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1D contaminant transport using element free Galerkin method with irregular nodes

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Abstract. The present study deals with the numerical modelling for the one dimensional contaminant transport through saturated homogeneous and stratified porous media using meshfree method. A numerical algorithm based on element free Galerkin method is developed. A one dimensional form of the advective-diffusive transport equation for homogeneous and stratified soil is considered for the analysis using irregular nodes. A Fortran program is developed to obtain numerical solution and the results are validated with the available results in the literature. A detailed parametric study is conducted to examine the effect of certain key parameters. Effect of change of dispersion, velocity, porosity, distribution coefficient and thickness of layer is studied on the concentration of the contaminant

Keywords: contaminant transport; irregular nodes; homogenous and stratified porous media; meshfree methods; element free Galerkin method

1. Introduction

The groundwater pollution due to contaminant transport from landfills and lagoons has been a major concern. Contamination is caused by landfills, lagoons, industries and factories. In order to adopt preventive measures, the extent of migration of contaminants must be known accurately. From the basic principles, the phenomenon of contaminant transport can be expressed in the form of governing differential equation, consisting of molecular diffusion, advection and sorption. Patil and Chore (2014) presented an overview of the various numerical and experimental studies on the contaminant transport. Analytical solutions find a very attractive approach for the flow equations (Ogata and Banks 1961, Van Genuchten 1981, Rowe and Booker 1989, Chen *et al.* 1989). Analytical solutions are very effective for homogenous isotropic medium and simple geometry; but they cannot be applied to complex groundwater problems encountered and hence, the need for an effective numerical technique arises.

The numerical methods like finite difference method (Mirbagheri 2004, Chakraborty and Ghosh 2010, Sharma *et al.* 2014) and the finite element method (Javadi and Al-Najjar 2007, Cooke and Rowe 2008) perform better while handling problems of complexity, heterogeneity and anisotropy.

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Finite difference method suffers from major disadvantage in modeling of irregular geometry and the studies are still going on for developing a robust method capable of using irregular geometry. Finite element method (FEM) requires mesh generation and it is time consuming. Further, solutions to advection-dominant transport problems by FEM are often corrupted by node-node oscillations which can only be removed by severe mesh and time step refinement. Moreover it cannot be used for irregular nodes. In recent years, meshfree methods are getting attention as their basic idea is to eliminate the structure of mesh and construct approximate solutions for the equation in terms of nodes. In the present study, governing differential equation is solved using element free Galerkin method (EFGM) for irregularly spaced nodes in the domain. Element free Galerkin method is a meshfree method developed recently in order to eliminate the structure of mesh and construct approximate solutions for the equation in terms of nodes (Liu and Gu 2005). The EFGM is the most successful meshfree method and has been used for solving boundary value problems related to various field study (Belytschko *et al*. 1994, Kumar and Dodagoudar 2008, 2009, Swati and Eldho 2013, Satavalekar and Sawant 2014). Few researchers developed finite difference method (FDM) with arbitrary grids, or the general FDM (Lizska and Orkisz 1980), meshless finite difference (Mikewski and Orkisz 2011) for solving various problems in the field of Engineering. Pepper and Stephenson (1995) developed unstructured adaptive meshing using finite element method. Wang *et al*. (2008) developed a common technique to develop support domains in regular as well as irregular nodes in meshless methods and it can be used with various meshless methods in the crack analysis. Mategaonkar and Eldho (2012) observed sensitivity of irregular nodes in point collocation method (PCM), a type of meshless method for contaminant transport modeling. In the present study, a Fortran program based on element free Galerkin method (EFGM) is developed for homogeneous and stratified media considering irregular spacing of the nodes. A parametric study is carried out to observe the effect on migration of contaminants by changing various parameters.

2. Formulation of element free Galerkin method

The one-dimensional form of the governing differential equation for contaminant migration through a saturated porous medium is expressed as

$$\left(1+\frac{\rho_d}{n}k_d\right)\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial x^2}-v_x\frac{\partial C}{\partial x}$$

$$\text{where, } R=\left(1+\frac{\rho_d}{n}k_d\right), D=\alpha v+D^* \tag{1}$$

in which, ρ_d and n are bulk density and porosity of porous medium, k_d is distribution constant, C is concentration of contaminant, D is dispersion coefficient, α is the dispersivity and D^* is diffusion coefficient and v_x is seepage velocity.

Initial Conditions:

at t=0 , $C(x,y,0)=0$

Boundary Conditions:

$C(0,y,t) = C_0$ on Γ_s (Dirichlet Boundary Condition)

$\nabla(C).n_s=g$ on Γ_E (Neumann Boundary Condition)

C_0 and g are concentrations at source and concentration gradient at exit boundary, n_s is the unit normal to the domain Ω and Γ_S and Γ_E are the portions of boundary where source concentration and concentration gradient are prescribed.

Element free Galerkin method (EFGM) uses only set of nodes to model the boundary and generate discrete equations. It employs moving least squares (MLS) approximants formulated by Lancaster and Salkauskas (1981) to approximate the function $C(x)$ with $C^h(x)$ in which $C(x)$ is the contaminant concentration at x, where x is a position coordinate. EFGM do not satisfy the Kronecker delta criterion and hence the Lagrangian multiplier technique (Dolbow and Belytschko 1998) is used to enforce the Dirichlet boundary condition.

2.1 Moving least squares approximations

According to the moving least squares proposed by Lancaster and Salkauskas (1981), the approximation $C^h(x)$ of $C(x)$ is

$$C(x) \cong C^h(x) = \sum_{i=1}^{m} p_i(x)a_i(x) = P^T(x)a(x) \qquad \forall x \in \Omega \tag{2}$$

in which,

$$P^T = [1\ x^2] \text{ and } a^T(x) = [a_0(x), a_1(x), a_2(x) \ldots . a_m(x)] \tag{3}$$

in which, $p(x)$ is a monomial basis function and $a(x)$ is a vector of undetermined coefficients, whose values can vary according to the position of x in Ω and m is the order of the basis.
The discrete L_2 norm is given by

$$J = \sum_{I=1}^{n} w(x - x_I)[C_L^h(x_I, x) - C_I]^2 = \sum_{I=1}^{n} w(x - x_I)[P^T(x_I)a(x) - C_I]^2 \tag{4}$$

where, n is the number of nodes in neighbourhood of x for which weight function $w(x\text{-}x_I)$ is non-zero and C_I refers to nodal parameter of C at $x=x_I$

A support domain of a point x_G determines the number of nodes that participate to approximate the function value at x_G and are usually common in circular or rectangular shape. A suitable support domain should be chosen as the accuracy of the approximation depends on it. For a point of interest at x_Q, the dimension of the support domain d_s is determined by

$$d_s = \alpha_c d_c$$

where d_s is the dimensionless size of the support domain, and d_c is the nodal spacing near the point at x_Q. If the nodes are uniformly distributed, d_c is simply the distance between two neighbouring nodes. The dimensionless size of the support domain d_s controls the actual dimension of the support domain. The actual number of nodes, n, can be determined by counting all the nodes included in the support domain.

The minimum of J in Eq. (4) with respect to $a(x)$ leads to the following set of linear equations

$$[A(x)]\{a(x)\} = [B(x)]\{C\} \tag{5}$$

in which,

$$A=\sum_{I=1}^{n} w(x-x_I)P(x_I)P^T(x)$$
$$= w(x-x_1)\begin{bmatrix} 1 & x_1 \\ x_1 & x_1^2 \end{bmatrix} + w(x-x_2)\begin{bmatrix} 1 & x_2 \\ x_2 & x_2^2 \end{bmatrix} + w(x-x_1)\begin{bmatrix} 1 & x_3 \\ x_3 & x_3^2 \end{bmatrix} + \dots\dots\dots w(x-x_1)\begin{bmatrix} 1 & x_n \\ x_n & x_n^2 \end{bmatrix} \tag{6}$$

$$B(x)=w(x-x_1)\begin{bmatrix} 1 \\ x_1 \end{bmatrix} w(x-x_2)\begin{bmatrix} 1 \\ x_2 \end{bmatrix} w(x-x_3)\begin{bmatrix} 1 \\ x_3 \end{bmatrix} \dots w(x-x_n)\begin{bmatrix} 1 \\ x_n \end{bmatrix} \tag{7}$$

$$C^T = \{C_1, C_2 \cdots\cdots C_n\} \tag{8}$$

Then vector of undetermined coefficients $a(x)$ is obtained by inverse operation.

$$\{a(x)\} = [A(x)]^{-1}[B(x)]\{C\}$$

By substituting Eq. (9) in Eq. (2), the MLS approximants can be defined in terms of shape function $\varphi_I(x)$ as

$$C^h(x) = \sum_{I=1}^{n} \varphi_I(x)C_I = \varphi(x)C \tag{9}$$

and

$$\varphi_I(x) = \sum_{j=0}^{m} p_j(x)(A^{-1}(x)B(x))_{JI} = P^T A^{-1} B_I \tag{10}$$

in which, m is the order of polynomial $p(x)$.

Derivative of shape function are obtained by

$$\varphi_{I,x} = (P^T A^{-1} B_I), x = P^T_{,x} A^{-1} B_I + P^T (A^{-1})_{,x} B_I + P^T A^{-1} B_{I,x} \tag{11}$$

in which, $B_{I,x}$ and $A^{-1}{}_{,x}$ are computed as

$$B_{I,x}(x) = \frac{dw}{dx}(x-x_I)\,p(x_I) \tag{12}$$

$$A^{-1}{}_{,x} = -A^{-1} A_{,x} A^{-1} \tag{13}$$

where,

$$A_{,x} = \sum_{I=1}^{n} \frac{dw}{dx}(x-x_I)\begin{bmatrix} 1 & x_I \\ x_I & x_I^2 \end{bmatrix} \tag{14}$$

EFGM shape functions do not satisfy the Kronecker delta criterion $\varphi_I(x_J) \neq \delta_{IJ}$. Therefore they are not interpolants, and the name approximants is used. For imposing essential boundary conditions Lagrangian multipliers are used (Belytschko *et al*. 1994)

2.2 Weight function description

Weight function is an important parameter for the calculation of shape functions in EFGM and is presented in Eq. (15). The weight function is non-zero over a small neighbourhood of x_I, called support domains. The weight function should be smooth and continuous. The choice of weight function affects the approximation results. Present study considers quartic spline function given by

$$w(x-x_I)=\begin{cases}1-6r_i^2+8r_i^3-3r_i^4 & r_i\le 1\\ 0 & r_i>1\end{cases} \tag{15}$$

where $d_i=\|x\text{-}x_I\|$ and $r=d_I/d_{mI}$, where d_{mI} is the size of domain of influence of I^{th} node. The size of the domain of influence at node, d_{mI} is computed by

$$d_{mI}=d_{max}z_I \tag{16}$$

where, d_{max} is a scaling parameter which is typically in the range of 2.0 to 3.0 for static analysis (Liu and Gu 2005). The distance z_I is determined by searching for enough neighbour nodes for a to be regular.

The derivatives for weight function are as follows

$$\frac{dw}{dx}(x-x_I)=\begin{cases}(-12r_i+24r_i^2-12r_i^3)\,sign(x\text{-}x_I) & r_i\le 1\\ 0 & r_i>1\end{cases} \tag{17}$$

The discretisation of the governing differential Eq. (1) is performed by observing following changes in implementation of boundary conditions.

$$\begin{aligned}&[K]\{C\}+[M]\{\dot{C}\}+[G]\{C\}=\{Q\}\\ &[G]\{C\}=\{q_k\}\end{aligned} \tag{18}$$

In the present study, Crank Nicolson time marching scheme is employed for discretization in time domain adopting constant β equal to 0.5.

$$K_{New}=M+\beta\,\Delta t\,K\ ;\ M_{New}=M-(1\text{-}\beta)\,(\Delta t)\,K \tag{19}$$

$$G_{IK}=\varphi_K\,|\Gamma_{SI}\ ;\ Q_I=\varphi_I D_g|\Gamma_E\ ;\ q_K=C_{oK} \tag{20}$$

$$f=M_{New}\times\{C_{n-1}\} \tag{21}$$

Individual matrices are defined by following relation

$$K^*_{IJ}=\int_0^L\left[\varphi_{I,x}^T D\varphi_{J,x}+\varphi_I^T v\varphi_{J,x}\right]dx \tag{22}$$

$$M_{IJ}=\int_0^L\left[\varphi_I R\varphi_J\right]dx \tag{23}$$

$$\begin{bmatrix}K & G\\ G^T & 0\end{bmatrix}\begin{Bmatrix}C_n\\ \lambda\end{Bmatrix}=\begin{bmatrix}f\\ q_k\end{bmatrix} \tag{24}$$

In the above expression, C_n and C_{n-1} are concentrations at present and previous time. K and G are global matrices.

2.3 Algorithm

Based on the above mathematical formulation following steps are considered in the algorithm.

1. Set up nodal points using the subroutine for irregular nodes and background cells
2. Set parameters for material properties like dispersion, velocity, retardation factor for each background cell of a given layer (for stratified case)
3. Set up initial concentrations C_I
4. Set up integration points and Jacobian for each cell
5. Loop over integration points

i) Calculate weights at each node for given integration point x_G

ii) Calculate shape functions and derivatives at points x_G

iii) Assemble stiffness matrix (K) and mass matrix (M)

iv) Assemble G matrix at 1st integration point

6. Apply the Crank Nicolson time marching scheme on stiffness matrix (K_{New}) and mass matrix (M_{New})
7. Assemble global stiffness matrix (KG) by adding stiffness matrix (K_{New}) and G matrix and inversing the global stiffness matrix (KGI)
8. Construct q_K vector
9. Loop over time

i) Construct F matrix (FMAT) by multiplying mass matrix (M_{New}) and concentrations of the previous time step

ii) Construct new concentrations (C_{new}) by multiplying inverse of global stiffness matrix (KGI) and F matrix (FMAT)

Based on the algorithm, a Fortran program is developed for the one dimensional analysis in domain. To consider irregularity of nodes in the domain, random numbers are generated between 0 and 1 in the subroutine. The subroutine consists of a multiplication number, a base number and a random base number given as 16807, 2147483647 and 4.65661287524579692 D-10. The seed is calculated using the multiplication number and base number and further SEED is multiplied with the random base number to get co-ordinates ranging in 0 to 1. These are sorted either in the increasing order and scaled in the domain of contaminant transport. SEED chosen is large and preferably a prime number.

3. Model verification

The numerical procedure developed using Fortran program is validated with three problems in the literature. Problem-1 is an advection-dispersion example discussed by Kumar and Dodagoudar (2008). In problem-2 experimental results from Rowe and Badv (1996) are used for comparison, whereas analytical results presented by Wang and Apperley (1994) are considered in problem-3.

3.1 Problem-1

The authors are going to demonstrate the accuracy of the numerical procedure developed with

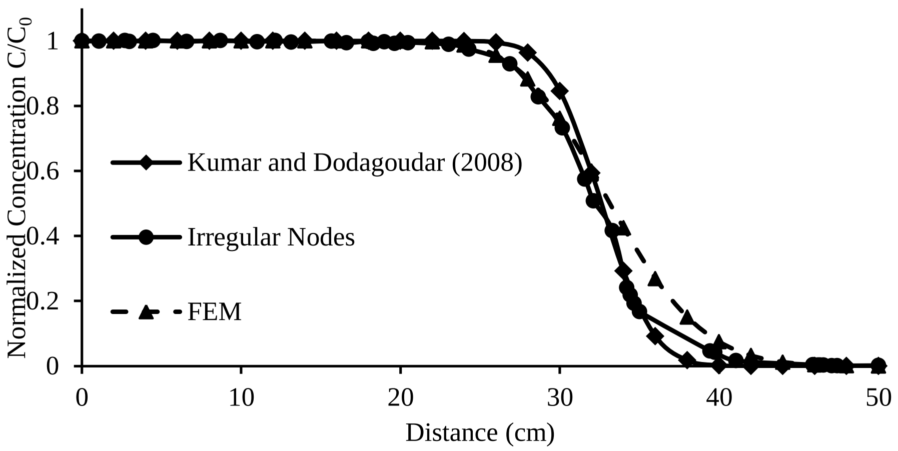


Fig. 1 Comparison of irregular nodes with Kumar and Dodagoudar (2008)

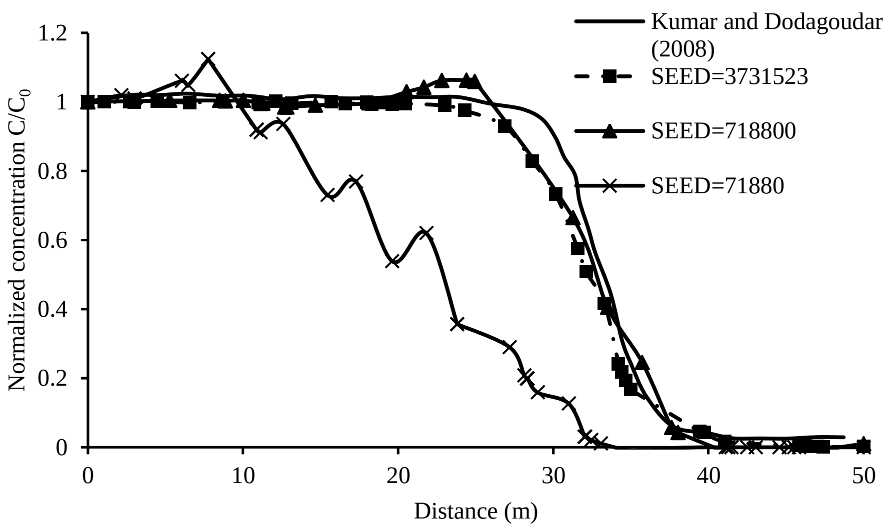


Fig. 2 Comparison of results using different SEED

the one done by previous researchers (Kumar and Dodagoudar 2008). The Péclet number is 20 that makes it advection dominant case. The properties like velocity v and dispersivity α are selected from Kumar and Dodagoudar (2008) as 1.6302 cm/min and 0.1 cm, respectively. The total length of domain is 50 cm and total time 20 mins. From Fig. 1, it can be observed that results for irregular nodes and regular nodes using EFGM are in good agreement and error in concentration is 12.8% at a distance of 29 cm from origin. The nodes for irregular spacing are 40, whereas for regular spacing nodes are 26. The SEED convergence study has been carried out by varying the SEEDs as 718800, 71880 and 3731523.

The results achieved are in good agreement with the available results, for the chosen SEED 3731523 (Fig. 2). The results are also compared with those from FEM. A good agreement is observed with the results reported by Kumar and Dodagoudar (2008).

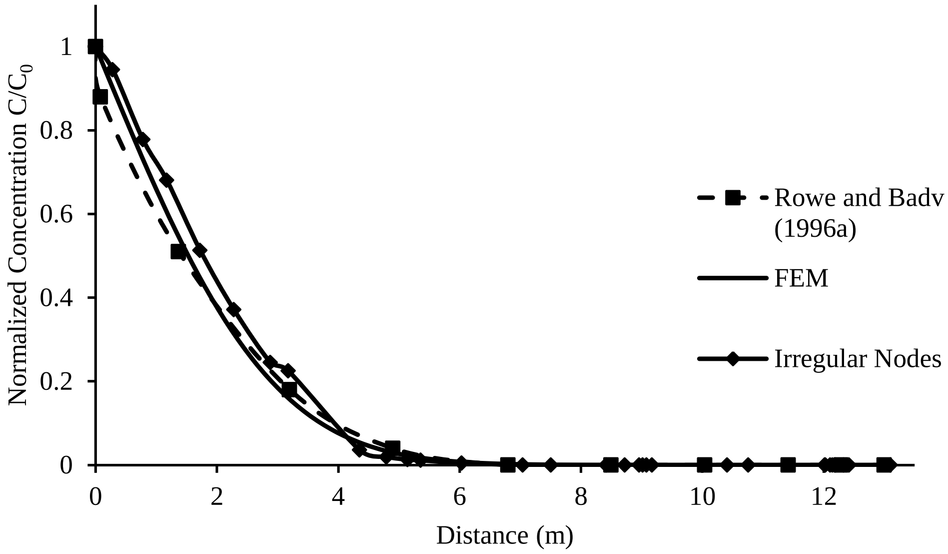


Fig. 3 Comparison of results from irregular nodes and Rowe and Badv (1996)

Table 1 Material properties from Wang and Apperley (1994)

Parameter	Value
Dispersion constant for 1st layer D (cm^2/hr)	0.18
Dispersion constant for 2nd layer D (cm^2/hr)	0.18
Average velocity for 1st layer (cm/hr)	0.10
Advection velocity for 2nd layer (cm/hr)	0.05
Length of clay liner and natural soil (cm)	7.5 & 24.5
Time (hours)	100
Time step (years)	0.5
Retardation factor	1.0

3.2 Problem-2

The accuracy of the numerical procedure for a pure diffusion case is demonstrated by comparing with results from Rowe and Badv (1996). The material properties of different parameters are as follows. Diffusion is 0.8536 cm^2/day, length of domain is 13.1 cm and total time is 2.96 days. The nodes considered are 31 and SEED is 3731523. It can be observed from Fig. 3 that the results for the irregular nodes using EFGM and those reported by Rowe and Badv (1996) are in good agreement and the error in concentration is 7.8% at a distance of 1.7 cm from origin it reduces with further increase in distance.

3.3 Problem-3

The authors are going to demonstrate the accuracy of the numerical procedure developed for stratified layer case discussed by Wang and Apperley (1994). In the first layer, 11 nodes are

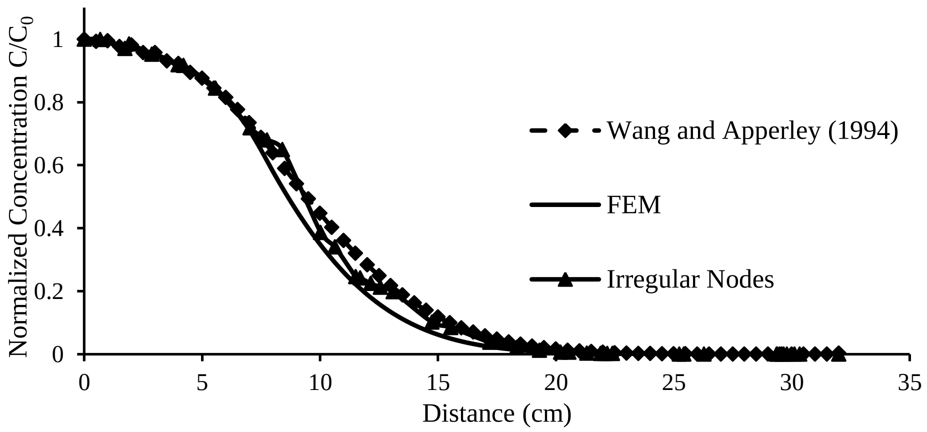


Fig. 4 Comparison of results from irregular nodes and Wang and Apperley (1994)

considered whereas 30 nodes are used for the second layer. SEED considered is 3731523. The material properties are reported in Table 1. The results are compared with those obtained by Wang and Apperley (1994) in Fig. 4. A good agreement is observed in the results obtained using EFGM developed in the present study for the irregular nodes and those reported by Wang and Apperley (1994). The maximum error in concentration is observed to be 10% at a distance of 8.5 cm from the origin which reduces with further increase in distance.

4. Parametric study

It is aimed to examine the effect of key parameters like diffusion/dispersion, thickness of layer, porosity, retardation factor and velocity on contaminant transport phenomena. In the detail parametric study carried out in the present investigation, selected parameter are varied to consider suitable range while others were set to the typical values reported in Tables 1-4.

4.1 Change in dispersivity/dispersion

The dispersion is the apparent mixing and spreading of the contaminant within the flow system. It is due to mechanical mixing and molecular diffusion. The mixing component, often called mechanical dispersion arises from velocity variations in porous media and dispersivity. Dispersivity varies from 0.1 to 100 m. An approximate value for dispersivity is 0.1 times the scale of test (Gelhar *et al*. 1992). The molecular diffusion is a process where ionic or molecular constituents move in the direction of their concentration gradients. The parameters are reported in Table 2 and Table 3. The parameters in Table 2 are meant for the homogeneous media having length 13.1 cm and total time 2.96 days, while the parameters in Table 3 are meant for stratified media having length of first layer and second layer as 7.5 cm and 24.5 cm, and the total time is 100 hours. The variation in concentration with respect to distance is obtained for four different cases as mentioned in Table 2 and is presented in Figs. 5 and 6. It is observed that with increase in dispersivity the concentration increases. Dispersivity causes the plume/contaminant to spread on the advective front and hence concentration increases. Similar phenomena can be observed for the

Table 1 Material properties for different dispersion on concentration for homogeneous media

Case	Velocity (cm/day)	Dispersivity (cm)	Dispersion (cm^2/day)
1	0.1	0.4	0.04
2	0.1	1	0.1
3	0.05	0.4	0.02
4	0.05	1	0.05

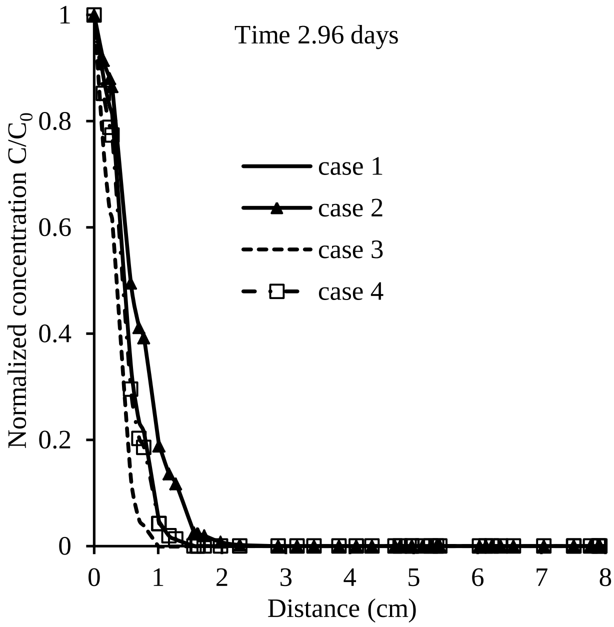


Fig. 5 Effect of change of dispersion on transport of concentration for homogeneous media

Table 2 Material properties for different dispersion on concentration for stratified media

Case	Velocity (cm/hr)		Dispersivity (cm)	
	first layer	second layer	first layer	second layer
1	0.1	0.05	0.4	0.02
2	0.1	0.05	1	0.05
3	0.1	0.05	2	0.1

case of stratified media where three cases are described. It is seen that with increase in dispersivity, the concentration from source travels at faster rate. Therefore, at a given distance and the time, the concentration migrated from the constant source will be more.

4.2 Change in hydraulic conductivity

The hydraulic conductivity is the ease with which flow takes place through porous medium. It has large values for permeable units like sand and gravel and relatively small values for poorly permeable materials like clay. The hydraulic conductivities of fine sand, silt and clay are in the

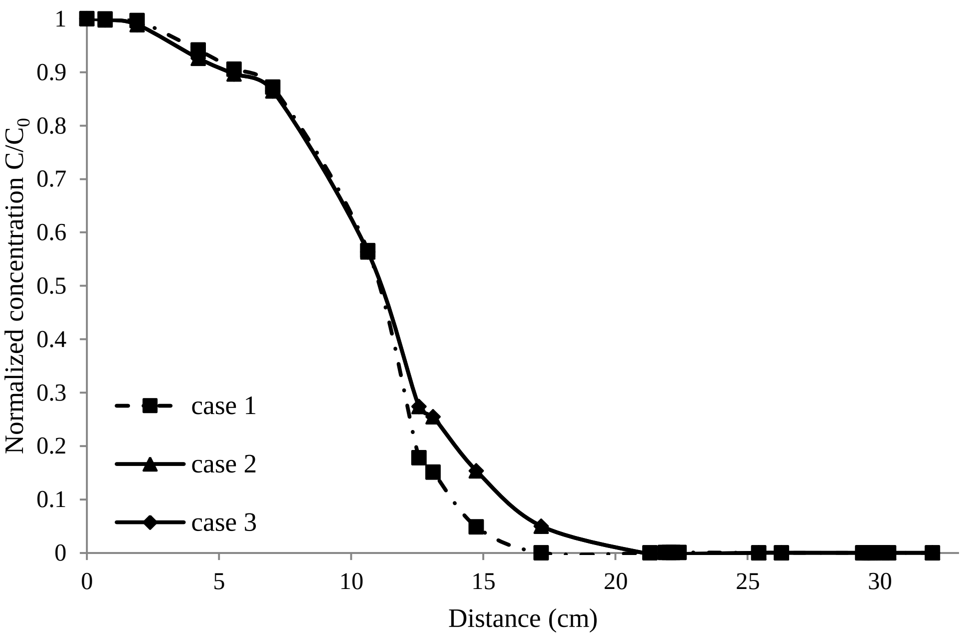
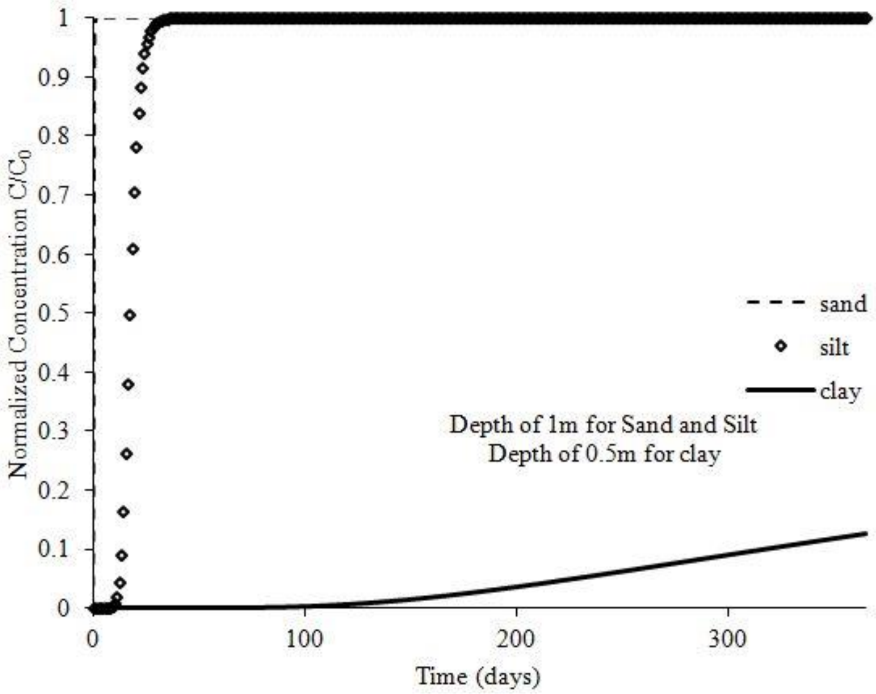
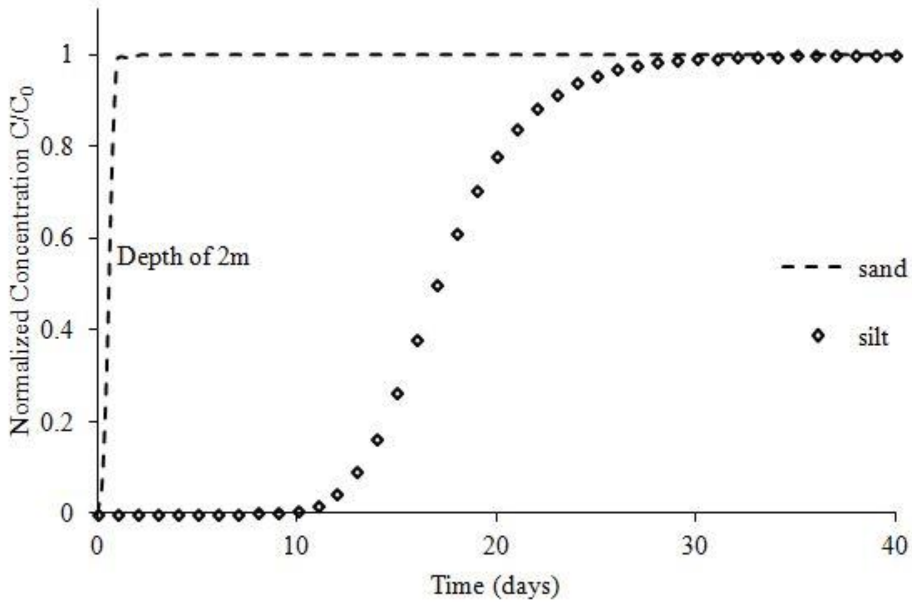


Fig. 6 Effect of change of dispersion on transport of concentration for stratified media



(a)

Fig. 7 Variation of concentration with time for different soils



(b)

Fig. 7 Continued

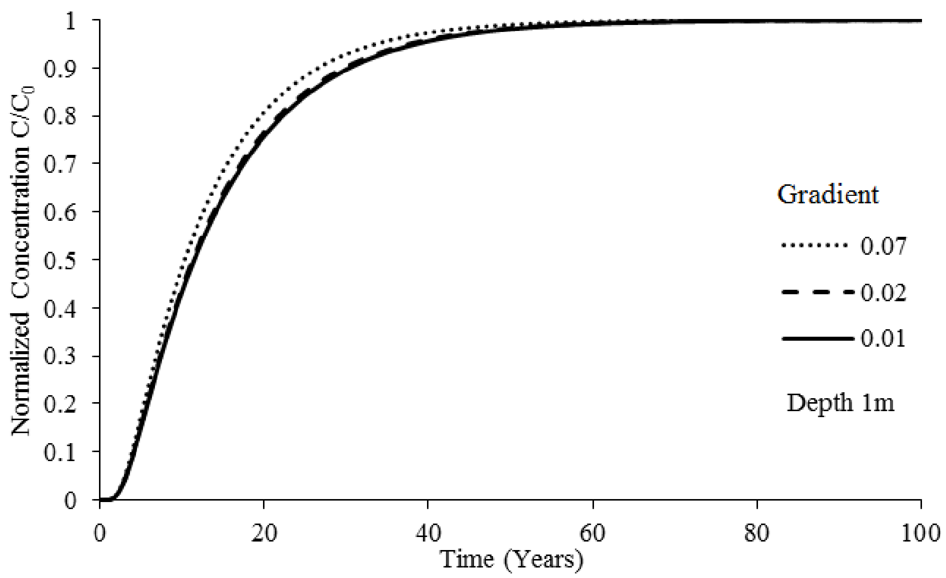


Fig. 8 Variation of concentration with time for clay having different gradients

range of 2×10^{-7}-2×10^{-4} m/s , 1×10^{-9}-2×10^{-5} m/s and 1×10^{-11}-4.7×10^{-9} m/s, respectively (Schwartz and Zhang 2012). The advection is the main process conveying dissolved mass from one point to another. For most practical problems, groundwater and dissolved mass will move at the same rate and in the same direction and is given by

$$v = -(K/n)\, grad\ h \tag{25}$$

where, v is advective velocity, K is hydraulic conductivity, n is porosity, grad h is gradient of soil.

Grad h is taken as 0.07, 0.02 and 0.01, diffusion coefficient D^* for chlorine Cl^- is 20.3×10^{-6} cm^2/s, dispersivity is 0.1m and therefore, the advective velocities for sand, silt and clay are 3.456 m/day, 0.3456 m/day and 8.12×10^{-5} m/day. The distribution coefficient K_d is 0.0, porosity is 0.35, density is 2 t/m^3. The depth is 1 m for the contaminant transport in homogeneous soils. Fig. 7 shows the variation of concentrations with time. It is observed that the concentrations reach the maximum in few days for sand and silt, whereas in clays the normalized concentrations that reach in 365 days is close to 0.1. From Fig. 8, it is observed that in order to reach the maximum concentration in clays, it takes close to 40 years. The gradient changes from 0.07 to 0.01 has a negligible effect on the transport of contaminants.

4.3 Effect of advective velocity on layered soil

The advective velocities mentioned for the different soils above are considered for observing the variation of the contaminant transport in a 2 m thick stratified strata, equally divided into two soil layers. The predicted response for variations in concentration with time and distance are presented graphically in Figs. 9-16.

Figs. 9 and 10 show the variation in contaminant transport with distance and time in clay-silt soils. The advective velocity for clay is very low as compared to other two types of soils (sand and silt). It is observed that the effect of concentration diminishes from 1 to 0 as the depth increases from 0 to 2 m. It can be seen from Fig. 10 that concentration is close to zero till 230 days after which it increases gradually.

Similar trend is observed for clay-sand layers (Figs. 11-12) as well. Figs. 13-14 show the variation in contaminant transport with distance and time in sand-clay soils. Advective velocity of

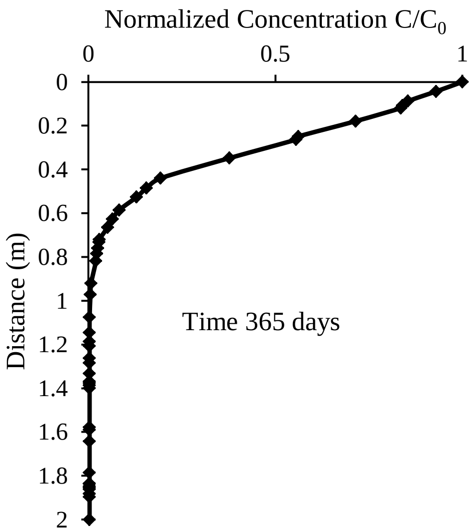


Fig. 9 Variation of concentration with distance for layered soil (clay-silt)

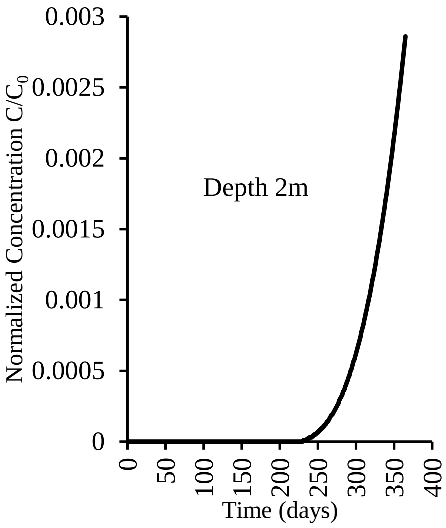


Fig. 10 Variation of concentration with time for layered soil (clay-silt)

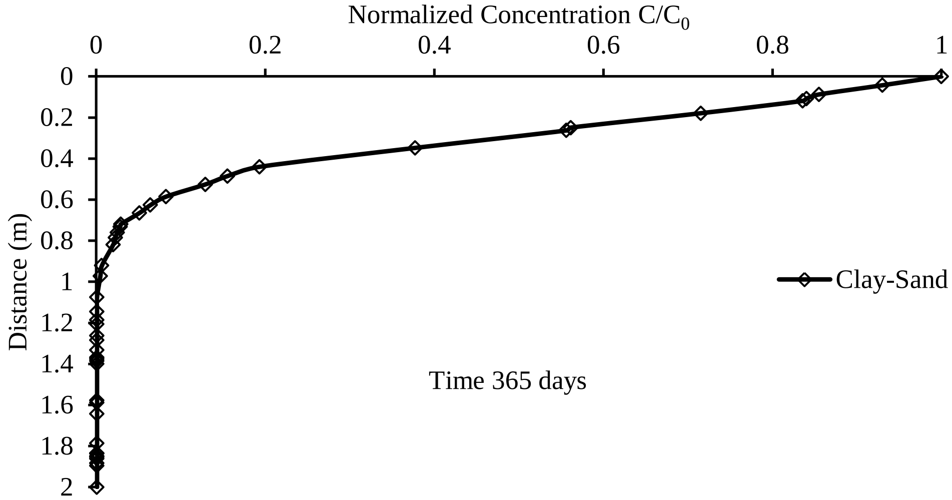


Fig. 11 Variation of concentration with distance for layered soil (clay-sand)

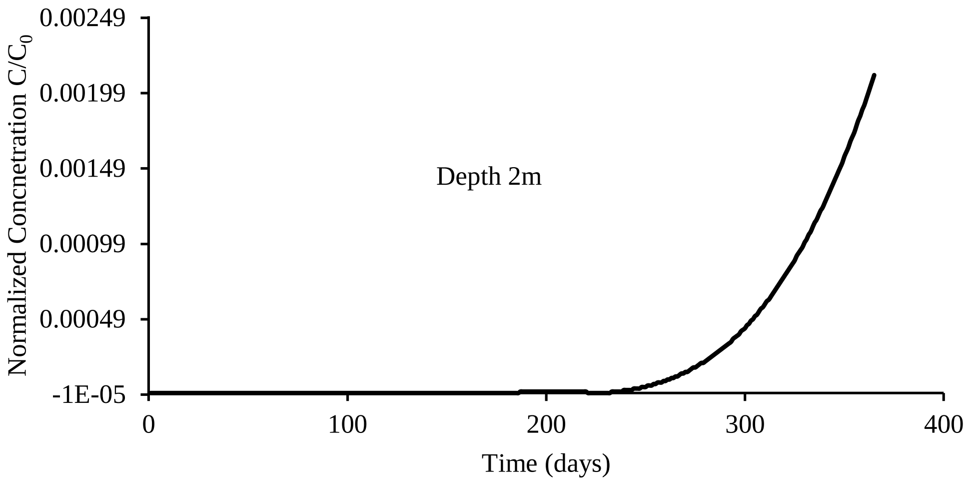


Fig. 12 Variation of concentration with time for layered soil (clay-sand)

silt is higher than clay, making the contaminant travel rapidly. Throughout the silt layer concentration is maximum (Fig. 13) and it starts diminishing as the clay layer is encountered at 1 m depth). Concentration is further observed to be diminishing gradually till it reaches the end of the depth (2 m). This is also evident from Fig. 14. Normalized concentration at the tip of clay layer after 275 days is negligible and then it increases at very slow rate. Similar trend is observed in case of silt clay combination (Fig. 15). Fig. 16 shows the variation in contaminant transport with time for sand and silt soils. Due to high advective velocities of both type of soil, the contaminant attains the maximum concentration in few days (7 days). Then it is maintaining the same concentration level till the last time step. This observation highlighted the effect of advective velocity on contaminant transport phenomenon. Therefore, in order to reduce contaminant transport the advective velocity should be very less and the gradient should not be high.

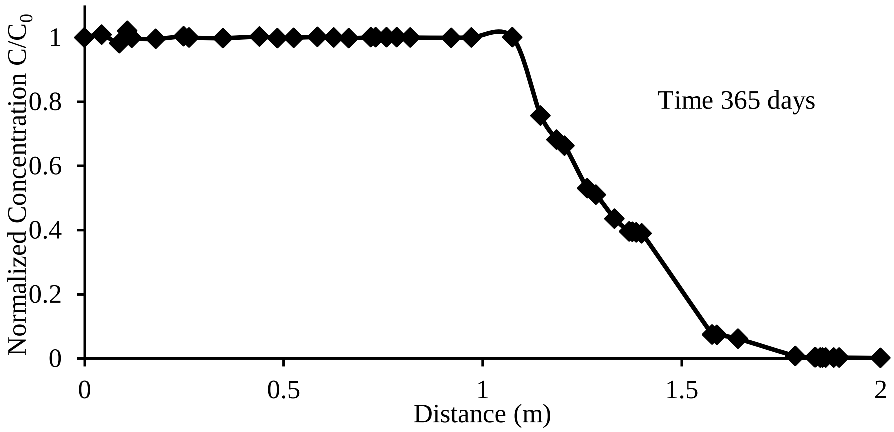


Fig. 13 Variation of concentration with distance for layered soil (sand-clay)

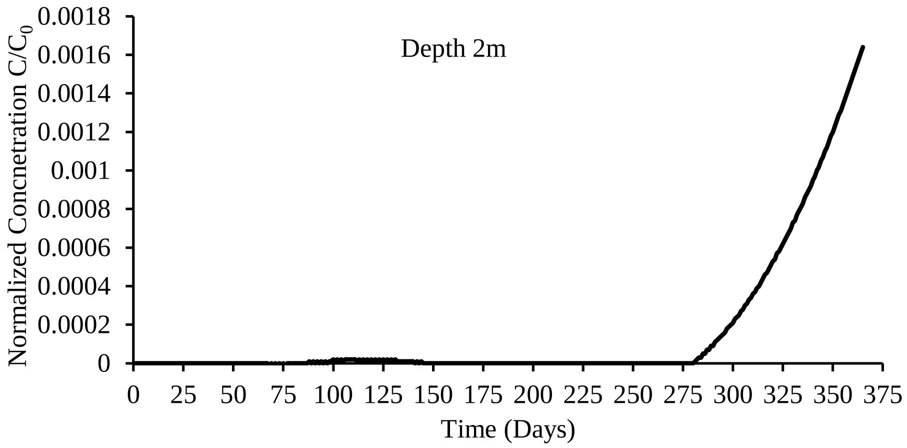


Fig. 14 Variation of concentration with time for layered soil (sand-clay)

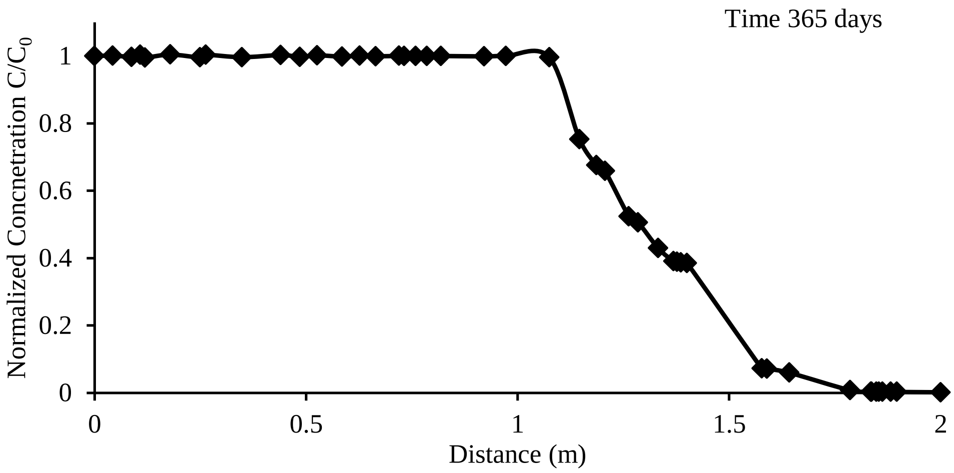


Fig. 15 Variation of concentration with time for layered soil (silt-clay)

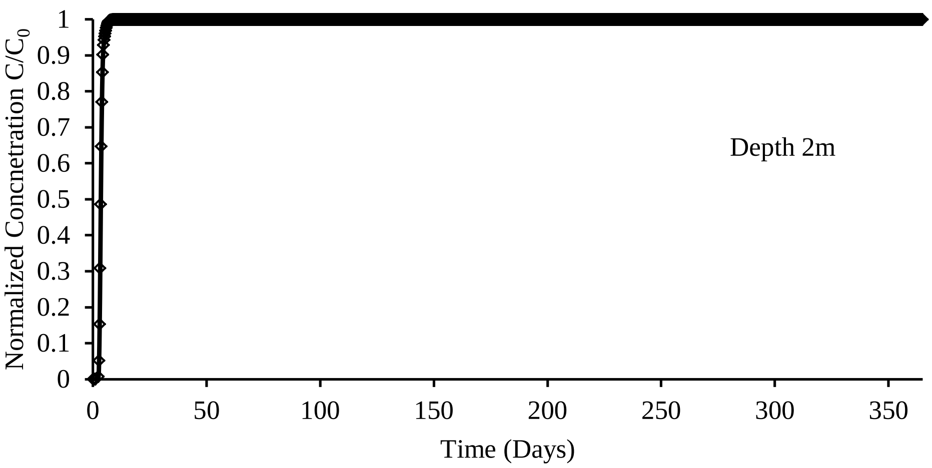


Fig. 16 Variation of concentration with time for layered soil (sand-silt)

Table 4 Retardation factor in stratified media

Case	First layer	Second layer
1	1	1
2	2	2
3	5	5
4	1	2
5	1	5
6	2	1
7	2	5
8	5	1
9	5	2

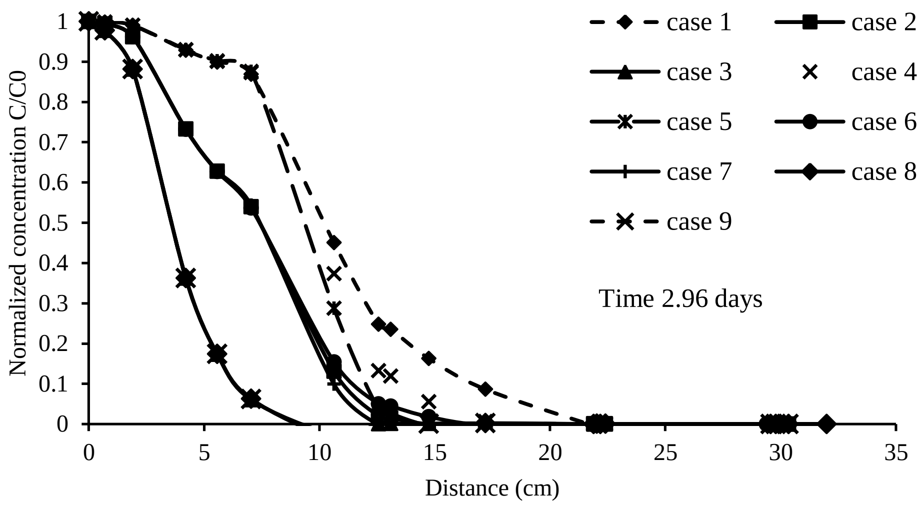


Fig. 17 Effect of change of retardation factor on transport of concentration for stratified media

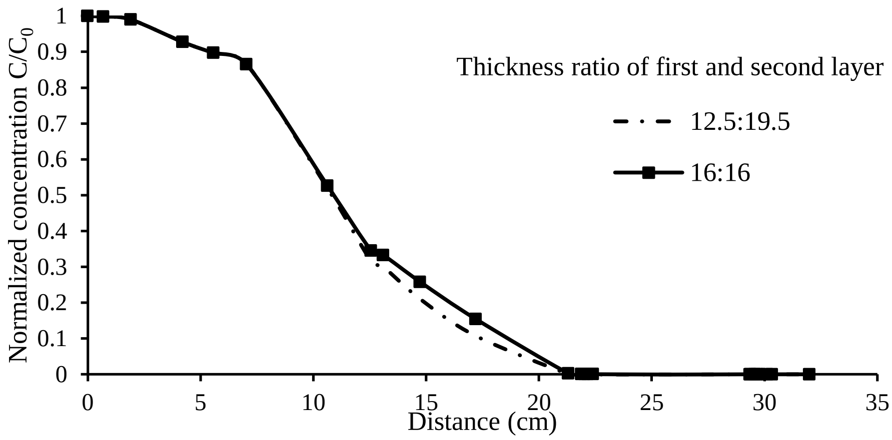


Fig. 18 Effect of change of thickness on transport of concentration

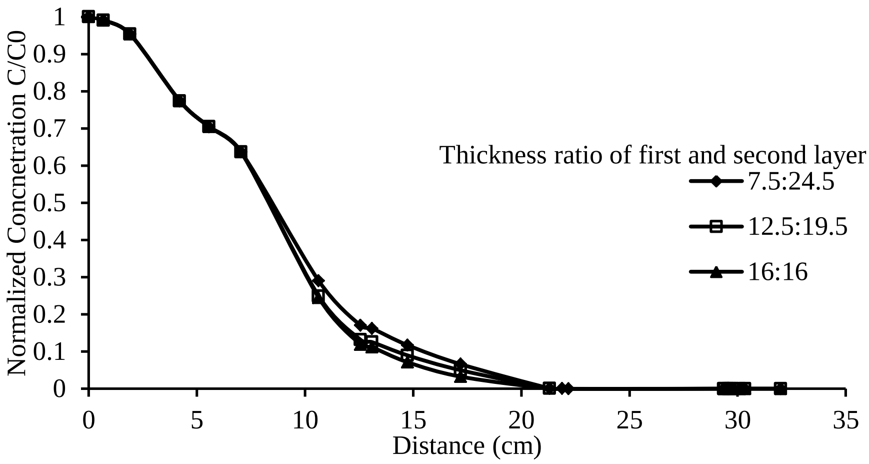


Fig. 19 Effect of change of thickness by changing velocity on transport of concentration

4.3 Change in retardation factor

The retardation factor R comprises of porosity (n), distribution coefficient (K_d) and density (ρ). It is used for determining the retardation of mass of solute moving while undergoing adsorption. K_d varies for different elements and type of soil. The porosity is usually ranging from 0.2 to 0.4 for good aquifers and ranges of density vary from 1.6 to 2 g/cc. The values of retardation factor for heterogeneous case are considered and are varied are reported in Table 4 to examine their impact on the transport. Other parameters were same as reported in Table 2. It can be observed from Fig. 17 that when the retardation factor increases for both the layers, transport of concentration reduces towards the end of the domain.

4.4 Change in thickness of layer

While designing the landfill, the variation in thickness is more appropriate in order to reduce the contaminant transport. By varying the thicknesses of the liner one can observe changes in the contaminant transport. The parameters are same as in Table 1 and the thickness for top layer is taken as 7.5 cm, 12.5 cm and 16 cm. The results are indicated in Figs. 18 and 19. Since the velocity of the top layer is 0.1 cm/hr and bottom layer is 0.05 cm/hr, the change in thickness doesn't reduce the transport of contaminants but when the velocities of top and bottom layer are reduced to 0.05 cm/hr and 0.1 cm/hr, a small change is observed. It can be concluded that for reducing the migration of contaminant by changing the thickness, the velocity also should be reduced whereby a considerable change can be observed in the transport.

5. Conclusions

From the present study it can be concluded that, irregular nodes can be used in Element free galerkin method which can be used for predicting contaminant transport for homogeneous as well as stratified porous media. The size of support domain (D_{max}) should be kept 1.25 for good results and the SEED considered is 3731523 for generating random numbers. Parametric study has been conducted by varying Dispersion, velocity, retardation factor and thickness of layer. Following observations are concluded from present study:

- The dispersivity of the medium has important bearing in the migration of contaminant. With increase in dispersivity, concentration from source travels at faster rate. So at a given distance and time, the concentration migrated from constant source will be more.
- Study highlighted the effect of advective velocity on contaminant transport phenomenon. In order to reduce contaminant transport the advective velocity should be very less and the gradient should not be high.
- Reduction in advective velocity and a low gradient helps in reducing the transport of contaminant in the field. Sand and Silt soils can be used effectively for removal of leachate and reduction in transport of contaminant.
- The retardation factor is inversely proportional to the porosity of the soil and directly proportional to the distribution coefficient. These properties can vary for different soil layers in the landfill. Increase in retardation factor for the top layer as well for the bottom layer decreases the migration of the contaminant transport and vice versa.
- Increasing the thickness of the top layer reduces the contaminant transport but it is also dependent on velocity of the flow, dispersion and retardation factor.

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