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Axisymmetric deformation of thick circular plate in microelongated thermoelastic solid

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Abstract. In the present work, a microelogated thermoelastic model based on Lord-Shulman (1967) and Green-Lindsay (1972) theories of thermoelasticity has been constructed. The governing equations for the simulated model are converted into two-dimensional case and made dimensionless for further simplification. Laplace and Hankel transforms followed by eigen value approach has been employed to solve the problem. The use of eigen value approach has the advantage of finding the solution of governing equations in matrix form notations. This approach is straight forward and convenient for numerical computation and avoids the complicate nature of the problem. The components of displacement, stress and temperature distribution are obtained in the transformed domain. Numerical inversion techniques have been used to invert the resulting quantities in the physical domain. Graphical representation of the resulting quantities for describing the effect of microelongation are presented. A special case is also deduced from the present investigation. The problem find application in many engineering problems like thick-walled pressure vessel such as a nuclear containment vessel, a cylindrical roller etc.

Keywords: eigen value approach; Laplace and Hankel transforms; microelongation; thermoelasticity

1. Introduction

Eringen (1971) extended the theory of micropolar elasticity (Eringen 1966) to include the effect of axial stretch during the rotation of molecules and developed the theory of micropolar elastic solids with stretch. These solids respond to intrinsic rotational motions and spin inertia and therefore can support couple stresses and body couples. The model introduced is conjectured to explain the motion of certain class of granular and composite materials in which grains and fibers are elastic along the direction of their major axis.

The theory of thermomicrostretch elastic solids was developed by Eringen (1990a). Eringen (1990b) also derived the equations of motion, constitutive equations, and boundary conditions for thermo-microstretch fluids and obtained the solution of the problem for acoustical waves in bubbly liquids. The microstretch continuum is a model for Bravais lattice with a basis on the atomic level

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and a two phase dipolar solid with a core on the macroscopic level.

(Sharma *et al.* 2013a with Sharma *et al.* 2013) and (Sharma *et al.* 2013b with Sharma *et al.* 2014) investigated the propagation of plane waves and fundamental solution in homogeneous and isotropic electro microstretch elastic solids and electro- microstretch viscoelastic solids. Marin *et al.* (2015) derived a relation of De Hoop-Knopoff type for displacement fields in thermomicrostretch elastic solid and obtained an explicit expression of the body loadings equivalent to a seismic dislocation. Sharma and Khator (2021, 2022) examined some problems of power generating due to renewable sources.

A microelongated elastic solid possesses four degrees of freedom: three for translation and one for microelongation. In microelongation theory, the material particles can perform only volumetric microelongation in addition to classical deformation of the medium. The material points of such medium can stretch and contract independently of their translations. Solid-liquid crystals, composite materials reinforced with chopped elastic fibers, porous media with pores filled with non-viscous fluid or gas can be categorized as a microelongated medium.

Kiris and Inan (2005) examined the problem of the Eshelby tensors for a spherical inclusion in a microelongated elastic solid field. Shaw and Mukhopadhyay (2012) studied the response due to periodically varying heat sources in a functionally graded microelongated medium. Shaw and Mukhopadhyay (2013) studied the response of moving heat source in a thermoelastic microelongated isotropic homogeneous medium. Ailawalia *et al.* (2015) presented a two-dimensional deformation problem in a thermoelastic microelongated medium with internal heat source. Sachdeva and Ailawalia (2015) investigated a problem in a thermoelastic microelongated elastic half space by using normal mode analysis to obtain the expressions for displacements components, stresses, and temperature distribution. Othman *et al.* (2020) examined plane waves in magneto-thermo-viscoelastic medium with voids under the effect of initial stress and laser pulse heating. Abo-Dahad *et al.* (2020) examined the problem of a thermoelastic functionally graded thin strip due to pulsed laser heating.

Othman *et al.* (2022) presented the dual phase lag model to investigate the influence of rotation on a two-dimensional problem in a microelongated thermoelastic half space. Lotfy (2022) discussed the problems of thermo-mechanical waves of excited microelongated semiconductor layer during photothermal transport processes. Ismail *et al.* (2022) explored the influence of variable thermal conductivity on thermal-plasma-elastic waves of excited microelongated semiconductor. Ismail *et al.* (2023) examined the effect of electron diffusion in the presence of microelongated parameters due to exponential laser-pulsed heat. El-Sapa *et al.* (2023) analysed the influence of microelongated parameters on displacements, stresses, temperature field and carrier density field in photothermoelastic medium with due to laser pulse. Raddadi *et al.* (2024) investigated the deformation in microelongated photothermoelastic under non-local rotating with variable thermal conductivity due to laser pulse.

In this study, two-dimensional problem in microelongated thermoelastic circular plate due to thermomechanical sources has been investigated. Laplace and Hankel transforms followed by eigen value approach has been applied to solve the problem. After using the inversion techniques of transforms, the results are obtained in the physical domain. The effect of microelongation on displacements, temperature distribution, normal stress and tangential stress have been shown graphically.

Premiliar equations

The constitutive equations for a linear isotropic microelongated thermoelastic solid are given by Eringen (1999), Lord-Shulman (1967), Green-Lindsay (1972) as

$$t_{ji} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + \lambda_0\psi\delta_{ij} - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij} \ , \tag{1}$$

$$\lambda_i^* = \alpha_0\psi, i, \tag{2}$$

$$\sigma = \lambda_0 e_{rr} + \lambda_1\psi - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{3}$$

and

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{4}$$

Balance of momentum and momentum moments in absence of body forces, body couples are given by

$$t_{ji,j} = \rho\frac{\partial^2 u_i}{\partial t^2}, \tag{5}$$

$$\lambda_{i,i}^* - \sigma = \frac{1}{2}\rho j_0\frac{\partial^2\psi}{\partial t^2}. \tag{6}$$

Heat conduction law involving heat flux and temperature gradient in absence of heat sources is given by

$$\left(1+\eta_0\tau_0\frac{\partial}{\partial t}\right)q_i = -K_1^*\,T_{,i}, \tag{7}$$

The energy equation in absence of heat source is given by Boley and Weiner (1960) as

$$\rho T_0 S = -q_{i,i}, \tag{8}$$

where (i) for Lord-Shulman (L-S) (1967)

$$\rho T_0 S = \rho C^*\,T + T_0\,(\nu e_{rr} + m_0\psi), \tag{9}$$

(ii) for Green-Lindsay (1972)

$$\rho T_0 S = \rho C^*\left(1+\tau_0\frac{\partial}{\partial t}\right)T + T_0\,\left(\nu e_{rr} + m_0\psi\right). \tag{10}$$

Making use of Eq. (1) in Eq. (5), also Eqs. (2) and (3) in Eq. (6) with the aid of Eq. (4) yield the equations of motion in microelogation generalized thermoelastic solid as

$$(\lambda+\mu)\nabla(\nabla.\vec{u}) + \mu\nabla^2\vec{u} + \lambda_0\nabla\psi - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{11}$$

$$\alpha_0\nabla^2\psi - \lambda_0(\nabla.\vec{u}) - \lambda_1\psi + m_0\left(1+\tau_1\frac{\partial}{\partial t}\right)T = \frac{1}{2}\rho j_0\frac{\partial^2\psi}{\partial t^2}, \tag{12}$$

Making use of Eqs. (7), (9) and (10) in Eq. (8) yield the heat conduction equation in compact form as

$$K_1^*\nabla^2 T - \nu T_0\left(\frac{\partial}{\partial t} + \eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u}) - m_0 T_0\left(\frac{\partial}{\partial t} + \eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\psi = \rho C^*\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T. \tag{13}$$

In the above equations, $\vec{u}$ is the displacement vector,ψ isscalar microelogational function, λ and μ are Lame's constant, α_0 ,$\lambda_0, \lambda_1, m_0$ are microelogational material parameters, ρ is the density of microelogated sample, j_0 is the microinertia of microelogation, λ_1^* is microstress vector, σ is microstress function, K_1^* is the coefficient of thermal conductivity, $\nu = (3\lambda+2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, T is the change in temperature of the medium at any time, T_0 is reference temperature of the body, S is the entropy per unit mass, q_i is heat flux vector, C^* is the specific heat at constant strain, τ_0, τ_1 are the thermal relaxation times, e_{ij} is the strain tensor, t_{ij} is the microelogational stress tensor, δ_{ij} is the Kronecker delta, ∇ is the gradient operator and ∇^2 is the Laplacian operator.

For L-S theory, $\tau_1 = 0,\ \tau_0 > 0$, and $\eta_0 = 1$,
For G-L theory, $\tau_1 \geq \tau_0 > 0$ and $\eta_0 = 0$.

2. Formulation of the problem

Consider a homogeneous, isotropic microelongated thermoelastic circular plate with thickness $2d$ and the circular plate occupied the region defined by $0 \leq r \leq \infty, -d \leq z \leq d$. The plate is acted upon by a transient axisymmetric temperature field and an instantaneous normal ring force, and the plate is thermally insulated. The cylindrical polar coordinate system (r, θ, z) with origin being taken in the middle surface of the plate and z-axis along the normal to the plate i.e., along the thickness of the plate. The problem becomes a two-dimensional axisymmetric problem with symmetry about z-axis. T_0 is the initial temperature of the thick circular plate which is taken as a constant temperature. For the present problem, we take

$$\vec{u} = (u_r, 0, u_z). \tag{14}$$

Eqs. (11)-(13) with the use of (14) take the form

$$(\lambda+\mu)\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)\frac{\partial}{\partial r}+\mu\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}\right)$$
$$+\lambda_0\frac{\partial\psi}{\partial r}-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial r}=\rho\frac{\partial^2 u_r}{\partial t^2}, \tag{15}$$

$$(\lambda+\mu)\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)\frac{\partial}{\partial z}+\mu\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial z^2}\right)+\lambda_0\frac{\partial\psi}{\partial z}-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}=\rho\frac{\partial^2 u_z}{\partial t^2}, \tag{16}$$

$$\alpha_0\left(\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}\right)-\lambda_0\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)$$
$$-\lambda_1\psi+m_0\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\frac{1}{2}\rho j_0\frac{\partial^2\psi}{\partial t^2}, \tag{17}$$

$$K_1^*\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)=\rho C^*\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)$$
$$+\nu T_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+m_0 T_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\psi. \tag{18}$$

The non-dimensional quantities are introduced as

$$r'=\frac{\omega^* r}{c_1},\quad z'=\frac{\omega^* z}{c_1},\quad u_r'=\frac{\rho c_1\omega^* u_r}{\nu T_0},\quad u_z'=\frac{\rho c_1\omega^* u_z}{\nu T_0},\quad \psi'=\frac{\rho c_1^2\psi}{\nu T_0},$$
$$T'=\frac{T}{T_0}, t'=\omega^* t,\quad \tau_0'=\omega^*\tau_0,\quad \tau_1'=\omega^*\tau_1,\quad t_{ij}'=\frac{t_{ij}}{\nu T_0},$$

where

$$\omega^*=\frac{\rho C^* c_1^2}{K_1^*}, c_1^2=\frac{\lambda+2\mu}{\rho}. \tag{19}$$

Eqs. (15)-(18) with the aid of dimensionless quantities (19) and after suppressing the primes, yield

$$\left(1-a_0\right)\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\partial^2 u_z}{\partial r\partial z}-\frac{u_r}{r^2}\right)+a_0\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}\right)$$

$$+a_1\frac{\partial\psi}{\partial r}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial r}=\frac{\partial^2 u_r}{\partial t^2}, \tag{20}$$

$$\left(1-a_0\right)\left(\frac{\partial^2 u_r}{\partial r\,\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right)+a_0\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial z^2}\right)$$
$$+a_1\frac{\partial\psi}{\partial z}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}=\frac{\partial^2 u_z}{\partial t^2}, \tag{21}$$

$$\left(\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}\right)\psi-a_2\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)-a_3\psi$$
$$+a_5\left(1+\tau_1\frac{\partial}{\partial t}\right)T\;=a_4\frac{\partial^2\psi}{\partial t^2}, \tag{22}$$

$$\nabla^2 T=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\epsilon\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+\bar{\nu}\epsilon\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\psi, \tag{23}$$

where

$a_0=\frac{c_2^2}{c_1^2}, a_1=\frac{\lambda_0}{\rho c_1^2}, a_2=\frac{\lambda_0\rho c_1^2}{\alpha_0\omega^{*2}},\ a_3=\frac{\lambda_1 c_1^2}{\alpha_0\omega^{*2}}, a_4=\frac{\rho j_0 c_1^2}{2\alpha_0}, a_5=\frac{\bar{\nu}\rho c_1^4}{\alpha_0\omega^{*2}}, \epsilon=\frac{\nu^2 T_0}{\rho K_1^*\omega^*}, \bar{\nu}=\frac{m_0}{\nu}, c_2^2=\frac{\mu}{\rho}$.

The Laplace and Hankel Transforms are defined as

$$\bar{f}(r,z,s)=L\{\bar{f}(r,z,t)\}=\int_0^\infty f(r,z,t)e^{-st}dt, \tag{24}$$

$$\tilde{f}\left(\eta,z,s\right)=H\{\bar{f}(x,z,s)\}=\int_0^\infty r\bar{f}(x,z,s)J_n\left(\eta r\right)dr. \tag{25}$$

Applying Laplace and Hankel transforms defined by (24) and (25) on the set of Eqs. (20)-(23), we obtain

$$\tilde{u}_r{}''=b_{11}\tilde{u}_r+b_{13}\tilde{\psi}+b_{14}\tilde{T}+\ f_{12}\tilde{u}_z{}', \tag{26}$$

$$\tilde{u}_z{}''=b_{22}\tilde{u}_z+f_{21}\tilde{u}_r{}'+f_{23}\tilde{\psi}'+f_{24}\tilde{T}', \tag{27}$$

$$\tilde{\psi}''=b_{31}\tilde{u}_r+b_{33}\tilde{\psi}+b_{34}\tilde{T}+\ f_{32}\tilde{u}_z{}', \tag{28}$$

$$\tilde{T}''=b_{41}\tilde{u}_r+b_{43}\tilde{\psi}+b_{44}\tilde{T}+\ f_{42}\tilde{u}_z{}', \tag{29}$$

Where

$$b_{11}=\left(\frac{\eta^2+s^2}{a_0}\right),\quad b_{12}=\frac{a_1\eta}{a_0},\quad b_{13}=-\frac{\eta}{a_0}(1+\tau_1 s),\quad b_{14}=\frac{\eta\left(1-a_0\right)}{a_0},$$
$$b_{21}=\left(\eta^2 a_0+s^2\right), b_{22}=\ \eta\left(a_0-1\right), b_{23}=-a_1,\ \ b_{24}=(1+\tau_1 s), b_{31}=a_2\eta,$$
$$b_{32}=\eta^2+a_4 s^2+a_3, b_{33}=-a_5(1+\tau_1 s),\ b_{34}=a_2,\ b_{41}=\eta\epsilon(s+\eta_0\tau_0 s^2),$$
$$b_{42}=\bar{\nu}\epsilon(s+\eta_0\tau_0 s^2), b_{43}=\ \eta^2+(s+\tau_0 s^2), b_{44}=\epsilon(s+\eta_0\tau_0 s^2).$$

The system of Eqs. (26)-(29) can be written as

$$\frac{d}{dz}W\left(\eta,z,s\right)=A\left(\eta,s\right)W\left(\eta,z,s\right), \tag{30}$$

where

$$W=\begin{bmatrix}U\\DU\end{bmatrix},\quad A=\begin{bmatrix}O & I\\A_2 & A_1\end{bmatrix},\quad U=\begin{bmatrix}\tilde{u}_r\\\tilde{u}_z\\\tilde{\psi}\\\tilde{T}\end{bmatrix},\ D=\frac{d}{dz}\ ,$$

$$O=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}, I=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\ A_2=\begin{bmatrix}b_{11}&0&b_{13}&b_{14}\\0&b_{22}&0&0\\b_{31}&0&b_{33}&b_{34}\\b_{41}&0&b_{43}&b_{44}\end{bmatrix},\quad A_1=\begin{bmatrix}0&f_{12}&0&0\\f_{21}&0&f_{23}&f_{24}\\0&f_{32}&0&0\\0&f_{42}&0&0\end{bmatrix}.$$

To solve Eq. (30), we assume

$$W(\eta,z,s)=X(\eta,s)e^{mz}, \tag{31}$$

for some parameter m, so that

$$A(\eta,s)W(\eta,z,s)=mW(\eta,z,s), \tag{32}$$

which leads to the solution of the problem through eigen value approach. Accordingly, the characteristic equation corresponding to the matrix A is given by

$$|\mathrm{A}-\mathrm{mI}|=0,$$

which an expansion provides

$$m^8-B_1m^6+B_2m^4-B_3m^2+B_4=0\ , \tag{33}$$

where

$$B_1=b_{11}+b_{22}+b_{33}+b_{44}+f_{12}f_{21}+f_{23}f_{32}+f_{24}f_{42},$$

$$B_2=b_{11}b_{22}+b_{11}b_{33}+b_{11}b_{44}+b_{22}b_{33}+b_{22}b_{44}+b_{33}b_{44}-b_{13}b_{31}-b_{14}b_{41}-b_{34}b_{43}$$
$$+(b_{11}+b_{33})f_{24}f_{42}+(b_{11}+b_{44})f_{23}f_{32}+(b_{33}+b_{44})f_{12}f_{21}-(b_{31}f_{23}+b_{41}f_{24})f_{12}$$
$$-(b_{13}f_{21}+b_{43}f_{24})f_{32}-(b_{14}f_{21}+b_{34}f_{23})f_{42}\ ,$$

$$B_3=b_{13}b_{34}b_{41}+b_{14}b_{31}b_{43}-(b_{11}+b_{22})b_{34}b_{43}+(b_{11}+b_{33})b_{22}b_{44}-(b_{22}+b_{33})b_{14}b_{41}$$
$$+(b_{11}b_{33}-b_{13}b_{31})b_{22}+(b_{11}b_{33}-b_{13}b_{31})b_{44}+(b_{33}b_{44}-b_{34}b_{43})f_{12}f_{21}$$
$$+(b_{34}b_{41}-b_{31}b_{44})f_{12}f_{23}+(b_{31}b_{43}-b_{33}b_{41})f_{12}f_{24}+(b_{14}b_{43}-b_{13}b_{44})f_{21}f_{32}$$
$$+(b_{13}b_{34}-b_{14}b_{33})f_{21}f_{42}+(b_{14}b_{44}-b_{14}b_{41})f_{23}f_{32}+(b_{14}b_{31}-b_{11}b_{34})f_{23}f_{42}$$
$$+(b_{13}b_{41}-b_{11}b_{43})f_{24}f_{32}+(b_{11}b_{33}-b_{13}b_{31})f_{24}f_{42}\ ,$$

$$B_4=(b_{13}b_{34}-b_{14}b_{33})b_{22}b_{41}+(b_{14}b_{31}-b_{11}b_{34})b_{22}b_{43}+(b_{11}b_{33}-b_{13}b_{31})b_{22}b_{44}\ .$$

Eq. (32) gives the roots as $\pm m_i,\ \ i=1,2,3,4$.

The eigenvectors $X_i(\eta,s)$ corresponding to the eigenvalues m_i may be obtained by solving the following equation.

$$[A-\mathrm{mI}]X_i(\eta,s)=0.$$

We write the set of eigen vector $X_i(\eta,s)$ as

$$X_i(\eta,s)=\begin{bmatrix}X_{i1}(\eta,s)\\X_{i2}(\eta,s)\end{bmatrix},$$

where

$$X_{i1}\left(\eta,s\right)=\begin{bmatrix}p_i m_i\\ q_i\\ e_i\\ f_i\end{bmatrix},\qquad X_{i2}\left(\eta,s\right)=\begin{bmatrix}p_i m_i^2\\ q_i m_i\\ e_i m_i\\ f_i m_i\end{bmatrix},\quad m=m_i;\, i=1,2,3,4,$$

$$X_{j1}\left(\eta,s\right)=\begin{bmatrix}-p_i m_i\\ q_i\\ e_i\\ f_i\end{bmatrix},\quad X_{j2}\left(\eta,s\right)=\begin{bmatrix}p_i m_i^2\\ -q_i m_i\\ -e_i m_i\\ -f_i m_i\end{bmatrix},\quad j=i+4,\;\; m=-m_i;\, i=1,2,3,4,$$

where

$$p_i=\frac{\eta}{a_0}[\left(\eta^2+(s+\tau_0 s^2)-m_i^2\right)^2\{a_1a_2-\left(1-a_0\right)\left(\eta^2+a_4s^2+a_3-m_i^2\right)\}+$$
$$\epsilon(1+,\text{we obtain }\tau_1 s)\;(s+\eta_0\tau_0 s^2)\left(\eta^2+(s+\tau_0 s^2)-m_i^2\right)\{\left(\eta^2+a_4s^2+a_3-m_i^2\right)-(1-a_0)\bar{v}^2\,a_4+2a_5\}],$$

$$q_i=\frac{1}{a_0}[\left(\eta^2+(s+\tau_0 s^2)-m_i^2\right)^2\{\left(\eta^2+a_4s^2+a_3-m_i^2\right)\left(\eta^2+s^2-m_i^2a_0\right)-a_1a_2\eta^2\}$$
$$+\epsilon(1+\tau_1 s)(s+\eta_0\tau_0 s^2)\left(\eta^2+(s+\tau_0 s^2)-m_i^2\right)$$
$$\{\eta^2\left(\eta^2+a_4s^2+a_3-m_i^2\right)+\bar{v}a_5\left(\eta^2+s^2-a_0m_i^2\right)-2a_1a_5\eta^2],$$
$$e_i=[\,a_2(\eta^2+(s+\tau_0 s^2)-m_i^2)+a_5\epsilon(1+\tau_1 s)(s+\eta_0\tau_0 s^2)](\eta p_i+q_i)m_i/G_{10},$$
$$f_i=[\epsilon(s+\eta_0\tau_0 s^2)\{\left(\eta^2+a_4s^2+a_3-m_i^2\right)$$
$$\left(\eta^2+(s+\tau_0 s^2)-m_i^2\right)+\epsilon\bar{v}^2\,a_4(1+\tau_1 s)(s+\eta_0\tau_0 s^2)\}-\epsilon\,a_5(s+\eta_0\tau_0 s^2)$$
$$\{\,a_1\left(\eta^2+(s+\tau_0 s^2)-m_i^2\right)+\epsilon\bar{v}(1+\tau_1 s)(s+\eta_0\tau_0 s^2)\}](\eta p_i+q_i)m_i/G_{20},$$

and

$$G_{10}=-[(\eta^2+(s+\tau_0 s^2)-m_i^2)\left(\eta^2+a_3+a_4s^2-m_i^2\right)-\epsilon\,a_5s^2(1+\tau_1 s)(s+\eta_0\tau_0 s^2)],$$
$$G_{20}=-[\left(\eta^2+(s+\tau_0 s^2)-m_i^2\right)(\eta^2+(s+\tau_0 s^2)-m_i^2)\left(\eta^2+a_3+a_4s^2-m_i^2\right)-$$
$$\epsilon a_5s^2(1+\tau_1 s)(s+\eta_0\tau_0 s^2)]\,.$$

The solution of (30) can be assumed as

$$W\left(\eta,z,s\right)=\sum_{i=1}^{4}S_i\,X_i\left(\eta,s\right)\cosh(m_i z), \tag{34}$$

where S_1, S_2, S_3 and S_4 are arbitrary constants.

3. Boundary conditions

The circular plate occupying the region is defined by $0\le r\le\infty,\; -d\le z\le d$ and the plate is acted upon by a transient axisymmetric temperature field dependent on the radial and axial direction of the cylindrical coordinates system and an instantaneous normal ring force as shown in Fig. 1. The tangential stress and gradient of microelongation field vanish at the surface $z=\pm d$. Also the plate is thermally insulated. Therefore, the non-dimension boundary conditions at the surface $z=\pm d$ of the plate are taken as

$$\frac{dT}{dz}=F_1(r,z,t), \tag{35}$$

$$t_{zz}=F_2(r,z,t), \tag{36}$$

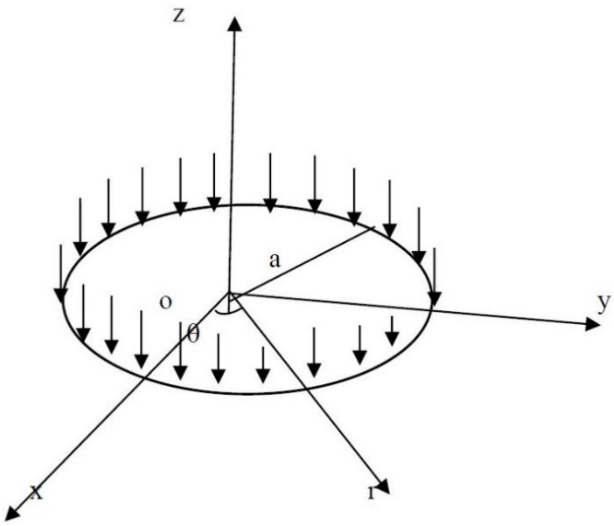

Fig. 1 Instantaneous and concentrated normal ring force

$$t_{zr} = 0, \tag{37}$$

$$\frac{d\psi}{dz} = 0, \tag{38}$$

where

$$F_1(r,z) = F_{10} z^2 e^{-\omega r} \delta(t), \quad \omega > 0, \tag{39}$$

$$F_1(r,t) = F_{20} \delta(t) \delta(a-r)\ . \tag{40}$$

The function $z^2 e^{-\omega r}$ is a function that increase in the axial direction symmetrically and falls off exponentially as one moves away from the centre of the plate along the radial direction.

F_{10} is the constant temperature applied on the boundary, F_{20} is the magnitude of the force and $\delta()$ is the dirac delta function.

The non dimensional stress components t_{zz} and t_{zr} are given by

$$t_{zz} = \frac{\partial u}{\partial z} + a_6 \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T + a_1 \psi, \tag{41}$$

$$t_{zr} = a_0 \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \tag{42}$$

where

$$a_6 = \frac{\lambda}{\rho c_1^2}.$$

Applying the Laplace and Hankel transforms defined by (24) and (25) on the boundary conditions (35)-(38), alongwith (39) and (40), give

$$\frac{d\tilde{T}}{dz} = \tilde{F}_1(\eta, z), \tag{43}$$

$$\widetilde{t_{zz}} = F_2\ (\tilde{\eta}, s), \tag{44}$$

$$\widetilde{t_{zr}} = 0, \tag{45}$$

$$\frac{d\tilde{\psi}}{dz} = 0, \tag{46}$$

where

$$\tilde{F}_1(\eta, z) = F_{10}\frac{z^2\omega}{(z^2+\omega^2)^{3/2}}, \tilde{F}_2(\eta, z) = F_{20}\ aJ_0\ (\eta\ a) \tag{47}$$

Applying Laplace and Hankel transforms defined (24)-(25) on Eqs. (41)-(42) and with the aid of (34), yield

$$\widetilde{t_{zz}} = R_1S_1\cosh(m_1z) + R_2S_2\cosh(m_2z) + R_3S_3\cosh(m_3z) + R_4S_4\cosh(m_4z) \tag{48}$$

$$\widetilde{t_{zr}} = T_1S_1\cosh(q_1z) + T_2S_2\cosh(q_2z) + T_3S_3\cosh(q_3z) + T_4S_4\cosh(q_4z), \tag{49}$$

where

$$R_i = [a_6\eta p_i m_i + a_1e_i - (1+\tau_1 s)f_i + q_i m_i], i = 1,2,3,4, \tag{50}$$

$$T_i = a_0\left[p_i m_i^2 - \eta q_i\right], i = 1, 2, 3, 4. \tag{51}$$

Making use of (34) and (48)-(49) in the boundary conditions (43)-(46), we obtain, the expressions of displacements, microelongation, temperature distribution and stresses in the transformed domain as

$$\left(\tilde{u}_r,\ \tilde{u}_z, \tilde{\psi}, \tilde{T}\right) = \frac{1}{\Delta}\sum_{i=1}^{4}(p_i m_i,\ q_i,\ e_i,\ f_i)\Delta_i\cosh(m_i z), \tag{52}$$

$$(\widetilde{t_{zz}},\ \widetilde{t_{zr}}) = \frac{1}{\Delta}\sum_{i=1}^{4}(R_i,\ T_i)\Delta_i\cosh(m_i z), \tag{53}$$

where

$$\Delta = \begin{vmatrix} E_1 & E_2 & E_3 & E_4 \\ F_1 & F_2 & F_3 & F_4 \\ G_1 & G_2 & G_3 & G_4 \\ H_1 & H_2 & H_3 & H_4 \end{vmatrix},$$

and $\Delta_i (i = 1,2,3,4)$ are obtained from Δ by replacing i^{th} column of Δ with $|F_{10}, F_{20}, 0, 0|^{tr}$, also

$$E_i = f_i m_i\cosh(m_i d),\ \ F_i = R_i\cosh(m_i d),\ \ G_i = T_i\cosh(m_i d),\ \ H_i = e_i m_i\cosh(m_i d),$$

$$(i = 1,2,3,4)$$

4. Particular case

In the absence of microelongation, the boundary conditions for thermoelastic medium are given by

$$\frac{dT}{dz} = F_1(r, z, t),$$
$$t_{zz} = F_2(r, z, t),$$

$$t_{zr} = 0.$$

The corresponding expressions for displacements, temperature distribution and stresses are obtained in thermoelastic medium as

$$\left(\tilde{u}_r,\ \tilde{u}_z, \tilde{T}\right) = \frac{1}{\Delta}\sum_{i=1}^{3}(\ p_i m_i,\ \ q_i,\ \ f_i)\Delta_i \cosh(m_i z),$$

$$(\ \widetilde{t_{zz}},\ \ \widetilde{t_{zr}}) = \frac{1}{\Delta}\sum_{i=1}^{3}(R_i,\ \ T_i)\Delta_i \cosh(m_i z),$$

where

$$\Delta_1 = (E_3F_{20} - F_3F_{10})G_2 + (F_2F_{10} - E_2F_{20})G_3,$$
$$\Delta_2 = (F_3F_{10} - E_3F_{20})G_1 + (E_1F_{20} - F_1F_{10})G_3,$$
$$\Delta_3 = (E_2F_{20} - F_2F_{10})G_1 + (F_1F_{10} - E_1F_{20})G_2,$$
$$\Delta = (E_2F_3 - E_3F_2)G_1 + (E_3F_1 - E_1F_3)G_2 + (E_1F_2 - E_2F_1)G_3,$$

and

$$E_i = f_i m_i \cosh(m_i d),\ \ F_i = R_i \cosh(m_i d),\ \ \ G_i = T_i \cosh(m_i d)\,,\ \ i = (1, 2, 3)$$

along with the changed values of p_i, q_i, f_i, R_i, T_i ,

$$p_i = \frac{\eta}{a_0}[\left(\eta^2 + (s + \tau_0 s^2) - m_i^2\right)^2\{-(1 - a_0)\left(\eta^2 - m_i^2\right)\}$$
$$+\epsilon(1 + \tau_1 s)(s + \eta_0\tau_0 s^2)\left(\eta^2 + (s + \tau_0 s^2) - m_i^2\right)\{\left(\eta^2 - m_i^2\right)\}],$$
$$q_i = \frac{1}{a_0}[\left(\eta^2 + (s + \tau_0 s^2) - m_i^2\right)^2\{\ \left(\eta^2 - m_i^2\right)\left(\eta^2 + s^2 - m_i^2 a_0\right)\}$$
$$+\epsilon(1 + \tau_1 s)(s + \eta_0\tau_0 s^2)\left(\eta^2 + (s + \tau_0 s^2) - m_i^2\right)\{\eta^2\left(\eta^2 - m_i^2\right),$$
$$f_i = [\epsilon(s + \eta_0\tau_0 s^2)\{\left(\eta^2 - m_i^2\right)\left(\eta^2 + (s + \tau_0 s^2) - m_i^2\right)\}](\eta p_i + q_i)m_i/G_{20},$$
$$R_i = [a_6\eta p_i m_i - (1 + \tau_1 s)f_i + q_i m_i],\ \ i = 1, 2, 3,$$
$$T_i = a_0\left(p_i m_i^2 - \eta q_i\right), i = 1, 2, 3\ ,$$

also m_i $(i = 1, 2, 3)$ are the root of the characteristic of the equation.

$$m^6 - B_1 m^4 + B_2 m^2 - B_3 = 0\ ,$$

with the changed values of B_1, B_2 and B_3

$$B_1 = b_{11} + b_{22} + b_{44} + f_{12}f_{21} + f_{24}f_{42}\ ,$$
$$B_2 = b_{11}b_{22} + b_{11}b_{44} + b_{22}b_{44} - b_{14}b_{41} + b_{11}f_{24}f_{42} + b_{44}f_{12}f_{21} - b_{14}f_{21}f_{42} - b_{41}f_{12}f_{24}\ ,$$
$$B_3 = b_{11}b_{22}b_{44} - b_{14}b_{22}b_{41}\ .$$

5. Inversion of transforms

To obtain the solution in the physical domain, we must invert the transform in (52)-(53). Here the displacements, stresses, temperature change and microelongation field are function of z, the parameters of Laplace and Hankel transforms s and η respectively and hence are the form $\tilde{f}(\eta, z, s)$. First we invert the Hankel transform, which give the Laplace transform expression $\bar{f}(r, z, s)$ of the function $f(r, z, t)$ as

$$\bar{f}(r, z, s) = \int_0^\infty \eta\ \tilde{f}(\eta, z, s)J_n(\eta r)d\eta, \tag{54}$$

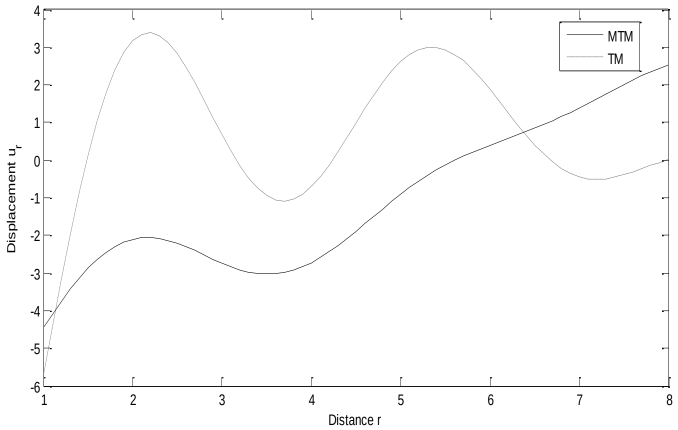

Fig. 2 Variations of displacement u_r

Now for fixed values of η, r, z, the function $\bar{f}(r,z,s)$ in (54) can be considered as the Laplace transform of $f(r,z,t)$. Following Hoing and Hirdes (1984), the Laplace transformed function $\bar{f}(r,z,s)$ can be inverted as given below

$$f(r,z,t) = \frac{1}{2\pi}\int_{c-i\infty}^{c+i\infty} \bar{f}(r,z,s)e^{st}ds, \tag{55}$$

where c is an arbitrary real number greater than all real parts of the singularities of $\bar{f}(r,z,s)$, taking $s = c + i\,y$

$$f(r,z,t) = \frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\bar{f}\big(r,z,c + i\,y\big)e^{st}dy. \tag{56}$$

The last step is to calculate the integral in Eq. (54). The method for calculate this integral is described by Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical results and discussion

Following Eringen (1984), we take the following values of relevant parameters for magnesium crystal like materials (microelongation elastic solid)

$$\lambda = 9.4\times10^{10}\ \mathrm{Nm^{-2}},\ \mu = 4.0\times10^{10}\ \mathrm{Nm^{-2}}, \rho = 1.74\times10^{3}\ \mathrm{Kgm^{-3}},$$
$$j_0 = 0.19\times10^{19}\ \mathrm{Nm^{-2}}, \alpha_0 = 0.779\times10^{-9}\ \mathrm{N}, \lambda_0 = 0.5\times10^{10}\ \mathrm{Nm^{-2}},$$
$$\lambda_1 = 6.5\times10\ \mathrm{Nm^{-2}}.$$

The values of thermal parameters are given by Dhaliwal and Singh (1980) as

$$K_1^* = 1.7\times10^{6}\ \mathrm{Jm^{-1}s^{-1}K^{-1}}, C^* = 1.04\times10^{3}\ \mathrm{JKg^{-1}K^{-1}}, \alpha_t = 2.33\times10^{-5}\ \mathrm{K^{-1}},$$
$$\tau_0 = 6.131\times10^{-13}\ \mathrm{sec}, \tau_1 = 8.765\times10^{-13}\ \mathrm{sec},$$
$$m = 1.13849\times10^{10}\ \mathrm{N/m^2}, T_0 = 0.298\times10^{3}\ \mathrm{K}.$$

The variations of displacements, normal stress, tangential stress and temperature distribution with distance r have been presented in case of microelongated thermoelastic medium (MTM) and

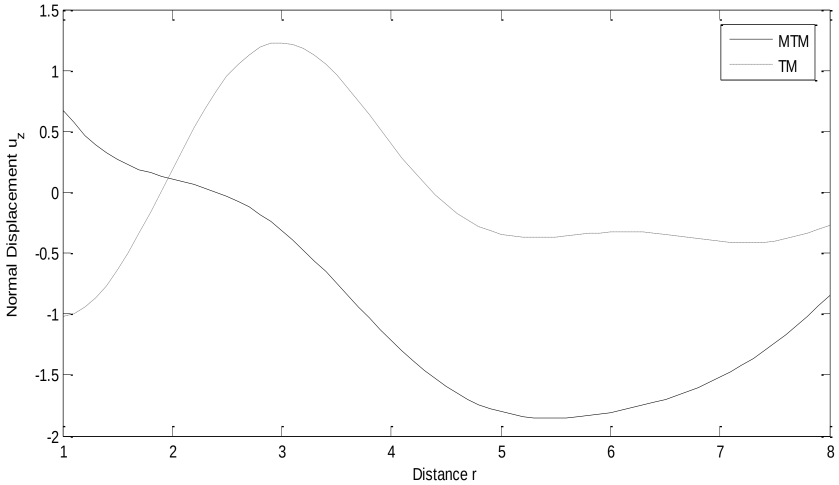

Fig. 3 Variations of normal displacement u_z

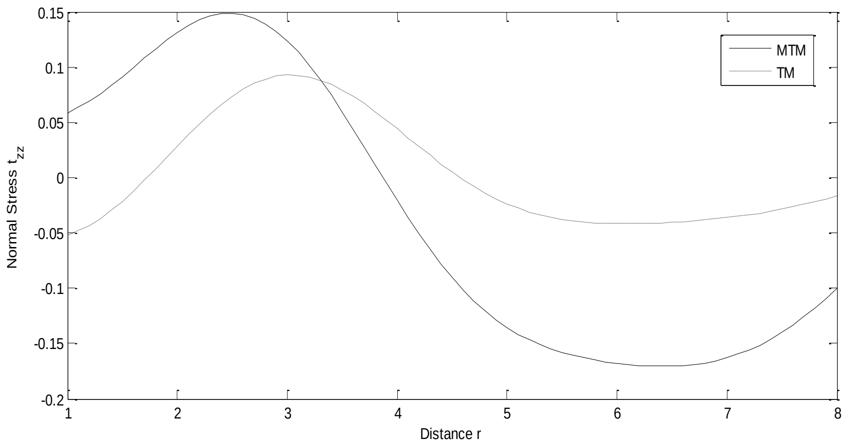

Fig. 4 Variations of normal stress t_{zz}

thermoelastic medium (TM) in Figs. 1-5 respectively. In these figures, MTM and TM corresponding to solid line (——), small dash line (- - - - -) respectively.

Fig. 2 indicates that the value of u_r oscillates for $1 \leq r \leq 4.5$ and increases for $4.5 \leq r \leq 8$ for MTM and the value for TM oscillates with different amplitude for $1 \leq r \leq 8$. The value of u_r is minimum for $1 \leq r \leq 1.2, 6.4 \leq r \leq 8$ and maximum for $1.2 \leq r \leq 6.4$ for TM in comparison to MTM.

Fig. 3 illustrates that the value of u_z decreases for $1 \leq r \leq 5.5$, increases for $5.5 \leq r \leq 8$ for MTM whereas its value for TM increases for $1 \leq r \leq 3$, decreases for $3 \leq r \leq 4.5$ and then oscillates with large amplitude. The value of u_z is large for $1 \leq r \leq 2$ and small for $2 \leq r \leq 8$ for MTM as compared to TM.

Fig. 4 shows that the values of t_{zz} initially increases for MTM and TM for the ranges $1 \leq r \leq 2.5$ and $1 \leq r \leq 3$ respectively. The value of t_{zz} for MTM and TM decreases for $2.5 \leq r \leq 6.5$ and $3 \leq r \leq 6.5$ and again increases for $6.5 \leq r \leq 8$. The value of t_{zz} is large for MTM for $1 \leq$

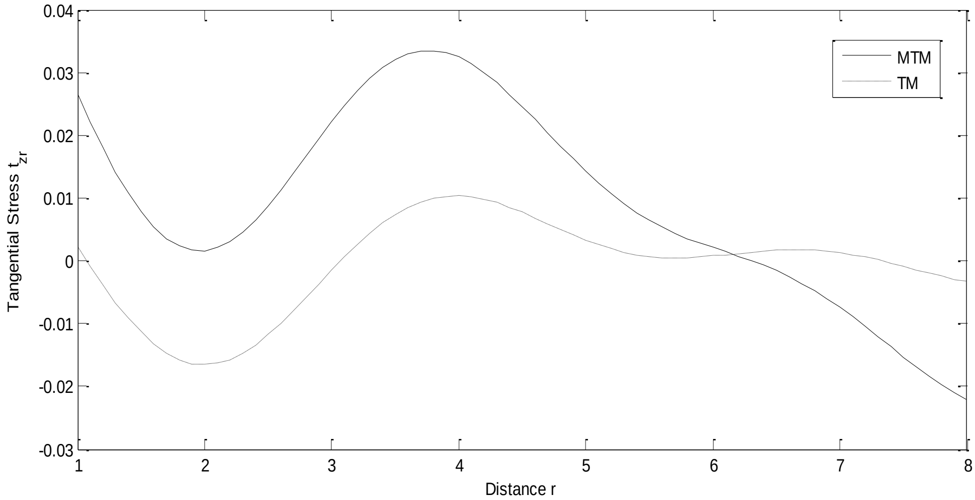

Fig. 5 Variations of tangential stress t_{zr}

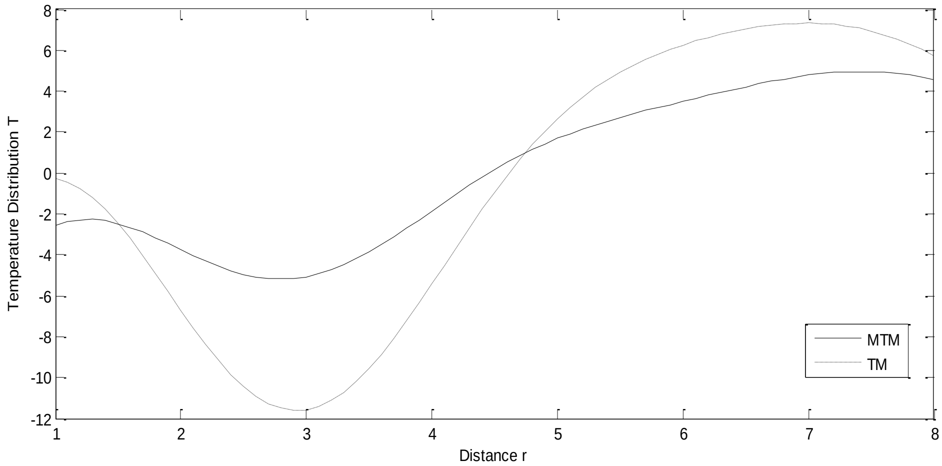

Fig. 6 Variations of temperature distribution T

$r \leq 2.3$ and small for $2.3 \leq r \leq 8$ as compared to the value for TM. A similar trend of variation is noticed for the entire range for MTM and TM.

Fig. 5 shows that the value of tangential stress t_{zr} for MTM oscillates for $1 \leq r \leq 6$ and then decreases for $6 \leq r \leq 8$. An oscillatory behaviour is noticed for t_{zr} with different amplitude over the whole range $1 \leq r \leq 8$ for TM. The value for t_{zr} is large near the application of the source and small away from the source for MTM in comparison to TM.

Fig. 6 exhibits that the value of T decreases for $1 \leq r \leq 3$, increases for $3 \leq r \leq 7$ and then again decreases for $7 \leq r \leq 8$ for MTM and TM. The value of T for TM is maximum as compared to the value for MTM near the application of the source and away from the source. T has maximum value for $1 \leq r \leq 1.7, 4.6 \leq r \leq 8$ and minimum value for $1.7 \leq r \leq 4.6$ for TM as compared to the value for MTM.

7. Conclusions

An axisymmetric problem in microelongated thermoelastic thick circular plate is examined due to thermomechanical sources. The governing equations are mode dimensionless for two-dimensional case. Laplace and Hankel transforms followed by eigen value approach has been employed to solve the problem with suitable boundary conditions. The physical quantities like displacements, stresses, microelongated, temperature distribution are obtained. From the above numerical values following conclusion are made.

It is observed that the values of all the physical quantities are close to each other for a particular inclination of the source. It is observed that the variations and behaviour of normal stress and temperature distribution are similar for MTM and TM except near the application of the source where the temperature distribution has slightly different behaviour. An oscillatory behaviour is observed for tangential stress and radial displacement for thermoelastic medium. The effect of microelongation plays an important role in processing and characterization to improve material properties. The measured displacements and stress provide unique informations leading to fundamental understanding of deformation mechanism in advanced structural materials.

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