1}{K_e} - \frac{1}{K_m}}{\frac{1}{K_c} - \frac{1}{K_m}} \tag{9}$$

$$\frac{\kappa_e - \kappa_m}{\kappa_c - \kappa_m} = \frac{V_c}{1 + V_m \frac{(\kappa_c - \kappa_m)}{3\kappa_m}} \tag{10}$$

Also, temperature-dependent coefficients of material phases can be expressed according to the following relation

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{11}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the temperature-dependent constants which are tabulated in Table 1 for Si_3N_4 and SUS304. The bottom and top surfaces of FG nanobeam are fully metal (SUS304) and fully ceramic (Si_3N_4), respectively.

2.2 Kinematic relations

In the framework of refined shear deformation beam theories, the displacement field of nonlocal FGM beam can be written as

$$u_x(x,z) = u(x) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{12}$$

$$u_z(x,z) = w_b(x) + w_s(x) \tag{13}$$

where, w_b, w_s indicates the components correspond to the bending and shear transverse displacements of a point on the mid-surface of the beam, respectively and u shows longitudinal displacement. Also, $f(z)$ denotes the shape function representing the shear stress/strain distribution through the thickness of the beam. The present theory has a trigonometric function in the form (Mantari *et al.* 2014)

$$f(z) = z - \sin(\xi z)/\xi \tag{14}$$

where, $\xi = \pi/h$. Non-zero strains of the present beam model can be expressed in the following form

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{15}$$

$$\gamma_{xz} = g\frac{\partial w_s}{\partial x} \tag{16}$$

where, $g(z)=1-df/dz$. Also, the Hamilton's principle states that

$$\int_0^t \delta(U+V-K)\,dt=0 \tag{17}$$

Here, U is strain energy, V is the work done by external forces and K is kinetic energy. The virtual strain energy can be written as

$$\delta U=\int_v \sigma_{ij}\delta\varepsilon_{ij}\,dV=\int_v(\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{xz}\delta\gamma_{xz})\,dV \tag{18}$$

Substituting Eqs. (15) and (16) into Eq. (18) yields

$$\delta U=\int_0^L\left(N\frac{d\delta u}{dx}-M_b\frac{d^2\delta w_b}{dx^2}-M_s\frac{d^2\delta w_s}{dx^2}+Q\frac{d\delta w_s}{dx}\right)dx \tag{19}$$

In which, the variables expressed in the above equation are defined as follows

$$\begin{aligned}&N=\int_A\sigma_{xx}\,dA,\; M_b=\int_A z\sigma_{xx}\,dA\\&M_s=\int_A f\sigma_{xx}dA,\; Q=\int_A g\sigma_{xz}\,dA\end{aligned} \tag{20}$$

The first variation of the work done by applied forces can be expressed in the following form

$$\delta V=\int_0^L\left(N^T\left(\frac{d(w_b+w_s)}{dx}\frac{d\delta(w_b+w_s)}{dx}\right)\right)dx \tag{21}$$

where, N^T and N^R indicate the applied force due to temperature and external force due to rotation that can be defined by

$$N^T=\int_{-h/2}^{h/2}E(z,T)\,\alpha(z,T)\,(T-T_0)\,dz \tag{22}$$

$$N^R=b\int_x^L\int_{-h/2}^{h/2}(\rho(z)A\Omega^2x)dxdz \tag{23a}$$

where, T_0 shows the reference temperature and Ω denotes the angular velocity. In this research, we assume a uniform rotating FG nanobeam and maximum axial force is considered (Narendar and Gopalakrishnan 2011)

$$N^R_{\max}=b\int_x^L\int_{-h/2}^{h/2}(\rho(z)A\Omega^2x)dxdz \tag{23b}$$

The variation of kinetic energy can be presented by

$$\begin{aligned}\delta K=\int_0^L\Big(&I_0\Big[\frac{du}{dt}\frac{d\delta u}{dt}+\Big(\frac{dw_b}{dt}+\frac{dw_s}{dt}\Big)\Big(\frac{d\delta w_b}{dt}+\frac{d\delta w_s}{dt}\Big)\Big]\\&-I_1\Big(\frac{du}{dt}\frac{d^2\delta w_b}{dxdt}+\frac{d^2w_b}{dxdt}\frac{d\delta u}{dt}\Big)+I_2\Big(\frac{d^2w_b}{dxdt}\frac{d^2\delta w_b}{dxdt}\Big)\\&-J_1\Big(\frac{du}{dt}\frac{d^2\delta w_s}{dxdt}+\frac{d^2w_s}{dxdt}\frac{d\delta u}{dt}\Big)+K_2\Big(\frac{d^2w_s}{dxdt}\frac{d^2\delta w_s}{dxdt}\Big)\\&+J_2\Big(\frac{d^2w_b}{dxdt}\frac{d^2\delta w_s}{dxdt}+\frac{d^2w_s}{dxdt}\frac{d^2\delta w_b}{dxdt}\Big)\Big)dx\end{aligned} \tag{24}$$

In which,

$$(I_0, I_1, J_1, I_2, J_2, K_2) = \int_A \rho(z)(1, z, f, z^2, zf, f^2)dA \tag{25}$$

The following equations are obtained by inserting Eqs. (19)-(23) in Eq. (17) when the coefficients of δu, δw_b and δw_s are equal to zero

$$\frac{\partial N}{\partial x} = I_0 \frac{d^2 u}{dt^2} - I_1 \frac{d^3 w_b}{dxdt^2} - J_1 \frac{d^3 w_s}{dxdt^2} \tag{26}$$

$$\begin{aligned} &\frac{d^2 M_b}{dx^2} + (N^T + N^R_{\max}) \frac{d^2(w_b + w_s)}{dx^2} \\ &= I_0 \left(\frac{d^2 w_b}{dt^2} + \frac{d^2 w_s}{dt^2}\right) + I_1 \frac{d^3 u}{dxdt^2} \\ &- I_2 \frac{d^4 w_b}{dx^2 dt^2} - J_2 \frac{d^4 w_s}{dx^2 dt^2} \end{aligned} \tag{27}$$

$$\begin{aligned} &\frac{d^2 M_s}{dx^2} + \frac{dQ}{dx} + (N^T + N^R_{\max}) \frac{d^2(w_b + w_s)}{dx^2} \\ &= I_0 \left(\frac{d^2 w_b}{dt^2} + \frac{d^2 w_s}{dt^2}\right) + J_1 \frac{d^3 u}{dxdt^2} \\ &- J_2 \frac{d^4 w_b}{dx^2 dt^2} - K_2 \frac{d^4 w_s}{dx^2 dt^2} \end{aligned} \tag{28}$$

2.3 The nonlocal FG nanobeam strain gradient model

Nonlocal strain gradient elasticity (Li *et al.* 2015) enumerates the stress for both nonlocal stress and strain fields. Therefore, the stress can be expressed by the following relations

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \frac{d\sigma_{ij}^{(1)}}{dx} \tag{29}$$

where, the stresses $\sigma_{xx}^{(0)}$ and $\sigma_{xx}^{(1)}$ are related to strain ε_{xx} and strain gradient $\varepsilon_{xx,x}$, respectively and are defined as

$$\sigma_{ij}^{(0)} = \int_0^L C_{ijkl}\alpha_0(x, x', e_0 a)\varepsilon'_{kl}(x')dx' \tag{30}$$

$$\sigma_{ij}^{(1)} = l^2 \int_0^L C_{ijkl}\alpha_1(x, x', e_1 a)\varepsilon'_{kl,x}(x')dx' \tag{31}$$

In which, C_{ijkl} are the elastic constants and e_0a and e_1a which are nonlocal parameters ate introduced to take into account the effect of nonlocal stress field (Narendar and Gopalakrishnan 2011) and l is the length scale parameter and represents the influence of higher order strain gradient stress field. When the nonlocal functions $\alpha_0(x, x', e_0a)$ and $\alpha_1(x, x', e_1a)$ satisfy the developed conditions by Eringen (1983), the constitutive relation for a FGM nanobeam can be stated as

$$[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]\sigma_{ij} \\ = C_{ijkl}[1-(e_1a)^2\nabla^2]\varepsilon_{kl} - C_{ijkl}\, l^2[1-(e_0a)^2\nabla^2]\nabla^2\varepsilon_{kl} \tag{32}$$

In which, ∇^2 denotes the Laplacian operator. Supposing e_1=e_0=e and discarding terms of order $O(\nabla^2)$, the general constitutive relation in Eq. (34) can be stated as (Li *et al.* 2015)

$$[1-(ea)^2\nabla^2]\sigma_{ij} = C_{ijkl}\,[1-l^2\nabla^2]\varepsilon_{kl} \tag{33}$$

Thus, the constitutive relations for a nonlocal refined shear deformable FG nanobeam can be expressed as

$$\sigma_{xx} - \mu\frac{\partial^2\sigma_{xx}}{\partial x^2} = E(z)(\,\varepsilon_{xx} - \lambda^2\,\frac{\partial^2\varepsilon_{xx}}{\partial x^2}) \tag{34}$$

$$\sigma_{xz} - \mu\frac{\partial^2\sigma_{xz}}{\partial x^2} = G(z)(\gamma_{xz} - \lambda^2\,\frac{\partial^2\gamma_{xz}}{\partial x^2}) \tag{35}$$

where μ=ea^2 and λ=l. By integrating Eqs. (34) and (35) over the cross-section area of nanobeam provides the following nonlocal relations for FGM beam model as

$$N - \mu\frac{\partial^2 N}{\partial x^2} = (1-\lambda^2\,\frac{\partial^2}{\partial x^2}) \\ (A\frac{\partial u}{\partial x} - B\frac{\partial^2 w_b}{\partial x^2} - B_s\,\frac{\partial^2 w_s}{\partial x^2}) - N_x^T \tag{36}$$

$$M_b - \mu\frac{\partial^2 M_b}{\partial x^2} = (1-\lambda^2\,\frac{\partial^2}{\partial x^2}) \\ (B\frac{\partial u}{\partial x} - D\frac{\partial^2 w_b}{\partial x^2} - D_s\,\frac{\partial^2 w_s}{\partial x^2}) - M_b^T \tag{37}$$

$$M_s - \mu\frac{\partial^2 M_s}{\partial x^2} = (1-\lambda^2\,\frac{\partial^2}{\partial x^2}) \\ (B_s\,\frac{\partial u}{\partial x} - D_s\,\frac{\partial^2 w_b}{\partial x^2} - H_s\,\frac{\partial^2 w_s}{\partial x^2}) - M_s^T \tag{38}$$

$$Q - \mu\frac{\partial^2 Q}{\partial x^2} = (1-\lambda^2\,\frac{\partial^2}{\partial x^2})(A_s\,\frac{\partial w_s}{\partial x}) \tag{39}$$

where, the cross-sectional rigidities are explained as

$$
\begin{aligned}
&(A, B, B_s, D, D_s, H_s) \\
&= \int_A E(z)\,(1, z, f, z^2, zf, f^2)\, dA
\end{aligned}
\tag{40}
$$

$$
A_s = \int_A g^2 G(z) dA \tag{41}
$$

The governing equations of refined shear deformable FGM nanobeams in terms of displacements are obtained by inserting for N, M_b, M_s and Q from Eqs. (36)-(39), respectively, into Eqs. (26)-(28) as follows

$$
\begin{aligned}
&A(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^2 u}{\partial x^2}) - B(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^3 w_b}{\partial x^3}) \\
&-B_s(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^3 w_s}{\partial x^3}) - I_0 \frac{\partial^2 u}{\partial t^2} \\
&+I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} + J_1 \frac{\partial^3 w_s}{\partial x \partial t^2} + \mu(I_0 \frac{\partial^4 u}{\partial x^2 \partial t^2} \\
&-I_1 \frac{\partial^5 w_b}{\partial x^3 \partial t^2} - J_1 \frac{\partial^5 w_s}{\partial x^3 \partial t^2}) = 0
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
&B(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^3 u}{\partial x^3}) - D(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^4 w_b}{\partial x^4}) \\
&-D_s(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^4 w_s}{\partial x^4}) - (N^T + N^R)\frac{\partial^2 (w_b + w_s)}{\partial x^2} \\
&-I_0(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) - I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} + J_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} \\
&+\mu((N^T + N^R)\frac{\partial^4 (w_b + w_s)}{\partial x^4} + I_0(\frac{\partial^4 w_b}{\partial x^2 \partial t^2} + \frac{\partial^4 w_s}{\partial x^2 \partial t^2}) \\
&+I_1 \frac{\partial^5 u}{\partial x^3 \partial t^2} - I_2 \frac{\partial^6 w_b}{\partial x^4 \partial t^2} - J_2 \frac{\partial^6 w_s}{\partial x^4 \partial t^2}) = 0
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
&B_s(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^3 u}{\partial x^3}) - D_s(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^4 w_b}{\partial x^4}) \\
&-H_s(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^4 w_s}{\partial x^4}) + A_s(1-\lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^2 w_s}{\partial x^2}) \\
&-(N^T + N^R)\frac{\partial^2 (w_b + w_s)}{\partial x^2} - I_0(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) \\
&-J_1 \frac{\partial^3 u}{\partial x \partial t^2} + J_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} + K_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} \\
&+\mu((N^T + N^R)\frac{\partial^4 (w_b + w_s)}{\partial x^4} + I_0(\frac{\partial^4 w_b}{\partial x^2 \partial t^2} \\
&+\frac{\partial^4 w_s}{\partial x^2 \partial t^2}) + J_1 \frac{\partial^5 u}{\partial x^3 \partial t^2} - J_2 \frac{\partial^6 w_b}{\partial x^4 \partial t^2} - K_2 \frac{\partial^6 w_s}{\partial x^4 \partial t^2}) = 0
\end{aligned}
\tag{44}
$$

3. Solution procedures

The solution of governing equations of nonlocal thermoelastic FGM nanobeam can be presented by

$$u(x,t) = U_n \exp[i(\beta x - \omega t)] \tag{45}$$

$$w_b(x,t) = W_{bn} \exp[i(\beta x - \omega t)] \tag{46}$$

$$w_s(x,t) = W_{sn} \exp[i(\beta x - \omega t)] \tag{47}$$

where (U_n, W_{bn}, W_{sn}) are the wave amplitudes; β and ω indicate the wave number and circular frequency, respectively. Inserting Eqs. (45)-(47) into Eqs. (42)-(44) gives

$$\left\{ \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} - \omega^2 \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \right\} \begin{Bmatrix} U_n \\ W_{bn} \\ W_{sn} \end{Bmatrix} = 0 \tag{48}$$

Where,

$$k_{11} = -A\beta^2(1+\lambda^2\beta^2), \quad k_{21} = -Bi\beta^3(1+\lambda^2\beta^2)$$
$$k_{22} = -D\beta^4(1+\lambda^2\beta^2) + (N_{\max}^R + N^T)\beta^2(1+\mu\beta^2)$$
$$k_{31} = -B_s i\beta^2(1+\lambda^2\beta^2), \quad k_{13} = -k_{31}, \quad k_{12} = -k_{21}$$
$$k_{32} = -D_s\beta^4(1+\lambda^2\beta^2) + (N_{\max}^R + N^T)\beta^2(1+\mu\beta^2)$$
$$k_{23} = -D_s\beta^4(1+\lambda^2\beta^2) + (N_{\max}^R + N^T)\beta^2(1+\mu\beta^2)$$
$$k_{33} = -(H_s\beta^4 + A_s\beta^2)(1+\lambda^2\beta^2) + (N_{\max}^R + N^T)\beta^2(1+\mu\beta^2)$$
$$m_{11} = -I_0(1+\mu\beta^2), \quad m_{21} = -I_1 i\beta(1+\mu\beta^2), \; m_{12} = -m_{21}$$
$$m_{31} = -iJ_1\beta(1+\mu\beta^2), \quad m_{13} = -m_{31}$$
$$m_{22} = -(I_0 + I_2\beta^2)(1+\mu\beta^2) \; m_{32} = -(I_0 + J_2\beta^2)(1+\mu\beta^2)$$
$$m_{23} = -(I_0 + J_2\beta^2)(1+\mu\beta^2), \quad m_{33} = -(I_0 + k_2\beta^2)(1+\mu\beta^2)$$

By setting the determinant of above matrix to zero, the circular frequency ω can be obtained. Hence, the roots of Eq. (43) can be written as

$$\omega_1 = M_0(\beta), \omega_2 = M_1(\beta), \omega_3 = M_2(\beta) \tag{49}$$

These roots are corresponded with the wave modes M_0, M_1 and M_2, respectively. The wave modes M_0 and M_2 are related to the flexural waves and mode M_1 is related to the extensional waves. Also, the phase velocity of waves can be calculated by the following relation

$$c_{p(i)} = \frac{M_i(\beta)}{\beta}, i = 1, 2, 3 \tag{50}$$

which displays the dispersion relation of phase velocity c_p and wave number β for the FGM nanobeam. Also, the escape frequencies of the FG nanobeam can be obtained by setting $\beta\rightarrow\infty$. It should be noted that after the escape frequency, the flexural waves will not propagate anymore.

4. Different types of thermal loading

4.1 Uniform temperature rise (UTR)

The temperature rise uniformly For a FG nanobeam with the reference temperature T_0, to the final temperature T which $\Delta T = T - T_0$.

4.2 Linear temperature rise (LTR)

In this state, the temperature varies linearly through the thickness of the nanobeam as follows

$$T = T_m + \Delta T\left(\frac{z}{h} + \frac{1}{2}\right) \tag{51}$$

Where, $\Delta T = T_c - T_m$ in which T_c and T_m are the temperature of bottom and top surface of the nanobeam, respectively.

4.3 Nonlinear temperature rise (NLTR)

In this case, the temperature varies nonlinearly through the thickness. Temperature distribution can be obtained by solving the steady-state heat conduction equation across the thickness, with the boundary conditions on bottom and top surfaces of the nanobeam

$$-\frac{d}{dz}\left(\kappa(z,\ T)\,\frac{dT}{dz}\right) = 0 \tag{52}$$

Considering the boundary conditions as follows

$$T\left(\frac{h}{2}\right) = T_c,\quad T\left(-\frac{h}{2}\right) = T_m \tag{53}$$

The solution of Eqs. (12) and (13) is

$$T = T_m + (T_c - T_m)\frac{\int_{-\frac{h}{2}}^{z}\frac{1}{\kappa(z,T)}dz}{\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{1}{\kappa(z,T)}dz} \tag{54}$$

where, $\Delta T = T_c - T_m$ indicates the temperature change.

5. Numerical results and discussions

This section is assigned to investigate the propagation characteristics of mentioned nanobeam.

This nanoscale beam is modeled based on Higher-Order shear deformable refined beam theory. The thickness of nanobeam is considered to be h=100 nm. Material properties of mentioned nanobeam are reported in Table 1. The wave frequencies of mentioned nanobeam is verified with those of Ebrahimi *et al.* [31]. For various wave number and angular velocity and a good agreement is observed as reported in Table 2.

Tables 3 and 4 reports the influence of angular velocity (**Ω**=1,3 and 5), length scale parameter or temperature change and material composition (p=0.2, 1 and 5) on phase velocity (c_p) of rotating refined FG nanobeam for various temperature distribution (UTR, LTR, NLTR) at L/h=20.

Table 1 Temperature-dependent coefficients for Si_3N_4 and SUS304

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	α(K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ρ(kg/m^3)	2370	0	0	0	0
	κ(W/mK)	13.723	0	-1.032e-3	5.466e-7	-7.876e-11
	v	0.24	0	0	0	0
SUS304	E (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α(K^{-1})	12.330e-6	0	8.086e-4	0	0
	ρ(kg/m^3)	8166	0	0	0	0
	κ(W/mK)	15.379	0	-1.264e-3	2.092e-6	-7.223e-10
	v	0.3262	0	-2.002e-4	3.797e-7	0

Table 2 Comparison of the wave frequency for rotating FG nanobeam. (NLTR, μ=1 nm, ΔT=800, l=1.5)

β	Ω	P=0		P=0.2		P=1		P=5	
		Ebrahimi *et al.* [31]	present	Ebrahimi *et al.* [31]	present	Ebrahimi *et al.* [31]	present	Ebrahimi *et al.* [31]	present
0.1	0	0.164617	0.106587	0.109812	0.0811336	0.0694308	0.0555902	0.058270	0.0405788
	1	0.164782	0.108673	0.109942	0.083689	0.0695951	0.0592566	0.0584509	0.0458823
	2	0.165275	0.114688	0.110329	0.0907568	0.070079	0.0674888	0.0589657	0.0578131
5	0	12.842553	12.8253	8.322701	8.16948	5.234263	5.231979	4.382287	4.359028
	1	12.842555	12.8255	8.322702	8.16952	5.234265	5.231980	4.382289	4.359033
	2	12.842562	12.8261	8.322707	8.16964	5.234271	5.231981	4.382296	4.359046
15	0	38.8989264	38.8931	25.208407	24.7478	15.853894	15.8469	13.273356	13.202798
	1	38.8989272	38.8932	25.208408	24.7478	15.853894	15.8469	13.273357	13.202800
	2	38.8989296	38.8934	25.208409	24.7479	15.853896	15.8469	13.273359	13.202804

In Table 3, it is found that, increasing angular velocity leads the increase in phase velocity for all three Tables. Also, in a constant angular velocity, gradient index and temperature distribution type, the phase velocity will increase due to increase in ΔT. Furthermore, with the increase in gradient index, the phase velocity will increase too. Finally, it can be seen that, for a constant value of **Ω**, gradient index and ΔT phase velocity decreases due to change in type of temperature

Table 3 variation of phase velocity of FG nanobeam for various gradient indices, angular velocity, temperature changes and thermal loadings. (L/h=20, μ=1 nm, β=0.08 (1/nm), l=1 nm)

	P=0.2			P=1			P=5		
	Ω=1	Ω=3	Ω=5	Ω=1	Ω=3	Ω=5	Ω=1	Ω=3	Ω=5
UTR									
ΔT=0	5.61695	6.70877	7.976	4.25386	5.47553	5.79173	3.48769	4.97612	5.10986
ΔT=200	5.5248	6.62856	7.85451	4.18573	5.39787	5.67595	3.43427	4.89626	5.00331
ΔT=500	5.3587	6.47548	7.57756	3.98648	5.10782	5.27783	3.22105	4.53325	4.57983
ΔT=800	5.08321	6.2028	7.05056	3.59779	4.42377	4.49009	2.80093	3.72275	3.73479
LTR									
ΔT=0	5.61726	6.709	7.97607	4.25414	5.47565	5.79173	3.48798	4.97619	5.10986
ΔT=200	5.51818	6.62351	7.85319	4.17951	5.39554	5.67587	3.42816	4.89508	5.0033
ΔT=500	5.33957	6.46137	7.57473	3.96935	5.10389	5.27768	3.20446	4.53218	4.5798
ΔT=800	5.05115	6.18121	7.0478	3.57258	4.42196	4.48998	2.77723	3.72252	3.73478
NLTR									
ΔT=0	5.61726	6.709	7.97607	4.25414	5.47565	5.79173	3.48798	4.97619	5.10986
ΔT=200	5.51798	6.62335	7.85314	4.17911	5.39539	5.67587	3.42786	4.89503	5.0033
ΔT=500	5.33832	6.46045	7.57455	3.96707	5.10336	5.27766	3.20277	4.53206	4.5798
ΔT=800	5.04794	6.17905	7.04752	3.56749	4.42159	4.48996	2.77358	3.72248	3.73478

Table 4 variation of phase velocity of FG nanobeam for various gradient indices, angular velocity, length scale parameter and thermal loadings. (L/h=20, μ=1 nm, β=0.08 (1/nm), ΔT=800)

	P=0.2			P=1			P=5		
	Ω=1	Ω=3	Ω=5	Ω=1	Ω=3	Ω=5	Ω=1	Ω=3	Ω=5
UTR									
λ=0	5.0685	6.18976	7.03039	3.58813	4.41063	4.47595	2.79443	3.71107	3.72293
λ=1	5.08321	6.2028	7.05056	3.59779	4.42377	4.49009	2.80093	3.72275	3.73479
λ=2	5.12707	6.24163	7.11042	3.62659	4.46284	4.53223	2.82036	3.75752	3.77017
LTR									
λ=0	5.03635	6.16819	7.02768	3.56287	4.40885	4.47585	2.77067	3.71085	3.72292
λ=1	5.05115	6.18121	7.0478	3.57258	4.42196	4.48998	2.77723	3.72252	3.73478
λ=2	5.09528	6.22001	7.10751	3.60156	4.46094	4.53211	2.79682	3.75728	3.77015
NLTR									
λ=0	5.03314	6.16603	7.02741	3.55777	4.40849	4.47582	2.76701	3.71081	3.72291
λ=1	5.04794	6.17905	7.04752	3.56749	4.42159	4.48996	2.77358	3.72248	3.73478
λ=2	5.0921	6.21784	7.10722	3.5965	4.46055	4.53209	2.79319	3.75724	3.77015

distribution (UTR<LTR<NLTR), but for the ΔT=0 this change is vice versa.

It is observable in Table 4 that, for a specific temperature distribution phase velocity will increase due to the increase in angular velocity and length scale parameter. However, with the

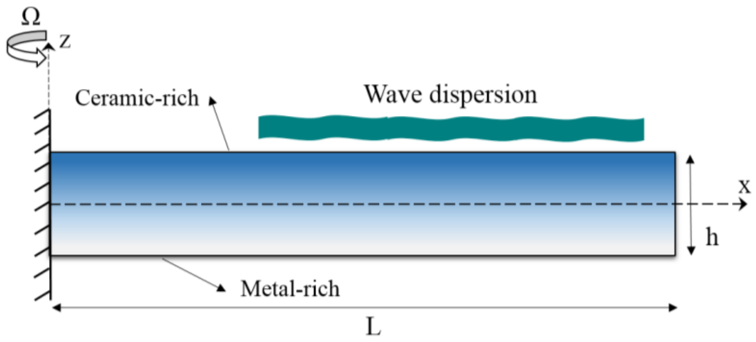


Fig. 1 Configuration of rotating FG nanobeam

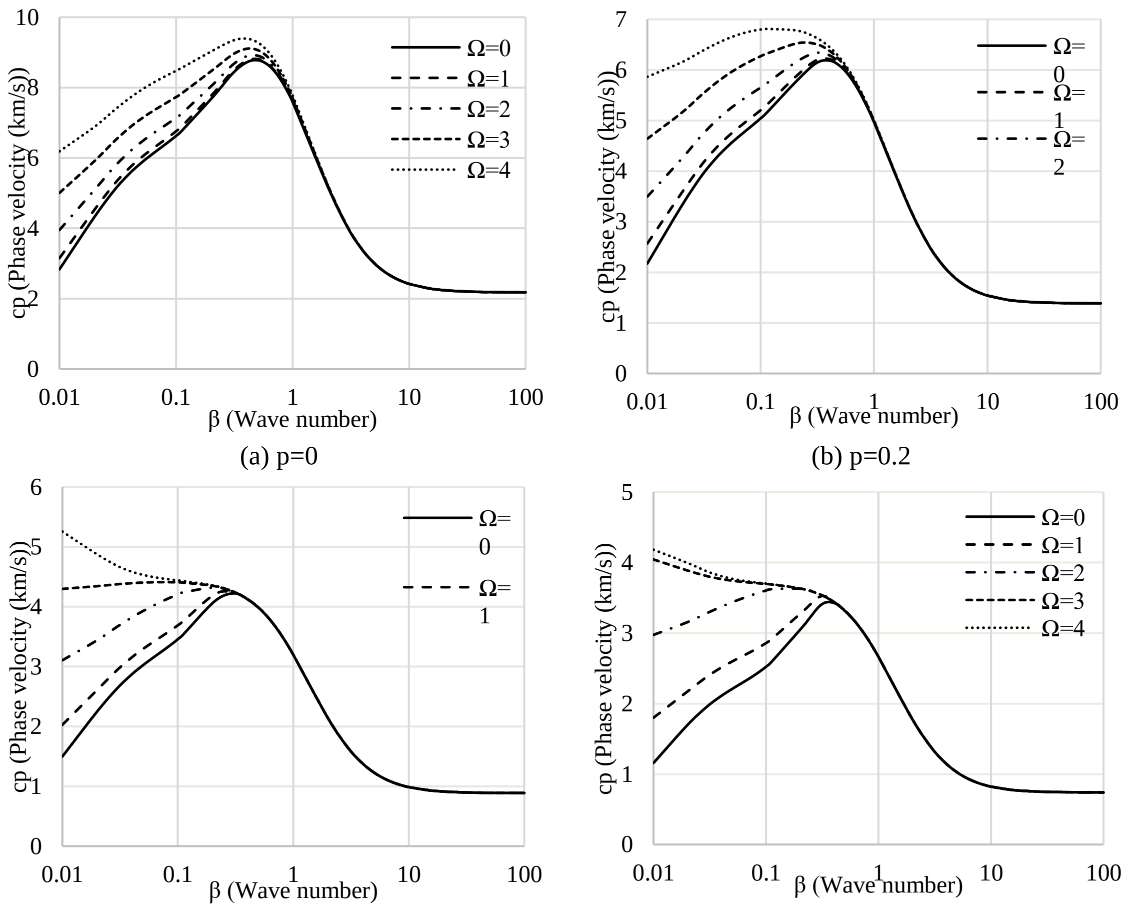
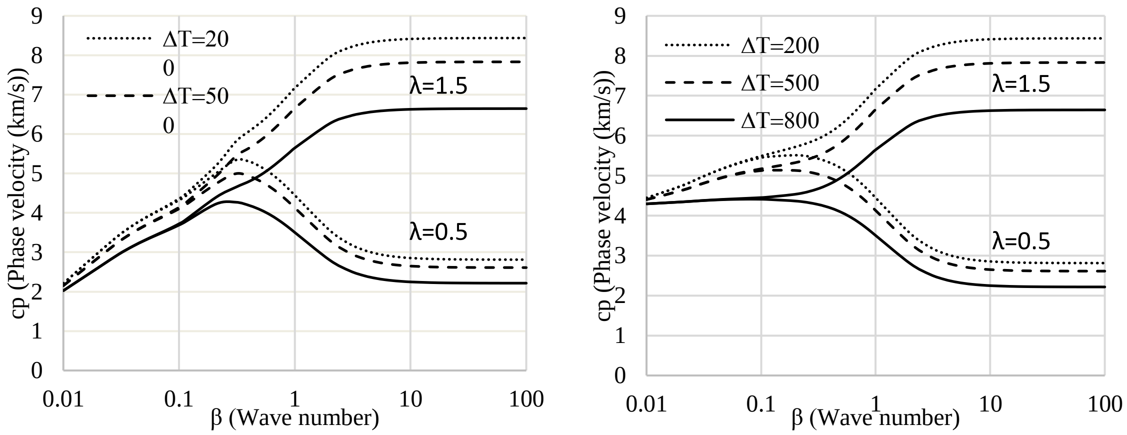


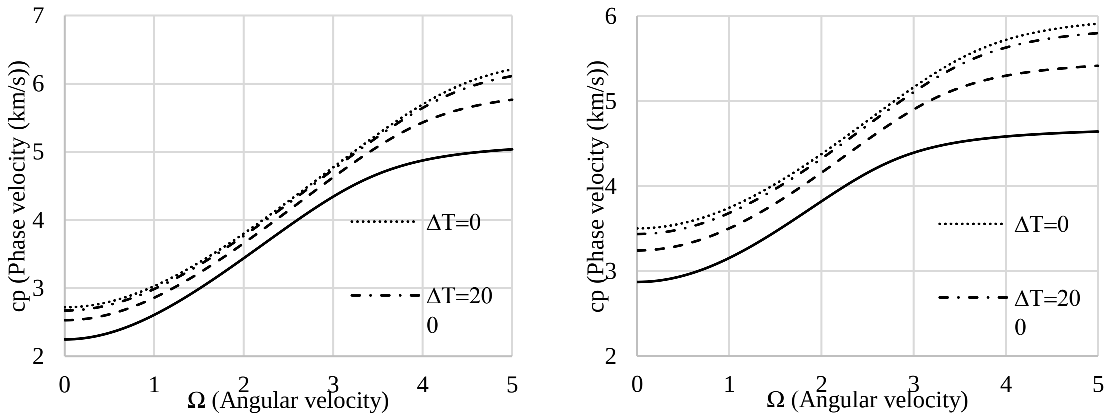
Fig. 2 Variation of phase velocity of rotating FG nanobeam versus wave number for various angular velocities and gradient indices (NLTR, μ=1 nm, λ=0.2 nm, ΔT=800)

increase in gradient index, the phase velocity will decrease. In addition, for a constant value of **Ω**, gradient index and ΔT phase velocity increases with the change in type of temperature distribution

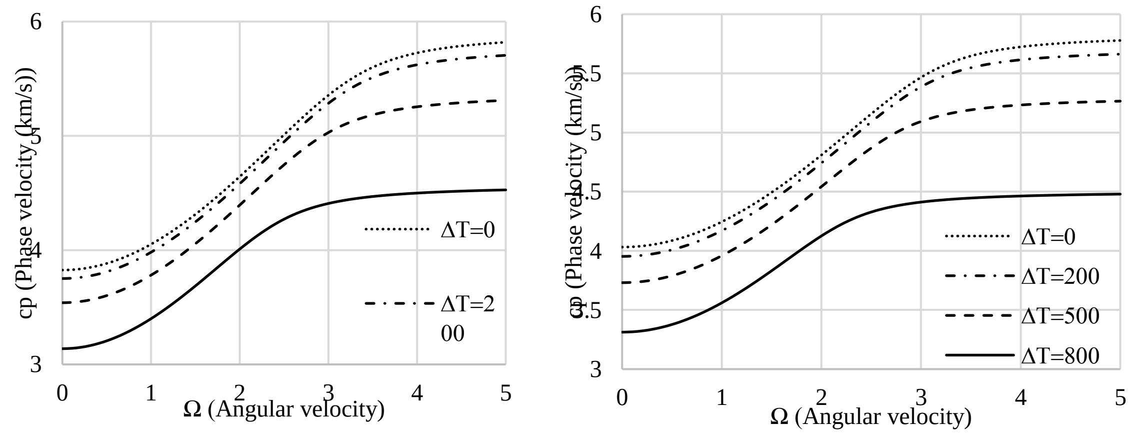


(a) Ω= 1 (b) Ω= 3

Fig. 3 Variation of phase velocity of rotating FG nanobeam versus wave number for various length scale parameters and temperature changes (NLTR, μ = 1 nm, p = 1)



(a) β = 0.02 (b) β = 0.04



(c) β = 0.06 (d) β = 0.08

Fig. 4 Variation of phase velocity of rotating FG nanobeam versus angular velocity for various temperature changes (NLTR, μ = 1 nm, λ= 0.5 nm, p = 1)

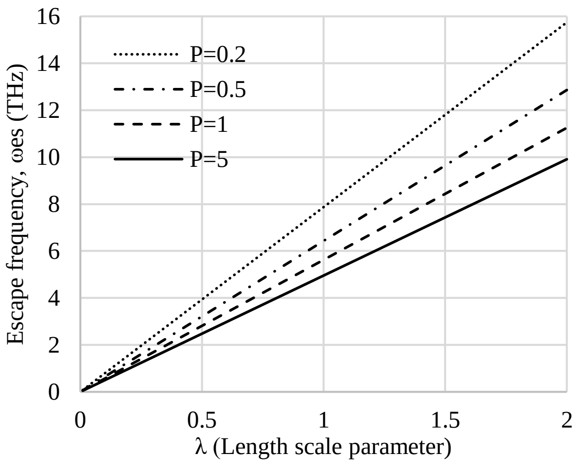


Fig. 5 Variation of escape frequency of rotating FG nanobeam versus length scale parameter for various gradient indices (μ = 1 nm, ΔT = 200 and Ω=2*10^9)

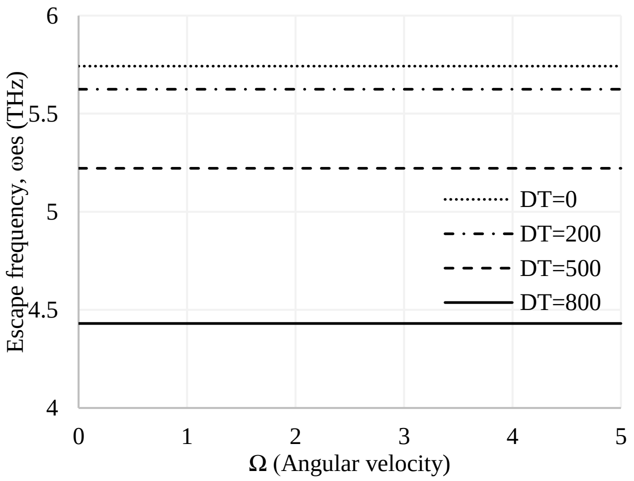


Fig. 6 Variation of escape frequency of rotating FG nanobeam versus angular velocity for various temperature changes (μ = 1 nm, λ = 0.2 nm, p = 1)

(UTR>LTR>NLTR).

In Fig. 2, variation of the phase velocity (c_p) of rotating FG nanobeam versus wave number (β) for various angular velocities (Ω) and different values of gradient indices for a constant value of nonlocality parameter (μ=1 nm), length scale parameter (λ=0.5 nm) and temperature (ΔT=800) for nonlinear temperature distribution model is plotted. It is observable that, in the lower values of wave number with an increase in wave number, the phase velocity will increase (it is not true Ω=3 and 4 for p=1 and 5). But for $\beta \geq 0.9$ (approximately), the phase velocity will decrease, then in $\beta \geq 10$ the phase velocity tends to a constant value and don't have sensible change with the increase in wave number. In addition, at a constant value of wave number, the phase velocity increases with the increase in angular velocity. However, diagram of different angular velocities in $\beta \leq 0.9$ nm are more distinguished and more observable. Thus, angular velocity of the mentioned nanobeam has no considerable influence on phase velocities at higher values of wave number. Moreover, phase velocity will decrease with increase in gradient index because of higher portion of metal phase by an increase in the gradient index.

Fig. 3 indicates the variation of phase velocity of rotating FG nanobeam versus wave number for various length scale parameters (λ=0.5 and 1.5) and temperature changes with the constant values of nonlocality parameter (μ = 1 nm) and gradient index (p = 1). It is clear that the phase velocity increases due to increase in wave number. But, for the λ=0.5 after $\beta \geq 0.8$ (approximately) the phase velocity decreases with the increase of wave number. In addition, increasing in length scale parameter leads to increase in phase velocity in higher values wave number. Also, diagrams of diagram various length scale parameter are distinguished. Moreover, for $\beta \geq 10$ the phase velocity tends to a constant value and don't change anymore. Finally, with the increase in temperature change and with constant wave number, the phase velocity will decrease.

Variation of escape frequency of rotating FG nanobeam versus length scale parameter for various gradient indices with the constant values of nonlocality parameter (μ = 1 nm), temperature change (ΔT = 200) and angular velocity (Ω=2*10^9) for the NLTR temperature distribution is plotted in Fig. 4. It is observable that, increase in length scale parameter leads to the increase of escape frequency in a linear way. Also, the escape frequency decreases due to increase in gradient index.

Variation of escape frequency of rotating FG nanobeam versus angular velocity for various temperature changes and a constant value of nonlocality parameter (μ = 1 nm), length scale parameter (λ = 0.2 nm) and gradient index (p = 1) for NLTR temperature distribution type in reported in Fig. 5. It can be seen that, escape frequency decreases due to increase in temperature change. Also, it is clear that the escape frequency has no sensible change with the increase in angular velocity.

6. Conclusions

In this paper, wave dispersion characteristics of a rotating thermos-elastic FG nanobeam is explored based on higher order refined shear deformable beam theory. Also, Mori-Tanaka distribution model and nonlocal strain gradient theory are applied. Finally, through some parametric study, the influence of different parameters such as angular velocity, gradient index, nonlocality parameter, wave number, temperature rise, and different temperature distribution on wave dispersion behavior of mentioned nanobeam is investigated. It is found that, increasing in angular velocity leads to the increase of phase velocity especially at higher values of angular velocity. However, the increase in angular velocity have no sensible effect on the escape frequency. Also, increasing in wave number causes the increase in phase velocity for lower values of wave number and angular velocity. In addition, phase velocity will increase due to increase in angular velocity, but the diagrams in lower values of wave number are distinguished. Moreover, increasing in length scale causes the increase of escape frequency, but in the constant value of length scale parameter, increasing in the gradient index leads to decreasing in escape frequency.

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