Coupled Systems Mechanics, *Vol. 8, No. 3 (2019) 247-257*
DOI: https://doi.org/10.12989/csm.2019.8.3.247

Nonlocal strain gradient thermal vibration analysis of double-coupled metal foam plate system with uniform and non-uniform porosities

Raad M. Fenjan, Ridha A. Ahmed, Abbas A. Alasadi and Nadhim M. Faleh*

Al-Mustansiriah University, Engineering Collage P.O. Box 46049, Bab-Muadum, Baghdad 10001, Iraq

(Received December 18, 2018, Revised January 9, 2019, Accepted March 25, 2019)

Abstract. Fee vibrational characteristics of porous steel double-coupled nanoplate system in thermo-elastic medium is studied via a refined plate model. Different pore dispersions called uniform, symmetric and asymmetric have been defined. Nonlocal strain gradient theory (NSGT) containing two scale parameters has been adopted to stablish size-dependent modeling of the system. Hamilton's principle has been adopted to stablish the governing equations. Obtained results from Galerkin's method are verified with those provided in the literature. The effects of nonlocal parameter, strain gradient, foundation parameters, porosity distributions and porosity coefficient on vibration frequencies of metal foam nanoscale plates have been examined.

Keywords: free vibration; refined plate theory; porous nanoplate; nonlocal elasticity; porosities

1. Introduction

The material structure in some metals is not perfect and there are porosities in them. This material imperfection or porosities might cause serious concerns about the accurate performance of metals in engineering applications. Also, there are many engineering structures which are constructed from porous metals or metal foams. As an example, metal (steel) plates are basic components of engineering structures and they may subjected to various sources of vibrations during their application period. Thus, studying vibrational characteristic of this components will be crucial and important. Looking for related researches in this subject reveals that there are some published papers (Chen *et al.* 2015, 2016, Rezaei and Saidi 2016).

Researches on metal foams state that pores might distribute with uniform or- non-uniform patterns. The terms uniform and non-uniform are related to the distribution of pores in thickness direction of plates. In the case of non-uniform pore distribution, the material can be placed in the category of functionally graded (FG) materials. The word FG is associated with a wide range of materials in which all material properties are position-dependent. So, there are also another type of FG materials which have ceramic and metal phases simultaneously. It means that the material dispersion is non-uniform thorough the plate thickness. In such materials, porosities might occur because of their imperfect production. This is another case study in the field of structural analysis,

*Corresponding author, Ph.D., E-mail: dr.nadhim@uomustansiriyah.edu.iq

Copyright © 2019 Techno-Press, Ltd.
http://www.techno-press.org/?journal=csm&subpage=8 ISSN: 2234-2184 (Print), 2234-2192 (Online)

for example porous FG beams and plates (Wattanasakulpong *et al.* 2014, Yahia *et al.* 2015, Atmane *et al.* 2015a,b, Barati and Zenkour 2016, Mechab *et al.* 2016, Ebrahimi and Barati 2017).

Recently, this kind of materials have found their applications in nano-scale structures. Vibration behavior of a nano-scale plate is not the same as a macro-scale plate. This is because small-size effects are not present at macro scale. So, mathematical modeling of a nanoplate can be done with the use of nonlocal elasticity (Eringen 1983) incorporating only one scale parameter (Natarajan *et al.* 2012, Belkorissat *et al.* 2015, Bounouara *et al.* 2016, Barati *et al.* 2016, Zenkour 2016, Barati 2017a, b, c, d, Ebrahimi and Daman 2016, Ebrahimi and Haghi 2018, Ebrahimi and Heidari 2018, Ebrahimi *et al.* 2018). Due to the ignorance of strain gradient effect in nonlocal elasticity theory, a more general theory wil be required. Strain gradients at nano-scale are observed by many researchers (Li *et al.* 2015). Thus, nonlocal-strain gradient theory was introduced as a general theory which contains an additional strain gradient parameter together with nonlocal parameter.

This paper uses NSGT for analyzing vibrational behavior of a double-coupled nanoplate system based on a refined plate theory (Zenkour 2009, Mehala *et al.* 2018, Sadoun *et al.* 2018, Mahmoudi *et al.* 2018). Two nanoplates are coupled with each other with the use of linear springs. These two nanoplates are made of metal foam with different pore distributions. Results will be illustrated to indicate the importance of pores, coupling springs and scale parameters.

2. Small scales based on NSGT

In its simplest form, NSGT contains two scale coefficients: one related to non-locality (ea) and one another related to strain gradients (l). By having elastic constants C_{ijkl} in hand, the NSGT the stresses σ_{ij} to strains as (Barati 2017b)

$$[1-(ea)^2\nabla^2]\sigma_{ij} = C_{ijkl}\,[1-l^2\nabla^2]\varepsilon_{kl} \tag{1}$$

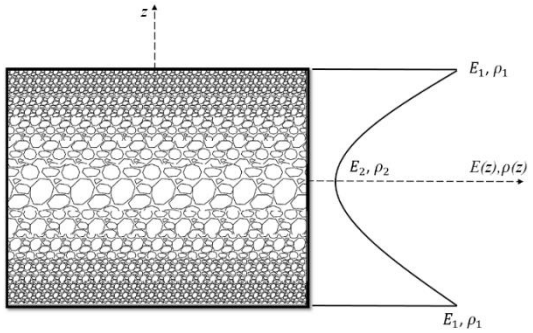
or

$$(1-(ea)^2\nabla^2)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix} = \frac{E(z)}{1-\nu^2}(1-l^2\nabla^2)\begin{pmatrix}1 & \nu & 0 & 0 & 0\\ \nu & 1 & 0 & 0 & 0\\ 0 & 0 & (1-\nu)/2 & 0 & 0\\ 0 & 0 & 0 & (1-\nu)/2 & 0\\ 0 & 0 & 0 & 0 & (1-\nu)/2\end{pmatrix}\begin{Bmatrix}\varepsilon_x-\alpha\Delta T\\ \varepsilon_y-\alpha\Delta T\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{2}$$

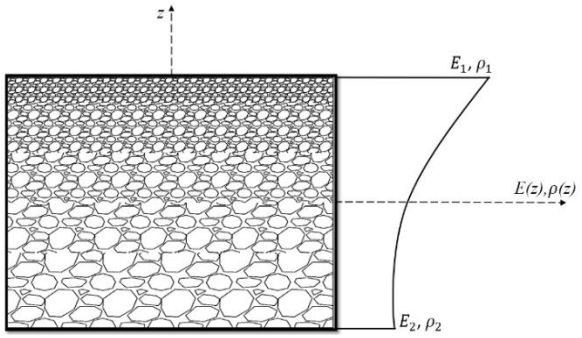
where $\alpha\Delta T$ is thermal strain in which α is thermal expansion coefficient and ΔT is temperature change.

3. Various porosity distributions

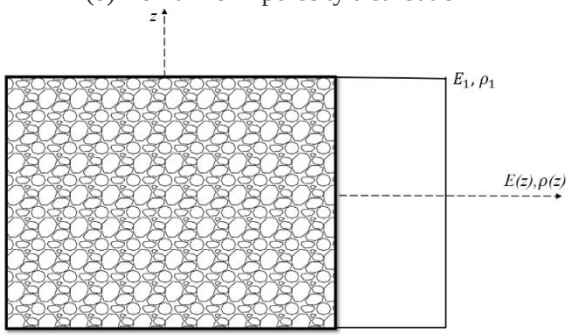
First, it must be mentioned that different kinds of through the thickness pores are shown in Fig. 1. Also, Fig. 2 illustrates a double-coupled nanoplate system with all parameters defined on it. Based upon these through the thickness pores, the material properties (elastic modulus E and mass density ρ) might be defined as (Barati 2017a)



(a) Non-uniform porosity distribution 1



(b) Non-uniform porosity distribution 2



(c) Uniform porosity distribution

Fig. 1 Various kinds of pore dispersions

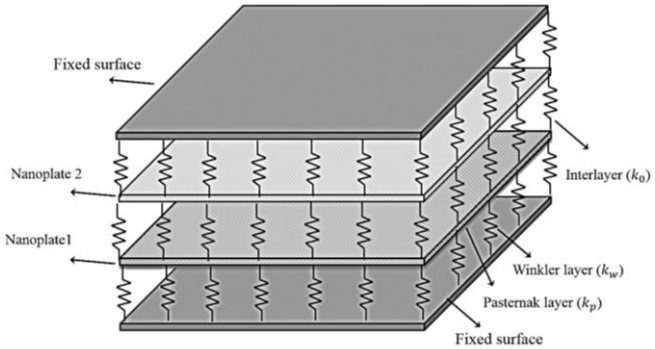


Fig. 2 A double-coupled nanoplate system

• Uniform porosity distribution

$$E = E_2(1-e_0\chi), \rho = \rho_2\sqrt{(1-e_0\chi)} \tag{3}$$

• Non-uniform distribution 1

$$E(z) = E_2(1-e_0\cos\left(\frac{\pi z}{h}\right)), \rho(z) = \rho_2(1-e_m\cos\left(\frac{\pi z}{h}\right)) \tag{4}$$

• Non-uniform distribution 2

$$E(z) = E_2(1-e_0\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)), \rho(z) = \rho_2(1-e_m\cos\left(\frac{\pi z}{2h}+\frac{\pi}{4}\right)) \tag{5}$$

in above relations E_2 and ρ_2 are corresponding to the highest material properties; e_0 is porosity parameter and

$$e_0 = 1-\frac{E_2}{E_1} = 1-\frac{G_2}{G_1}, e_m = 1-\frac{\rho_2}{\rho_1} = 1-\sqrt{1-e_0} \tag{6}$$

$$\chi = \frac{1}{e_0}-\frac{1}{e_0}\left(\frac{2}{\pi}\sqrt{1-e_0}-\frac{2}{\pi}+1\right)^2 \tag{7}$$

Here, E_2 = 200 GPa, ρ_2=7850 kg/m^3, v =0.33. For mathematical modeling each nanoplate of the system, refined plate theory having shear function *f(z)* can be used which introduces fields components in the form

$$u_1(x,y,z,t) = u(x,y,t)-(z-z^*)\frac{\partial w_b}{\partial x}-[f(z)-z^{**}]\frac{\partial w_s}{\partial x} \tag{8}$$

$$u_2(x,y,z,t) = v(x,y,t)-(z-z^*)\frac{\partial w_b}{\partial y}-[f(z)-z^{**}]\frac{\partial w_s}{\partial y} \tag{9}$$

$$u_3(x,y,z,t) = w(x,y,t) = w_b(x,y,t)+w_s(x,y,t) \tag{10}$$

where

$$z^* = \frac{\int_{-h/2}^{h/2}E(z)\,z dz}{\int_{-h/2}^{h/2}E(z)\,dz}, \quad z^{**} = \frac{\int_{-h/2}^{h/2}E(z)\,f(z)dz}{\int_{-h/2}^{h/2}E(z)\,dz}, \quad f(z) = -\frac{z}{4}+\frac{5z^3}{3h^2} \tag{11}$$

Also, *u* and *v* are membrane displacements and w_b and w_s are associated with the bending and shear displacement, respectively. The derivation of governing equations might be done based on Hamiltons' principle and the whole procedure can be find in the work of Barati (2017b)

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y} = I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w_b}{\partial x\partial t^2}-I_3\frac{\partial^3 w_s}{\partial x\partial t^2} \tag{12a}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y} = I_0\frac{\partial^2 v}{\partial t^2}-I_1\frac{\partial^3 w_b}{\partial y\partial t^2}-I_3\frac{\partial^3 w_s}{\partial y\partial t^2} \tag{12b}$$

$$\frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}-k_w(w_b+w_s)+k_p\nabla^2(w_b+w_s)$$
$$=I_0\frac{\partial^2(w_b+w_s)}{\partial t^2}+I_1(\frac{\partial^3 u}{\partial x\partial t^2}+\frac{\partial^3 v}{\partial y\partial t^2})-I_2\nabla^2(\frac{\partial^2 w_b}{\partial t^2})-I_4\nabla^2(\frac{\partial^2 w_s}{\partial t^2}) \tag{12c}$$

$$\frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial Q_{xz}}{\partial x}+\frac{\partial Q_{yz}}{\partial y}-k_w(w_b+w_s)$$
$$+(k_p-N^T)\nabla^2(w_b+w_s)=I_0\frac{\partial^2(w_b+w_s)}{\partial t^2}+I_3(\frac{\partial^3 u}{\partial x\partial t^2}+\frac{\partial^3 v}{\partial y\partial t^2})-I_4\nabla^2(\frac{\partial^2 w_b}{\partial t^2})-I_5\nabla^2(\frac{\partial^2 w_s}{\partial t^2}) \tag{12d}$$

where membrane loads and moments are N_x, N_{xy}, N_y, M_x, M_{xy} and M_y; and kw and kp are Winkler and Pasternak constants; N^T is thermal loading. Complete expressions for these loads and moments can be found in Barati (2017b) and there is no need to re-publish them in the present paper. The mass moments of inertia in above equations can be defined as

$$(I_0,I_1,I_2,I_3,I_4,I_5)=\int_{-h/2}^{h/2}(1,z-z^*,(z-z^*)^2,f-z^{**},(z-z^*)(f-z^{**}),(f-z^{**})^2)\rho(z)dz \tag{13}$$

The governing equations can be stablished in terms of displacement components. So, after computing the membrane loads and moments the governing equations will become

$$-D(1-\lambda\nabla^2)(\frac{\partial^4 w_{1,b}}{\partial x^4}+2\frac{\partial^4 w_{1,b}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{1,b}}{\partial y^4})-E(1-\lambda\nabla^2)(\frac{\partial^4 w_{1,s}}{\partial x^4}+2\frac{\partial^4 w_{1,s}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{1,s}}{\partial y^4})$$
$$+(1-\mu\nabla^2)(-I_0\frac{\partial^2(w_{1,b}+w_{1,s})}{\partial t^2}+I_2\nabla^2(\frac{\partial^2 w_{1,b}}{\partial t^2})$$
$$+I_4\nabla^2(\frac{\partial^2 w_{1,s}}{\partial t^2})-k_w(w_{1,b}+w_{1,s})+k_p\nabla^2(w_{1,b}+w_{1,s}))$$
$$-k_0[(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s})-\mu(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s}))]=0 \tag{14}$$

$$-E(1-\lambda\nabla^2)(\frac{\partial^4 w_{1,b}}{\partial x^4}+2\frac{\partial^4 w_{1,b}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{1,b}}{\partial y^4})-F(1-\lambda\nabla^2)(\frac{\partial^4 w_{1,s}}{\partial x^4}+2\frac{\partial^4 w_{1,s}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{1,s}}{\partial y^4})$$
$$+A_{44}(1-\lambda\nabla^2)(\frac{\partial^2 w_{1,s}}{\partial x^2}+\frac{\partial^2 w_{1,s}}{\partial y^2})+(1-\mu\nabla^2)(-I_0\frac{\partial^2(w_{1,b}+w_{1,s})}{\partial t^2}$$
$$+I_4\nabla^2(\frac{\partial^2 w_{1,b}}{\partial t^2})+I_5\nabla^2(\frac{\partial^2 w_{1,s}}{\partial t^2})-k_w(w_{1,b}+w_{1,s})+k_p\nabla^2(w_{1,b}+w_{1,s}))$$
$$-k_0[(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s})-\mu(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s}))]=0 \tag{15}$$

$$-D(1-\lambda\nabla^2)(\frac{\partial^4 w_{2,b}}{\partial x^4}+2\frac{\partial^4 w_{2,b}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{2,b}}{\partial y^4})-E(1-\lambda\nabla^2)(\frac{\partial^4 w_{2,s}}{\partial x^4}+2\frac{\partial^4 w_{2,s}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{2,s}}{\partial y^4})$$
$$+(1-\mu\nabla^2)(-I_0\frac{\partial^2(w_{2,b}+w_{2,s})}{\partial t^2}+I_2\nabla^2(\frac{\partial^2 w_{2,b}}{\partial t^2})$$
$$+I_4\nabla^2(\frac{\partial^2 w_{2,s}}{\partial t^2})-k_w(w_{2,b}+w_{2,s})+k_p\nabla^2(w_{2,b}+w_{2,s}))$$
$$+k_0[(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s})-\mu(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s}))]=0 \tag{16}$$

$$
\begin{aligned}
&-E(1-\lambda\nabla^2)(\frac{\partial^4 w_{2,b}}{\partial x^4}+2\frac{\partial^4 w_{2,b}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{2,b}}{\partial y^4})-F(1-\lambda\nabla^2)(\frac{\partial^4 w_{2,s}}{\partial x^4}+2\frac{\partial^4 w_{2,s}}{\partial x^2\partial y^2}+\frac{\partial^4 w_{2,s}}{\partial y^4})\\
&+A_{44}(1-\lambda\nabla^2)(\frac{\partial^2 w_{2,s}}{\partial x^2}+\frac{\partial^2 w_{2,s}}{\partial y^2})+(1-\mu\nabla^2)(-I_0\frac{\partial^2(w_{2,b}+w_{2,s})}{\partial t^2}\\
&+I_4\nabla^2(\frac{\partial^2 w_{2,b}}{\partial t^2})+I_5\nabla^2(\frac{\partial^2 w_{2,s}}{\partial t^2})-k_w(w_{2,b}+w_{2,s})+k_p\nabla^2(w_{2,b}+w_{2,s}))\\
&+k_0[(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s})-\mu(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(w_{1,b}+w_{1,s}-w_{2,b}-w_{2,s}))]=0
\end{aligned}
\tag{17}
$$

4. Method of solution

With the help of double Fourier series and the concept of Galerkin's method, the governing differential equations can be solved numerically. However, it must be considered that the system undergoes the following types of vibration as

- Out of phase vibration: $w_b=w_{1,b}-w_{2,b}\neq 0$ and $w_s=w_{1,s}-w_{2,s}\neq 0$
- In-phase vibration: $w_b=w_{1,b}-w_{2,b}=0$ and $w_s=w_{1,s}-w_{2,s}=0$
- One nanoplate fixed: $w_b=w_{1,b}=0$ and $w_s=w_{1,s}=0$

Since exact location of neutral surface is considered in this research, the in-plane and out-of-plane displacements have been decoupled. So, only the last two of governing equations will be solved. Based on double Fourier series, the transverse displacements will be assumed as

$$w_b=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{bmn}X_m(x)Y_n(y)e^{i\omega_n t} \tag{18}$$

$$w_s=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{smn}X_m(x)Y_n(y)e^{i\omega_n t} \tag{19}$$

where (W_{bmn}, W_{smn}) are the maximum deflections and the functions X_m=Sin($m\pi x/a$) and Y_n=Sin($n\pi y/b$) satisfy simply-supported edge conditions. Now, Eqs. (18) and (19) must be placed into governing equations and the coefficients of maximum deflections must be collected

$$\left\{[K]+\bar{\omega}_n^{\,2}[M]\right\}\begin{Bmatrix}W_{bmn}\\W_{smn}\end{Bmatrix}=0 \tag{20}$$

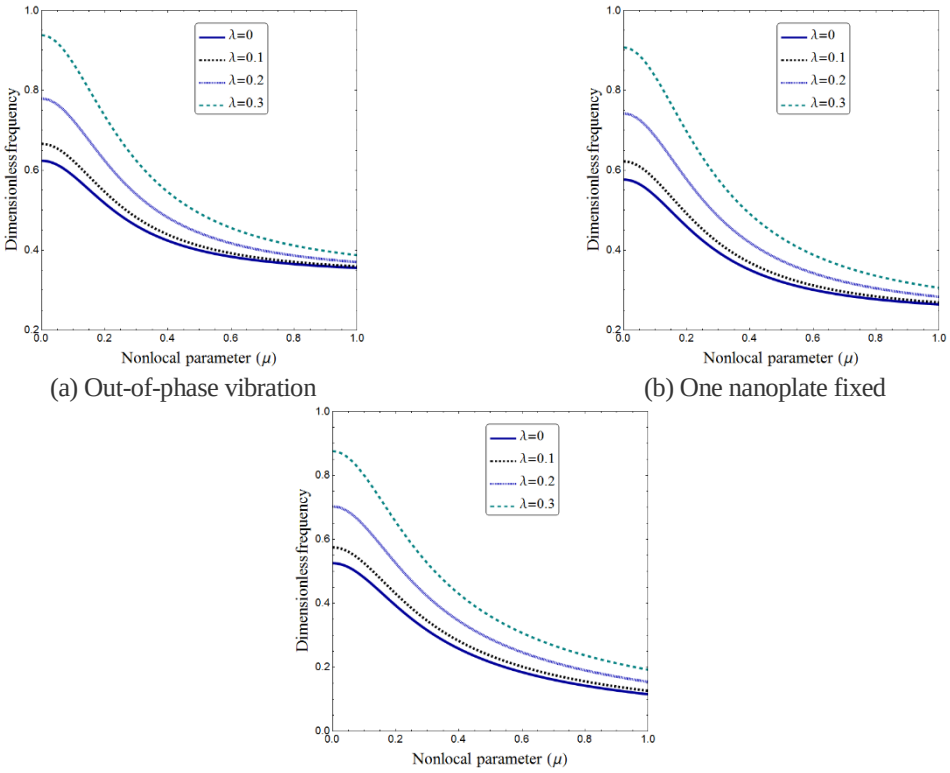
where the procedure of computing stiffness and mass matrices is very similar to that introduced in Barati (2017b). Here, the determinant of coefficient matrix is selected to be zero and then natural frequencies will be found. In this research, in-put and out-put parameters are normalized as

$$
\begin{aligned}
&\hat{\omega}=\omega a\sqrt{\frac{\rho_2}{E_2}},\ K_w=\frac{k_w a^4}{D_2},K_0=\frac{k_0 a^4}{D_2},K_p=\frac{k_p a^2}{D_2},\\
&D_1=\frac{E_2 h^3}{12(1-\nu^2)},\ \mu=\frac{ea}{a},\ \lambda=\frac{l}{a}
\end{aligned}
\tag{21}
$$

5. Numerical results and discussions

This paper uses NSGT for analyzing vibrational behavior of a double-coupled nanoplate system based on a refined plate theory. Two nanoplates are coupled with each other with the use of linear springs. These two nanoplates are made of metal foam with different pore distributions. Results will be illustrated to indicate the importance of pores, coupling springs and scale parameters.

The validation is performed based on a comparison of vibrational frequencies with those stablished by Natarajan *et al.* (2012) for a FG nanoplate. It can be seen from Table 1 that presented solution is previous section can predict the vibration frequency of a nanoplate with high accuracy.



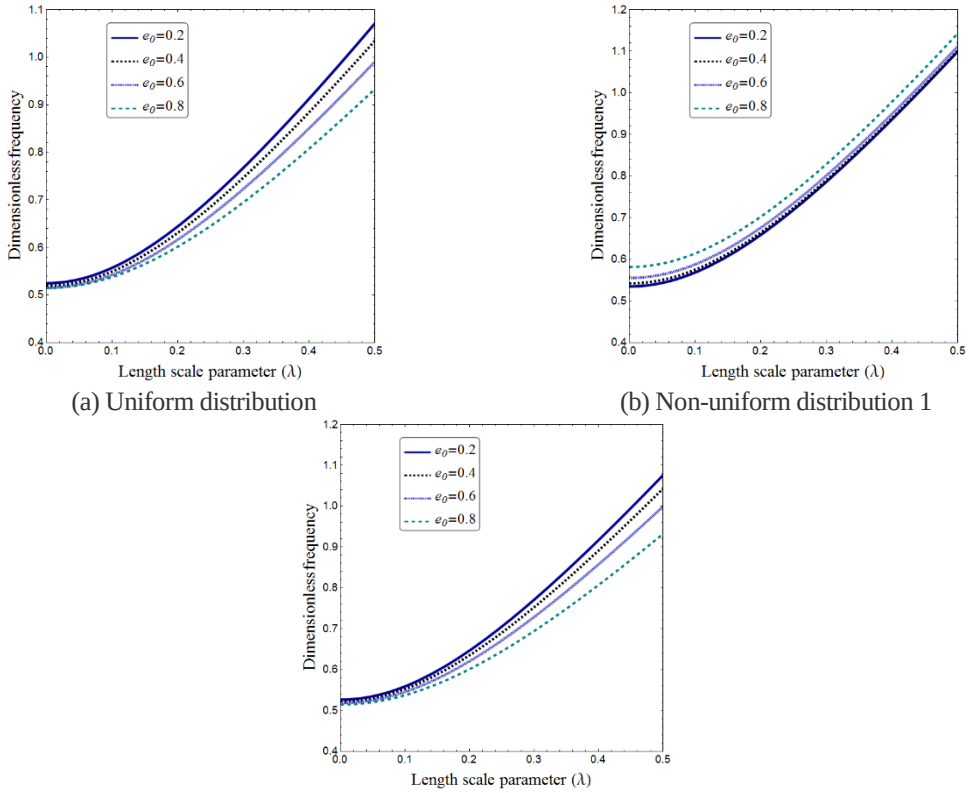
(a) Out-of-phase vibration (b) One nanoplate fixed

(c) In phase vibration

Fig. 3 Normalized frequency of double-coupled nanoplate against nonlocal and strain gradient parameters (a/h=10, K_w=0, K_p=0, ΔT=0, K_0=50, e_0=0.5)

Table 1 Frequency verification of simply-supported FG nanoplates

a/h	μ	a/b=1		a/b=2	
		Natarajan *et al.* (2012)	present	Natarajan *et al.* (2012)	present
10	0	0.0441	0.043823	0.1055	0.104329
	1	0.0403	0.04007	0.0863	0.085493
	2	0.0374	0.037141	0.0748	0.074174
	4	0.0330	0.032806	0.0612	0.060673



(a) Uniform distribution (b) Non-uniform distribution 1

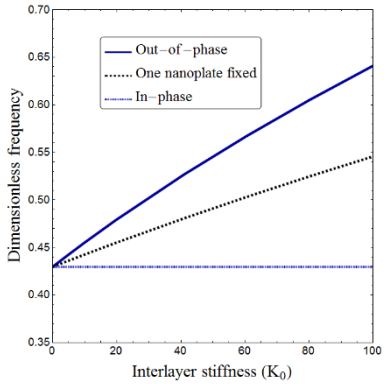
(c) Non-uniform distribution 2

Fig. 4 Normalized frequency of double-coupled nanoplate against strain gradient parameter for different porosity distributions (K_0=50, a/h=10, ΔT=0, K_w=0, K_p=0, μ=0.2)

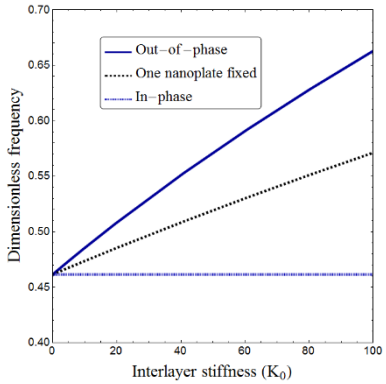
In Fig. 3, one can see the variation of vibrational frequency for a variety of both nonlocal and strain gradient coefficients. This figure has three parts and each part is related to one type of motion for double-coupled system. Porosity parameter for nanoplates is chosen to be e_0=0.5. It can be understood from Fig. 3 that vibration frequency of system will rise with strain gradient coefficient and will reduce with nonlocality coefficient. This observation is valid for all kinds of coupled system motion. So, vibration behavior of double nanoplate system is dependent on both scale effects.

In Fig. 4 one can see the variation of vibrational frequency of double-nanoplate system with different porosity coefficients and dispersions. Effect of surrounding medium is neglected for this figure. It can be understood from Fig. 4 that vibration frequency of system will reduce or increase with pore coefficient. But, this variation relies on the type of pore dispersion in thickness of nanoplates. Pore type 1 gives higher vibrational frequencies than other pore types.

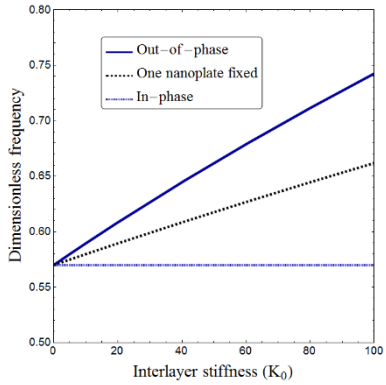
Fig. 5 presents the stiffness effect of coupling springs on vibrational frequency of double-nanoplate system. As an example, pore coefficient is chosen as e_0=0.5 with uniform type. Vibrational frequency is found to be independent of coupling springs in the case of in-phase motion. But, in other two cases, vibrational frequency will rise with the coupling spring stiffness. So, the double nanoplate system will be more rigid as the coupling spring or Winkler/Pasternak parameters become stiffer.



(a) K_w=0, K_p=0

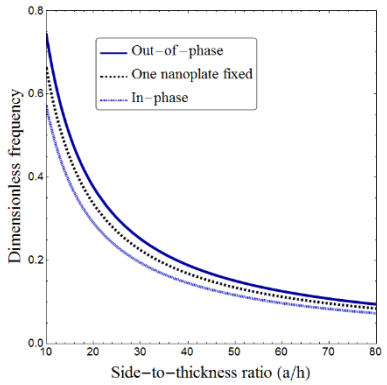


(b) K_w=25, K_p=0

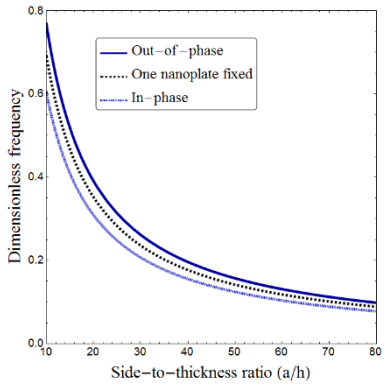


(c) K_w=25, K_p=5

Fig. 5 Normalized frequency of double-coupled nanoplate against interlayer stiffness for various elastic foundation parameters (a/h=10 e_0=0.5, μ=0.2, ΔT=0.1, λ=0.1)



(a) Uniform distribution



(b) Non-uniform distribution 1

Fig. 6 Normalized frequency of double-coupled nanoplate against side-to-thickness ratio for different types of vibration (e_0=0.5, K_w=25, K_p=5, μ=0.2, ΔT=0.1, K_0=100)

In Fig. 6, the change of vibration frequency of double-coupled nanoplate with side-to-thickness ratios (a/h) is illustrated for all kinds of system motion as well as uniform and non-uniform pores

1. One can see that double-nanoplate system is less rigid for greater side-to-thickness ratios. Thus, derived vibrational frequency becomes lower by an increase of a/h. Since material properties are constant over the thickness in the case of uniform pore type, the vibration frequency in this case is smaller than the case of non-uniform pore type 1.

6. Conclusions

This article focused on vibration characteristic of a double-coupled nanoplate system modeled by NSGT and refined plate theories. Nanoplates were considered to be porosity-dependent accounting for different pore types. It was understood that vibration frequency of system raised with strain gradient coefficient and reduced with nonlocality coefficient. It was also found that vibration frequency of system might reduce or increase with pore coefficient. Also, pore type 1 gave highest vibration frequency among considered pore types. Vibrational frequency was found to be independent of coupling springs in the case of in-phase motion. But, in the case of out-of-phase motion, vibrational frequency raised with the coupling spring stiffness.

Acknowledgments

The researchers extend their thanks and appreciation to Mustansiriya University and the collage of Engineering.

References

Atmane, H.A., Tounsi, A., Bernard, F and Mahmoud, S.R. (2015b), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct.*, **19**(2), 369-384. https://doi.org/10.12989/scs.2015.19.2.369.

Atmane, H.A., Tounsi, A and Bernard, F. (2015a), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *Int. J. Mech. Mater. Des.*, **13**(1), 71-84. https://doi.org/10.1007/s10999-015-9318-x.

Barati, M.R and Zenkour, A.M. (2016), "Electro-thermoelastic vibration of plates made of porous functionally graded piezoelectric materials under various boundary conditions", *J. Vib. Control*, **24**(10), 1910-1926. https://doi.org/10.1177%2F1077546316672788.

Barati, M.R. (2017a), "Nonlocal-strain gradient forced vibration analysis of metal foam nanoplates with uniform and graded porosities", *Adv. Nano Res.*, **5**(4), 393-414. https://doi.org/10.12989/anr.2017.5.4.393.

Barati, M.R. (2017b), "Vibration analysis of FG nanoplates with nanovoids on viscoelastic substrate under hygro-thermo-mechanical loading using nonlocal strain gradient theory", *Struct. Eng. Mech.*, **64**(6), 683-693. https://doi.org/10.12989/sem.2017.64.6.683.

Barati, M.R. (2017c), "Investigating dynamic response of porous inhomogeneous nanobeams on hybrid Kerr foundation under hygro-thermal loading", *Appl. Phys. A*, **123**(5), 332. https://doi.org/10.1007/s00339-017-0908-3.

Barati, M.R. (2017d), "On wave propagation in nanoporous materials", *Int. J. Eng. Sci.*, **116**, 1-11. https://doi.org/10.1016/j.ijengsci.2017.03.007.

Barati, M.R., Zenkour, A.M and Shahverdi, H. (2016), "Thermo-mechanical buckling analysis of embedded nanosize FG plates in thermal environments via an inverse cotangential theory", *Compos. Struct.*, **141**, 203-212. https://doi.org/10.1016/j.compstruct.2016.01.056.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081. https://doi.org/10.12989/scs.2015.18.4.1063.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249. https://doi.org/10.12989/scs.2016.20.2.227.

Chen, D., Yang, J. and Kitipornchai, S. (2015), "Elastic buckling and static bending of shear deformable functionally graded porous beam", *Compos. Struct.*, **133**, 54-61. https://doi.org/10.1016/j.compstruct.2015.07.052.

Chen, D., Yang, J. and Kitipornchai, S. (2016), "Free and forced vibrations of shear deformable functionally graded porous beams", *Int. J. Mech. Sci.*, **108**, 14-22. https://doi.org/10.1016/j.ijmecsci.2016.01.025.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710. https://doi.org/10.1063/1.332803.

Han, S.C., Park, W.T. and Jung, W.Y. (2015), "A four-variable refined plate theory for dynamic stability analysis of S-FGM plates based on physical neutral surface", *Compos. Struct.*, **131**, 1081-1089. https://doi.org/10.1016/j.compstruct.2015.06.025.

Li, L., Hu, Y. and Ling, L. (2015), "Flexural wave propagation in small-scaled functionally graded beams via a nonlocal strain gradient theory", *Compos. Struct.*, **133**, 1079-1092. https://doi.org/10.1016/j.compstruct.2015.08.014.

Mahmoudi, A., Benyoucef, S., Tounsi, A., Benachour, A. and Bedia, E.A.A. (2018), "On the effect of the micromechanical models on the free vibration of rectangular FGM plate resting on elastic foundation", *Earthq. Struct.*, **14**(2), 117-128. https://doi.org/10.12989/eas.2018.14.2.117.

Mechab, B., Mechab, I., Benaissa, S., Ameri, M. and Serier, B. (2016), "Probabilistic analysis of effect of the porosities in functionally graded material nanoplate resting on Winkler-Pasternak elastic foundations", *Appl. Math. Model.*, **40**(2), 738-749. https://doi.org/10.1016/j.apm.2015.09.093.

Mehala, T., Belabed, Z., Tounsi, A. and Beg, O.A. (2018), "Investigation of influence of homogenization models on stability and dynamics of FGM plates on elastic foundations", *Geomech. Eng.*, **16**(3), 257-271. https://doi.org/10.12989/gae.2018.16.3.257.

Natarajan, S., Chakraborty, S., Thangavel, M., Bordas, S. and Rabczuk, T. (2012), "Size-dependent free flexural vibration behavior of functionally graded nanoplates", *Comput. Mater. Sci.*, **65**, 74-80. https://doi.org/10.1016/j.commatsci.2012.06.031.

Rezaei, A.S. and Saidi, A.R. (2016), "Application of Carrera unified formulation to study the effect of porosity on natural frequencies of thick porous-cellular plates", *Compos. Part B Eng.*, **91**, 361-370. https://doi.org/10.1016/j.compositesb.2015.12.050.

Sadoun, M., Houari, M.S.A., Bakora, A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2018), "Vibration analysis of thick orthotropic plates using quasi 3D sinusoidal shear deformation theory", *Geomech. Eng.*, **16**(2), 141-150. https://doi.org/10.12989/gae.2018.16.2.141.

Wattanasakulpong, N and Ungbhakorn, V. (2014), "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", *Aerosp. Sci. Technol.*, **32**(1), 111-120. https://doi.org/10.1016/j.ast.2013.12.002.

Yahia, S.A., Atmane, H.A., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, **53**(6), 1143-1165. https://doi.org/10.12989/sem.2015.53.6.1143.

Zenkour, A.M. (2009), "The refined sinusoidal theory for FGM plates on elastic foundations", *Int. J. Mech. Sci.*, **51**(11), 869-880. https://doi.org/10.1016/j.ijmecsci.2009.09.026

Zenkour, A.M. (2016), "Nonlocal transient thermal analysis of a single-layered graphene sheet embedded in viscoelastic medium", *Physica E Low-dimen. Syst. Nanostruct.*, **79**, 87-97. https://doi.org/10.1016/j.physe.2015.12.003.

DC