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Dynamic response analysis of a foam-based nanoscale plate based on finite strip method

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Abstract. The present article deals with a dynamic response analysis of a foam-based nanoscale plate based on finite strip method (FSM). The nanoscale plate formulation has been adopted based upon a higher order plate theory and then, a higher order finite strip has been used to solve the problem. The considered finite strip is capable of considering the bending displacement and also shear deformation effects. The foam-based material has been treated as a porous material with some particular pore distribution. The non-uniformity of strain field as well as the nonlocality of stress field have been incorporated with the usage of nonlocal strain gradient elasticity. It is clearly showen that the proposed solution based on finite strip method can accurately simulate the dynamic response of considered plate under external forces. The scale factors due to small size of the plate and foam-based material will show a remarkable impact on the dynamic response.

Keywords: dynamic response; finite strip; foams; higher-order plate; nanoscale

1. Introduction

Foams are in the category of smart and porous materials with low weight due to possessing different variations of porosities in them. Applying electric field to piezoelectric material structures yields elastic deformations and changed vibrational properties. The variation of porosities in this material causes a significant difference between metal foams and other perfect metals. In a non-perfect metal, the material characteristics are notably influenced by pore variations. Also, this variation in pores can affect the vibration frequencies of engineering structures made of metal foams. This issue can be understood from the works done by Chen *et al*. (2015, 2016). Different from metal foams, there are also functionally graded (FG) or ceramic-metal materials in which pore variation effect is very important (Ahmed *et al*. 2020a, b, Fenjan *et al*. 2020a, b, Abdulrazzaq *et al*. 2020). In this material, pores may be produced in a phase between ceramic and material. Engineering structures made of this materials are studied to understand their vibration behaviors as reported in the works of Wattanasakulpong *et al*. (2014), Yahia *et al*. (2015). This type of material is used in different structures such as beams, plates and shells. There are some studies on different structures in the literature (Singh *et al*. 2018, Singhal and Chaudhary 2019, Forsat *et al*. 2020, Mirjavadi *et al*. 2020 a-l, Barati and Shahverdi 2018a, b).

Recent studies focus on engineering structures at nano-scales due to their involvement in nano-

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mechanical systems or devices (Barati and Shahverdi 2017, Ebrahimi and Barati 2017, Barati and Zenkour 2019a, b). However, the main issue in these studies is to select an appropriate elasticity theory accounting for small scale impacts (Ebrahimi and Barati 2018a-h). The impact of size-dependency might be considered with the help of a scale parameter involved in non-local theory of elasticity Eringen (1983). The word "non-local" means that the stresses are not local anymore (Ebrahimi and Barati 2019a-d, Ebrahimi *et al*. 2019a, b, Shariati *et al*. 2020a, b, Muhammad *et al*. 2019, Kunbar *et al*. 2020). This is because we are talking about a stress field of nano-scale structure. Many authors are aware of these facts and they are using this theory to analysis mechanical characteristics of small size engineering structures (Natarajan *et al*. 2012, Elmerabet *et al*. 2017, Zenkour and Abouelregal 2015, Sobhy and Radwan 2017, Li *et al*. 2016a, Sayyad and Ghugal 2018). Related to the mechanics of porous functionally graded nano-size structures, there are some studies about their vibrations or buckling in the literature such as the paper of Mechab *et al*. (2016). These papers showed that pores inside FG material can cause extraordinary dynamic and static properties.

Strain gradients at nano-scale are observed by many researchers (Lim *et al*. 2015). Thus, nonlocal-strain gradient theory was introduced as a general theory which contains an additional strain gradient parameter together with nonlocal parameter (Li and Hu 2016, Li *et al*. 2016b, Xiao *et al*. 2017, Zhou and Li 2017). The scale parameters used in nonlocal strain gradient theory can be obtained by fitting obtained theoretical results with available experimental data and even molecular dynamic (MD).

The present article deals with a dynamic response analysis of a foam-based nanoscale plate based on finite strip method. The nanoscale plate formulation has been adopted based upon a higher order plate theory and then, a higher order finite strip has been used to solve the problem. The considered finite strip is capable of considering the bending displacement and also shear deformation effects. The foam-based material has been treated as a porous material with some particular pore distribution. The non-uniformity of strain field as well as the nonlocality of stress field have been incorporated with the usage of nonlocal strain gradient elasticity. It is clearly showen that the proposed solution based on finite strip method can accurately simulate the dynamic response of considered plate under external forces. The scale factors due to small size of the plate and foam-based material will show a remarkable impact on the dynamic response.

2. Basic formulation for nanoplates

In the well-known nonlocal strain gradient theory (Lim *et al* 2015), strain gradient impacts are taken into accounting together with nonlocal stress influences defined in below relation (Barati 2017, Barati 2018a-c)

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla \sigma_{ij}^{(1)} \tag{1}$$

in such a way that stress $\sigma_{ij}^{(0)}$ is corresponding to strain components ε_{kl} and a higher order stress is related to strain gradient components $\nabla \varepsilon_{kl}$ which are (Lim *et al*. 2015)

$$\sigma_{ij}^{(0)} = \int_V C_{ijkl}\, \alpha_0(x, x', e_0 a) \varepsilon'_{kl}(x') dx' \tag{2a}$$

$$\sigma_{ij}^{(1)} = l^2 \int_V C_{ijkl}\, \alpha_1(x, x', e_1 a) \nabla \varepsilon'_{kl}(x') dx' \tag{2b}$$

in which C_{ijkl} express the elastic properties; Also, e_0a and e_1a are corresponding to nonlocality impacts and l is related to strains gradients. Whenever two nonlocality functions $\alpha_0(x,x',e_0a)$ and $\alpha_1(x,x',e_1a)$ verify Eringen's announced conditions, NSGT constitutive relation may be written as follows

$$\begin{aligned}&[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]\sigma_{ij}\\&=C_{ijkl}[1-(e_1a)^2\nabla^2]\varepsilon_{kl}\\&-C_{ijkl}l^2[1-(e_0a)^2\nabla^2]\nabla^2\varepsilon_{kl}\end{aligned} \tag{3}$$

so that ∇^2defines the operator for Laplacian; by selecting $e_1=e_0=e$, above relationship decreases to

$$[1-(ea)^2\nabla^2]\sigma_{ij}=C_{ijkl}[1-l^2\nabla^2]\varepsilon_{kl} \tag{4}$$

3. Modeling a porous foam material

A porous material, for instance a steel foam, might be placed in the category of lightweight materials and can be applied in several structures such as sandwich panels. Often, pore variation along the thickness of panels/plates results in a notable alteration in every kind of material property. When the pore distribution inside the material is selected to be non-uniform, the metal foam might be defined as a functionally graded material since its properties obey some specified functions. Herein, the following types of pore dispersion will be employed:

• Uniform kind

$$\boldsymbol{E}=\boldsymbol{E}_2(\mathbf{1}-\boldsymbol{e}_0\boldsymbol{\chi}) \tag{5a}$$

$$\boldsymbol{G}=\boldsymbol{G}_2(\mathbf{1}-\boldsymbol{e}_0\boldsymbol{\chi}) \tag{5b}$$

$$\boldsymbol{\rho}=\boldsymbol{\rho}_2\sqrt{(\mathbf{1}-\boldsymbol{e}_0\boldsymbol{\chi})} \tag{5c}$$

• Non-uniform kind

$$\boldsymbol{E}(\boldsymbol{z})=\boldsymbol{E}_2(\mathbf{1}-\boldsymbol{e}_0\,\boldsymbol{cos}\left(\frac{\boldsymbol{\pi z}}{\boldsymbol{h}}\right)) \tag{6a}$$

$$\boldsymbol{G}(\boldsymbol{z})=\boldsymbol{G}_2(\mathbf{1}-\boldsymbol{e}_0\,\boldsymbol{cos}\left(\frac{\boldsymbol{\pi z}}{\boldsymbol{h}}\right)) \tag{6b}$$

$$\boldsymbol{\rho}(\boldsymbol{z})=\boldsymbol{\rho}_2(\mathbf{1}-\boldsymbol{e}_m\,\boldsymbol{cos}\left(\frac{\boldsymbol{\pi z}}{\boldsymbol{h}}\right)) \tag{6c}$$

The most important factors in above relations are the greatest values of material properties E_2, G_2 and ρ_2. For a piezoelectric foam all material properties (P) including elastic constants (c_{ij}), piezoelectric constants (e_{ij}) and dielectric constants (k_{ij}) can be described via the function $P=P_2(1-e_0\chi)$ for uniform porosities and $P=P_2(1-e_0\,cos\left(\frac{\pi z}{h}\right))$ for non-uniform porosities. Also, there are two important factors related to pores and mass which are e_0 and e_m as

$$\boldsymbol{e}_0=\mathbf{1}-\frac{\boldsymbol{E}_2}{\boldsymbol{E}_1}=\mathbf{1}-\frac{\boldsymbol{G}_2}{\boldsymbol{G}_1} \tag{7}$$

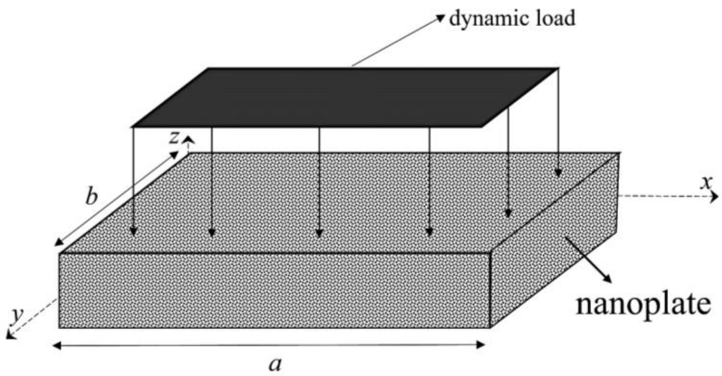

Fig. 1 Configuration of foam nanoplate under dynamical loading.

$$\boldsymbol{e_m = 1 - \frac{\rho_2}{\rho_1}} \tag{8}$$

There is a relationship between elastic modulus and mass density of porous materials with open cells as

$$\boldsymbol{\frac{E_2}{E_1} = \left(\frac{\rho_2}{\rho_1}\right)^2} \tag{9a}$$

$$\boldsymbol{e_m = 1 - \sqrt{1 - e_0}} \tag{9b}$$

Based on uniformly distributed pores, the following parameter is used in Eq. (5) as

$$\chi = \boldsymbol{\frac{1}{e_0} - \frac{1}{e_0}\left(\frac{2}{\pi}\sqrt{1 - e_0} - \frac{2}{\pi} + 1\right)^2} \tag{10}$$

By defining exact location of neutral surface, the displacement components based on axial u, lateral v, bending w_b and shear w_s displacements may be introduced as

$$\boldsymbol{u_x(x, y, z, t) = u(x, y, t) - (z - r^*)\frac{\partial w_b}{\partial x} - [\Upsilon(z) - r^{**}]\frac{\partial w_s}{\partial x}} \tag{11a}$$

$$\boldsymbol{u_y(x, y, z, t) = v(x, y, t) - (z - r^*)\frac{\partial w_b}{\partial y} - [\Upsilon(z) - r^{**}]\frac{\partial w_s}{\partial y}} \tag{11b}$$

$$\boldsymbol{u_z(x, y, z, t) = w(x, y, t) = w_b + w_s} \tag{11c}$$

so that

$$r^* = \int_{-h/2}^{h/2} E(z)z\,dz \Big/ \int_{-h/2}^{h/2} E(z)\,dz, \qquad r^{**} = \int_{-h/2}^{h/2} E(z)\Upsilon(z)dz \Big/ \int_{-h/2}^{h/2} E(z)\,dz \tag{12}$$

The shear deformation function has been selected for the nanoplate shown in Fig. 1 as

$$\Upsilon(z) = z - z[1 + \frac{3\pi}{2}sech^2(0.5)] + \frac{3\pi}{2}h\,tanh(\frac{z}{h}) \tag{13}$$

Finally, the strains based on the four-unknown plate model have been obtained as

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} - (z - r^*)\frac{\partial^2 w_b}{\partial x^2} - [\Upsilon(z) - r^{**}]\frac{\partial^2 w_s}{\partial x^2} \\
\varepsilon_y &= \frac{\partial v}{\partial y} - (z - r^*)\frac{\partial^2 w_b}{\partial y^2} - [\Upsilon(z) - r^{**}]\frac{\partial^2 w_s}{\partial y^2} \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2(z - r^*)\frac{\partial^2 w_b}{\partial x \partial y} - 2[\Upsilon(z) - r^{**}]\frac{\partial^2 w_s}{\partial x \partial y} \\
\gamma_{yz} &= g(z)\frac{\partial w_s}{\partial y}, \qquad \gamma_{xz} = g(z)\frac{\partial w_s}{\partial x}
\end{aligned} \tag{14}$$

Next, one might express the total potential energy as follows based on strain energy (U) and kinetic energy (T)

$$\Pi = U + T + V \tag{15}$$

and V is the work of non-conservative loads. Based on above relation we have

$$\begin{aligned}
U = 0.5\int_V (&\sigma_{xx}\varepsilon_{xx} + \sigma_{xx}^{(1)}\nabla\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{yy}^{(1)}\nabla\varepsilon_{yy} + \sigma_{xy}\gamma_{xy} + \sigma_{xy}^{(1)}\nabla\gamma_{xy} + \sigma_{yz}\gamma_{yz} \\
&+ \sigma_{yz}^{(1)}\nabla\gamma_{yz} + \sigma_{xz}\gamma_{xz} + \sigma_{xz}^{(1)}\nabla\gamma_{xz})dV
\end{aligned} \tag{16}$$

Placing Eq. (14) in Eq. (16) leads to

$$\begin{aligned}
U = 0.5\int_0^a\int_0^b [&N_{xx}[\frac{\partial u}{\partial x}] - M_{xx}^b\frac{\partial^2 w_b}{\partial x^2} - M_{xx}^s\frac{\partial^2 w_s}{\partial x^2} + N_{yy}[\frac{\partial v}{\partial y}] - M_{yy}^b\frac{\partial^2 w_b}{\partial y^2} \\
-M_{yy}^s\frac{\partial^2 w_s}{\partial y^2} &+ N_{xy}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) - 2M_{xy}^b\frac{\partial^2 w_b}{\partial x \partial y} - 2M_{xy}^s\frac{\partial^2 w_s}{\partial x \partial y} + Q_{yz}\frac{\partial w_s}{\partial y} + Q_{xz}\frac{\partial w_s}{\partial x}]dydx
\end{aligned} \tag{17}$$

in which

$$\begin{aligned}
N_{xx} &= \int_{-h/2}^{h/2}(\sigma_{xx}^0 - \nabla\sigma_{xx}^{(1)})dz = N_{xx}^{(0)} - \nabla N_{xx}^{(1)} \\
N_{xy} &= \int_{-h/2}^{h/2}(\sigma_{xy}^0 - \nabla\sigma_{xy}^{(1)})dz = N_{xy}^{(0)} - \nabla N_{xy}^{(1)} \\
N_{yy} &= \int_{-h/2}^{h/2}(\sigma_{yy}^0 - \nabla\sigma_{yy}^{(1)})dz = N_{yy}^{(0)} - \nabla N_{yy}^{(1)} \\
M_{xx}^b &= \int_{-h/2}^{h/2}z(\sigma_{xx}^0 - \nabla\sigma_{xx}^{(1)})dz = M_{xx}^{b(0)} - \nabla M_{xx}^{b(1)} \\
M_{xx}^s &= \int_{-h/2}^{h/2}\Upsilon(\sigma_{xx}^0 - \nabla\sigma_{xx}^{(1)})dz = M_{xx}^{s(0)} - \nabla M_{xx}^{s(1)} \\
M_{yy}^b &= \int_{-h/2}^{h/2}z(\sigma_{yy}^0 - \nabla\sigma_{yy}^{(1)})dz = M_{yy}^{b(0)} - \nabla M_{yy}^{b(1)} \\
M_{yy}^s &= \int_{-h/2}^{h/2}f(\sigma_{yy}^0 - \nabla\sigma_{yy}^{(1)})dz = M_{yy}^{s(0)} - \nabla M_{yy}^{s(1)}
\end{aligned}$$

$$M_{xy}^{b} = \int_{-\hbar/2}^{\hbar/2} z(\sigma_{xy}^{0} - \nabla\sigma_{xy}^{(1)})dz = M_{xy}^{b(0)} - \nabla M_{xy}^{b(1)}$$
$$M_{xy}^{s} = \int_{-\hbar/2}^{\hbar/2} f(\sigma_{xy}^{0} - \nabla\sigma_{xy}^{(1)})dz = M_{xy}^{s(0)} - \nabla M_{xy}^{s(1)}$$
$$Q_{xz} = \int_{-\hbar/2}^{\hbar/2} g(\sigma_{xz}^{0} - \nabla\sigma_{xz}^{(1)})dz = Q_{xz}^{(0)} - \nabla Q_{xz}^{(1)} \tag{18}$$
$$Q_{yz} = \int_{-\hbar/2}^{\hbar/2} g(\sigma_{yz}^{0} - \nabla\sigma_{yz}^{(1)})dz = Q_{yz}^{(0)} - \nabla Q_{yz}^{(1)}$$

where

$$N_{ij}^{(0)} = \int_{-\hbar/2}^{\hbar/2} (\sigma_{ij}^{(0)})dz,$$
$$N_{ij}^{(1)} = \int_{-\hbar/2}^{\hbar/2} (\sigma_{ij}^{(1)})dz$$
$$M_{ij}^{b(0)} = \int_{-\hbar/2}^{\hbar/2} z(\sigma_{ij}^{b(0)})dz,$$
$$M_{ij}^{b(1)} = \int_{-\hbar/2}^{\hbar/2} z(\sigma_{ij}^{b(1)})dz$$
$$M_{ij}^{s(0)} = \int_{-\hbar/2}^{\hbar/2} \Upsilon(\sigma_{ij}^{s(0)})dz,$$
$$M_{ij}^{s(1)} = \int_{-\hbar/2}^{\hbar/2} \Upsilon(\sigma_{ij}^{s(1)})dz \tag{19}$$
$$Q_{xz}^{(0)} = \int_{-\hbar/2}^{\hbar/2} g(\sigma_{xz}^{i(0)})dz,$$
$$Q_{xz}^{(1)} = \int_{-\hbar/2}^{\hbar/2} g(\sigma_{xz}^{i(1)})dz$$
$$Q_{yz}^{(0)} = \int_{-\hbar/2}^{\hbar/2} g(\sigma_{yz}^{i(0)})dz,$$
$$Q_{yz}^{(1)} = \int_{-\hbar/2}^{\hbar/2} g(\sigma_{yz}^{i(1)})dz$$

for which (*ij*=*xx*, *xy*, *yy*). The work of non-conservative force is expressed by

$$V = \int_0^a \int_0^b (N_x^0 \frac{\partial^2 (w_b + w_s)}{\partial x^2} + N_y^0 \frac{\partial^2 (w_b + w_s)}{\partial y^2}$$
$$+2N_{xy}^0 \frac{\partial^2 (w_b + w_s)}{\partial x \partial y} + (q_{dynamic})(w_b + w_s) \tag{20}$$
$$-(N^T)(\frac{\partial^2 (w_b + w_s)}{\partial x^2} + \frac{\partial^2 (w_b + w_s)}{\partial y^2}))dydx$$

where N_x^0, N_y^0, N_{xy}^0 denote membrane forces; N^T is the load due to thermal environment. Moreover, $q_{dynamic}$ is the applied force from periodic mechanical loading. Also, the kinetic energy is obtained as

$$K = 0.5\int_0^a \int_0^b [m_0(\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 v}{\partial t^2} + \frac{\partial^2 (w_b + w_s)}{\partial t^2}) - m_1(2\frac{\partial u}{\partial t}\frac{\partial w_b}{\partial x\partial t} + 2\frac{\partial v}{\partial t}\frac{\partial w_b}{\partial y\partial t}) -m_2(2\frac{\partial u}{\partial t}\frac{\partial w_s}{\partial x\partial t} + 2\frac{\partial v}{\partial t}\frac{\partial w_s}{\partial y\partial t}) + m_3(2\frac{\partial w_b}{\partial x\partial t}\frac{\partial w_b}{\partial x\partial t}) + m_4(2\frac{\partial w_s}{\partial x\partial t}\frac{\partial w_s}{\partial x\partial t}) +m_5(2\frac{\partial w_b}{\partial x\partial t}\frac{\partial w_s}{\partial x\partial t} + 2\frac{\partial w_b}{\partial y\partial t}\frac{\partial w_s}{\partial y\partial t})]dydx \tag{21}$$

in which

$$(m_0, m_1, m_2, m_3, m_4, m_5) = \int_{-h/2}^{h/2} \begin{matrix}(1, z - r^*, (z - r^*)^2, \Upsilon - z^{**}, \\ (z - r^*)(\Upsilon - r^{**}), (\Upsilon - r^{**})^2)\rho\end{matrix} (z)dz \tag{22}$$

the thermal load can be defined as $N^T = \int_{-h/2}^{h/2} E\alpha\Delta T dz$.

Finally, the nonlocal strain gradient constitutive relations based on refined FG plate model for metal foam plate can be expressed by

$$(1 - \mu\nabla^2)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix} = \frac{E(z)}{1-v^2}(1 - \lambda\nabla^2) * \begin{pmatrix}1 & v & 0 & 0 & 0\\ v & 1 & 0 & 0 & 0\\ 0 & 0 & (1-v)/2 & 0 & 0\\ 0 & 0 & 0 & (1-v)/2 & 0\\ 0 & 0 & 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{23}$$

After integrating Eq. (24) in thickness direction, we get the following relationships

$$(1 - \mu\nabla^2)\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} = A(1 - \lambda\nabla^2) * \begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\end{Bmatrix} \tag{24}$$

$$(1 - \mu\nabla^2)\begin{Bmatrix}M_x^b\\ M_y^b\\ M_{xy}^b\end{Bmatrix} = D(1 - \lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\frac{\partial^2 w_b}{\partial x^2}\\ -\frac{\partial^2 w_b}{\partial y^2}\\ -2\frac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix}$$

$$+E(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix} \quad (25)$$

$$(1-\mu\nabla^2)\begin{Bmatrix}M_x^s\\ M_y^s\\ M_{xy}^s\end{Bmatrix}=E(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix}$$
$$+F(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix} \quad (26)$$

$$(1-\mu\nabla^2)\begin{Bmatrix}Q_x\\ Q_y\end{Bmatrix}=A_{44}(1-\lambda\nabla^2)*\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{Bmatrix}\dfrac{\partial w_s}{\partial x}\\ \dfrac{\partial w_s}{\partial y}\end{Bmatrix} \quad (27)$$

in which

$$\begin{aligned}
A &= \int_{-h/2}^{h/2}\frac{E(z)}{1-v^2}dz, D=\int_{-h/2}^{h/2}\frac{E(z)(z-r^*)^2}{1-v^2}dz,\\
E &= \int_{-h/2}^{h/2}\frac{E(z)(z-r^*)(\Upsilon-r^{**})}{1-v^2}dz\\
F &= \int_{-h/2}^{h/2}\frac{E(z)(\Upsilon-r^{**})^2}{1-v^2}dz,\\
A_{44} &= \int_{-h/2}^{h/2}\frac{E(z)}{2(1+v)}g^2\,dz
\end{aligned} \quad (28)$$

4. Finite Strip Method (FSM)

In this section, the finite strip method based upon proposed refined plate model, which is known as refined finite strip method, has been used for investigating the transient vibrational behavior of GOP-reinforced plate. Fig. 2 illustrates a single strip having length a_s and width b_s with two nodal

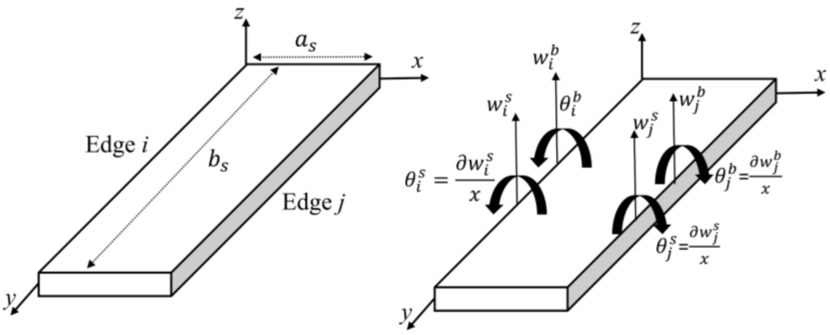

Fig. 2 Refined finite strip and nodal coordinates

lines of i and j. This figure also shows the strip nodal degrees of freedom. As the first step, the bending and shear displacements of the strip can be introduced by

$$w_b = \sum_{e=1}^{r} f_e^{w_b}(x)\, g_e^{w_b}(y) \tag{29}$$

$$w_s = \sum_{e=1}^{r} f_e^{w_s}(x)\, g_e^{w_s}(y) \tag{30}$$

where r defines the number of harmonic modes, $f_e^{w_b}(x)$ and $f_e^{w_s}(x)$ are appropriate Hermitian shape functions; $g_e^{w_b}(y)$ and $g_e^{w_s}(y)$ are trigonometric functions satisfying boundary conditions at y direction.

Based on Hermitian shape functions, Eqs. (29) and (30) can be re-written as

$$w_b = \sum_{e=1}^{r} \begin{bmatrix} (1-3\zeta^2+2\zeta^3)(w_i^b)_e \\ +a_s(\zeta-2\zeta^2+\zeta^3)(\theta_i^b)_e \\ +(3\zeta^2-2\zeta^3)(w_j^b)_e \\ +a_s(-\zeta^2+\zeta^3)(\theta_j^b)_e \end{bmatrix} g_e^{w_b}(y) \tag{31}$$

$$w_s = \sum_{e=1}^{r} \begin{bmatrix} (1-3\zeta^2+2\zeta^3)(w_i^s)_e \\ +a_s(\zeta-2\zeta^2+\zeta^3)(\theta_i^s)_e \\ +(3\zeta^2-2\zeta^3)(w_j^s)_e \\ +a_s(-\zeta^2+\zeta^3)(\theta_j^s)_e \end{bmatrix} g_e^{w_s}(y) \tag{32}$$

in which $\zeta = x/a_s$ and $\{w_i^b\ \theta_i^b\ w_j^b\ \theta_j^b\}$ are bending degrees of freedom of each nodal line, whereas $\{w_i^s\ \theta_i^s\ w_j^s\ \theta_j^s\}$ are the shear degrees of freedom. Eqs. (31) and (32) can then be re-written in vector forms as

$$w_b = \sum_{e=1}^{r} N_e^b\, \delta_e^b \tag{33}$$

$$w_s = \sum_{e=1}^{r} N_e^s \, \delta_e^s \tag{34}$$

where

$$N_e^b = N_e^s = [1 - 3\zeta^2 + 2\zeta^3 \;\; a_s(\zeta - 2\zeta^2 + \zeta^3) \;\; 3\zeta^2 - 2\zeta^3 \;\; a_s(-\zeta^2 + \zeta^3)] g_e^{w_b}(y) \tag{35}$$

Also, δ_e^b and δ_e^s are the displacement vectors related to mode e and has the following form

$$\delta_e^b = \left\{ w_i^b \; \theta_i^b \; w_j^b \; \theta_j^b \right\}_e^T = \begin{Bmatrix} \left(\delta_i^b\right)_e \\ \left(\delta_j^b\right)_e \end{Bmatrix} \tag{36}$$

$$\delta_e^s = \left\{ w_i^s \; \theta_i^s \; w_j^s \; \theta_j^s \right\}_e^T = \begin{Bmatrix} (\delta_i^s)_e \\ \left(\delta_j^s\right)_e \end{Bmatrix} \tag{37}$$

Then, the complete displacement vector for a strip can be represented as

$$\delta = \left\{ w_i^b \; \theta_i^b \; w_j^b \; \theta_j^b \; w_i^s \; \theta_i^s \; w_j^s \; \theta_j^s \right\}_e^T = \begin{Bmatrix} \delta_i^b \\ \delta_j^b \\ \delta_i^s \\ \delta_j^s \end{Bmatrix} \tag{38}$$

Using the developed total potential energy and minimizing it to field coefficients results in below relation containing simultaneous algebraic equations

$$\frac{\partial \Pi}{\partial \delta} = 0 \tag{39}$$

Introducing Eqs. (33)-(34) to obtained relations yields

$$[K]\{\delta\} + [M]\{\ddot{\delta}\} = \{F(t)\} \tag{40}$$

Then, it is possible to express obtained boundary conditions as

$$\begin{aligned} & w_b = w_s = 0, \\ & \frac{\partial^2 w_b}{\partial x^2} = \frac{\partial^2 w_s}{\partial x^2} = \frac{\partial^2 w_b}{\partial y^2} = \frac{\partial^2 w_s}{\partial y^2} = 0 \\ & \frac{\partial^4 w_b}{\partial x^4} = \frac{\partial^4 w_s}{\partial x^4} = \frac{\partial^4 w_b}{\partial y^4} = \frac{\partial^4 w_s}{\partial y^4} = 0 \end{aligned} \tag{41}$$

The presented results are based on the below dimensionless factors

$$\begin{aligned} & \mu = \frac{ea}{a}, \lambda = \frac{l}{a} \\ & \Omega = \omega_{ex} a \sqrt{\frac{\rho_2}{E_2}} \end{aligned} \tag{42}$$

Table 1 Verification study on normalized vibrational frequencies of nano-scale graded plates with various nonlocal factors

a/h	μ	*a/b*=1		*a/b*=2	
		Natarajan *et al*. (2012)	present	Natarajan *et al*. (2012)	present
10	0	0.0441	0.0439	0.1055	0.1044
	1	0.0403	0.0401	0.0863	0.0856
	2	0.0374	0.0372	0.0748	0.0742
	4	0.0330	0.0329	0.0612	0.0605
20	0	0.0113	0.0113	0.0279	0.0277
	1	0.0103	0.0102	0.0229	0.0228
	2	0.0096	0.0095	0.0198	0.0197
	4	0.0085	0.008418	0.0162	0.016111

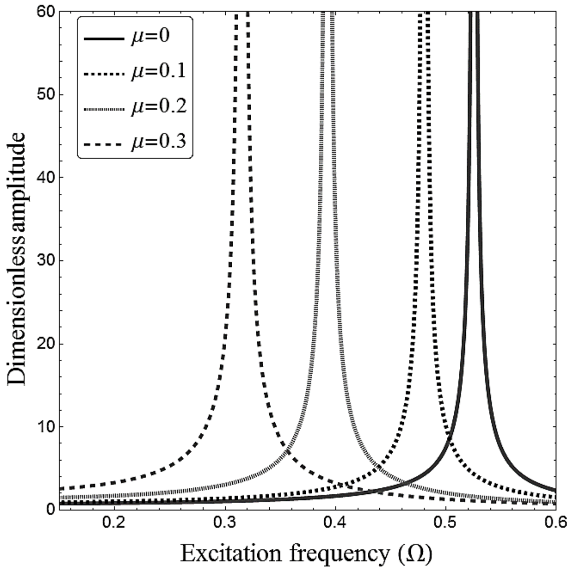

Fig. 3 Effect of nonlocal and strain gradient factors on response curves of nano-size plate (λ=0, *a/h*=10, e_0=0.5)

5. Discussions on results

Thorough the present section, results are provided for forced vibration investigation of scale-dependent foam plates formulated by a four-unknown plate theory and NSGT. The nano-size foam plate under a periodic dynamical loading has been depicted in Fig.1. Table 1 provides validation study for vibrational frequency of a functionally graded small scale plate with the results obtained by Natarajan *et al*. (2012). Accordingly, the present formulation and finite strip solution is capable of giving accurate results of nanoplates. In this research, obtained results based on metal foam material are presented using the below properties:

• $E_2 = 200\ GPa, \rho_2 = 7850\ kg/m^3, v = 0.33,$

In Figs. 3 and 4, the variation of normalized deflections of a metal foam nano-dimension plate versus excitation frequency of mechanical loading is represented for several nonlocality (μ) and

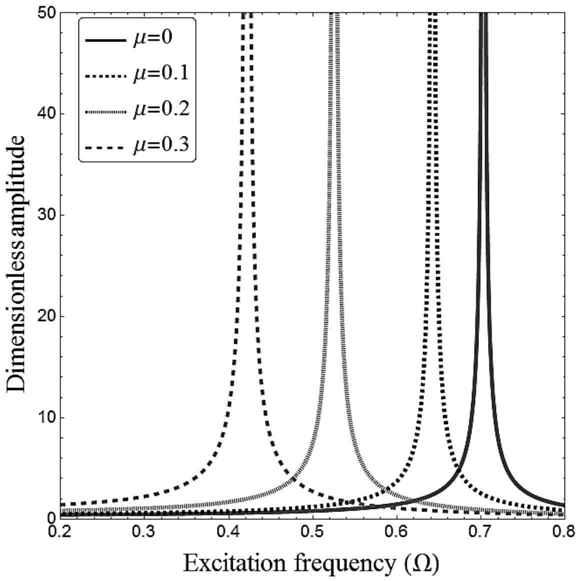

Fig. 4 Effect of nonlocal and strain gradient factors on response curves of nano-size plate (λ=0.2, a/h=10, e_0=0.5)

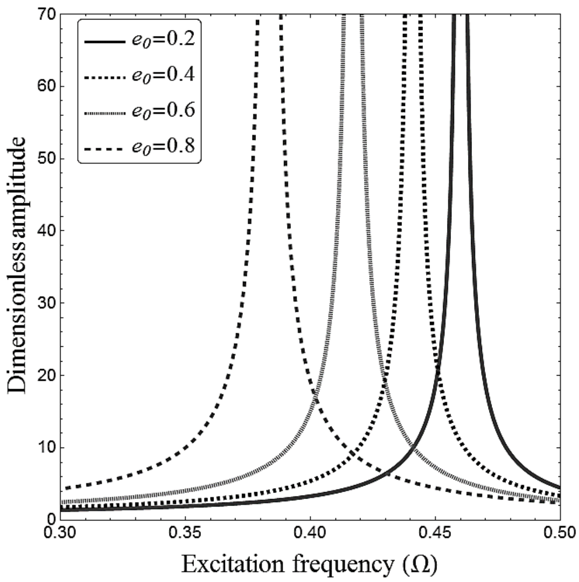

Fig. 5 Effect of pore factor with uniform distribution on response curves of the nano-size metal foam plate (μ=0.2, λ=0.1, a=10h)

stain gradients (λ) coefficients when a/h=10. By selecting $\mu=\lambda=0$, the deflections and vibrational frequencies based upon classic plate assumption will be derived. Actually, selecting λ=0 gives the deflections in the context of nonlocal elasticity theory and discarding strain gradients impacts. Exerting higher values of excitation frequency leads to larger deflections and finally resonance of the plate. It can be understood from Figs. 3 and 4 that normalized deflection of system will reduce

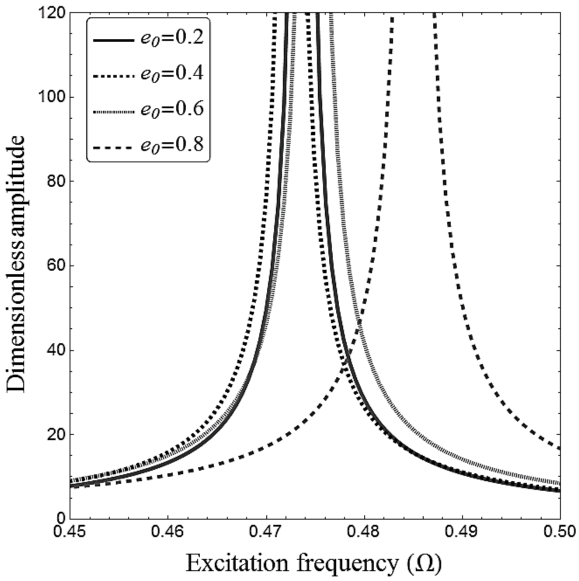

Fig. 6 Effect of pore factor with non-uniform distribution on response curves of the nano-size metal foam plate (μ=0.2, λ=0.1, a=10h)

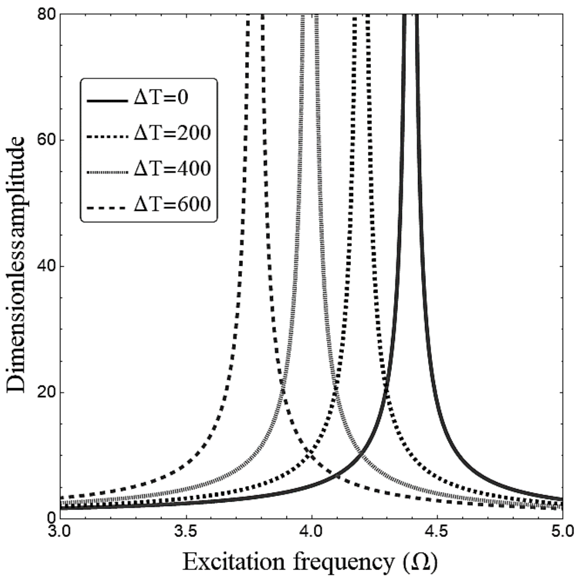

Fig. 7 Effect of thermal environments on response curves of nano-size plate (a/h=20, μ=0.2, e_0=0.2)

with strain gradient coefficient and will rise with nonlocality coefficient. This observation is valid for excitation frequencies before resonance. So, forced vibration behavior of the nanoplate system is dependent on both scale effects. An important finding is that the resonance frequencies of metal foam plate are outstandingly affected by the values of nonlocal and strain gradient coefficients.

In Figs. 5 and 6 one can see the response curves of metal foam plate system with different porosity coefficients and dispersions. Effect of surrounding medium is neglected for this figure. It can be understood from the figures that resonance frequency of system will reduce or increase with pore coefficient. But, this variation relies on the type of pore dispersion in thickness of nanoplates. Uniform pore type gives higher resonance frequencies than other pore types.

Thermal effects on dynamic response of a porous PZT nanoplate has been plotted in Fig. 7 assuming that nonlocal factor is μ=0.2. Actually, this figure shows nanoplate center deflection against applied frequency of excitation. At first step, nanoplate center deflection (amplitude) is increasing with the growth of plied frequency of excitation. After the shift frequent (at which the center deflection is infinitive), the center deflection will reduce. Thermal environment has an important impact on the frequency curves and the location of shift frequency. Note that increase of temperature will reduce the magnitude of shift frequency because of the reduced stiffness of the nano-sized plate.

6. Conclusions

In this research, dynamic responses of a porous nano-sized plate modeled by a nonlocal higher-order refined plate model were explored in detail. The porous material considered in this research had uniform or non-uniform porosity distribution across the cross section. Stain gradient effects were also considered for more accurate modeling of the scale-dependent plate. It was realized that resonance vibration frequency of system raised with strain gradient coefficient and reduced with nonlocality coefficient. It was also found that resonance vibration frequency and dynamic deflection of system might reduce or increase with pore coefficient. Also, uniform pore type gave highest resonance frequency among considered pore types. The resonance frequency is also declined with the rise of temperature.

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