Coupled Systems Mechanics, *Vol. 11, No. 3 (2022) 199-215*
https://doi.org/10.12989/csm.2022.11.3.199

Effect of two temperature and energy dissipation in an axisymmetric modified couple stress isotropic thermoelastic solid

Parveen Lata[a] and Harpreet Kaur*

Department of Basic and Applied Sciences, Punjabi University, Patiala, Punjab, India

(Received August 21, 2021, Revised October 8, 2021, Accepted October 9, 2021)

Abstract. The present paper deals with the axisymmetric deformation in homogeneous isotropic thermoelastic solid with two temperatures, with and without energy dissipation using modified couple stress theory. The effect of energy dissipation and two temperature is studied due to the concentrated normal force, normal force over the circular region, thermal point source and thermal source over the circular region. The Laplace and Hankel transform techniques have been used to find the solution to the problem. The displacement components, conductive temperature distribution, stress components and couple stress are computed in the transformed domain and further calculated in the physical domain using numerical inversion techniques. Effects of two temperature and energy dissipation on the conductive temperature, stress components and couple stress are depicted graphically.

Keywords: energy dissipation; isotropic thermoelastic solid; Laplace and Hankel transform; modified couple stress theory; two temperature

1. Introduction

Isotropic materials are helpful since they are simpler to shape, and their conduct is simpler to anticipate. One basic illustration of the homogeneous isotropic material is the water. All cells in the human body are made mostly of water content in their cytoplasm. Important functions of water in the body include supporting the cellular metabolism, molecular transport, biochemical reactions, and the physical properties of water, such as surface tension. So, study of deformation in homogeneous isotropic materials is required. Classical continuum theory predicts the behavior of structures under loads at macro scale in which the forces are transmitted at an infinitesimal element surface as tractions, but careful experiments have shown that it deviates in capturing behavior of materials at micro/nano scale. As a result size dependent theories came into existence In these theories moments are transmitted on an infinitesimal element surface as moment or couple tractions in addition to force tractions. Couple-stress theory is an extended continuum theory that includes the effects of a couple per unit area on a material volume, in addition to the classical normal and shear forces per unit area. One other branch of theories based on Voigt (1887) was developed by Toupin

*Corresponding author, Ph.D., E-mail: mehrokpreet93@gmail.com
[a]Associate Professor, E-mail: parveenlata@pbi.ac.in

http://www.techno-press.org/?journal=csm&subpage=8 ISSN: 2234-2184 (Print), 2234-2192 (Online)

(1962), Mindlin and Tiersten (1962) and Koiter (1964) in which displacements and macrorotations were taken as the kinematical quantities. Koiter introduced the constitutive relationships for couple stress theory, involving length scale parameters to predict the size effects. It involves four material constants for isotropic elastic materials which are very difficult to determine (1964). So, modified couple stress theory (M-CST) with one length scale parameter was presented by Yang *et al.* (2002), in which only one couple stress parameter is involved. But these theories had some indeterminacy/ inconsistencies in the couple stress and force stress tensors due to the limited number of relations. So, Hadjesfandiari and Dargush (2011) offered consistent couple stress theory (C-CST) with the skew-symmetric couple-stresses to resolve the inconsistencies in previous models. This theory was not applicable to anisotropic materials. Therefore, Chen and Li (2013) introduced the new modified couple stress theory (NM-CST) for anisotropic materials containing three length scale parameters. Marin (2010) studied Some estimates on vibrations in thermoelasticity of dipolar bodies. Marin *et al*. (2013) consider a theory of thermoelasticity for modeling a microstretch thermo-elastic body with two temperatures. Based on a modified couple stress theory, the static mechanical properties of Euler-Bernoulli beam model were investigated by Park and Gao (2006), and also bending test outcomes of an epoxy polymeric beam were explained, a Timoshenko beam model was introduced by Ma *et al*. (2008) to study the size-dependent static bending and free vibration properties. The modified couple stress theory was also utilized by Kong *et al*. (2008) to obtain the governing equation and boundary and initial conditions of an Euler-Bernoulli microbeam. Based on the couple stress theory, Reddy (2011) defined nonlinear size-dependent Euler-Bernoulli and Timoshenko functionally graded beam models and by using the Navier solution determined the natural frequency, buckling load, and deflection for beams with simply supported boundary conditions. Utilizing the modified couple stress Euler-Bernoulli beam theory, Wang *et al*. (2013) analyzed the nonlinear free vibration behavior of microbeams. They concluded that the nonlinear vibration frequency obtained by their model is higher than that predicted by the classical continuum theory. Based on modified couple stress theory a model for sigmoid functionally graded material (S-FGM) nanoplates on elastic medium is developed by Jung *et al*. (2014), the vibrational analysis of piezoelectric microbeams based on the modified couple stress theory is studied by Ansari *et al*. (2014). By using the modified couple stress theory, the vibration analysis of composite laminated beams in order of micron is developed and effect of shear deformation is studied by considering different beam theories by Mohammad-Abadi and Daneshmehr (2014). Zhou and Gao (2014) developed a non classical model for circular Mindlin plates subjected to axisymmetric loading, by deriving the equations of motion and boundary conditions through a variational formulation based on Hamilton's principle. Transient analysis of coupled thermoelasticity of micro-beams by applying MCS theory and LS heat conduction model by Kakhki *et al*. (2016), dynamic characteristics of electrostatically actuated micro-beams resting on squeeze-film damping under mechanical shock are investigated by Ghiasi(2016) using modified couple stress theory. Borjalilou and Asghari (2018) studied the thermoelastic damping of Kirchhoff microplatesbased using the non-classical continuum theory of modified couple stress and non-classical heat conduction model of dual-phase-lag. This theory has been utilized in many studies to investigate the mechanical behavior of microstructures such as microbeams, microplates, and microshells. Small-scale thermoelastic damping phenomenon is studied in micro-beams utilizing the modified couple stress theory and the dual-phase-lag heat conduction model by Borjalilou *et al*. (2019). Kaur and Lata (2020) studied axisymmetric deformation in transversely isotropic magneto-thermoelastic solid with Green -Naghdi III due to inclined load. Hobiny and Abbas (2021) established the bio-heat model with fractional derivative to study the variations of temperature and the thermal damage in spherical tissues during thermal therapy. The effects of a fractional parameter, the blood perfusion and the laser exposure time on the temperature of living tissue and the resulting of thermal damages are studied. Marin (1998) studied a temporally evolutionary equation in elasticity of micropolar bodies with void. Zhang *et al*. (2020) performed the entropy analysis of the blood flow through an anisotropically tapered arteries under the suspension of magnetic Zinc-oxide (ZnO) nanoparticles (NPs). Abbas and Marin (2018) adopted the analytic solutions of a two dimensional generalized thermoelastic diffusions problem due to laser pulse. Abbas *et al*. (2016) obtained a dispersion relation for Rayleigh-Lamb wave propagation in a plate of thermoelastic material. Abbas *et al*. (2008) studied the combined effect of thermal dispersion and thermal radiation on the non-Darcy natural convection flow over a vertical flat plate kept at higher and constant temperature in a fluid saturated porous medium. Abd-Alla and Abbas (2011) investigated the a transversely isotropic, elastic cylinder of infinite length and perfectly conducting material placed in a primary constant magnetic field while the curved surface of the cylinder is subjected to periodic loading. Hobiny and Abbas (2018) studied the the wave propagation on non-homogenous semiconductor through photo-thermal process by using the theory of coupled plasma and thermoelastic wave. Hobiny and Abbas (2017) investigated the wave propagation on semiconductor material with cylindrical cavity during photo-thermoelastic process. Mohamed *et al*. (2009) studied the flow, chemical reaction and mass transfer of a steady laminar boundary layer of an electrically conducting and heat generating fluid driven by a continuously moving porous surface embedded in a non-Darcian porous medium in the presence of a transfer magnetic field.

The present investigation deals with the deformation in isotropic thermoelastic solid using modified couple stress theory with and without energy dissipation and with two temperatures within context of modified couple stress theory proposed by Yang (2002). This theory contains one couple stress parameter to determine the size effects. Concentrate normal force, normal force over the circular region, thermal point source and thermal source over the circular region have been applied to find the utility of the problem. Laplace and Hankel transform technique are applied to obtain the solutions of the governing equations. The displacement components, conductive temperature, stress components and couple stress are obtained in the transformed domain. A numerical inversion technique has been used to obtain the solutions in the physical domain. The effect of two temperature and energy dissipation is depicted graphically on the resulted quantities.

2. Basic equations

Following Kumar *et al.* (2017), Youssef (2006), and the field equations for isotropic modified couple stress thermoelastic medium with two temperatures and with and without energy dissipation in the absence of body forces are given by

(a) Constitutive relations

$$t_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \frac{1}{2}e_{kij}m_{lk,l} - \beta_1 T\delta_{ij}, \tag{1}$$

$$m_{ij} = 2\alpha\chi_{ij}, \tag{2}$$

$$\chi_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right), \tag{3}$$

$$\omega_i = \frac{1}{2} e_{ijk} u_{k,j}. \tag{4}$$

(b) Equation of motion

$$\left(\lambda + \mu + \frac{\alpha}{4}\Delta\right) \nabla(\nabla.\vec{u}) + \left(\mu - \frac{\alpha}{4}\Delta\right) \nabla^2\vec{u} - \beta_1 \nabla T = \rho\ddot{\vec{u}}, \tag{5}$$

(c) Equation of heat conduction

$$K\nabla^2\varphi + K^*\nabla^2\dot{\varphi} = \rho C^* \ddot{T} + \beta_1 T_0 \nabla.\ddot{\vec{u}}, \tag{6}$$

where

$$T = (1 - a\nabla^2)\varphi. \tag{7}$$

Here $u = (u_r, u_\theta, u_z)$ is the components of displacement vector, t_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress, a is the two temperature parameter, T is the thermodynamical temperature, φ is the conductive temperature, K^* is the coefficient of thermal conductivity, χ_{ij} is curvature, ω_i is the rotational vector, ρ is the density, K isthe materialistic constant, C^* is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that $T/_{T_0} \ll 1$, and, $\beta_1 = (3\lambda + 2\mu)\alpha_t$. Here α_t is the coefficients of linear thermal expansion, α is the couple stress parameter, Δ is the Laplacian operator, ∇ is del/nabla operator, δ_{ij} is Kronecker's delta.

3. Formulation and solution of the problem

We consider a two dimensional homogeneous isotropic modified couple stress thermoelastic medium initially at uniform temperature T_0 occupying the region of a half space $z \geq 0$ .A cylinderical coordinate system (r, θ, z) having origin on the surface $z = 0$ has been taken. All the field quantities depend on (r, z, t).

$$\begin{aligned} u_r &= u_r(r, z, t), \\ u_z &= u_z(r, z, t), \\ \varphi &= \varphi(r, z, t). \end{aligned} \tag{8}$$

The initial and regularity conditions are given by

$$u_r(r, z, 0) = 0 = \dot{u_r}(r, z, 0),$$
$$u_z(r, z, 0) = 0 = \dot{u_z}(r, z, 0),$$
$$\varphi(r, z, 0) = 0 = \dot{\varphi}(r, z, 0) for\ z \geq 0, -\infty < r < \infty,$$
$$u_r(x, z, t) = u_z(x, z, t) = \varphi(x, z, t) = 0\ for\ t > 0\ when\ z \to \infty.$$

We introduce the dimensionless quantities

$$x' = \frac{\omega^*}{c_1}x, z' = \frac{\omega^*}{c_1}z,\ u' = \frac{\omega^*}{c_1}u,\ w' = \frac{\omega^*}{c_1}w, t' = \omega^* t, t_{ij}' = \frac{t_{ij}}{\beta_{1T_0}}, m_{ij}' = \frac{m_{ij}}{c_1\beta_{1T_0}}, T' = \frac{\beta_1 T}{\rho c_1^2}, \varphi' = \frac{\beta_1\varphi}{\rho c_1^2}, c_1^2 = \frac{\lambda+2\mu}{\rho},\ \omega^{*2} = \frac{\lambda}{\mu t^2 + \rho\alpha},\ a' = \left(\frac{\omega^*}{c_1}\right)^2 a\ . \tag{9}$$

where ω^* and c_1 are the characteristic frequency and longitudinal wave velocity.

Upon using (7) & introducing (8), (9) in Eqs. (5)-(6), after suppressing the primes, we obtain

$$a_1\frac{\partial e}{\partial r}+a_2\left(\nabla^2-\frac{1}{r^2}\right)u_r+a_3\Delta\left(\frac{\partial e}{\partial r}-\left(\nabla^2-\frac{1}{r^2}\right)u_r\right)-\frac{\partial}{\partial r}(1-a\nabla^2)\varphi=\frac{\partial^2 u_r}{\partial t^2}, \tag{10}$$

$$a_1\frac{\partial e}{\partial z}+a_2\nabla^2 u_z+a_3\Delta\left(\frac{\partial e}{\partial r}-\nabla^2 u_z\right)-\frac{\partial}{\partial z}(1-a\nabla^2)\varphi=\frac{\partial^2 u_z}{\partial t^2}, \tag{11}$$

$$-a_6\frac{\partial^2 e}{\partial t^2}+\nabla^2\varphi+a_4\nabla^2\frac{\partial\varphi}{\partial t}-a_5(1-a\nabla^2)\frac{\partial^2\varphi}{\partial t^2}=0. \tag{12}$$

Where

$a_1=\frac{(\lambda+\mu)}{\rho c_1^2}$, $a_2=\frac{\mu}{\rho c_1^2}$, $a_3=\frac{\alpha\omega^{*2}}{4\rho c_1^4}$, $a_4=\frac{K^*\omega^*}{K}$, $a_5=\frac{\rho c_1^2 C^*}{K}$, $a_6=\frac{\beta_1^2 T_0}{\rho K}$, $e=\frac{u_r}{r}+\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}$, $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{\partial}{r\partial r}+\frac{\partial^2}{\partial z^2}$

The displacement components u_r and u_z in terms of potential functions q and Ψ in a dimensionless form are given by

$$u_r=\frac{\partial\Phi_1}{\partial r}+\frac{\partial^2\Phi_2}{\partial r\partial z},\quad u_z=\frac{\partial\Phi_1}{\partial z}-\left(\frac{\partial^2\Phi_2}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi_2}{\partial r}\right). \tag{13}$$

With the aid of (13) Eqs. (10)-(12) yield

$$(\nabla^2-\frac{\partial^2}{\partial t^2})\Phi_1-(1-a\nabla^2)\varphi=0, \tag{14}$$

$$(a_2\nabla^2-a_3\nabla^4-\frac{\partial^2}{\partial t^2})\Phi_2=0, \tag{15}$$

$$-a_6\frac{\partial^2}{\partial t^2}(\nabla^2\Phi_1)+\left(\nabla^2+a_4\nabla^2\frac{\partial}{\partial t}-a_5(1-a\nabla^2)\frac{\partial^2}{\partial t^2}\right)\varphi=0. \tag{16}$$

Where $\nabla^2\Phi_1=e$.

We define laplace and Hankel transform as

$$\hat{f}(r,z,s)=\int_0^\infty f(r,z,t)e^{-st}dt, \tag{17}$$

$$\tilde{f}(\xi,z,s)=\int_0^\infty \hat{f}(r,z,s)rJ_n(r\xi)dr.. \tag{18}$$

Where ξ is Hankel transform parameter and $J_n()$ is Bessel function of first kind

Applying the and Laplace Hankel transforms defined by (17)-(18) to the Eqs. (14)-(15), we obtain

$$(-\xi^2+D^2-s^2)\widetilde{\Phi_1}-\left(1-a(-\xi^2+D^2)\right)\tilde{\varphi}=0, \tag{19}$$

$$\left((-a_2-a_3\xi^2)\xi^2-s^2+(a_2+2a_3\xi^2)D^2-a_3D^4\right)\widetilde{\Phi_2}=0, \tag{20}$$

$$\left(a_6 s^2(\xi^2-D^2)\right)\Phi_1+(-(1+a_4 s)\xi^2-a_5 s^2(1+a\xi^2)+(1+a_4 s+aa_5 s^2)D^2)\tilde{\varphi}=0. \tag{21}$$

The non trivial solution of the system of Eqs. (16)-(18) yields

$$(PD^8 + QD^6 + RD^4 + SD^2 + T))(\widetilde{\Phi_1}, \widetilde{\Phi_2}, \tilde{\varphi}) = 0. \tag{22}$$

Where

$$P = -a_3(\delta_4 + aa_5 s^2) - aa_3 a_6 s^2,$$
$$Q = -(\delta_1 a_3 + \delta_3)(\delta_4 + aa_5 s^2) + a_3\big(\delta_4 \xi^2 + a_5 s^2(1 + a\xi^2)\big) + a_6 s^2(a_3 + 2aa_3\xi^2 + a\delta_3),$$
$$R = -\Big((\delta_1\delta_3 + \delta_2)(\delta_4 + aa_5 s^2) + (\delta_1 a_3 + \delta_3)\big(\delta_4\xi^2 + a_5 s^2(1 + a\xi^2)\big) - a_6 s^2\big((1 + a\xi^2)a_3\xi^2 + \delta_3 - a\delta_2\big)\Big),$$
$$S = \delta_1\delta_2(\delta_4 + aa_5 s^2) + (\delta_1\delta_3 + \delta_2)\big(\delta_4\xi^2 + a_5 s^2(1 + a\xi^2)\big) + a_6 s^2\big((1 + a\xi^2)\delta_3\xi^2 + \delta_2(1 + 2a\xi^2)\big),$$
$$T = -\delta_1\delta_2\big(\delta_4\xi^2 + a_5 s^2(1 + a\xi^2)\big) - a_6 s^2\delta_2(1 + a\xi^2)\xi^2,$$
$$\delta_1 = \xi^2 + s^2, \delta_2 = a_2\xi^2 + a_3\xi^4 + s^2, \delta_3 = a_2 + 2a_3\xi^2, \delta_4 = 1 + a_4 s.$$

The roots of the Eq. (24) are $\pm\lambda_i (i = 1, 2, 3, 4)$, using the radiation condition that $\hat{q}, \hat{\Psi}, \hat{\varphi} \to 0$ as $z \to \infty$ the solution of Eq. (19) may be written as

$$(\widetilde{\Phi_1}, \widetilde{\Phi_2}, \tilde{\varphi}) = \sum_{i=1}^{4} (1, R_i, S_i) A_i e^{-\lambda_i z}, \tag{23}$$

Where

$$R_i = \frac{P^* + Q^*\lambda_i^2 + R^*\lambda_i^4}{A^* + B^*\lambda_i^2 + C^*\lambda_i^4 + D^*\lambda_i^6},$$

$$S_i = \frac{P^{**} + Q^{**}\lambda_i^2 + R^{**}\lambda_i^4 + S^{**}\lambda_i^6}{A^* + B^*\lambda_i^2 + C^*\lambda_i^4 + D^*\lambda_i^6}.$$

Where

$$P^* = \delta_1\big(\delta_4\xi^2 + a_5 s^2(1 + a\xi^2)\big) + a_6 s^2\xi^2(1 + a\xi^2), Q^* = -\delta_1(\delta_4 + aa_5 s^2) - \big(\delta_4\xi^2 + a_5 s^2(1 + a\xi^2)\big) - a_6 s^2(1 + 2a\xi^2),$$
$$R^* = \delta_4 + aa_5 s^2 + aa_6 s^2,$$
$$A^* = \delta_2\big(\delta_4\xi^2 - a_5 s^2(1 + a\xi^2)\big), \quad B^* = -\delta_2(\delta_4 + aa_5 s^2 - \delta_3\xi^2\delta_4) - \delta_3 a_5 s^2(1 + a\xi^2),$$
$$C^* = \delta_3(\delta_4 + +aa_5 s^2) + a_3\delta_4\xi^2 - a_3 a_5 s^2(1 + a\xi^2), \qquad D^* = -a_3(\delta_4 + aa_5 s^2),$$
$$P^{**} = \delta_1\delta_2, Q^{**} = -\delta_1\delta_3 - \delta\delta_2,$$
$$R^{**} = \delta_1 a_3 + \delta\delta_3, \qquad S^{**} = -\delta a_3.$$

4. Boundary conditions

The appropriate boundary conditions are

$$t_{zz}(r, z, t) = -P_1(r, t), \tag{23}$$

$$t_{zr}(r, z, t) = 0, \tag{24}$$

$$\frac{\partial\varphi}{\partial r}(r, z, t) = P_2(r, t), \tag{25}$$

$$m_{\theta z} = 0\,. \tag{26}$$

$P_1(r,t)$ and $P_2(r,t)$ are well behaved functions.

Here $P_2(r,t)$ =0 corresponds to plane boundary subjected to normal force and $P_1(r,t)$ =0 corresponds to plane boundary subjected to thermal point source.

The non-dimensional values of t_{zz}, t_{zr} and $m_{\theta z}$ are given by

$$t_{zz} = \frac{\lambda e}{\beta_1 T_0} + \frac{2\mu}{\beta_1 T_0} e_{zz} - \frac{\rho c_1^2}{\beta_1^2 T_0}(1 - a\nabla^2)\varphi, \tag{27}$$

$$t_{zr} = \frac{2\mu}{\beta_1 T_0} e_{rz} + \frac{\alpha}{\beta_1 T_0}\left(\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right), \tag{28}$$

$$m_{\theta z} = \frac{\alpha \omega^*}{2\beta_1 T_0 c_1^2}\left(\frac{\partial^2 u_r}{\partial z^2} - \frac{\partial^2 u_z}{\partial r \partial z}\right). \tag{29}$$

Where $e_{rr} = \frac{\partial u}{\partial r}$, $e_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)$, $e_{\theta\theta} = \frac{u}{r}$, $e_{zz} = \frac{\partial w}{\partial z}$.

<u>Applications</u>

Case 1. Concentrated normal force/ Thermal point source

When plane boundary is subjected to concentrated normal force/ thermal point force, then $P_1(r,t), P_2(r,t)$ take the form

$$(P_1(r,t), P_2(r,t)) = \left(\frac{P_1 \delta(r)\delta(t)}{2\pi r}, \frac{P_2 \delta(r)\delta(t)}{2\pi r}\right). \tag{30}$$

P_1 is the magnitude of the force applied, P_2 is the magnitude of the constant temperature applied on the boundary and $\delta(\quad)$ is the Dirac delta function.

Making use of Eqs. (30) in the boundary conditions (23)-(26) with the aid of (17)-(18), (26)-(28) the components of displacement, conductive temperature, components of stress and couple stress are given by (33)-(38).

Case 2. Normal force over the circular region/ Thermal source over the circular region

Let a uniform pressure of total magnitude / constant temperature applied over a uniform circular region of radius a^* is obtained by setting

$$(P_1(r,t), P_2(r,t)) = \left(\frac{P_1}{\pi {a^*}^2} H(a^* - r)\delta(t), \frac{P_2}{\pi {a^*}^2} H(a^* - r)\delta(t)\right), \tag{31}$$

where $H(a^* - r)$ is the Heaviside unit step function.

Making use of dimensionless quantities defined by (9) and then applying Laplace and Hankel transforms defined by (17)-(18) on (30)

$$\left(\widetilde{P_1}(\xi,s), \widetilde{P_2}(\xi,s)\right) = \left(\frac{P_1}{\pi a^* \xi} J_1(a^*\xi), \frac{P_2}{\pi a^* \xi} J_1(a^*\xi)\right). \tag{32}$$

The expressions for the components of displacements, stress, couple stress and conductive temperature are obtained by replacing $\frac{P_1}{2\pi}$ with $\frac{P_1 J_1(a^*\xi)}{\pi a^*\xi}$ and by replacing $\frac{P_2}{2\pi}$ with $\frac{P_2 J_1(a^*\xi)}{\pi a^*\xi}$ in Eqs. (33)-(38) respectively and are given by (39)-(44).

Concentrated normal force/ Thermal point source

$$\widetilde{u_r} = -\frac{P_1}{2\pi\Delta}\sum_{i=1}^{4} B_{1i}e^{-\lambda_i z} + \frac{P_2}{2\pi\Delta}\sum_{i=1}^{4} B_{3i}e^{-\lambda_i z}, \tag{33}$$

$$\widetilde{u_z} = -\frac{P_1}{2\pi\Delta}\sum_{i=1}^{4} B_{1i}R_i e^{-\lambda_i z} + \frac{P_2}{2\pi\Delta}\sum_{i=1}^{4} B_{3i}R_i e^{-\lambda_i z}, \tag{34}$$

$$\tilde{\varphi} = -\frac{P_1}{2\pi\Delta}\sum_{i=1}^{4} B_{1i}S_i e^{-\lambda_i z} + \frac{P_2}{2\pi\Delta}\sum_{i=1}^{4} B_{3i}S_i e^{-\lambda_i z}, \tag{35}$$

$$\widetilde{t_{zz}} = -\frac{P_1}{2\pi\Delta}\sum_{i=1}^{4} A_{1i}B_{1i}e^{-\lambda_i z} + \frac{P_2}{2\pi\Delta}\sum_{i=1}^{4} A_{1i}B_{3i}e^{e^{-\lambda_i z}}, \tag{36}$$

$$\widetilde{t_{zr}} = -\frac{P_1}{2\pi\Delta}\sum_{i=1}^{4} A_{2i}B_{1i}e^{e^{-\lambda_i z}} + \frac{P_2}{2\pi\Delta}\sum_{i=1}^{4} A_{2i}B_{3i}e^{-\lambda_i z}, \tag{37}$$

$$\widetilde{m_{\theta z}} = -\frac{P_1}{2\pi\Delta}\sum_{i=1}^{4} A_{4i}B_{1i}e^{e^{-\lambda_i z}} + \frac{P_2}{2\pi\Delta}\sum_{i=1}^{4} A_{4i}B_{3i}e^{-\lambda_i z}. \tag{38}$$

For circular region

$$\widetilde{u_r} = -\frac{P_1}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} B_{1i}e^{-\lambda_i z} + \frac{P_2}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} B_{3i}e^{-\lambda_i z}, \tag{39}$$

$$\widetilde{u_z} = -\frac{P_1}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} B_{1i}R_i e^{-\lambda_i z} + \frac{P_2}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} B_{3i}R_i e^{-\lambda_i z} \tag{40}$$

$$\tilde{\varphi} = -\frac{P_1}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} B_{1i}S_i e^{-\lambda_i z} + \frac{P_2}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} B_{3i}S_i e^{-\lambda_i z}, \tag{41}$$

$$\widetilde{t_{zz}} = -\frac{P_1}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} A_{1i}B_{1i}e^{-\lambda_i z} + \frac{P_2}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} A_{1i}B_{3i}e^{-\lambda_i z} \tag{42}$$

$$\widetilde{t_{zr}} = -\frac{P_1}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} A_{2i}B_{1i}e^{-\lambda_i z} + \frac{P_2}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} A_{2i}B_{3i}e^{-\lambda_i z} \tag{43}$$

$$\widetilde{m_{\theta z}} = -\frac{P_1}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} A_{4i}B_{1i}e^{e^{-\lambda_i z}} + \frac{P_2}{\pi a^*\xi\Delta}J_1(a^*\xi)\sum_{i=1}^{4} A_{4i}B_{3i}e^{e^{-\lambda_i z}}. \tag{44}$$

where

$$A_{1i} = \frac{\lambda}{\beta_1 T_0}\left(-\xi^2 + \lambda_i^2\right) + \frac{2\mu}{\beta_1 T_0}\left(\lambda_i^2 - \xi^2\lambda_i R_i\right) - \frac{\rho c_1^2}{\beta_1^2 T_0}\left(1 - a\left(-\xi^2 + \lambda_i^2\right)S_i\right),$$

$$A_{2i} = \frac{1}{\beta_1 T_0}\left(\mu(2\xi\lambda_i + \left(-\xi\lambda_i^2 - \xi^3\right)R_i) + \frac{\alpha\omega^{*2}}{4c_1^2}\left(-\xi^2 + \lambda_i^2\right)\left(-\xi^2\lambda_i^2 + \xi^3\right)R_i\right),$$

$$A_{3i} = -\lambda_i S_i,$$

$$A_{4i} = \frac{\alpha\omega^*\xi\lambda_i}{2\beta_1 T_0 c_1^2}\left(\lambda_i^2 - \xi^2\right)R_i,$$

$$\Delta = \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4,$$
$$\Delta_1 = A_{11}A_{22}(A_{33}A_{44} - A_{43}A_{34}) - A_{11}A_{23}(A_{32}A_{44} - A_{42}A_{34}) + A_{11}A_{24}(A_{32}A_{43} - A_{42}A_{33}),$$
$$\Delta_2 = A_{12}A_{21}(A_{33}A_{44} - A_{43}A_{34}) - A_{12}A_{23}(A_{31}A_{44} - A_{41}A_{34}) + A_{24}A_{12}(A_{31}A_{43} - A_{41}A_{33}),$$
$$\Delta_3 = A_{13}A_{21}(A_{32}A_{44} - A_{42}A_{34}) - A_{22}A_{13}(A_{31}A_{44} - A_{41}A_{34}) + A_{13}A_{24}(A_{31}A_{42} - A_{41}A_{32}),$$
$$\Delta_4 = A_{14}A_{21}(A_{32}A_{43} - A_{42}A_{33}) - A_{22}A_{14}(A_{31}A_{43} - A_{41}A_{33}) + A_{14}A_{23}(A_{31}A_{42} - A_{41}A_{32}),$$
$$B_{1i} = \frac{(-1)^{1+i}\Delta_i}{A_{1i}},\ A_i = \frac{1}{\Delta}(-\widetilde{P_1}(\xi,s)B_{1i} + (\widetilde{P_2}(\xi,s)B_{3i}),$$

$$B_{31} = A_{12}(A_{23}A_{44} - A_{43}A_{24}) - A_{13}(A_{22}A_{44} - A_{42}A_{24}) + A_{14}(A_{22}A_{43} - A_{42}A_{23}),$$
$$B_{32} = -A_{11}(A_{23}A_{44} - A_{43}A_{24}) + A_{13}(A_{21}A_{44} - A_{41}A_{24}) - A_{14}(A_{21}A_{43} - A_{41}A_{23}),$$
$$B_{33} = A_{11}(A_{22}A_{44} - A_{42}A_{24}) - A_{12}(A_{21}A_{44} - A_{41}A_{24}) + A_{14}(A_{21}A_{42} - A_{41}A_{22}),$$
$$B_{34} = -A_{11}(A_{22}A_{43} - A_{42}A_{23}) + A_{12}(A_{21}A_{43} - A_{41}A_{23}) - A_{13}(A_{21}A_{42} - A_{41}A_{22}).$$

6. Particular cases

If $K^* = 0$ and $a = 0$ in Eqs. (33)-(44), we obtain the resulting expressions for isotropic thermoelastic solid for GN-II theory and without two temperature.

If $K^* = 0$ and $a = 0$ in Eqs. (33)-(44), we obtain the resulting expressions for isotropic thermoelastic solid for GN-II theory and with two temperature.

If $K^* \neq 0$ and $a = 0$ in Eqs. (33)-(44), we obtain the resulting expressions for isotropic thermoelastic solid for GN-III theory and without two temperature.

If $K^* \neq 0$ and $a = 0$ in Eqs. (33)-(44), we obtain the resulting expressions for isotropic thermoelastic solid for GN-III theory and with two temperature.

7. Inversion of the transformations

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (33)-(44). Here the distance components ,conductive temperature, stress components and couple stress are functions of z, the parameters of Hankel and laplace transforms are ξ and s respectively and hence are of the form $\tilde{f}\ (\xi\ , z, s)$. To obtain the function $f(r, z, t)$ in the physical domain, we first invert the Hankel transform using

$$\hat{f}(r,z,s) = \int_0^\infty \xi \tilde{f}(\xi\ , z\ , s) J_n(\xi r)\, d\xi. \tag{45}$$

Now for the fixed values of ξ, r and z the function $\hat{f}(r\ , z\ , s)$ in the expression above can be considered as the Laplace transform $\hat{g}\ (s)$ of $g\ (t)$. Following Honig and Hirdes (1984), the Laplace transform function $\hat{g}\ (s)$ can be inverted.

The last step is to calculate the integral in Eq. (45). The method for evaluating this integral is described in Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

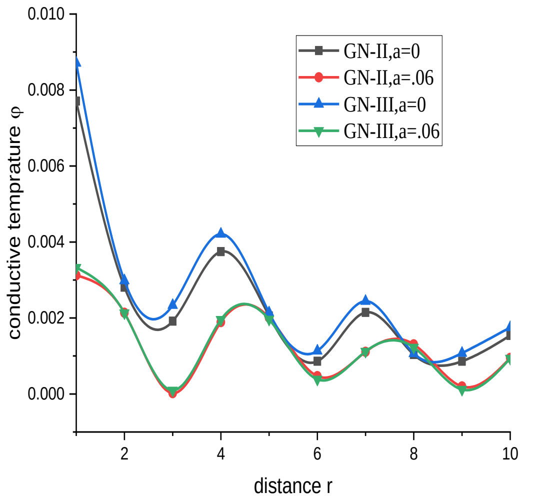

Fig. 1 Variation of conductive temperature φ with the radial distance r (concentrated normal force)

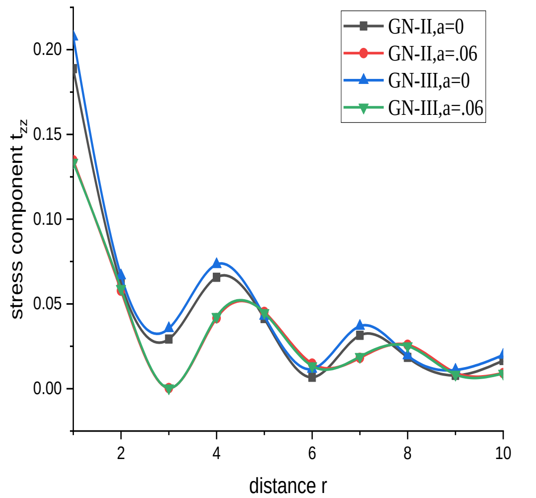

Fig. 2 Variation of stress component t_{zz} with the radial distance r (concentrated normal force)

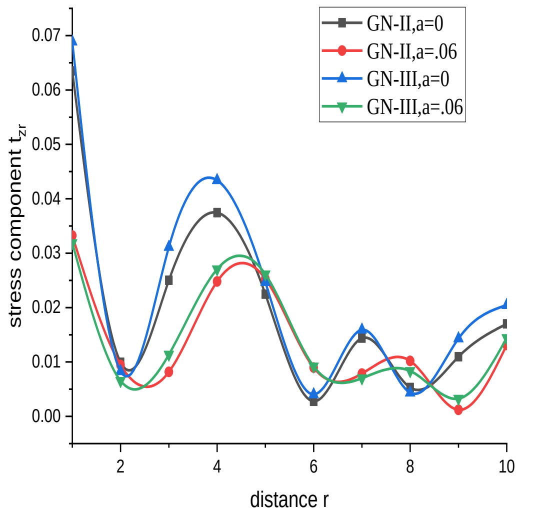

Fig. 3 Variation of stress component t_{zr} with the radial distance r (concentrated normal force)

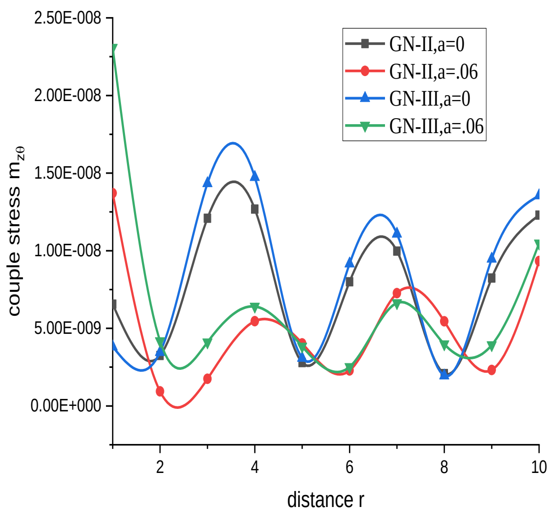

Fig. 4 Variation of couple stress $m_{z\theta}$ with the radial distance r (concentrated normal force)

8. Results and discussions

For numerical computations following Devi (2017), we take the copper material which is isotropic as

$$\lambda = 7.76 \times 10^{10} Kgm^{-1}s^{-1}, \mu = 3.86 \times 10^{10} Kgm^{-1}s^{-1}, T_0 = 293K,$$
$$C^* = .3831 \times 10^3 JKg^{-1}K^{-1}, \alpha_t = 1.78 \times 10^{-5} K^{-1}, \rho = 8.954 \times 10^3 Kgm^{-3},$$
$$K = .383 \times 10^3 Wm^{-1}K^{-1}, \alpha = .05 Kgms^{-2}, t = 0.5\ s$$

Software GNU octave has been used to determine the components of displacements, conductive temperature, normal stress, tangential stress and couple stress for homogeneous isotropic

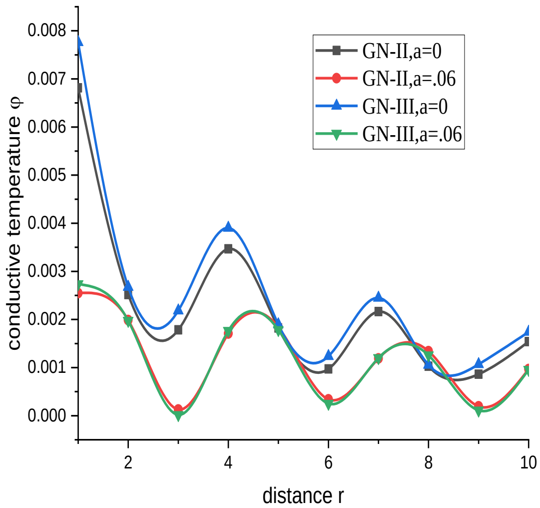

Fig. 5 variation of conductive temperature φ with the radial distance r (normal force over the circular region)

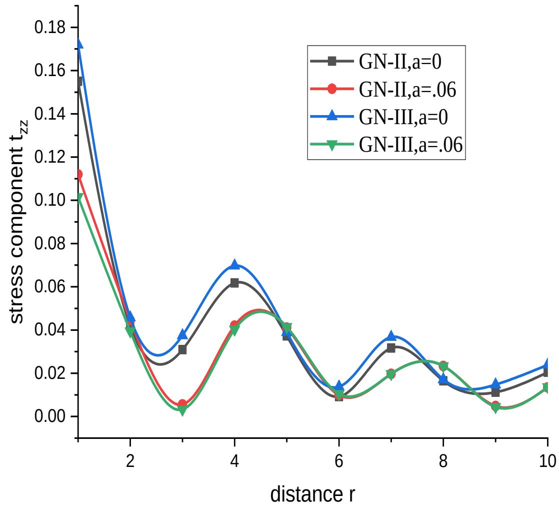

Fig. 6 variation of stress component t_{zz} with the radial distance r (normal force over the circular region)

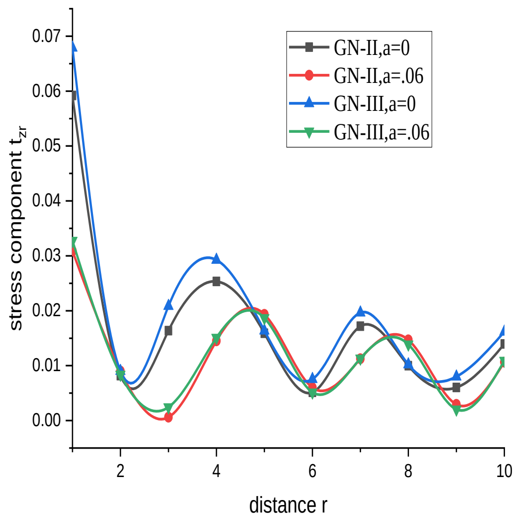

Fig. 7 Variation of stress component t_{zr} with the radial distance r (normal force over the circular region)

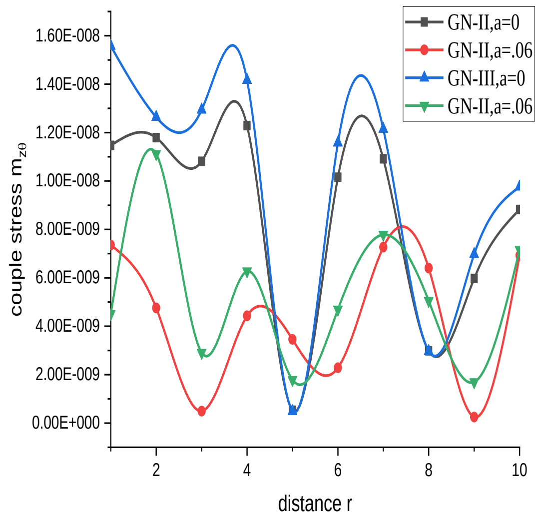

Fig. 8 Variation of couple stress $m_{z\theta}$ with the radial distance r (normal force over the circular region)

thermoelastic medium with radial distance r for GN-II theory $(K^* = 0N\,sec^{-2}K^{-1})$ and GN-III $(K^* = .3N\,sec^{-2}K^{-1})$ theory with two temperature and without two temperature..

The solid lines in black with centre symbol square, red with centre symbol circle, blue with centre symbol triangle and green centre symbol inverted triangle respectively corresponds to the $\mathrm{GN-II}, a = 0$, $\mathrm{GN-II}, a = .06$, $\mathrm{GN-III}, a = 0$ and $\mathrm{GN-III}, a = .06$. Figs. 1-6 corresponds to the variations of u_r,u_z, φ, t_{zz} $t_{zr}$and $m_{z\theta}$ respectively for concentrated normal force. Figs. 7-12

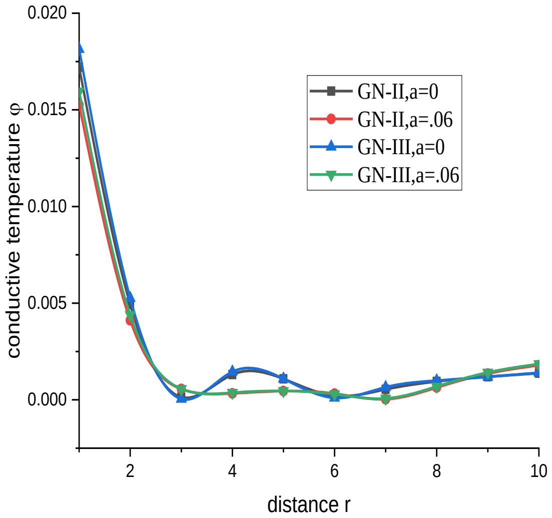

Fig. 9 Variation of conductive temperature φ with the radial distance r (thermal point source)

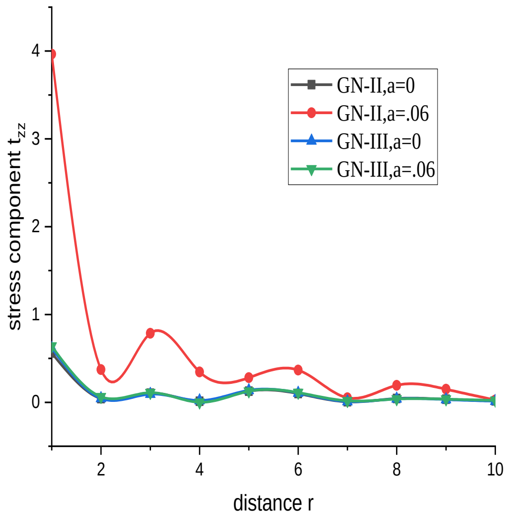

Fig. 10 Variation of stress component t_{zz} with the radial distance r (thermal point source)

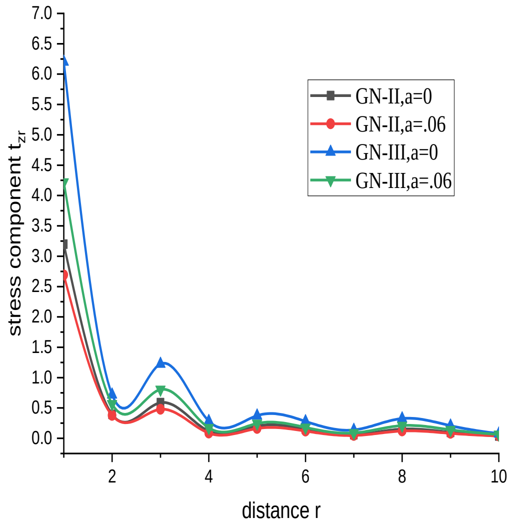

Fig. 11 Variation of stress component t_{zr} with the distance r (thermal point source)

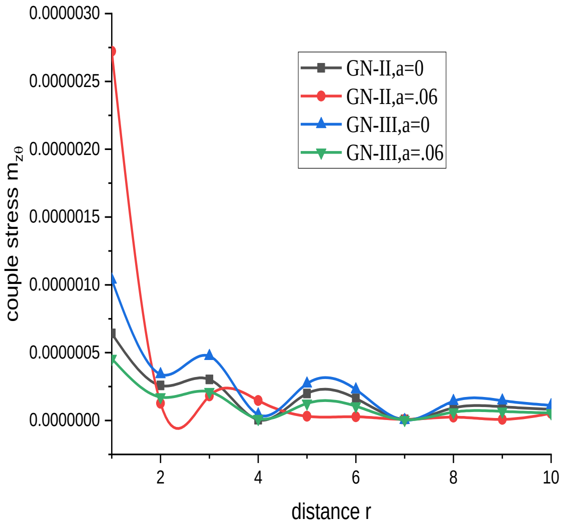

Fig. 12 Variation of couple stress $m_{z\theta}$ with the radial distance r (thermal point source)

corresponds to the variations of u_r,u_z, φ, t_{zz} t_{zr} and $m_{z\theta}$ respectively for normal force over the circular region. Figs. 13-18 corresponds to the variations of u_r,u_z, φ, t_{zz} t_{zr} and $m_{z\theta}$ respectively for thermal point source. Figs. 19-21 corresponds to the variations of u_r,u_z, φ, t_{zz} t_{zr} and $m_{z\theta}$ respectively for thermal source over the circular region.

Concentrated normal force

In Fig. 1 variations corresponding to the φ are oscillatory in nature. Amplitude of variations decreses as we move away from the pole. In Fig. 2 variations corresponding to the t_{zz} are oscillatory

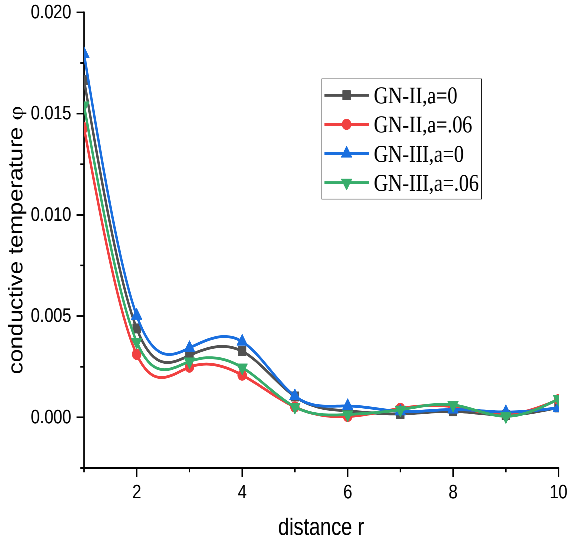

Fig. 13 Variation of conductive temperature φ with the radial distance r (thermal source over the circular region)

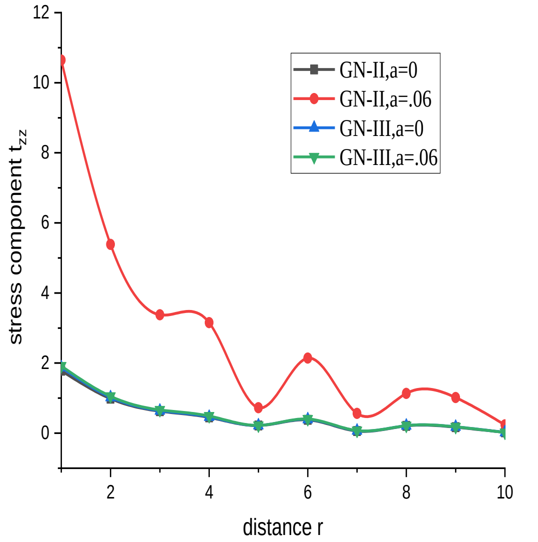

Fig. 14 Variation of stress component σ_{zz} with the radial distance r (thermal source over the circular region)

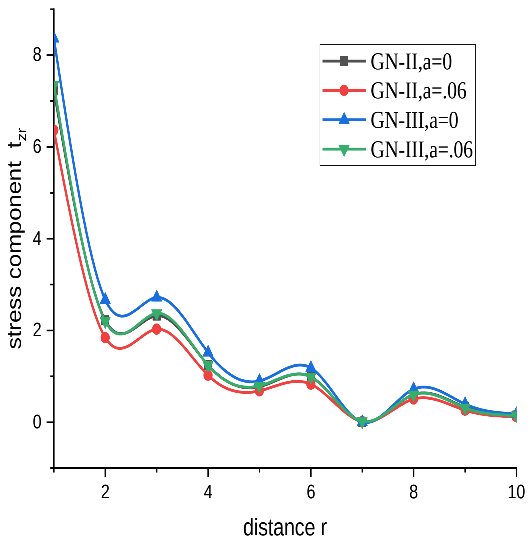

Fig. 15 Variation of stress component t_{zr} with the radial distance r (thermal source over the circular region)

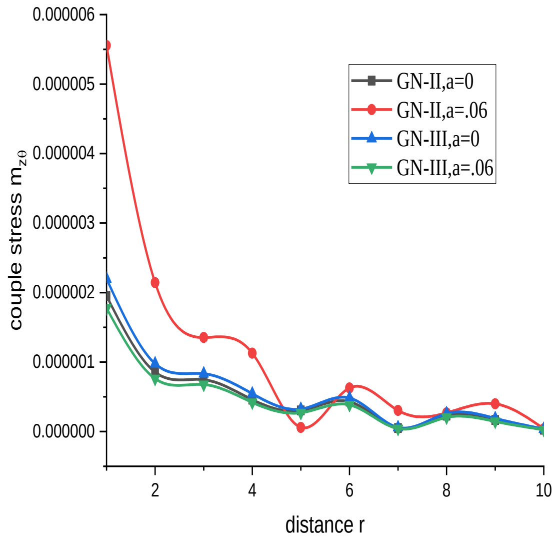

Fig. 16 Variation of couple stress $m_{z\theta}$ with the radial distance r (thermal source over the circular region)

in nature. Presence of two temperature and energy dissipation changes the magnitude of variation. Value of t_{zz} is highest at $x = 1$. In Fig. 2 variations corresponding to the t_{zr} are oscillatory in nature. Amplitude of variations decreases in the first half range and increases in the remaining half. Values of GN-II theory are lower than GN-III theory for both the cases with and without two temperature. In Fig. 4 Variations of $m_{z\theta}$ are similar to the corresponding variations of t_{zr} except the magnitude.

Normal force over the circular region

In Fig. 5 variations corresponding to the φ are oscillatory in nature.For both cases with and without energy dissipation amplitude of the variation is smaller for GN-II theory than GN-III theory. In Fig. 6 variations for t_{zz} are similar to the corresponding variations of φ, except the magnitude. In Fig. 7 variations for t_{zr} are oscillatory in trend. In Fig. 8 variations for $m_{z\theta}$ are oscillatory in trend. Appreciable effect of two temperature and energy dissipation is seen on the magnitude of $m_{z\theta}$.

Thermal point source

In Fig. 9 variations of corresponding to the φ are oscillatory in nature. For both cases with and without energy dissipation magnitude of the variation is different for GN-II theory than GN-III theory. In Fig. 10 variations of corresponding to the t_{zz} are oscillatory in nature. Magnitude of variation is the highest for GN-II, *a*=.06. In Fig. 11 variations of corresponding to the t_{zr} are oscillatory in nature. Magnitude of variation is higher for GN-III theory than GN-II theory. For given GN-theory (GN-II or GN-III) presence of two temperature decreases the magnitude t_{zr}. variations for $m_{z\theta}$ are oscillatory in trend. Trend of oscillation is different for GN=II, *a*=.06.

Thermal source over the circular region

In Fig. 13 variations of corresponding to the φ are oscillatory in nature. For both cases with and without energy dissipation magnitude of the variation is greater for GN-III theory than GN-II theory. In Fig. 14 variations of corresponding to the t_{zz} are oscillatory in nature. Magnitude of variation is the highest for GN-II, *a*=.06. In Fig. 15 variations of corresponding to the t_{zr} are oscillatory in nature. Magnitude of variation is higher for GN-III theory than GN-II theory. For given GN-theory (GN-II or GN-III) presence of two temperature decreases the magnitude t_{zr}. In Fig. 16 variations for $m_{z\theta}$ are oscillatory in trend. Trend of oscillation is different for GN=II, *a*=.06 than others.

9. Conclusions

Effect of energy dissipation and two temperature is significant on components of displacement, conductive temperature, normal stress, tangential stress and couple stress. As disturbance travels through the constituents of the medium, it suffers sudden changes and result in an non-uniform pattern of graphs. Pattern of the variations of GN-II theory is different from GN-III theory in most of the cases. Examinig from graphs we conclude that two temperature and energy dissipation changes the magnitude of the physical quantity. Appreciable effect of two temperature and energy dissipation is observed on the physical quantities. The results obtained in the study should be beneficial for people working on modified couple stress thermoelastic solid with two temperature and energy dissipation.

References

Abbas, I.A. and Marin, M. (2018), “Analytical solutions of a two-dimensional generalized thermoelastic diffusions problem due to laser pulse”, *Iran. J. Sci. Technol., Trans. Mech. Eng.*, **42**(1), 57-71.

https://doi.org/10.1007/s40997-017-0077-1.
Abbas, I.A., Abdalla, A.E.N.N., Alzahrani, F.S. and Spagnuolo, M. (2016), "Wave propagation in a generalized thermoelastic plate using eigenvalue approach", *J. Therm. Stress.*, **39**(11), 1329-1334. https://doi.org/10.1080/01495739.2016.1218229.
Abbas, I.A., El-Amin, M.F. and Salama, A. (2008), "Effect of thermal dispersion on free convection in a fluid saturated porous medium", *Int. J. Heat Fluid Flow*, **30**(2), 229-236. https://doi.org/10.1016/j.ijheatfluidflow.2009.01.004.
Abd-Alla, A.E.N.N. and Abbas, I.A. (2011), "A problem of generalized magnetothermoelasticity for an infinitely long, perfectly conducting cylinder", *J. Therm. Stress.*, **25**(11), 1009-1025. https://doi.org/10.1080/01495730290074612.
Ansari, R., Ashrafi, M.A. and Hosseinzadeh, S. (2014), "Vibration characteristics of Piezoelectric microbeams based on modified couple stress theory", *Shock Vib.*, **2014**, 1-12. https://doi.org/10.1155/2014/598292.
Bhatti, M.M. and Michaelides, E.E. (2021), "Study of Arrhenius activation energy on the thermo-bioconvection nanofluid flow over a Riga plate", *J. Therm. Anal. Calorim.*, **143**, 2029-2038. https://doi.org/10.1007/s10973-020-09492-3.
Borjalilou, V., Asghari, M. and Bagheri, E. (2019), "Small-scale thermoelastic damping in micro-beams utilizing the modified couple stress theory and the dual-phase-lag heat conduction model", *J. Therm. Stress.*, **42**(7), 801-814. https://doi.org/10.1080/01495739.2019.1590168.
Borjalilou. V. and Asghari, M. (2018), "Small-scale analysis of plates with thermoelastic damping based on the modified couple stress theory and the dual-phase-lag heat conduction model", *Acta Mechanica*, **229**, 3869-3884. https://doi.org/10.1007/s00707-018-2197-0.
Chen, W. and Li, X. (2013), "Size-dependent free vibration analysis of composite laminated Timoshenko beam based on new modified couple stress theory", *Arch. Appl. Mech.*, **83**(3), 431-444. https://doi.org/10.1007/s00419-012-0689-2.
Cosserat, E. and Cosserat, F. (1909), *Theory of Deformable Bodies*, Hermann et Fils, Paris, France.
Eringen, A.C. (1999), *Theory of Micropolar Elasticity*, *Microcontinuum Field Theories*, Springer, New York, NY.
Ghiasi, E.K. (2016), "Application of modified couple stress theory to study dynamic characteristics of electrostatically actuated micro-beams resting upon squeeze-film damping under mechanical shock", *Int. J. Adv. Mech. Eng.*, **6**(1), 1-15.
Hobiny, A. and Abbas, I.A. (2017), "A study on photothermal waves in an unbounded semiconductor medium with cylindrical cavity", *Mech. Time-Depend. Mater.*, **21**(1), 61-72. https://doi.org/10.1007/s11043-016-9318-8.
Hobiny, A. and Abbas, I.A. (2018), "Analytical solutions of photo-thermo-elastic waves in a non-homogenous semiconducting material", *Result. Phys.*, **10**(1), 385-390. https://doi.org/10.1016/j.rinp.2018.06.035.
Hobiny, A. and Abbas, I.A. (2021), "Analytic solutions of Fractional bioheat model in a spherical tissue", *Mech. Bas. Des. Struct. Mach.*, **49(**3), 430-439. https://doi.org/10.1080/15397734.2019.1702055.
Honig, G. and Hirdes, U. (1984), "A method for the numerical inversion of the Laplace transforms", *J. Comput. Appl. Math.*, **10**, 113-132. https://doi.org/10.1016/0377-0427(84)90075-X.
Jung, W.Y., Han, S.C. and Park, W.T. (2014), "A modified couple stress theory for coupling analysis of S-FGM nanoplates embedded in Pasternak elastic medium", *Compos. Part B: Eng.*, **60**, 746-756. https://doi.org/10.1016/j.compositesb.2013.12.058.
Kakhki, E.K., Hosseini, S.M. and Tahani, M. (2016), "An analytical solution for thermoelastic damping in a micro-beam based on generalized theory of thermoelasticity and modified couple stress theory", *Appl. Math. Model.*, **40**(4), 3164-3174. https://doi.org/10.1016/j.apm.2015.10.019.
Kaur, I. and Lata. P. (2020), "Axisymmetric deformation in transversely isotropic magneto-thermoelastic solid with Green-Naghdi III due to inclined load", *Int. J. Mech. Mater. Eng.*, **15**(1), 3. https://doi.org/10.1186/s40712-019-0111-8.
Koiter, W.T. (1964), "Couple stresses in the theory of elasticity, I and II", *Proceedings Series B, Koninklijke Nederlandse Akademie van Wetenschappen*, **67**, 17-29.
Kong, S., Zhou, S., Nie, Z. and Wang, K. (2008), "The size-dependent natural frequency of Bernoulli-Euler

micro-beams", *Int. J. Eng. Sci.*, **46**(5), 427-437. https://doi.org/10.1016/j.ijengsci.2007.10.002.
Kumar, R., Devi, S. and Sharma, V. (2017), "Effect of Hall current and rotation in modified couple stress generalized thermoelastic half space due to ramp type heating", *J. Solid Mech.*, **9**(3), 527-542.
Ma, H.M. Gao, X.L. and Reddy, J.N. (2008), "A microstructure-dependent Timoshenko beam model based on a modified couple stress theory", *J. Mech. Phys. Solid.*, **56**(12), 3379-3391. https://doi.org/10.1016/j.jmps.2008.09.007.
Marin, M. (1998), "A temporally evolutionary equation in elasticity of micropolar bodies with void", *UPB Scientif. Bull., Ser. A: Appl. Math. Phys.*, **60** (3-4), 3-12, 1998.
Marin, M. (2010), "Some estimates on vibrations in thermoelasticity of dipolar bodies", *J. Vib. Control*, **16**(1), 33-47. https://doi.org/10.1177/1077546309103419.
Marin, M., Agarwal, R.P. and Mahmoud, S.R. (2013), "Modeling a microstretch thermo-elastic body with two temperatures", *Abstr. Appl. Anal.*, **2013**, 1-7. https://doi.org/10.1155/2013/583464.
Mindlin, R.D. (1964), "Micro-structure in linear elasticity", *Arch. Rat. Mech. Anal.*, **16**, 51-78.
Mindlin, R.D. and Tiersten, H.F. (1962), "Effects of couple-stress in linear elasticity", *Arch. Rat. Mech. Anal.*, **11**(1), 415-448. https://doi.org/10.1007/BF00253946.
Mohamed, R.A., Abbas, I.A. and Abo-Dahab, S. (2009), "Finite element analysis of hydromagnetic flow and heat transfer of a heat generation fluid over a surface embedded in a non-Darcian porous medium in the presence of chemical reaction", *Commun. Nonlin. Sci. Numer. Simul.*, **14**(4), 1385-1395. https://doi.org/10.1016/j.cnsns.2008.04.006.
Mohammad-Abadi, M. and Daneshmehr, A.R. (2015), "Modified couple stress theory applied to dynamic analysis of composite laminated beams by considering different beam theories", *Int. J. Eng. Sci.*, **87**, 83-102. https://doi.org/10.1016/j.ijengsci.2014.11.003.
Park, S.K. and Gao, X.L. (2006), "Bernoulli-Euler beam model based on a modified couple stress theory", *J. Micromech. Microeng.*, **16**(11), 2355-2359.
Press, W.H., Teukolsky, S.A., Vellerling, W.T. and Flannery, B.P. (1986), *Numerical Recipe*, Cambridge University Press.
Razavilar, R., Alashti, R.A. and Fathi, A. (2016), "Investigation of thermoelastic damping in rectangular microplate resonator using modified couple stress theory", *Int. J. Mech. Mater. Des.*, **12**(1), 39-51. https://doi.org/10.1007/s10999-014-9286-6.
Reddy, J.N. (2011), "Microstructure-dependent couple stress theories of functionally graded beams", *J. Mech. Phys. Solid.*, **59**(11), 2382-2399. https://doi.org/10.1016/j.jmps.2011.06.008.
Rezazadeh, G., Vahdat, A.S., Tayefeh-rezaei, S. and Cetinkaya, C. (2012), "Thermoelastic damping in a micro-beam resonator using modified couple stress theory", *Acta Mechanica*, **223**(6), 1137-1152. https://doi.org/10.1007/s00707-012-0622-3.
Simsek, M., Aydin, M., Yurtcu, H. and Reddy, J. (2015), "Size-dependent vibration of a microplate under the action of a moving load based on the modified couple stress theory", *Acta Mechanica*, **226**(11), 3807-3822. https://doi.org/10.1007/s00707-015-1437-9.
Taati, E., Najafabadi, M.M. and Tabrizi, H.B. (2014), "Size-dependent generalized thermoelasticity model for Timoshenko micro-beams", *Acta Mechanica*, **225**(7), 1823-1842. https://doi.org/10.1007/s00707-013-1027-7.
Tahani, M., Askari, A.R., Mohandes, Y. and Hassani, B. (2015), "Size-dependent free vibration analysis of electrostatically pre-deformed rectangular micro-plates based on the modified couple stress theory", *Int. J. Mech. Sci.*, **94**, 185-198. https://doi.org/10.1016/j.ijmecsci.2015.03.004.
Tsiatas, G.C. and Yiotis, A.J. (2015), "Size effect on the static, dynamic and buckling analysis of orthotropic Kirchhoff-type skew micro-plates based on a modified couple stress theory: comparison with the nonlocal elasticity theory", *Acta Mechanica*, **226**(4), 1267-1281. https://doi.org/10.1007/s00707-014-1249-3.
Voigt, W. (1887), *Theoretische Studien ¨uber die Elasticit¨atsverh¨altnisse der Krystalle"(Theoretical studies on the elasticity relationships of crystals)*, Abhandlungen der königlichen Gesellschaft der Wissenschaften in Göttingen, Dieterichsche Verlags-Buchhandlung.
Wang, Y.G., Lin, W.H. and Liu, N. (2013), "Nonlinear free vibration of a microscale beam based on modified couple stress theory", *Physica E: Low-Dimens. Syst. Nanostruct.*, **47**, 80-85.

https://doi.org/10.1016/j.physe.2012.10.020.
Yang, F., Chong, A.C.M., Lam, D.C.C. and Tong, P. (2002), "Couple stress based strain gradient theory for elasticity", *Int. J. Solid. Struct.*, **39**(10), 2731-2743. https://doi.org/10.1016/S0020-7683(02)00152-X.
Youssef, H.M. (2006), "Theory of two temperature generalized thermoelasticity", *IMA J. Appl. Math.*, **71**(3), 383-390. https://doi.org/10.1093/imamat/hxh101.
Zhang, L., Bhatti, M.M. and Michaelides, E.E. (2021), "Electro-magnetohydrodynamic flow and heat transfer of a third-grade fluid using a Darcy-Brinkman-Forchheimer model", *Int. J. Numer. Meth. Heat Fluid Flow*, **31**(8), 2623-2639. https://doi.org/10.1108/HFF-09-2020-0566.
Zhang, L., Bhatti, M.M., Marin, M. and Mekheimer, K.S. (2020), "Entropy analysis on the blood flow through anisotropically tapered arteries filled with magnetic zinc-oxide (ZnO) nanoparticles", *Entropy*, **22**(10), 1070. https://doi.org/10.3390/e22101070.
Zhang, L., Bhatti, M.M., Shahid, Ellahi, R., Bég, O.A. and Sadiq, S.M. (2021), "Nonlinear nanofluid fluid flow under the consequences of Lorentz forces and Arrhenius kinetics through a permeable surface: a robust spectral approach", *J. Taiwan Inst. Chem. Eng.*, **124** 98-105. https://doi.org/10.1016/j.jtice.2021.04.065.
Zhong, Z.Y., Zhang, W.M., Meng, G. and Wang, M.Y. (2015), "Thermoelastic damping in the size-dependent microplate resonators based on modified couple stress theory", *J. Microelectromech. Syst.*, **24**(2), 431-445. https://doi.org/10.1109/JMEMS.2014.2332757.
Zhou, S.S. and Gao, X.L. (2014), "A nonclassical model for circular Mindlin plates based on modified couple stress theory", *J. Appl. Mech*., **81**(5), 217-235. https://doi.org/10.1115/1.4026274.

DC