Coupled Systems Mechanics, *Vol. 5, No. 3 (2016) 223-234*
DOI: https://doi.org/10.12989/csm.2016.5.3.223

Analytical assessment of elevated tank natural period considering soil effects

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(Received August 2, 2016, Revised November 21, 2016, Accepted November 25, 2016)

Abstract. The main purpose of current study is to find the soil effects on natural period of elevated tank. The coupled analytical method is used to assess in this study. The current study presented models which are capable to consider the soil dynamic stiffness changes and fluid- structure interaction effects on natural period of elevated tanks. The basic of mentioned models is extracted from elastic beam and lumped mass theory. The finite element is used to verify the results. It is observed that, external excitation can change the natural period of elevated tanks. Considering the increase of excitation frequency, the natural period will be decreased. The concluded values of natural period in case of soft and very soft soil are more affected from excitation frequency values. The high range of excitation frequency may reduce the natural period values. In addition it is observed that the excitation frequency has no significant effect on convective period compare with impulsive period.

Keywords: ground excitation; analytical model; natural period; elevated tank; impulsive; convective

1. Introduction

The elevated tanks are the most important and necessary structures which used in water distribution, Water storage system, oil storage system and firefighting (ACI 350 2006, ACI-317 2008, Dutta 1995, 2004). They are also working as a life line structures. They should be stable during and after the earthquake and natural disasters (Livaoglu and Dogangum 2006, Ghanbari and Maedeh 2015). To design these sensitive structures, the role of natural period is one of more necessary parameters to consider the dynamic behavior of elevated tanks (Housner 1963, Haroun and Temraz 1992, Livaoglu and Dogangum 2006, Livaoglu and Dogangum 2007). One of the major topics in natural period determination is to consider the effects of soil and fluid on the body of structure (Livaoglu and Dogangum 2006).

The method which consider both fluid and soil effects on natural period is called as fluid-structure- soil interaction (Livaoglu and Dogangum 2006, Ghanbari and Maedeh 2015). Most of

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http://www.techno-press.org/?journal=csm&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

pervious researches have considered, only the effects of liquid on vessels which called as fluid-structure interaction theory (Goudarzi and Sabbagh-Yazdi 2008, 2009, Ghaemmaghami *et al.* 2010, 2013). To careful examination of natural period, pervious researchers recommended to separate the liquid part into two different phases as convective and impulsive phases (Housner 1963). The convective phase works as a fluctuated part of the liquid and the impulsive phase works as a rigid mass which connected to the body of vessel (Housner 1963, Marashi and Shakib 2008, Moslemi *et al.* 2011).

According to the effects of soil and liquid on response of elevated tanks natural period, various research methods have been applied to the study of elevated tank in recent years. Haroun and Ellaithy (1985), Resheidat and Sunna (1990), Haroun and Temraz (1992), Dutta *et al.* (2004), Livaoglu and Dogangun (2006), Livaoglu and Dogangun (2007), Marashi and Shakib (2008), Goudarzi and Sabbagh-Yazdi (2008), Livaoglu *et al.* (2011), Livaoglu (2013), Moeindarbari *et al.* (2014), Soraces *et al.* (2015) and Ghanbari and Maedeh (2015) had different assessment of vessels dynamic behavior.

There are many different methods to find the natural period of structures, but regarding the analytical assessment structures dynamic behavior, the most famous method is called as equivalent mass-spring (Wolf 1985, Chopra 2000, Jahankhah *et al.* 2013). Majority of developed analytical soil- structure interaction methods are capable to determine the natural period in a constant condition of soil stiffness (Lysmer 1979, Livaoglu 2006). Considering the frequency changes during the earthquake evidence, the dynamic stiffness of soil will be change (Novak *et al.* 1978, Gazetas 1991, Gazetas and Stokoe 1991, Pacheco 2008, Shirgir *et al.* 2015). In this reason, find the natural period of an elevated tank system considering the soil dynamic stiffness changes in different cases of excitation frequency will be important (Pacheco 2008, Shirgir *et al.* 2015).

In this study by using two different analytical models which cable to consider the effects of dynamic stiffness of soil and fluid- structure interaction on elevated tank, the natural period will be evaluated. The base of analytical models is extracted from elastic beam and added mass theory (Novak and Abloul-Ella 1978, Novak *et al.* 1978, Pacheco 2007, 2008, Shirgir *et al.* 2015). Results of statically condition are verified by results of numerical model. The effects of excitation frequency on the impulsive and convective natural period will be evaluated.

2. Basic assumptions of analytical model

To find hydrodynamic effects on liquid tanks natural period, many different analytical and semi-analytical methods have developed (Moslemi *et al.* 2011). The most useful and applicable analytical method was developed by Westergaard (1933). Several regular approximate methods were also developed by Housner (1963) and Haroun and Ellaithy (1985) which, they are used in international codes such as Eurocode 8 (2006) and ACI 350 (2006). In this study the Housner (1963) recommended model is used for the liquid part evaluation.

To analytical soil model in current study it is assumed that, the layer of soil which placed under of elevated tank foundation has a similar behavior of a continuous pile. The soil layers are considered homogeneous, isotropic, and linear-elastic. The soil-pile is assumed to be vertical, cylindrical and moving as a rigid body (a hypothesis that is consistent with the Naiver-Bernoulli beam theory) (Novak and Abloul-Ella 1978, Novak *et al.* 1978, Pacheco 2008, Shirgir *et al.* 2015). The complex horizontal stiffness of the soil associated with a unit length of the cylinder, k_h, (or the dynamic soil reaction per unit length of pile to a unit horizontal harmonic displacement of the rigid

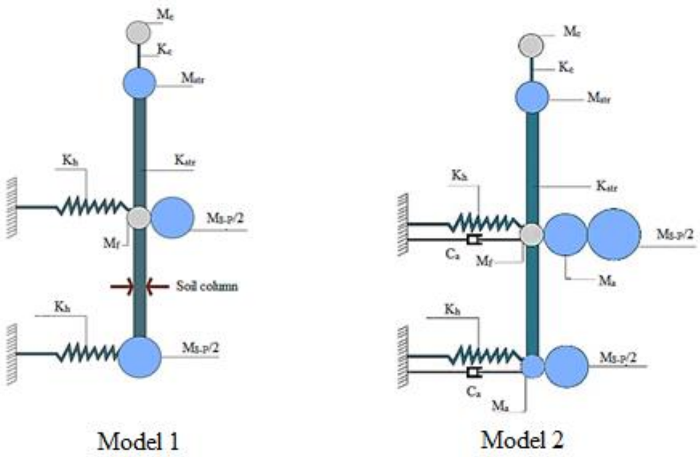

Fig. 1 Schematics of current study analytical models

disc) is given by the following Eq. (1) (Pacheco 2008, Shirgir *et al.* 2015)

$$K_h = G\pi f(a_0, \nu, D) = G\pi\{\mathrm{Real}[f(a_0, \nu, D)] + i\ \mathrm{Imag}[f(a_0, \nu, D)]\} \tag{1}$$

Where G is the shear modulus of soil and the $f(a_0, v, D)$ is a dynamic variable which calculated by using the Bessel function (Novak and Abloul-Ella 1978, Novak *et al.* 1978, Pacheco 2008). a_o is a dimensionless frequency$=\omega r_o/V_S$, ω is the vibration frequency in rad/sec, r_o is the pile (foundation) radius, V_s is the shear wave velocity of the soil; v is the Poisson's ratio of the soil and also D is the damping ratio of the soil. Basic information of this method is reported in pervious and original literature (Novak *et al.* 1978, Novak 1974).

The coefficients α_k, α_m, and α_c were determined by pervious researcher (Novak and Abloul-Ella 1978, Novak *et al.* 1978, Pacheco 2007, 2008). In this study the natural period considering dynamic excitation will be evaluated with and without considering the dynamic damping (Pacheco 2008, Shirgir 2015). The schematic picture of presented analytical models is shown in Fig. 1. Regarding the schematic the superstructure part assumption is constant in both recommended models. The analytical soil modeling of model 1 just considers the real part of Eq. (1) and it is neglected from dynamic damping. Model 2 will consider both the real and imaginary parts of the Eq. (1).

The assembled developed stiffness and mass matrix for an elevated tank system considering mentioned models show as following respectively

$$[K] = \begin{bmatrix} K_c & -K_c & 0 & 0 & 0 & 0 \\ -K_c & K_c + K_{str} & -K_{str} & 0 & 0 & 0 \\ 0 & -K_{str} & K_{str} + K_h + \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ 0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} + K_h & -\frac{6EI}{L^2} \\ 0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$[M] = \begin{bmatrix} m_c & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{str} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_f + \frac{m_{S-P}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_f + m_{str}(h_i + h_{str})^2 + m_c(h_c + h_{str})^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{m_{S-P}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{s-p} \end{bmatrix}$$

Where:
M_c: mass of convective liquid
M_{str}: mass of impulsive liquid+mass of vessel+66% of shaft structure
M_f: mass of foundation
I_f: moment inertia of foundation
K_c: convective liquid stiffness
K_{str}: stiffness of structure

The basic information of above values are described in literature (Housner 1963, ACI-350 2006). In this study, model 1 is defined as analytical model without the dynamic damping effects and model 2 defined analytical model to consider damping effects in dynamic soil stiffness. The numerical model with an emphasis on fluid structure soil interaction is made in an advanced finite element software. To model the vessel and soil the plain 82 elements and for liquid the fluid 79 elements are used. There are many experiences of mentioned FEM method to verify the vessel model (Goudarzi and Sabbagh-Yazdi 2009, Hacıefendioğlu 2012).

3. The case study properties

An RC shaft elevated tank with capacity of 486 m^3 is considered in current study. The elevated tank has a shaft supporting structure with a total height of 20 meter from ground surface. Container is assumed filled 4 meters to final safe capacity of vessel with water to a density of 1000 kg/m^3. The density of concrete is assumed 25 kN/m^3 and additional information of shaft supporting, vessel are show in Fig. 2.

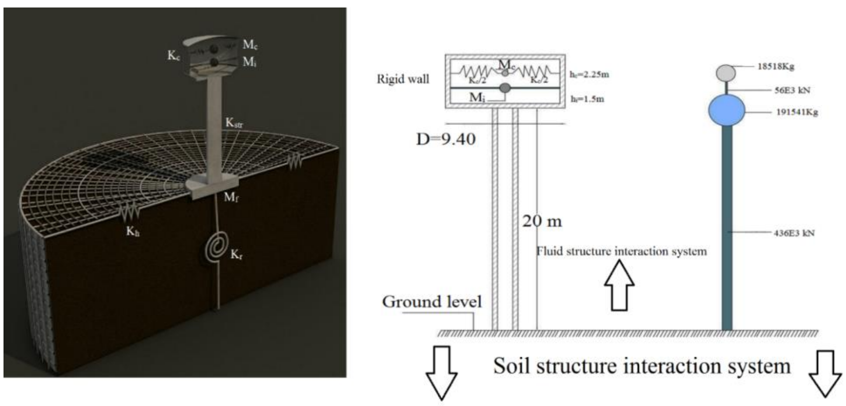

Fig. 2 Simplified geometry of elevated tank and the simplest analytical model

Table 1 Soil classification and general properties

Soil category	*v*	*γ*(kN/m^3)	*G*(kN/m^2)	*Vs*(m/s)
Very hard	0.2	19	2041667	1026.71
Hard	0.3	18	293461.5	399.92
Soft	0.35	17	35666.67	143.46
Very soft	0.4	13	11428.57	92.86

Different categories of current study soil properties show in Table 1.

4. Results and discussion

Considering the current study assumptions analytical and numerical models are made. The finite element model by using the direct method technique is made in general software ANSYS 15 (ANSYS 2015). The direct method estimation allows the performer to analyze the considered soil foundation- structure system as a complete system in a single step, in which the free field input motions are specified along the base and sides of the model (Livaoglu and Dogangun 2007, Torabi and Reyhani 2014). This method have been particularly employed for solution of the tank- soil system in this study, because it is the robust method that remains valid for all kinds of problems involving material linearity, contact problems, different loading cases and complex geometries (Preisig and Jeremic 2005, Li *et al.* 2014). The convective and impulsive mode in numerical model are shown in Fig. 3.

It is observe that there is no shaft deformation in case of convective mode and only the liquid has deformation. Both liquid and shaft will be deformed in impulsive mode. The shape of liquid deformation in impulsive mode is different compare with convective mode. Also, the maximum liquid wave height of impulsive mode is lower than convective mode.

To find the natural convective and impulsive period the Block Lanczos method (ANSYS 2015) was chosen. The pervious researcher recommended to find the convective period of liquid maximum mass participation of liquid should be considered (Moslemi *et al.* 2011). Results of numerical direct method considering different condition of soil cases are reported in Fig. 4.

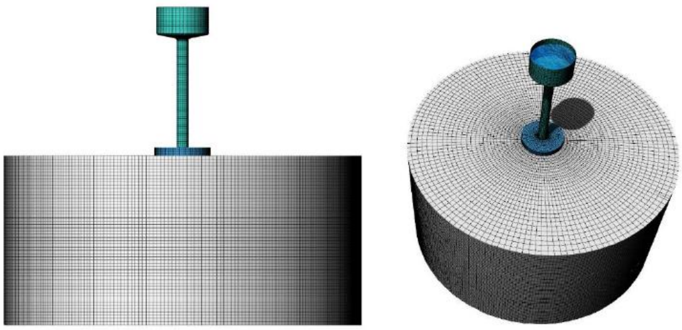

Fig. 3 The numerical model of elevated tank by using the 3D direct method

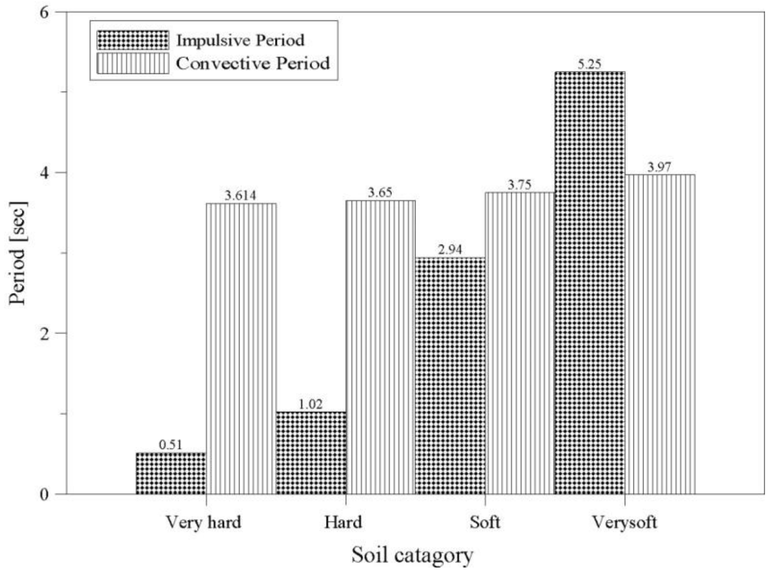

Fig. 4 Results of finite element method with an emphasis on fluid- structure-soil interaction

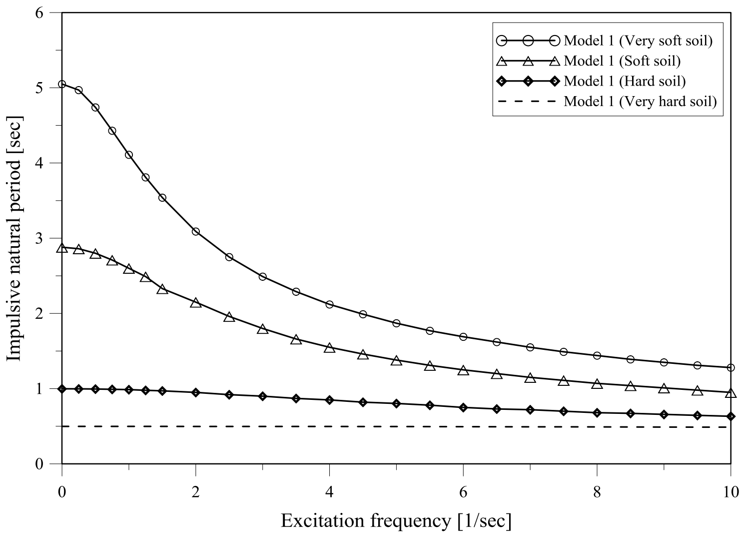

Fig. 5 Results of impulsive period values concluded from model 1

It is observe that in case of very soft soil the impulsive period values will change to more convective period values. One of the most reasons of this event is soil stiffness value changes. Considering the soil softening, the maximum displacement of the base and foundation would be increased. In this case according to the super position theory, the maximum displacement of vessel will be added to maximum displacement of ground and foundation. Increasing the total displacement of the super structure system, will be a cause of the impulsive period increase. In addition, it is observed that in case of very hard soil, the maximum displacement of ground is low and the impulsive period has lower values compare with convective period.

Table 2 Results of impulsive period concluded from model 1 and finite element

Soil	Model 1 (sec)	FEM (sec)	Error %
Very hard	0.55	0.51	7.84
Hard	1.088	1.02	6.67
Soft	3.01	2.94	2.38
Very soft	5.2	5.25	-0.95

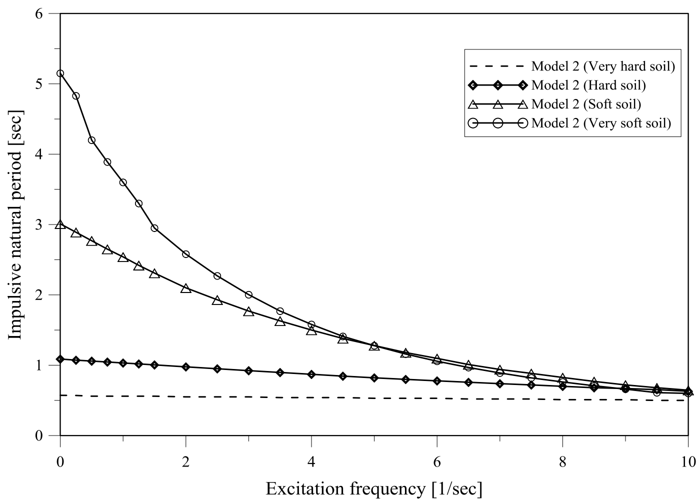

Fig. 6 Results of impulsive period by using model 2 considering different soil category and excitation frequency

The analytical model results are given by substituting the assembled mass and stiffness matrix in eigenvalue equation (Chopra 2000). To consider the ground mass participation, regarding empirical experiences (Pacheco 2008, Shirgir *et al.* 2015) the suggested soil pile length for the case of hard and very hard soils is recommended 30 meters. The recommended value in case of soft and very soft soil is changed to 25 meters. It is observe that the mass participation for hard and very hard soil is higher that soft and very soft soil.

The excitation frequency domain is chosen from 0 to 10 (1/sec). The normal range of construction natural frequency is from 0.1 to 3 (1/sec) (Kramer 1996), but the frequency content of excitation is included more different frequency values. The bed rock material properties, depth of failure occurrence and many other reasons (Kramer 1996) can take effects on earthquake frequency content. Regarding to frequency excitation changes, the value of a_0 will be changed (Pacheco 2008, Shirgir *et al.* 2015). In this reason the values of mass and stiffness matrix would change considering different frequency of excitation (Pacheco 2008, Shirgir *et al.* 2015).

Results of impulsive period extracted from model 1 considering different case of soil properties and excitation frequency values are plotted in Fig. 5. It is observed that by increasing the excitation frequency the impulsive natural period of elevated tank will be decreased. Results show that the dependency of natural period to excitation frequency in case of hard and very hard is not significant compare with the results of soft and very soft soil cases.

The low range of excitation frequency (Less than four 1/sec) is more effective on impulsive

Table 3 Results of impulsive period concluded from model 2 and finite element

Soil	Model 2 (sec)	FEM (kN/m^3)	Error %
Very hard	0.5	0.51	-1.96
Hard	0.998	1.02	-2.16
Soft	2.88	2.94	-2.04
Very soft	5.05	5.25	-3.81

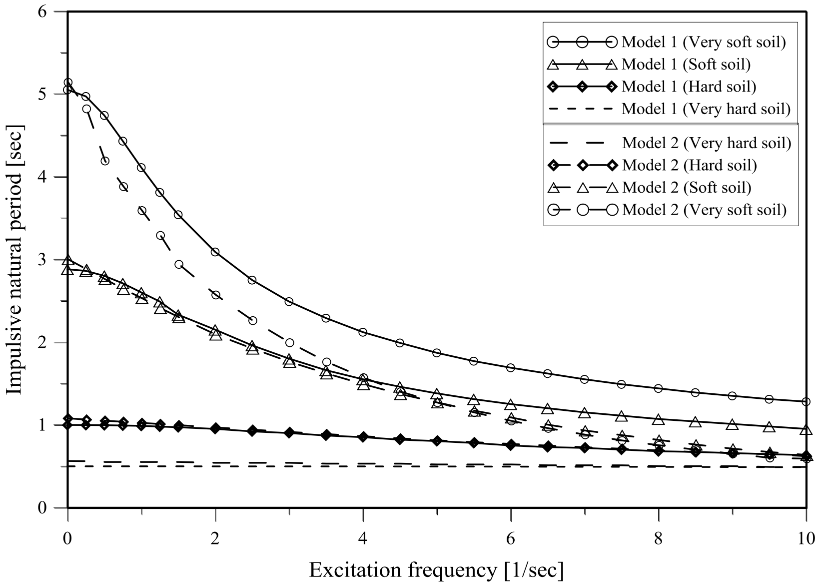

Fig. 7 Comparison results of impulsive period extracted from models 1 and 2

natural period in cases of soft and very soft soil. The effects of high range frequency excitation will be reduced. The effects of excitation frequency in range of more than 10 (1/sec) can be ignored.

To compare the results of model 1 and numerical model, both mentioned models values are reported to Table 2. Maximum difference will be occurred in case of hard and very hard soil. The values of ground mass participation will be a more effective reason of this difference. Neglecting from the geometry and moment inertia of the vessel and shaft and their effects on rocking displacement of vessel will be the second reason of results differences. The next reason of this evidence is the uncertainty in soil pile length choosing as ground mass participation in analytical model. In addition, in this analytical model it is neglected from dynamic damping effects on the response of natural period.

The results of model 2 considering the dynamic damping of soil and its effects on soil dynamic stiffness show in Fig. 6. It is observed that in case of high frequency excitation the results of different soil cases will be converged. The low range frequency of excitations have more effects on natural impulsive period in case of soft and very soft soil. Same as model 1 it is observed that in model 2 the frequency dependency of natural impulsive period will be significant in case of soft and very soft soil condition.

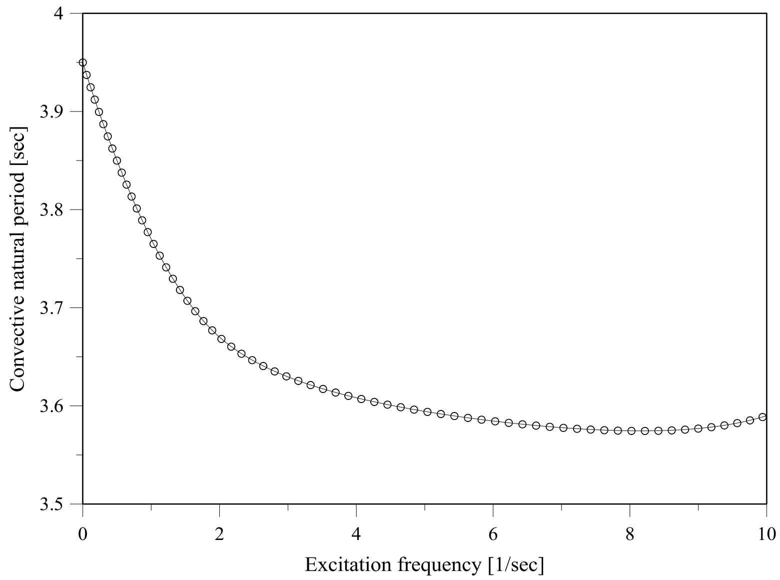

Fig. 8 Results of convective period with an emphasis on very soft soil case

The compared results of model 2 and FEM are reported in Table 3. It is observed that the model 2 have less error compare to results of model 1. Considering the dynamic damping in this model is the cause of fewer error compare with results of model 1.

The compared results of model 1 and model 2 are shown in Fig. 7. It is observed that generally model 2 has a lower estimation of natural impulsive period in case of very soft soil. Additional results show that there are negligible differences according to results of model 1 and model 2 in case of soft to very hard soil. According to extracted results of impulsive natural period in case of soft and very soft soils, it is observed that, the results values are located on the region of displacement sensitive in response spectrum. The displacement controlling in this area will be necessary for superstructure designing. Also in case of hard and very hard soils the natural impulsive period values are located in acceleration sensitive part of response spectrum. It is observed that the statically condition of soil have a higher estimation of natural impulsive period compare with the high frequency excitation condition. It means the high frequency excitation is capable to change the dynamic soil stiffness and decrease the impulsive natural period.

The natural convective period results show that the excitation frequency effects in convective period is not significant. Maximum variation of convective period in different case of soil and excitation frequency reported less than 6 percent. The value of convective period changes in case of very soft soil are shown in Fig. 8.

Complimentary evaluation on the concluded results show that, the dependency of impulsive period to soil category, ground mass participation and excitation frequency will be more important than convective period. Also results show that regarding to decrease the natural period considering excitation frequency the base shear and over turning moment can be increase. Also results of natural impulsive period show that, its value is place on acceleration sensitive part in response spectrum in case of hard and very hard soil but also the values will be moved to displacement sensitive part in case of soft and very soft soils. The high frequency excitation will be changed the sensitive part of impulsive period in case of soft and very soft soil. Generally the convective period values are placed on displacement sensitive of response spectrum.

5. Conclusions

The main propose of current study is to find the natural period of elevated tank system considering dynamic soil stiffness and the inside liquid effects. Two analytical models are used in current study which are capable to consider dynamic stiffness of soil, soil damping ratio and liquid effects on natural period. Regarding current study evaluation the following conclusions are reported:

• Two presented analytical models in this study are capable to estimate natural impulsive and convective period of elevated tank considering dynamic soil stiffness and the ground excitation frequency. The mentioned models have good estimation of impulsive and convective period compare with finite element results in case of statically mode.

• In case of very soft soil generally the model 2 has a much lower estimate compare with results of model 2. Considering different soils cases the estimation difference will be decreased. The dynamic damping effect on soil stiffness values is one of the important reason of this occurrence.

• Maximum error of both presented models compare with FEM results in case of static condition is determined lower than 8 percent. In this reason both presented models can use in analytical assessment of elevated reservoir.

• Regarding both analytical models results, considering increase of excitation frequency the natural impulsive period will be decreased. It means in the high range of excitation frequency the dynamic soil stiffness will be increased.

• The excitation frequency effects on convective period are not significant compare with the effects on impulsive period. Maximum 6 percent difference will be occurred in case of very low to very high range of excitation frequency.

Acknowledgments

The authors wish to appreciate institute of geotechnical engineering for University of Bodenkultur, Wien, Austria for generous supports to complete this work.

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