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Rayleigh waves in anisotropic magnetothermoelastic medium

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Abstract. The present paper is concerned with the investigation of Rayleigh waves in a homogeneous transversely isotropic magnetothermoelastic medium with two temperature, in the presence of Hall current and rotation. The formulation is applied to the thermoelasticity theories developed by Green-Naghdi theories of Type-II and Type-III. Secular equations are derived mathematically at the stress free and thermally insulated boundaries. The values of Determinant of secular equations, phase velocity and Attenuation coefficient with respect to wave number are computed numerically. Cobalt material has been chosen for transversely isotropic medium and magnesium material is chosen for isotropic solid. The effects of rotation, magnetic field and phase velocity on the resulting quantities and on particular case of isotropic solid are depicted graphically. Some special cases are also deduced from the present investigation.

Keywords: transversely isotropic thermoelastic; phase velocity; attenuation coefficient; rotation; hall current; secular equations

1. Introduction

Rayleigh waves are always generated when a free surface exists in a continuous body. Rayleigh (1885) firstly introduced them as solution of the free vibration problem for an elastic half space (On waves propagated on the plane surface of an elastic solid). Rayleigh waves play an important role in the study of earthquakes, seismology, geo-physics and geodynamics. During earthquake, Rayleigh waves play more drastic role than other seismic waves because these waves are responsible for destruction of buildings, plants and loss of human lives etc. Lockett (1958) studied the problem of Rayleigh waves in thermoelastic medium. Propagation of Rayleigh waves alongwith isothermal and insulated boundaries was discussed by Chadwick and Windle (1964). Kumar and Kansal (2010) presented the problem of Rayleigh waves in an isotropic generalized thermoelastic diffusive half space medium. Kumar and Gupta (2015) investigated Rayleigh waves in generalized thermoelastic medium with mass diffusion. Recently influence of new parameters on surface waves has been investigated by many researchers (Ahmed and Abo-Dahab 2012, Abo-Dahab 2015, Abd-Alla *et al.* 2015, Kakkar and Kakkar 2016, Abo-Dahab *et al.* 2016).

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Chen and Gurtin (1968), Chen *et al*. (1968) and Chen *et al.* (1969) have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins 1962).

Green and Naghdi (1993) postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical thermoelastic model (based on Fourier's law). The linearised version of model-II and III permit propagation of thermal waves at finite speed. Green-Naghdi's second model (GN-II), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy (1993). In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green-Naghdi's third model (GN-III) admits dissipation of energy. In this model the constitutive equations are [i]derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient, are among the constitutive variables. Green and Naghdi (1992) included the derivation of a complete set of governing equations of a linearised version of the theory for homogeneous and isotropic materials in terms of the displacement and temperature fields and a proof of the uniqueness of the solution for the corresponding initial boundary value problem.

A comprehensive work has been done in thermoelasticity theory with and without energy dissipation and thermoelasticity with two temperature. Youssef (2013), constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Quintanilla (2002) investigated thermoelasticity without energy dissipation of materials with microstructure. Several researchers studied various problems involving two temperature e.g., (Youssef and AI-Lehaibi 2007, Youssef 2011, Youssef 2006, Kaushal *et al.* 2011, Ezzat and Awad 2010, Sharma and Marin 2013, Sharma and Bhargav 2014, Sharma *et al*. 2013, Sharma and Kumar 2013).

When the magnetic field is very strong, the conductivity will be a tensor and the effect of Hall current and rotation cannot be neglected. (Zakaria 2014, Ezzat and Bary 2016, Hosseini and Dini 2016, Marin 2010, 1996, Karamany and Ezzat 2014, 2015, 2016) considered various problems due to Hall current, magnetic field and rotation.

In this paper, propagation of Rayleigh waves in a transversely isotropic thermoelastic solid with the combined effects of Hall current, rotation and two temperature has been investigated. Secular equations are derived mathematically at the stress free and thermally insulated boundaries. The values of determinant of secular equations, phase velocity and Attenuation coefficient with respect to wave number are computed numerically and effect of various quantities on the resulting quantities are shown graphically.

2. Basic equations

Following Kumar *et al*. (2016), the constitutive relations for a transversely isotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T \tag{1}$$

Equation of motion for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega n$, where n is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i \ = \ \rho\{\ddot{u}_i + \big(\Omega \times (\Omega \times u)\big)_i + (2\Omega \times \dot{u})_i \tag{2}$$

Following Chandrasekharaiah (1998) and Youssef (2006), the heat conduction equation with two temperature and with and without energy dissipation is given by

$$K_{ij}\varphi_{,ij} + K_{IJ}^{*}\dot{\varphi}_{ij} = \beta_{ij}T_0\dot{e_{ij}} + \rho C_E\ddot{T} \tag{3}$$

The above equations are supplemented by generalized Ohm's law for media with finite conductivity and including the Hall current effect

$$\boldsymbol{J} = \frac{\sigma_0}{1+m^2}\left(\boldsymbol{E} + \mu_0\left(\dot{\boldsymbol{u}} \times \boldsymbol{H} - \frac{1}{en_e}\boldsymbol{J} \times \boldsymbol{H}_0\right)\right) \tag{4}$$

and the strain displacement relations are

$$e_{ij} = \frac{1}{2}\big(u_{i,j} + u_{j,i}\big) i,j = 1,2,3 \tag{5}$$

$F_i = \mu_0(\boldsymbol{J} \times \boldsymbol{H}_0)_i, \beta_{ij} = C_{ijkl}\alpha_{ij}$ and $T = \varphi - a_{ij}\varphi_{,ij}$
$\beta_{ij} = \beta_i\delta_{ij},\ K_{ij} = K_i\delta_{ij}\ ,\ K_{ij}{}^{*} = K_i{}^{*}\delta_{ij}\ ,\ i$ is not summed
where, F_i are the components of Lorentz force, $C_{ijkl}(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the thermal conductivity, K_{IJ}^{*} is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion, Ω is the angular velocity of the solid, H is the magnetic strength, $\dot{\boldsymbol{u}}$ is the velocity vector, **E** is the intensity vector of the electric field, $\boldsymbol{J}$ is the current density vector, $m(= \omega_e t_e = \frac{\sigma_0\mu_0 H_0}{en_e})$ is the Hall parameter, t_e is the electron collision time, $\omega_e = \frac{e\mu_0 H_0}{m_e}$ is the electronic frequency, e is the charge of an electron, m_e is the mass of the electron, $\sigma_0 = \frac{e^2 t_e n_e}{m_e}$, is the electrical conductivity and n_e is the number of density of electrons.

3. Formulation and solution of the problem

We consider a homogeneous perfectly conducting transversely isotropic thermoelastic medium which is rotating uniformly with an angular velocity $\boldsymbol{\Omega}$ initially at uniform temperature T_0. The rectangular Cartesian co-ordinate system (u_1, u_2, u_3) having origin on the surface (x_3=0) with x_3-axis pointing vertically downwards into the medium is introduced. The surface of the half-space is subjected to thermomechanical sources. For two dimensional problem in *xz*-plane, we take

$$\boldsymbol{u} = (u_1, 0, u_3). \tag{6}$$

We also assume that

$$\boldsymbol{E}=0,\ \boldsymbol{\Omega}=(0,\Omega,0). \tag{7}$$

The generalized Ohm's law

$$J_2=0 \tag{8}$$

the current density components J_1 and J_3 using Eq. (4) are given as

$$J_1=\frac{\sigma_0\mu_0H_0}{1+m^2}\left(m\frac{\partial u_1}{\partial t}-\frac{\partial u_3}{\partial t}\right) \tag{9}$$

$$J_3=\frac{\sigma_0\mu_0H_0}{1+m^2}\left(\frac{\partial u_1}{\partial t}+m\frac{\partial u_3}{\partial t}\right) \tag{10}$$

Following Slaughter (2002), using appropriate transformations

$x_1'=x_1\cos\psi^*+x_2\sin\psi^*$, $x_2'=-x_1\sin\psi^*+x_2\cos\psi^*$, $x_3'=x_3$, on the set of Eqs. (2) and (3) and with the aid of Eqs. (6)-(10), we obtain the equations for transversely isotropic thermoelastic solid as

$$c_{11}\frac{\partial^2u_1}{\partial x^2}+c_{13}\frac{\partial^2u_3}{\partial x_1\partial x_3}+c_{44}\left(\frac{\partial^2u_1}{\partial {x_3}^2}+\frac{\partial^2u_3}{\partial x_1\partial x_3}\right)-\beta_1\frac{\partial}{\partial x_1}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial {x_1}^2}+a_3\frac{\partial^2\varphi}{\partial {x_3}^2}\right)\right\}$$
$$-\mu_0J_3H_0=\rho\left(\frac{\partial^2u_1}{\partial t^2}-\Omega^2u_1+2\Omega\frac{\partial u_3}{\partial t}\right) \tag{11}$$

$$(c_{13}+c_{44})\frac{\partial^2u_1}{\partial x_1\partial x_3}+c_{44}\frac{\partial^2u_3}{\partial {x_1}^2}+c_{33}\frac{\partial^2u_3}{\partial {x_3}^2}-\beta_3\frac{\partial}{\partial x_3}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial {x_1}^2}+a_3\frac{\partial^2\varphi}{\partial {x_3}^2}\right)\right\}$$
$$+\mu_0J_1H_0=\rho\left(\frac{\partial^2u_3}{\partial t^2}-\Omega^2u_3-2\Omega\frac{\partial u_1}{\partial t}\right) \tag{12}$$

$$\left(k_1+{k_1}^*\frac{\partial}{\partial t}\right)\frac{\partial^2\varphi}{\partial {x_1}^2}+\left(k_3+{k_3}^*\frac{\partial}{\partial t}\right)\frac{\partial^2\varphi}{\partial {x_3}^2}=T_0\frac{\partial^2}{\partial t^2}\left\{\beta_1\frac{\partial u_1}{\partial x_1}+\beta_3\frac{\partial u_3}{\partial x_3}\right\}+\rho C_E\ddot{T} \tag{13}$$

and

$$t_{11}=c_{11}e_{11}+c_{13}e_{33}-\beta_1T \tag{14}$$

$$t_{33}=c_{13}e_{11}+c_{33}e_{33}-\beta_3T \tag{15}$$

$$t_{13}=2c_{44}e_{13}. \tag{16}$$

where

$$T=\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x_1^2}+a_3\frac{\partial^2\varphi}{\partial x_3^2}\right)$$

$$\beta_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_3\ ,\qquad \beta_3=2c_{13}\alpha_1+c_{33}\alpha_3$$

In the above equations we use the contracting subscript notations $(1\to 11,2\to 22,3\to 33,4\to$

$23{,}5 \rightarrow 31{,}6 \rightarrow 12$) to relate $c_{ijkl} to\ c_{mn}$.

We assume that medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have the initial and regularity conditions are given by

$$\begin{gathered} u_1(x_1,x_3,0) = 0 = \dot{u_1}(x_1,x_3,0) \\ u_3(x_1,x_3,0) = 0 = \dot{u_3}(x_1,x_3,0) \\ \varphi(x_1,x_3,0) = 0 = \dot{\varphi}(x_1,x_3,0) \text{ For } x_3 \geq 0, \quad -\infty < x_1 < \infty \\ u_1(x_1,x_3,t) = u_3(x_1,x_3,t) = \varphi(x_1,x_3,t) = 0 \ For\ t > 0 \quad when\ x_3 \rightarrow \infty \end{gathered} \tag{17}$$

To facilitate the solution, following dimensionless quantities are introduced

$$\begin{gathered} x_1{}' = \frac{x_1}{L},\ x_3{}' = \frac{x_3}{L},\ u_1{}' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1,\ u_3' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3,\ T' = \frac{T}{T_0},\ t' = \frac{c_1}{L}t,\ t_{11}' = \frac{t_{11}}{\beta_1 T_0}, \\ J' = \frac{\rho c_1^2}{\beta_1 T_0}J,\ t_{33}' = \frac{t_{33}}{\beta_1 T_0},\ t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \varphi' = \frac{\varphi}{T_0}, a_1' = \frac{a_1}{L}, a_3' = \frac{a_3}{L},\ h' = \frac{h}{H_0},\quad M = \frac{\sigma_0\mu_0 H_0}{\rho c_1 L}, \\ \Omega' = \frac{L}{c_1}\Omega \end{gathered} \tag{18}$$

Making use of Eq. (18) in Eqs. (11)-(13), after suppressing the primes, yield

$$\begin{gathered} \frac{\partial^2 u_1}{\partial x_1{}^2} + \delta_4\frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \delta_2\left(\frac{\partial^2 u_1}{\partial x_3{}^2} + \frac{\partial^2 u_3}{\partial x_1\partial x_3}\right) - \frac{\partial}{\partial x_1}\left\{\varphi - \left(\frac{a_1}{L}\frac{\partial^2\varphi}{\partial x_1{}^2} + \frac{a_3}{L}\frac{\partial^2\varphi}{\partial x_3{}^2}\right)\right\} \\ -\frac{M}{1+m^2}\mu_0 H_0\left(\frac{\partial u_1}{\partial t} + m\frac{\partial u_3}{\partial t}\right) = \frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + \quad 2\Omega\frac{\partial u_3}{\partial t} \end{gathered} \tag{19}$$

$$\begin{gathered} \delta_1\frac{\partial^2 u_1}{\partial x_1\partial x_3} + \delta_2\frac{\partial^2 u_3}{\partial x_1{}^2} + \delta_3\frac{\partial^2 u_3}{\partial x_3{}^2} - \frac{\beta_3}{\beta_1}\frac{\partial}{\partial x_3}\left\{\varphi - \left(\frac{a_1}{L}\frac{\partial^2\varphi}{\partial x_1{}^2} + \frac{a_3}{L}\frac{\partial^2\varphi}{\partial x_3{}^2}\right)\right\} \\ +\frac{M}{1+m^2}\mu_0 H_0\left(m\frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t}\right) = \frac{\partial^2 u_3}{\partial t^2} - \ \Omega^2 u_3 - 2\Omega\frac{\partial u_1}{\partial t} \end{gathered} \tag{20}$$

$$\begin{gathered} \varepsilon_1\left(1 + \frac{\varepsilon_3}{\varepsilon_1}\frac{\partial}{\partial t}\right)\frac{\partial^2\varphi}{\partial x_1{}^2} + \varepsilon_2\left(1 + \frac{\varepsilon_4}{\varepsilon_2}\frac{\partial}{\partial t}\right)\frac{\partial^2\varphi}{\partial x_3{}^2} = \varepsilon_5{}'\beta_1{}^2\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_1}{\partial x_1} + \frac{\beta_3}{\beta_1}\frac{\partial u_3}{\partial x_3}\right) \\ + \frac{\partial^2}{\partial t^2}\left(\left\{\varphi - \frac{a_1}{L}\frac{\partial^2\varphi}{\partial x_1{}^2} + \frac{a_3}{L}\frac{\partial^2\varphi}{\partial x_3{}^2}\right\}\right) \end{gathered} \tag{21}$$

$\delta_1 = \frac{(c_{13}+c_{44})}{c_{11}}$, $\delta_2 = \frac{c_{44}}{c_{11}}$, $\delta_3 = \frac{c_{33}}{c_{11}}$, $\delta_4 = \frac{c_{13}}{c_{11}}$, $\varepsilon_1 = \frac{k_1}{\rho C_E c_1^2}$,
$\varepsilon_2 = \frac{k_3}{\rho C_E c_1^2}$, $\varepsilon_3 = \frac{k_1{}^*}{L\rho C_E c_1}$, $\varepsilon_4 = \frac{k_3{}^*}{L\rho C_E c_1}$, $\varepsilon_5{}' = \frac{T_0}{\rho^2 C_E c_1^2}$

We assume the solution of the form

$$(u_1, u_3, \varphi) = (u_1{}^*, u_3{}^*, \varphi^*)e^{i\xi(x_1-ct)} \tag{22}$$

where ξ is the wave number, $\omega = \xi c$ is the angular frequency and c is the phase velocity of the wave.

Making use of Eq. (22) on Eqs. (19)-(21), we obtain a system of homogeneous equations in terms of $u_1{}^*, u_3{}^*$ and φ^*, which yield a non-trivial solution if determinant of coefficient $\{u_1{}^*, u_3{}^*, \varphi^*\}^t$ vanishes and we obtain the following characteristic equation

$$PD^6 + QD^4 + RD^2 + S = 0 \tag{23}$$

where

P, Q, R and S are given as

$$P = \delta_2\delta_3\zeta_7 + \varepsilon_5\zeta_5\delta_2$$

$$Q = \zeta_{10}\zeta_7\delta_3 + \delta_2\zeta_7\zeta_{11} - \delta_3\delta_2\zeta_6 - \varepsilon_5{\beta_3}^2\zeta_4\delta_2 - \zeta_3^2\zeta_7 - i\xi\zeta_3 p_3\zeta_5\varepsilon_5{\beta_1}^2 + \xi^2\varepsilon_5\delta_3\zeta_5{\beta_1}^2 - i\xi\varepsilon_5\zeta_5\beta_1\beta_3\zeta_3 - \xi^2\varepsilon_5{\beta_1}^2\delta_3\zeta_5$$

$$R = \zeta_{10}\zeta_{11}\zeta_7 - \zeta_{10}\delta_3\zeta_6 - \varepsilon_5{\beta_3}^2\zeta_{10}\zeta_4 - \delta_2\zeta_6\zeta_{11} + \zeta_3^2\zeta_6 + \zeta_2^2\zeta_7 + p_3\zeta_3\zeta_4\varepsilon_5{\beta_1}^2 i\xi + \mathrm{i}\xi\zeta_3\zeta_4\beta_1\beta_3\varepsilon_5 - \xi^2\zeta_5\zeta_{11}{\beta_1}^2\varepsilon_5$$

$$S = -\zeta_{10}\zeta_{11}\zeta_6 - \zeta_2^2\zeta_6 + \xi^2\zeta_{11}{\beta_1}^2\zeta_4\varepsilon_5 \tag{23}$$

$$\zeta_1 = \left(\frac{M}{1+m^2}\mu_0 H_0 i\xi c + (\xi c)^2\right) + \Omega^2,\ \zeta_2 = \frac{M}{1+m^2}\mu_0 H_0 m i\xi c + 2\Omega i\xi c\ ,\zeta_3 = (\delta_4 + \delta_2)i\xi\ ,\zeta_4 = 1 + \frac{a_1}{L}\xi^2,\ \zeta_5 = \frac{a_3}{L}$$

$$\zeta_6 = (\varepsilon_1 - \varepsilon_3 i\xi c)\xi^2 + s^2(1 + \frac{a_1}{L}\xi^2)\ ,\ \zeta_7 = \varepsilon_2 - \varepsilon_4 i\xi c + \frac{a_3}{L}(\xi c)^2\ ,\ \zeta_{10} = \zeta_1 - \xi^2,$$

$$\zeta_{11} = \zeta_1 - \delta_2\xi^2\ ,\varepsilon_5 = \frac{-T_0(\xi c)^2}{\rho^2 C_E c_1^2}$$

The characteristic equation in Eq. (23) is cubic in λ_j^2 $(j = 1,2,3)$. Therefore the former solution which satisfy the radiation conditions that u_1 ,u_3 , $\varphi \to 0$ as $x_3 \to \infty$ is given by

$$u_1 = (A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3})e^{i\xi(x_1 - ct)} \tag{24}$$

$$u_3 = (d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3})e^{i\xi(x_1 - ct)} \tag{25}$$

$$\varphi = (l_1 A_1 e^{-\lambda_1 x_3} + l_2 A_2 e^{-\lambda_2 x_3} + l_3 A_3 e^{-\lambda_3 x_3})e^{i\xi(x_1 - ct)} \tag{26}$$

where $\pm\lambda_i$, $(i = 1,2,3)$, are the roots of Eq. (23) and d_i and l_i are given as

$$d_i = \frac{\lambda_i^2(\varepsilon_5 i\xi\delta_3 - \zeta_3\beta_3\beta_1\xi) - \lambda_i(\xi\beta_1\beta_3\zeta_2) + \varepsilon_5 i\xi\zeta_{11}}{\lambda_1^4(\delta_3\zeta_7 + \varepsilon_5\zeta_5{\beta_3}^2) + \lambda_1^2(\zeta_7\zeta_{11} - \delta_3\zeta_6 - {\beta_3}^2\varepsilon_5\zeta_4) - \zeta_6\zeta_{11}}\ i = 1,2,3$$

$$l_i = \frac{-\lambda_i^3(\zeta_3\zeta_7 + \varepsilon_5 i\xi\zeta_5 \mathrm{p}_3) - \zeta_2\zeta_7\lambda_i^2 + (\zeta_3\zeta_6 + \varepsilon_5 i\xi \mathrm{p}_3\zeta_4)\lambda_i + \zeta_6\zeta_2}{\lambda_1^4(\delta_3\zeta_7 + \varepsilon_5\zeta_5{\beta_3}^2) + \lambda_1^2(\zeta_7\zeta_{11} - \delta_3\zeta_6 - {\beta_3}^2\varepsilon_5\zeta_4) - \zeta_6\zeta_{11}}\ i = 1,2,3$$

4. Boundary conditions

The appropriate boundary conditions at the interface $x_3 = 0$ are

$$\text{i) } t_{33} = 0 \tag{27}$$

$$\text{ii) } t_{31} = 0 \tag{28}$$

$$\text{iii) } \frac{\partial\varphi}{\partial x_3} = 0 \quad \text{or } \varphi = 0 \qquad \text{at } x_3 = 0$$

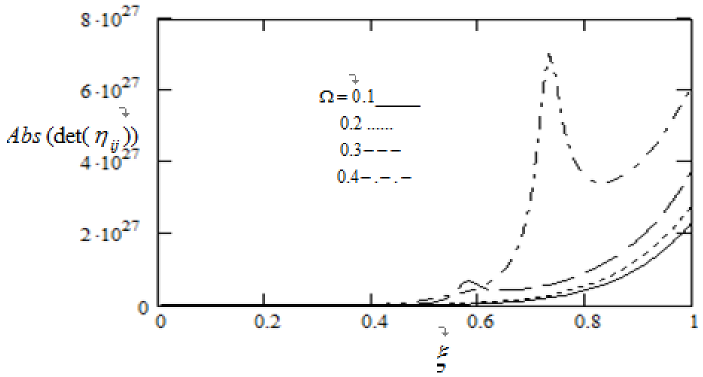

Fig. 1 Determinant of Rayleigh waves secular equation with varies values of rotation with respect to ξ

5. Derivation of secular equations

Making se of Eqs. (14)-(16), (18), (24)-(26) in Eqs. (27)-(29), we obtain a system of simultaneous homogeneous equations

$$\sum_{j=1}^{3}\eta_{qj}A_j = 0 \quad (q = 1,2,3) \tag{30}$$

$$\eta_{1j} = \frac{c_{31}}{\rho c_1^2} i\xi - \frac{c_{33}}{\rho c_1^2} d_j\lambda_j - \frac{\beta_3}{\beta_1} l_j - \frac{\beta_3}{\beta_1 T_0} a_1 l_j \xi^{\,2} + \frac{\beta_3}{\beta_1} l_j a_3 \lambda_j^{\,2} \ , \ j = 1,2,3 \tag{31}$$

$$\eta_{2j} = -\frac{c_{44}}{\rho c_1^2}\lambda_j + \frac{c_{44}}{\rho c_1^2} i\xi d_j \ , j = 1,2,3 \tag{32}$$

$$\eta_{3j} = -\lambda_j l_j \quad j = 1,2,3 \ \text{ or } \ \eta_{3j} = l_j \ , j = 1,2,3 \tag{33}$$

The system of Eq. (30) has a non-trivial solution if the determinant of unknowns (A_j,j=123) vanishes i.e.,

$$|\eta_{ij}|_{3\times 3} = 0$$

6. Particular cases

(i) If $k_1{}^* = k_3{}^* = 0$, then from Eq. (23) and Eqs. (30)-(33), we obtain the corresponding expressions for transversely isotropic magneto-thermoelastic solid without energy dissipation and with two temperature with Hall current effect and rotation.

(ii) If $a_1 = a_3 = 0$, then from Eq. (23) and Eqs. (30)-(33), we obtain the corresponding expressions for transversely isotropic magneto-thermoelastic solid with and without energy dissipation along with with Hall current effect and rotation.

(iii) If we take $c_{11} = \lambda + 2\mu = c_{33}$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu$, $\beta_1 = \beta_3 = \beta$, $\alpha_1 = \alpha_3 = \alpha$, $K_1 = K_3 = K$, $a_1 = a_3 =$a in Eqs. (23) and Eqs. (30)-(33), we obtain the corresponding

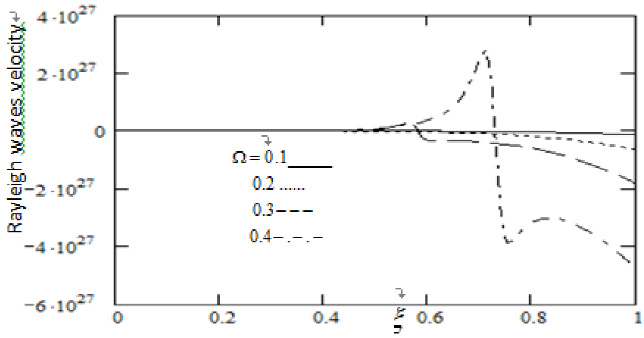

Fig. 2 Rayleigh waves velocity with varies values of rotation with respect to ξ

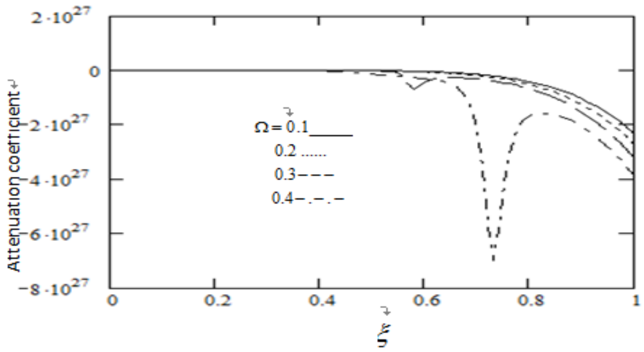

Fig. 3 Attenuation coefficient with varies values of rotation with respect to ξ

expressions isotropic magneto-thermoelastic solid with two temperature and with and without energy dissipation along with combined effects of Hall current and rotation.

(iv) If m=0, in Eq. (23), we obtain the values for transversely isotropic magneto-thermoelastic solid and with and without energy dissipation and with two temperature along with rotation.

7. Numerical results and discussion

Fig. 1 shows variations of Determinant of Rayleigh waves secular equation with varies values of rotation with respect to ξ.Here, we notice that for Ω=0.1,0.2 and 0.3, initially the values lie at boundary surface but as ξ approaches 0.4, the variations increase monotonically till end whereas for Ω=0.4 as ξ goes beyond 0.4 , a slow increase is followed by sharp increase attaining maximum value at ξ =0.7 ,from where the values decrease very fast upto the range ξ =0.8 and then increase smoothly till ξ =1.

Fig. 2 depicts Rayleigh waves velocity with varies values of rotation with respect to ξ. Here, we notice that initially there are no variations but as ξ approaches 0.4,the variations decrease slowly and smoothly corresponding to Ω=0.2 and 0.3 whereas corresponding to Ω =0.4, for $\xi \geq 4$, the variations are increasing and as ξ approaches 0.6, a jump in the values of Rayleigh wave velocity is noticed which is followed by a sudden fall and then rising up to small amount, the values start decreasing.

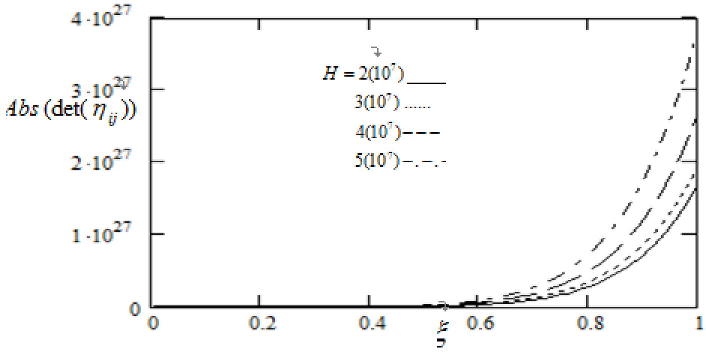

Fig. 4 Determinant of Rayleigh waves secular equation with aries values of magnetic field with respect to ξ

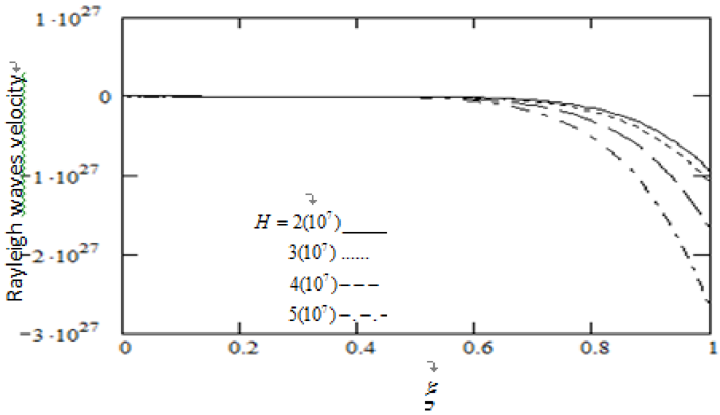

Fig. 5 Rayleigh waves velocity with varies values of magnetic field with respect to ξ

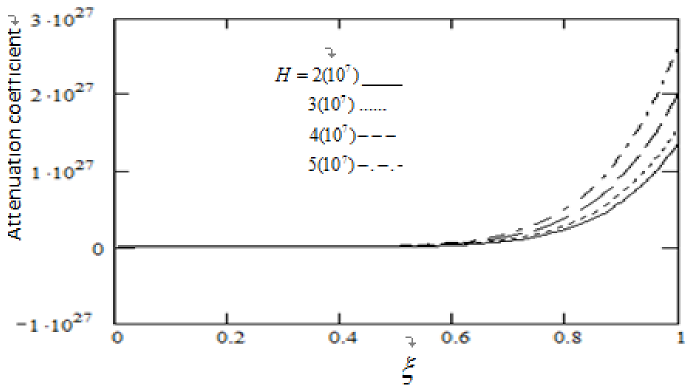

Fig. 6 Attenuation coefficient with varies values of magnetic field with respect to ξ

Fig. 3 shows variations of Attenuation coefficient with varies values of rotation with respect to ξ. Here also variations are noticed only in the range $0.4 \le \xi \le 1$, which are decreasing slowly and smoothly corresponding to Ω=0.1,0.2 and 0.3 whereas for $\Omega = 0.4$, an instant fall of the variations is noticed in the range $0.6 \le \xi \le 0.8$, which is followed by smooth decrease in the rest. Fig. 4 depicts the variations of Determinant of Rayleigh waves secular equation with varies values

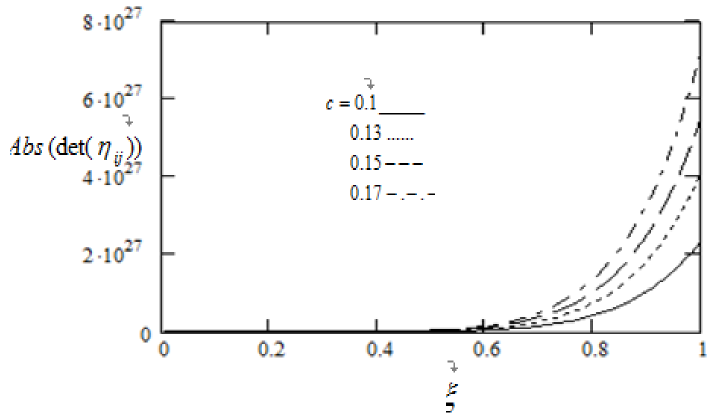

Fig. 7 Determinant of Rayleigh waves secular equation with varies values of phase velocity with respect to ξ

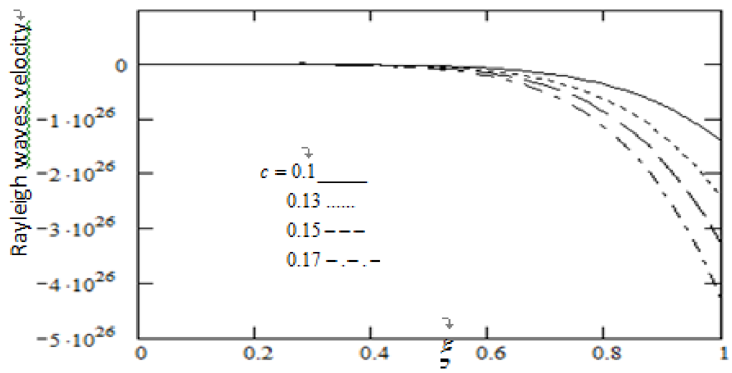

Fig. 8 Rayleigh waves velocity with varies values of phase velocity with respect to ξ

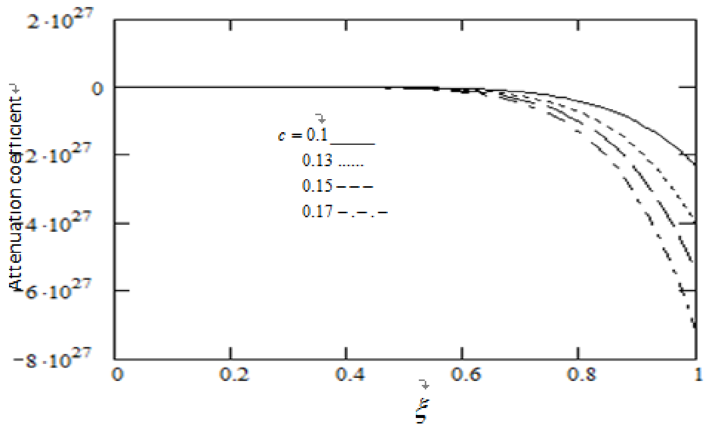

Fig. 9 Attenuation coefficient with varies values of phase velocity with respect to ξ

of magnetic field with respect to ξ. Here it is noticed that, the values of the Determinant are increasing for the range $0.5 \leq \xi \leq 1$ and there are no variations in the rest. The trends are similar corresponding to all values of Magnetic field.

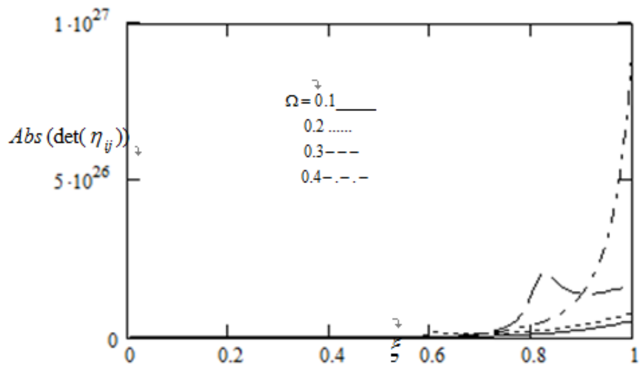

Fig. 10 Determinant of Rayleigh waves secular equation with varies values of rotation with respect to ξ

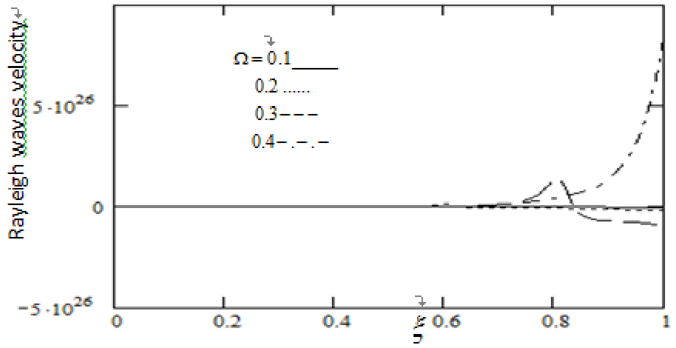

Fig. 11 Rayleigh waves velocity with varies values of rotation with respect to ξ

Fig. 5, shows the variations of Rayleigh waves velocity with varies values of magnetic field with respect to ξ . Here the trend of variations is decreasing in the range $0.5 \leq \xi \leq 1$ corresponding to all the cases and no variations are noticed in the rest.

Figs. 6-7 show the variations of Attenuation coefficient withvaries values of magnetic field with respect to $\xi$and Determinant of Rayleigh waves secular equation withvaries values of phase velocity with respect to ξ respectively. Here, in both the figures, we notice similar trends of variations with change in magnitudes corresponding to all the values of magnetic field. Also the trends are noticed to be monotonically increasing for the range $0.5 \leq \xi \leq 1$ whereas no variations are noticed for the initial range.

Figs. 8-9 exhibit the variations of Rayleigh waves velocity withvaries values of phase velocity with respect to ξ and Attenuation coefficient withvaries values of phase velocity. Here in both the figures opposite trends are noticed as discussed in Figs. 6-7.

For isotropic case

Fig. 10 exhibits the variations of Determinant of Rayleigh waves secular equation with varies values of ξ. Here, we notice that the values of the Determinant increase very slowly for the range $0.5 \leq \xi \leq 1$ corresponding to Ω=0.1 and 0.2 and a high increase is assumed for $\Omega = 0.3$ whereas corresponding to $\Omega = 0.4$, an increase is found in the range $0.6 \leq \xi \leq 0.8$ which is followed by the decreasing trend.

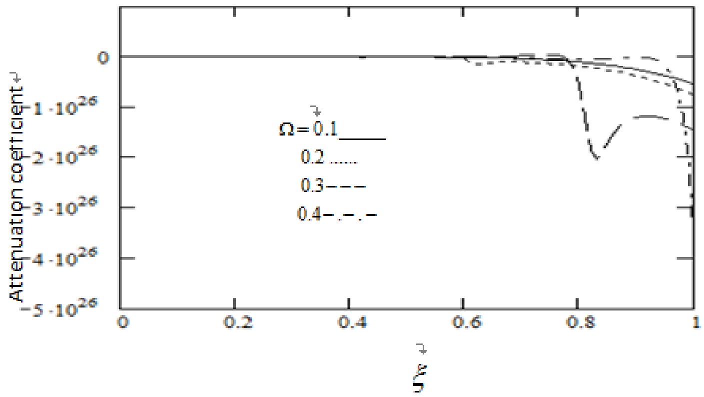

Fig. 12 Attenuation coefficient with varies values of rotation with respect to ξ

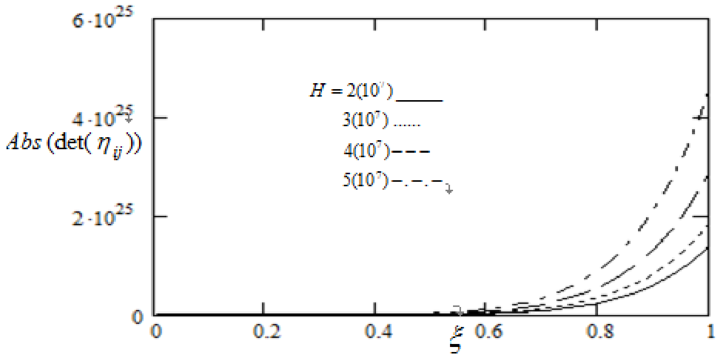

Fig. 13 Determinant of Rayleigh waves secular equation with varies values of magnetic field with respect to ξ

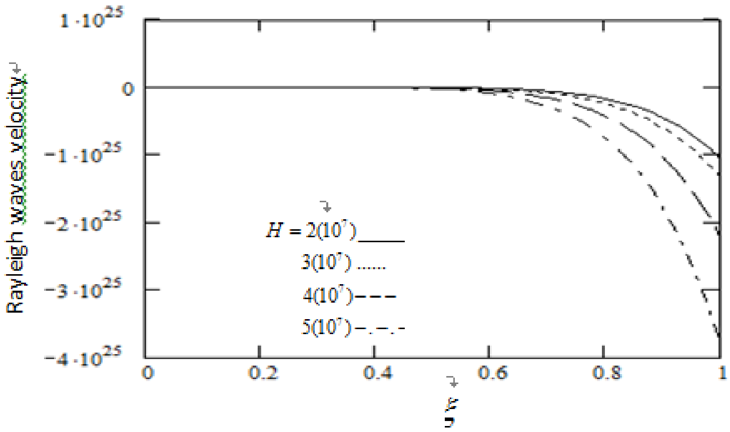

Fig. 14 Rayleigh waves velocity with varies values of magnetic field with respect to ξ

Fig. 11 exhibits the values of Rayleigh waves velocity with varies values of rotation with respect to ξ. Here the trends are similar with lessvariations as discussed in Fig. 10.

Fig. 12 shows the variations of Attenuation coefficient with varies values of rotation with

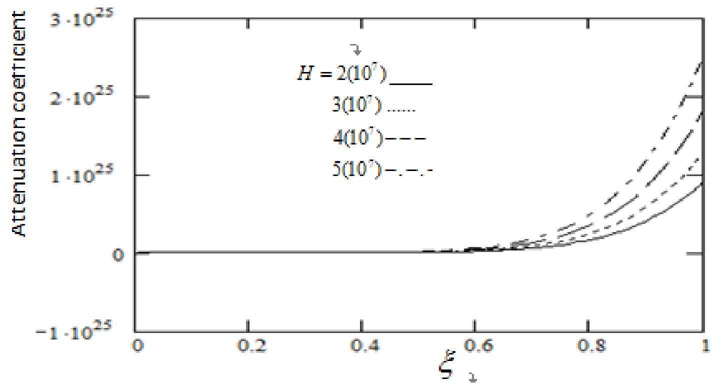

Fig. 15 Attenuation coefficient with varies values of magnetic field with respect to ξ

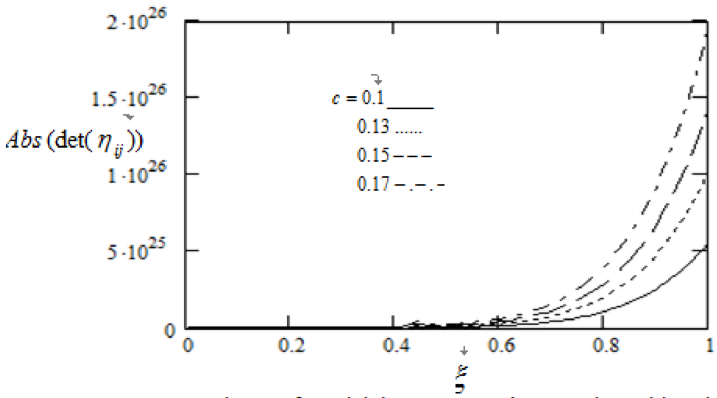

Fig. 16 Determinant of Rayleigh waves secular equation with varies values of phase velocity with respect to ξ

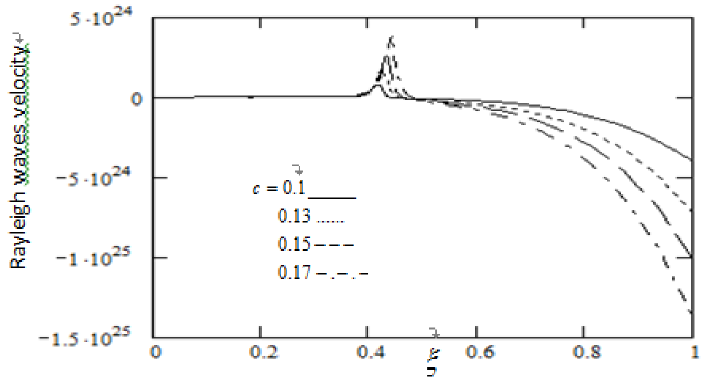

Fig. 17 Rayleigh waves velocity with varies values of phase velocity with respect to ξ

respect to ξ. Here the trends of variations are opposite to the trends discussed in Fig. 10.

Figs. 13-14. explain the trends of variations of Determinant of Rayleigh waves secular equation with varies values of magnetic field with respect to ξ and Rayleigh waves velocity with varies values of magnetic field with respect to ξ. Here in both the figures, we find that the trends are

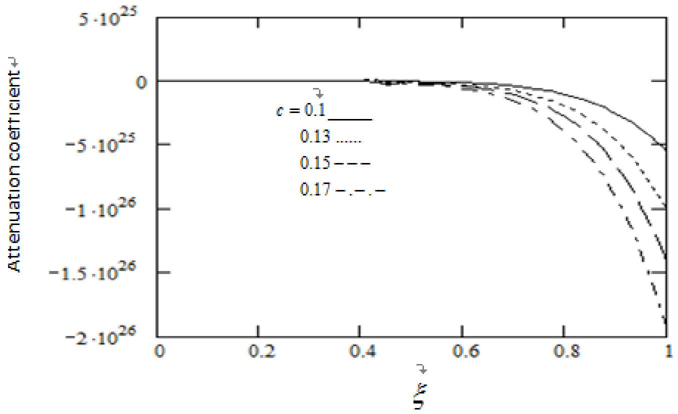

Fig. 18 Attenuation coefficient with varies values of phase velocity with respect to ξ

decreasing corresponding to all the cases for the range $0.6 \leq \xi \leq 1$ and no variations are found for the initial range.

Figs. 15-16 show the variations of Attenuation coefficient with varies values of magnetic field with respect to $\xi$and Determinant of Rayleigh waves secular equation with varies values of phase velocity with respect to ξ. Here in both the figures opposite trends are noticed as discussed in the Figs. 13-14.

Fig. 17 shows Rayleigh waves velocity with varies values of phase velocity with respect to ξ. Here, for the initial range, there are no variations but as ξapproaches 0.4, a jump in the variations is noticed followed by a smooth decrease in the rest.

Fig. 18 shows the variations of Attenuation coefficient with varies values of phase velocity with respect to ξ. Here the trends are similar as discussed in the Figs. 13-14.

8. Conclusions

The graphs permit us some concluding remarks

(i) Rotation has significant impact on the variations of various quantities. By assigning small values to rotation i.e., for Ω=0.1,0.2 and 0.3, the variations are similar with change in magnitude whereas for Ω=0.4, either sudden jumps or sudden falls in the variations appear.

(ii) Change in magnetic effect on the various quantities varies the magnitudes of values in the resulting quantities showing its own impact.

(iii) Change in the wave velocity effects the trends of variations by keeping them same in the beginning but creating a lot of change with increase in the wave number.

(iv) While comparing the effects of rotation in transversely isotropic and isotropic medium, more variations in various quantities in transversely isotropic medium are found.

(IV) The results presented in this paper are very useful in the study of earthquakes, seismology and geo-physics because during earthquakes Rayleigh waves are responsible for destruction of buildings, plants etc.

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Appendix

For the purpose of numerical evaluation, cobalt material has been chosen following Dhaliwal and Singh (1980), as

$c_{11} = 3.071 \times 10^{11} Nm^{-2}$, $c_{33} = 3.581 \times 10^{11} Nm^{-2}$, $c_{13} = 1.027 \times 10^{11} Nm^{-2}$, $c_{44} = 1.510 \times 10^{11} Nm^{-2}, \rho = 8.836 \times 10^{3} Kgm^{-3}$, $T_0 = 298°K$, $C_E = 4.27 \times 10^{2} JKg^{-1}deg^{-1}$, $K_1 = .690 \times 10^{2} wm^{-1}deg^{-1}$, $K_3 = .690 \times 10^{2} wm^{-1}deg^{-1}$, $\beta_1 = 7.04 \times 10^{6} Nm^{-2}deg^{-1}$, $\beta_3 = 6.90 \times 10^{6} Nm^{-2}deg^{-1}$, $K_1^* = 0.02 \times 10^{2} Nsec^{-2}deg^{-1}$, $K_3^* = 0.04 \times 10^{2} Nsec^{-2}deg^{-1}$, $\mu_0 = 1.2571 \times 10^{-6} Hm^{-1}$, $\varepsilon_0 = 8.838 \times 10^{-12} Fm^{-1}$, p_3= 0.98, with non-dimensional parameter L=1 and $\sigma_0 = 9.36 \times 10^{5} col^2/Cal.cm.$sec, $t_0 = 0.02$, M=3 and two temperature parameters is taken as a_1=0.03 and a_3=0.06.

Following Dhaliwal and Singh (1980), magnesium crystal is chosen for the purpose of numerical calculation (isotropic solid). The physical constants used are

$\lambda = 2.17 \times 10^{10} Nm^2$, $\mu = 3.278 \times 10^{10} Nm^2$, $K = 0.02 \times 10^{2} Nsec^{-2}deg^{-1}$,$K^* = 1.7 \times 10^{2} Wm^{-1}deg^{-1}$, s=0.1, $\omega_1 = 3.58 \times 10^{11} S^{-1}$, $\beta = 2.68 \times 10^{6}\ Nm^{-2}deg^{-1}$, $\rho = 1.74 \times 10^{3} Kgm^{-3}, T_0 =$ 298K, $C_E = 1.04 \times 10^{3} Jkg^{-1}deg^{-1}, \sigma_0 = 9.36 \times 10^{5} col^2/Cal.cm.$sec, Ω=3, $t_0 = 0.02$, M=3 and two temperature parameters is taken as a_1=0.03 =a_3, L=1.