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Theoretical and experimental research of external prestressed timber beams in variable moisture conditions

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Abstract. Hybrid girders can be constructed in different geometrical forms and from different materials. Selection of beam's effective constellation represents a complex process considering the variations of geometrical parameters, changes of built in material characteristics and their mutual relations, which has important effect on the behavior of the girder. This paper presents the theoretical and experimental research on behavior of the timber-steel hybrid girders' different geometrical constellation with external prestressing and in different conditions of timber moisture. These researches are based on linear elastic analysis, and further refine by using the plasticity and damage models.

Keywords: hybrid girder; glued laminated timber; cable; effective force of prestressing; geometrical rigidity/stiffness; self-rigidity/stiffness

1. Introduction

Previous research have shown that the application of hybrid systems can achieve an increase in capacity and decrease the deformability of simple girder, which results the use on longer spans.

The basic idea of prestressing is that in the carrier system or single-carrier element, prior to applying the load of exploitation, artificially causes forces which should optimize the final behavior of the system under the total load. Prestressing is possible only in the case that the carrier system or element of the system have sufficient strength or rigidity, or a reaction to imposed prestressing forces.

Direct impact of prestressing on the behavior of loaded system, in certain budgetary situations, provides:

- reducing deflection by increasing the rigidity of the system or by including an expanded zone of the section in the reception of force,
- increase of capacity considering the opposite effect of entered force of prestressing in relation to the future load of the system,
- preventing the cancellation of certain elements of the system, or changes of the system,
- reducing deformability of movable systems by increasing the resistance of the system as a

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result of geometric changes in prestressing,
changing the own frequency of prestressed elements.

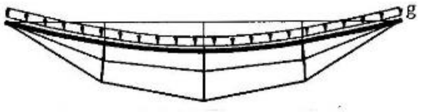
2. Application of prestressing in hybrid systems

Previous entering the prestressing force in the cable of hybrid system may have a positive impact on the avarage size of a distributed force system when the cable is set in the affine position corresponding to simple girder's bending line. For the selection of the prestressing force's size can be observed the girder under the constant load and a model of continuous beam, as in the principle of supporting (Fig. 1).

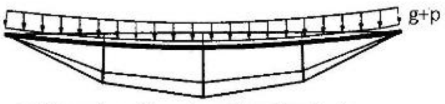
2.1 Impacts on the hybrid system's constellation for prestressing from the standpoint of capacity

The characteristics of the hybrid system suitable for prestressing from the standpoint of capacity for (Wagner, Schl 172/13-1) are:

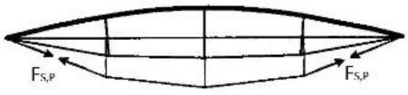
- On the capacity of hybrid girders affects the number of grants, with or without prestressing;
- Capacity of hybrid girder exclusively depends on the ratio of girder's stiffness for bending and stretching cables;
- With the increase of f/l increases the rigidity of cable;
- With the increase of h/l increases the rigidity of girder;
- Changing the girder's cross-section from rectangular through the segmented (I profile), cross-sectional area of the girder is reduced, and a permanent relationship A_S/A_n rapidly decreases the rigidity of the cable, while the rigidity of the girder seamlessly drops;
- With the increasing relations E_S/E_n the rigidity of cable increases.



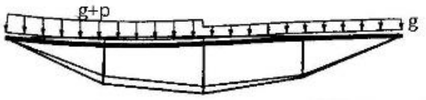
deformation caused by permanent load



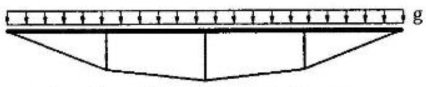
deformation of prestressed system by the permanent and live load



deformation caused by prestressing

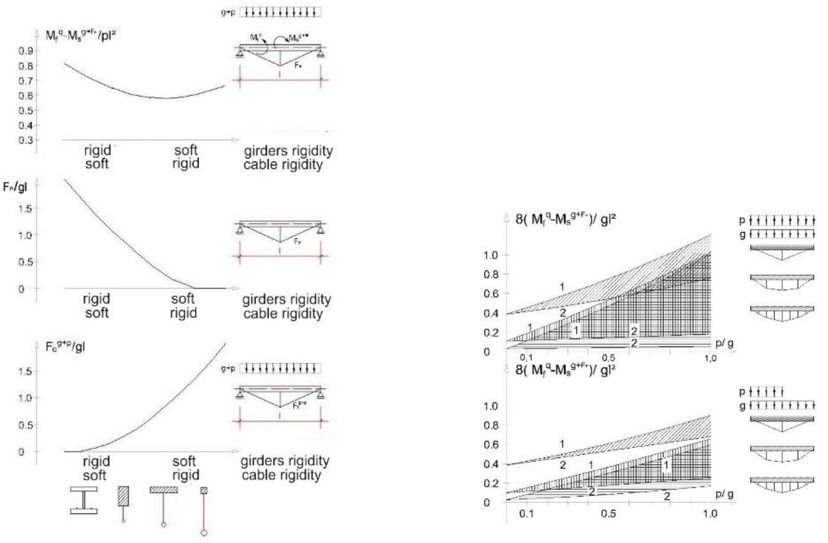


deformation of prestressed system caused by ansymetrical live load and permanent load



deformation of the prestressed sistem by permanent load

Fig. 1 Affine position of the cable corresponding to simple girder's bending line (condition of the effectively supporting)



(a) Utilization of prestressed girder on the total load compared to a simply supported beam under constant load

(b) Utilization of the bending moments, depending on the number of struts

Fig. 2 Redistribution of moments on girder (Wagner, Schl 172/13-1)

Prestressing of hybrid systems is one of the ways to increase the rigidity of the whole system. Prestressing is effective only when the hybrid system composed of the bar elements, wherein the resistivity of a cable system to the normal force is in balance with the resistance to bending of the girder. According to (Wagner, Schl 172/13-1), in hybrid systems, where is more equal redistribution of loads between the girder and cable, later addition of the prestressing force gives positive effects.

If we look at the hybrid systems with different girders stiffness, it can be concluded that:

- the uneven distribution of moments is expressed at a very solid steel girders, as well as for soft wooden girders inside the hybrid system with a marked rigidity of supporting cable,
- prestressing efficiently causes an increase in the rigidity of the system and for uniform distribution of bending moments under various schemes of the load, which is better the mutual compatibility of partial rigidity (the bending rigidity of the girder and strain rigidity of the cable). The more number of struts the more balanced utilization of bending moments is (Fig. 2(b)).

2.2 Selection of the optimal constellation of hybrid systems for prestressing

If we look at the hybrid girder within the hybrid system, where prestressing has the role of equalization of partial stiffness, but also the uniformity of moment’s impact on the girder, it can be concluded that for prestressing is suitable the following constellation systems:

- rectangular cross-section with highlighted hight h – bigger stiffness of the girder,

- shallow under suspensory , or small relationship f/l – simple steel cable,
- deeper under suspensory (steel cable for prestressing or less h girders, but then the moments are more uneven),
- ratio of partial stiffness such that prestressing surrender cable greater role in carrying,
- the influence of moisture and creep softens girder, meaning that it needs more support.

In hybrid systems performed as glued laminated timber beam and steel cable of different geometric forms is expressed deformability of the system. To analyze the behavior of these systems in real environments in which they are located, as well as the possibility of prestressing, and the determination of the effective prestressing force, it is required additional parametric analysis.

3. Parametric analysis of the hybrid system glulam – steel cable

Parametric analysis of hybrid system was performed for the following parameters: number of struts, the relationship between geometric height (arrow) and span f/l, the ratio of the girder's hight and span h/l, the cross section of girder, the ratio between cross-sectional area of the girder and cable A_n/A_S, the ratio of elastic module dry / wet wood and steel[*] E_n/E_S.

All calculational analysis in this paper are specifically implemented for the reference hybrid girder, range l= 4000 mm, which is made from glued laminated beams with constant cross section b/h=80/220 mm and of steel cord S500 and constant d=22 mm.

3.1 Approximate procedure – procedure to Dietz

Based on the idea (Dietz 2008) for reference hybrid system of specific geometric constellations, an analysis of redistribution of loads from the condition of compatibility of deformation in the contact section of individual system by strut. For evenly distributed load q, determined from the condition of load capacity, it is studied the behavior of the girder, which were varied: the form of a hybrid system, the relationship between arrow and span f/l and module of the wooden beam elasticity.

If the hybrid system of certain geometric constellation, under the known evenly load q, sets the condition of compatibility set by the general Eq. (1), it would be possible to approximately estimate the deformability of the system. If the Eq. (1) is satisfied, given in the general term

$$\frac{F_n}{\omega_n}+\frac{F_S}{\omega_S}\geq\frac{F_{uk}}{\omega_{dop}} \tag{1}$$

then the deflection of a hybrid girder is within the permitted value. However, if not satisfied, deflections of the system are higher than permitted, so this system demands prestressing to reduce deformability. It means that the term can be checked by the known constellation system, load q and total corresponding actual deflection of the system, should be the equation between the left and right side in the Eq. (1).

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[*] Girder's elasticity modulus of glued laminated wood is taken nominally for dry wood $E_{0,mean}=11600\,N/mm^2$, or $E_{0,mean}=3866{,}7\,N/mm^2$ for wet wood, whereas for steel cable is $E_S=210000\,N/mm^2$- according to EC 5 (exploitation class 3, short-term load).

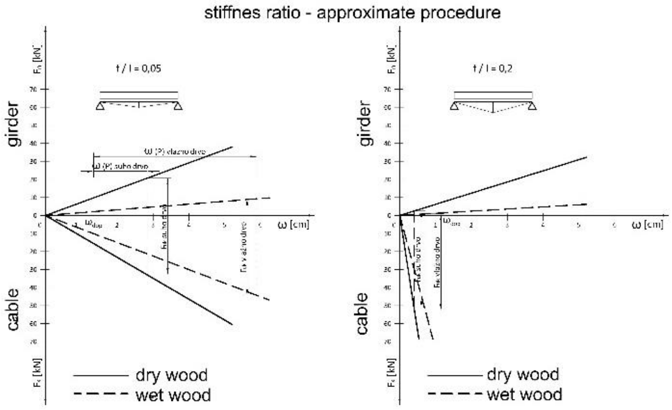


Fig. 3 Ratio of stiffness, that is, load redistribution of hybrid system with the glulam girder, of different geometric stiffness for different moisture of glulam

From the developed form, on the basis of the above requirements for well-known even load q, and varying the parameters f/l and E_n, obtained a percentage share of the individual stiffness of the overall stiffness of the system, that is, participation of individual elements of the hybrid system in carrying the load.

In Fig. 3 are given diagrams of approximate stiffness estimate of individual elements within a hybrid system, where are given the value of the actual deflection systems on abscissa for dry and wet wood and permissible deflection values of given system according to EC 5. On ordinate are given values of corresponding load according to percentage redistribution of the total system's load, applied so that the abscissa is mapping axis.

The analysis found that for the used approximate expressions , the difference with the results after more accurate calculation procedures significantly increase with the increase of geometric height of the hybrid girder, as well as reducing the stiffness of the wooden girder due to an increase in the percentage of moisture[†].

Thus, this approach does not give a more accurate insight into the possibility of pre-stressing of the hybrid system, because it refers only to a narrow area of constellation system where is a small percentage of deviations from the claims in the work (Dietz 2008).

The analyzes can be concluded that the overall stiffness of the system can not be expressed just by summing the individual rigidity of the system, and that the geometric rigidity given the approximate expressions overall rigidity of the cable according to equation $\Delta f = \frac{q \cdot l^4}{32 \cdot \cos^3\alpha \cdot E_S \cdot A_S}$ can be used with sufficient accuracy by side approach where it will be taken into account any geometric "adjustment" of the system to given load (Miljanovic 2012).

[†] For single strut f/l = 0,05 is 26,17% for dry wood, and 52,1% for wet wood, as well as for f/l = 0,2 is 37,1% and 57,8%, respectively. For double strut and f/l = 0,05 is 1,32% for dry wood and 30,4% for wet wood, while the f/l = 0,2 amounts to 0,9% and 30,64% respectively. For multiple strut the f/l = 0,05 error is 24% for dry wood and 50,3% for wet wood, and for f/l = 0,2 is 44,7% and 6159% respectively.

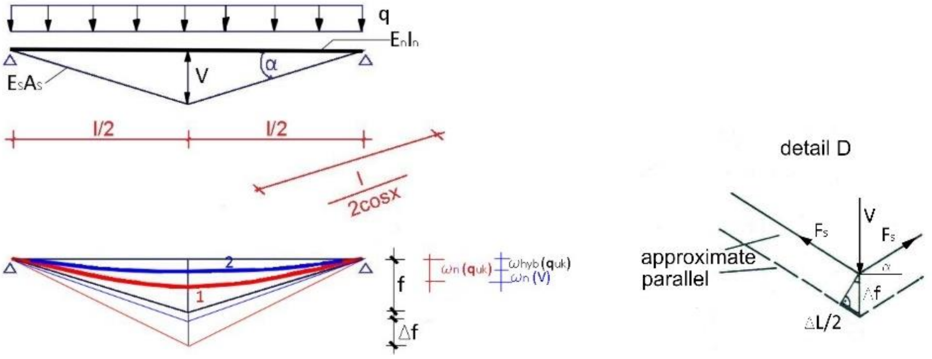


Fig. 4 Force in the vertical strut of single supported by evenly divided load q on the girder depends on the stiffness of the system

3.2 The exact procedure - own parametric analysis of a hybrid system

Reducing deformation behavior of the hybrid system in relation to the girder with bending stiffness is caused by the activation of the cable’s stiffness consisting of geometric rigidity and stiffness of the strain.

Deformability of the hybrid system can be analyzed by observing the hybrid system in deformability’s limit cases of individual systems it is made of. The typical sections observed in those that provide the connection between the individual basic systems, or the cross-sections where can be established compatibility of deformations of the individual system within the hybrid system.

If we apply this approach to a variety of geometric forms of hybrid systems it is possible to obtain exact expressions for finding forces in strut and deflection values of the hybrid system.

If the hybrid system, burdened with an even load q_{uk}, is seen as a system with a completely strain flexible cable, then the deflection of the system in place of strut is equal to deflection of girder, or girder transmits a total load, while the cable is not taking part in the transmission of loads.

If the hybrid system is composed of elements limited flexural and strain own stiffnesses, then deflection of the system is

$$\omega_{hyb}(q_{uk})=\omega_n(q_{uk})-\omega_n(V)+\Delta f \tag{2}$$

where:

$\omega_{hyb}(q_{uk})$ - deflection of hybrid system single-type supported is in the middle of the span,

$\omega_n(q_{uk})$ - girder’s deflection of the total load with strain flexible cable,

$\omega_n(V)$ - girder’s deflection in the middle of the span of the total activated force in strut of the strain unflexible cable,

Δf -vertical deformation of the cable due to effect of the full force of pressure in strut, or $V=q_{uk}\cdot l$.

From Eq. (2), we can say that the deformation of the system can be represented as a deformation of girder corrected at the place of a connection between the strut by activating the full force of pressure in strut, which then stretches the cable of certain geometry.

Correlation of deflections the characteristic girder sections and cable from the total load q_{uk} makes it the actual value of the force of pressure in strut, achieved after deforming individual basic hybrid systems at the connection

$$V_{hyb,stv}(q_{uk})=\frac{q_{uk}\cdot l^2\cdot \sin\alpha}{4\cdot f}\cdot\left(1-\frac{48\cdot E_n\cdot I_n}{32\cdot f^2\cdot \cos^3\alpha\cdot E_S\cdot A_S}\right) \tag{3}$$

Total actual deflection of hybrid girder then is

$$\omega_{hyb}(q_{uk})=\frac{5}{384}\cdot\frac{q_{uk}\cdot l^4}{E_n\cdot I_n}-\frac{V_{hyb,stv}\cdot l^3}{48\cdot E_n\cdot I_n} \tag{4}$$

where the other article qualitative represents the effect of strut.

Parametric analysis was performed for the limit values of the parameters appropriate for the hybrid system, and with focus on the material combination wood-steel cable, as follows:

- relationship between arrow and span: f/l, according to analysis of suitable constellations for aspect of capacity for hybrid systems as recommended by the (Wagner, Schl 172/13-1)[‡],
- ratio of girder's height and span: h/l, by analogy of the hybrid system's behaviour with free systems of wooden girders and associated referential values for the height of the cross-section of the wooden girder of the system of continuous and simple beam[§] ,
- relationship of cross-sectional area of girder and cable: A_n/A_S, according to the condition $V_{hyb,stv}=0$ for different limit values of previous parameters[**].

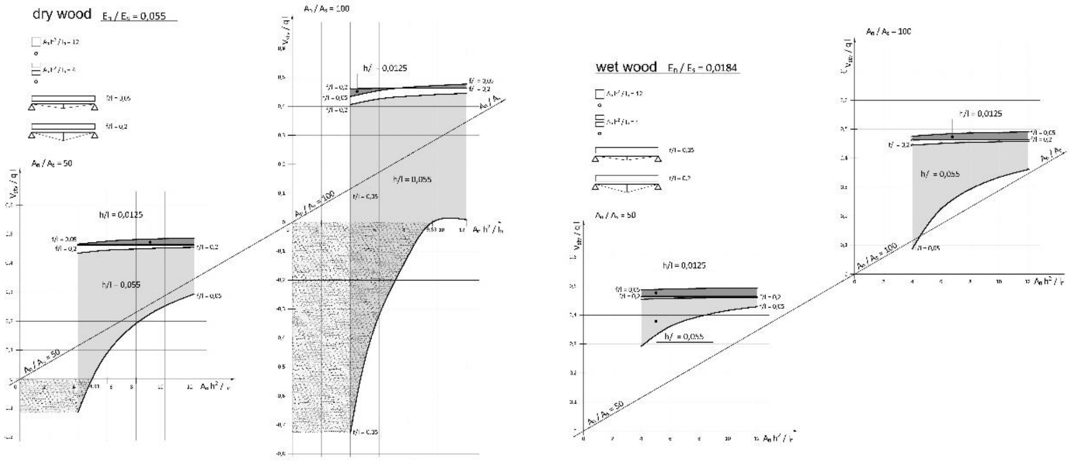
The condition $V_{hyb,stv}=0$ from Eq. (4) means that support has no influence on the deformability of the system. In that case, from Eq. (3) this condition satisfies equality partial stiffness, bending girders and cable strain in its geometric form, which means that for equal redistribution of loads between girders and cable hybrid system should prestress to reduce the deformability.

It is assumed that with an increase in the percentage of moisture content reduces the stiffness of the bending girder, and thus increased the share of the load transmitted by the cable. It means that for hybrid systems of small geometric height and high stiffness of bending girders the increase of percentage of moisture content has a positive effect, and causes the activation of the system as a hybrid. For hybrid systems with low height girder's cross-section and small geometric height of hybrid, deformability of not supported parts of the girder is further increased, whereas with large geometric height of hybrids cable takes over almost all the load.

[‡] … in limited/border values $f/l=0{,}05\div0{,}2$.

[§] … where is the analogy of hybrid system of simple strut the extreme case of system's behavior of continuous girder, when the supporting is vertically indivertible, and then is h/l = 0,0125, or for the analogy with the simple girder, when fully indulgent/yielding cable, h/l = 0,055.

[**] … where the upper limit values are determined in accordance with the lowest ratio for different forms of the hybrid systems, and for the constellations f/l = 0,05 and h/l = 0,055, and amounts: $An / A_S = 122{,}0$ for single, $An / A_S = 394{,}01$ for a double and $An / A_S = 117{,}54$ multiple strut.



(a) dry wood (b) wet wood

Fig. 5 Size of compressive force in strut of a single-type hybrid system depending on the shape of the cross section of the girder

3.3 Effective prestressing force and its impact on glulam grder

- According to parameter analysis from the point of dimensioning (Wagner, Schl 172/13-1), where are given changes of cross-section forces depending on constellation of system and its materialization, it is possible to determine the condition for determining the effective prestressing force from the viewpoint of increasing the system's capacity. As the carrier within the hybrid system is exposed to the effect of the normal forces of pressure and bending moments systems in which the application of prestressing preferred are those in which:
- activating the cable from the effects of external load on the girder carries a little value of compressive force, and at the same time bending moments are unambiguous[††], where the prestressing increase proportion of the normal force of pressure on the girder, and the moments of the sections equalize,
- for high altitude girders $h/l \geq 0{,}055$ and geometric high of hybrid $f/l \geq 0{,}1$, where the force of pressure in the girder has a little value, and for all the height of the girder and $f/l \geq 0{,}2$ has a constant value, where with the increase in geometric height of hybrid and constant height of the girder falls the value of moments on the girder,
- uneven distribution of the moments is expressed in a very rigid girders with segmented sections, as well as "soft" wooden girders within a hybrid system with a marked geometrical stiffness of the cable, where the prestressing can affect the equalization of the

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[††] For small geometric height of hybrid the force of pressure in the girder rapidly grows in more rigid girders, while in small girder's rigid receives a constant stream. Bending moments on strut of rigid girder for all geometric heights and high girders has a positive sign, where for the girders of the "softer" materials drops sharply, and large geometric heights and low heights of girder achieves a negative sign.

moments,

- in hybrid systems by reducing the height of the girder, the effect of geometric height of hybrid is more expressed, to finally be the most expressed for rectangular sections of girder, where increasing geometric heights of hybrid force pressures slightly grow compared to systems with stiffer girders, and a supporting moments are reduced. Faster growth of normal force can be obtained with these systems by prestressing.

3.3.1 The effect of prestressing on the failure of glulam girder

In determining the effective prestressing force from the viewpoint of the enhancement of capacity of the hybrid system with a wooden girder, in addition to the above features of the system, where practically adjust the values of cross-section forces and the typical cross-section, it is possible to apply the theoretical basis of distribution of stress at break in cross-section loaded in bending (Bohannan 1964).

In his work (Bohannan, 1964) used the assumption that the prestressing wooden beam of inferior quality class can delay the failure of section in bending, and reduce the variability of bending strength. The explanation for this approach lies in the distribution of wooden beams power at fracture and mechanical properties that influence the distribution of power in section.

The ideal wooden beam would be the one in which the inside section, for a balanced mechanical properties, fracture achieves at the same time yield the tensile strength and compressive strength parallel to the fibers. Such a redistribution of stresses at failure would be achieved at perfect real fiber material. Failure of section should first cause plastic redistribution of stresses in compressed part, after which would follow the failure of tensioned zone releasing the final fiber yield strength of wood in tension parallel to fibers.

However, the failure of section on bending characterized stress distribution which is a plastic character in the compressed and the elastic nature in the tensile zone section. The presence of defects in the growth of wood in a tensile zone makes tensile strength parallel to fibers less reliable size from compressive strength, as in lower quality class of wood results in a sudden fracture of a tight area in the cross section before it reaches the compressive strength.

By entering the prestressing force in the tensile zone of section can be increased the component of tension force in the inner coupling force, which would balanced out with the maximum pressure force that can be achieved in cross-section. In this way, the failure of section is delayed and occurs by forcing a plastic coating pressed zone yield compressive strength and finally cancel of section by suddenly cancellation of tensile zone.

It means that the effective prestressing force is obtained from the condition of order cancellation section compression-tension. To take the advantage of compressive strength parallel to fibers, the induced pressure stress in the tensioned zone should be at least equal to the difference of compressive and tension strength parallel to fibers for established quality class wood.

The effective prestressing force is then as follows

$$\left(-\frac{F_P}{A_n}+\frac{F_P\cdot e}{W_n}\right)+\left(-\frac{F_P}{A_n}-\frac{F_P\cdot e}{W_n}\right)=f_{c,0,k}+f_{t,0,k} \tag{5}$$

or

$$F_P=-\left(\frac{f_{c,0,k}+f_{t,0,k}}{2}\cdot A_n\right) \tag{6}$$

where is:

F_P - effective prestressing force,

$f_{c,0,k}$ - characteristic value of compressive strength parallel to fiber,
$f_{t,0,k}$ - characteristic value of tensile strength parallel to fiber,
A_n - cross sectional area of the girder,
W_n - resistant moment of cross-section beams,
e - eccentricity of the prestressing force to the gravity center of the cross section of the girder.

According to Eq. (6) the effective prestressing force depends only on the quality of wood and size of girder's cross-section which is pre-tensioned. Effective pre-stress is obtained from the condition that the eccentricity of the prestressing force is at the edge of the core section, as well as the value of the prestressing force given in the Eq. (6). The effect of such pre-stressing is to increase flexural strength, and bearing capacity of girder.

3.3.2 The results of own research

Previously presented approach to (Bohannan 1964) can be applied to hybrid systems as well. This approach can be used to initial controls of choosing the correct constellation of system taking into concern that the typical cross-sections due to the effect of external load already burdened by bending moment and normal force. Size of cross section forces depends on the load and the relationship of rigidities in the system, and for each section is then possible to control the eccentricity of force and the stress level of edge lamellas. For the purpose of increasing the capacity on the principle of forcing order cancellations sectional compression-tension, the effective prestressing force is practically "tightening" the cable for value

$$\Delta F_{S,hyb}=F_{P,hyb}=\left(\frac{f_{c,0,k}+f_{t,0,k}}{2\cdot\cos\alpha}\cdot A_n\right)+F_{S,hyb} \tag{7}$$

where the strength are given again in the algebraic sum.

Here is:

$\Delta F_{S,hyb}=F_{P,hyb}$ - size force "tightening" of cable, or effective prestressing force of the hybrid system,

$F_{S,hyb}$- force in the cable of hybrid system due to the effect of external load,

α - angle of the cable relative to the girder's axis in the supporting section.

If watching in terms of capacity of hybrid system loaded only evenly external load q based on analyzes carried out in the work (Wagner, Schl 172/13-1) and on the basis of its own analysis, can be concluded that the systems in which the geometric height is constant with decreasing of girder's height moments are reduced and normal force is growing. Then with the constellation of the system already achieves access of increasing the girder's flexural strength by effect of external loads without prestressing. If on such a girder operates external load at midspan, the normal force will decline, and the moments in the field can significantly increase, which significantly reduces the load capacity of the section in the field if the bending moment has dominant influence on the normal force.

Surely, for the application of the pre-stressing is again the most favorable constellation of more rigid girder and small geometric height, where due to greater participation of girders in transferring loads more dominant bending moments in comparison to the low normal force because of weak cable's activation. Prestressing of this system obtaines on the increase of the normal forces, and bending moments are becoming aligned along the length of the girder.

If the hybrid system loaded with an external load without prestressing increase the percentage of girder's moisture, it comes to a decline in the value of moments and force in the cable grows, and suddenly increase the normal force on the girder. The total capacity of the system is reduced in comparison with the same girder of dry wood.

For the same system with the prestressing force and increasing the wood's moisture content, the cable's force and its changes can be observed through a change of its components, or part of the force of the external loads and the prestressing force. Then the force in the cable of the external load increases, and the prestressing force partly decreases, but now the total normal force is slightly smaller compared to the normal force for dry wood and prestressed system. However, its value remains close to the corresponding values in Eq. (6) for a smaller algebraic sum of compressive strength and tension of wet wood parallel to fibers[‡‡]. Bending moments for prestressed system and wet wood are increased, so that the maximum value of moment in the field, and the failure of section still takes place in the order the compression-tension, but at lower external load.

Finally, it can be concluded that the principle of increasing flexural rigidity is applicable in hybrid systems with and without prestressing and for the real conditions in which the system is located, but the determination of the effective prestressing force in hybrid wood-steel cable is complicated due to the changes in the redistribution of loads depending from the influence of the environment in which the system is located.

3.3.3 Reducing the deformability of prestressing

For finding the size of the effective prestressing force to reduce the deformability of the hybrid system it is necessary to observe the system again in limit cases of deformability of the individual elements that make up due to the fact of only the prestressing force to the system.

Typical sections observed are those that provide the connection between the individual basic elements, or sections which can establish the compatibility of deformation of the individual "clean" systems within the hybrid system.

Deflection of Prestressed Hybrid System

If the hybrid system is loaded only with the prestressing force entered in the cable by stretching activates a pressure force in strut of which causes a vertical deformation of the girder and cable. If this deformations are observed for extreme cases of behavior of the two pure systems depending on the stiffness obtaines direct dependence of the size of the cable elongation and deflection of hybrid system.

If this approach apply to different geometric forms of hybrid systems it is possible to obtain exact expressions for finding the effective size of the cable's elongation depending on the necessary deflection of girder in place of strut. Size of stretching cable represents prestressing without losses due to elastic and plastic deformation of elements and geometric changes of system.

Fig. 6 shows the single-strut system loaded only with prestressing force in the cable. The vertical deformation of the cable occurs due to the fact of activated vertical force in the compressed strut, and to streamline of the budget with sufficient accuracy can be taken that due to the deformation the cable remains in parallel position with the deformed shape. This deformation is now imposed by the cable's elongation on hybrid's supports and on the system acts as a shortening of the cable, so it is

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[‡‡] Taking into account also the possibility of wood's creep, improved by percenage of humidity, a component of the normal force of prestressing has a loss, and component of the normal force of redistribution of loads on the cable increases.

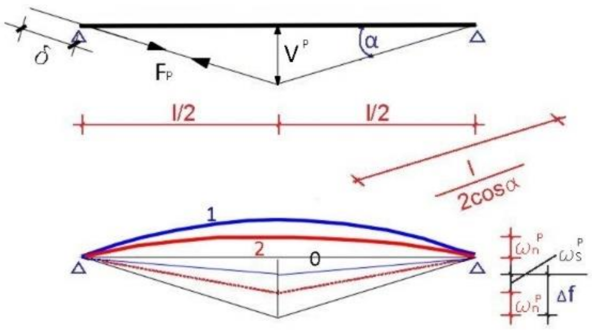


Fig. 6 Deflection lines of single strut hybrid system by the prestressing force

$$\Delta f = \frac{\Delta L}{2\cdot\sin\alpha} = \frac{\delta}{2\cdot\sin\alpha} \tag{8}$$

where is:

Δf- hybrid's deflection of prestressing force,

ΔL- size elongation / shortening of cable,

δ - stretching the cable for entering the prestressing force.

Fig.6 are given the deformation of the system due to the force of the prestressing, assuming infinitely rigid strut, and for the cases:

line 1 - full prestressing force entered by stretching the cable with real stiffness gives deflection of the system when the carrier has the infinitely small flexural stiffness $E_n\cdot I_n\cong 0$,

line 2 – activation of the full girder's rigidity on bending activates the pressure force in strut of what happens after stretching cable of the force in the strut, which reduces the total initial full vertical deformation Δf on the actual system's deflection from the effects of the prestressing force.

This observed behavior of the system for effect of the prestressing force can be represented by the expression:$\Delta f - \omega_n^P = \omega_S^P$, where is:

$\Delta f = \frac{\delta}{2\cdot\sin\alpha}$ - deflection of the system due to the force of the prestressing provided that $E_n\cdot I_n\cong 0$,

$\omega_n^P = \frac{V^P\cdot l^3}{48\cdot E_n\cdot I_n}$ - girder's deflection in real stiffness of the fact of force in support due to prestressing system,

$\omega_S^P = \frac{F_S^H\cdot f}{\frac{E_S\cdot A_S}{G}} \cong \frac{V^P\cdot l}{4\cdot f}\cdot f\cdot \frac{l^2}{4\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S} = \frac{V^P\cdot l^3}{16\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S}$ - vertical deformation caused by stretching a cable from the force in support of the prestressing force

or

$$\frac{\delta}{2\cdot\sin\alpha} - \frac{V^P\cdot l^3}{48\cdot E_n\cdot I_n} = \frac{V^P\cdot l^3}{16\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S} \tag{9}$$

where is:

- δ - cable's elongation to achieve the effective prestressing force, and effective system deflection,

- V^P- pressure force in support of activated due to the force of prestressing of hybrid system with elements of real stiffness,
- $E_n·I_n, E_S·A_S$- real stiffness of elements of hybrids - the girder of the bending and stretching of the cable.

$$\delta=2\cdot\sin\alpha\cdot\frac{V^P\cdot l^3}{48\cdot E_n\cdot I_n}\cdot\left(1+\frac{48\cdot E_n\cdot I_n}{16\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S}\right) \tag{10}$$

or

$$\delta=2\cdot\sin\alpha\cdot\omega_n\cdot\left(1+\frac{48\cdot E_n\cdot I_n}{16\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S}\right) \tag{11}$$

The Eq.(11) can be used with sufficient accuracy for determining the required cable's stretching from the condition of necessary deflection in place of strut, and to achieve reduction of system's deformation[§§].

If set to a condition that the supporting effect of the single-strut hybrid system uniformly loaded by the load q such that there is no vertical displacement in the supporting section, it is

$$\omega_{hyb}\left(\frac{1}{2}\right)=\frac{5}{384}\cdot\frac{q\cdot l^4}{E_n\cdot I_n}-\frac{V^q_{hyb,stv}\cdot l^3}{48\cdot E_n\cdot I_n}=0 \tag{12}$$

following $V^q_{hyb,stv}=\frac{5}{8}\cdot q\cdot l$, practically the same expression as in the previous practice and represents fully activated force in strut. The real value of force in strut depends on the relation of partial stiffness of the system.

If the same condition is set for hybrid system loaded by even load q with prestressing force, which provides vertical immovability of typical sections, the Eq. (12) means

$$\omega^P_{hyb}\left(\frac{1}{2}\right)=\frac{5}{384}\cdot\frac{q\cdot l^4}{E_n\cdot I_n}-\frac{V^q_{hyb,stv}\cdot l^3}{48\cdot E_n\cdot I_n}-\frac{V^P_{hyb,stv}\cdot l^3}{48\cdot E_n\cdot I_n}=0 \tag{13}$$

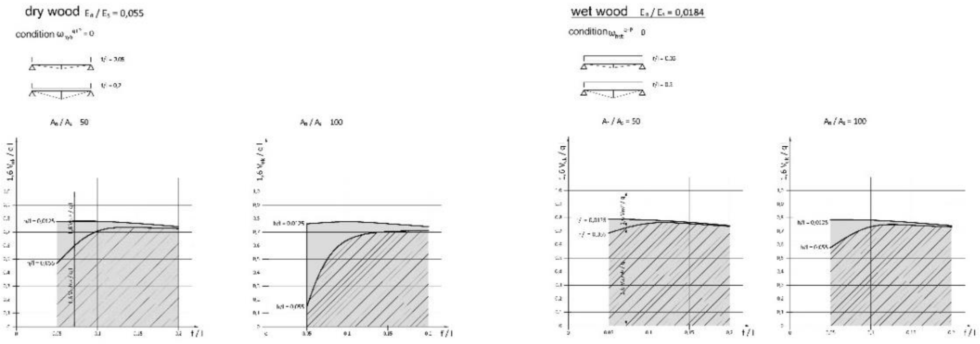
following $V^q_{hyb,stv}+V^P_{hyb,stv}=\frac{5}{8}\cdot q\cdot l$, or $\frac{1{,}6\cdot V^{q+P}_{hyb,stv}}{q\cdot l}=1$.

In Fig. 7 are given the achieved real value forces in strut of prestressed hybrid system loaded with even load q, such as the values of the real force in strut only for external load given to the expression

$$\frac{V_{hyb,stv}}{q_{uk}\cdot l}=\frac{1}{4\cdot f}\cdot\frac{1}{\sqrt{1+\frac{l^2}{4\cdot f^2}}}\cdot\left(1-\frac{3}{2}\cdot\frac{E_n}{E_S}\cdot\frac{I_n}{A_n\cdot h^2}\cdot\frac{A_n}{A_S}\cdot\frac{h^2}{l^2}\cdot\frac{l^2}{f^2}\cdot\frac{1}{\sqrt{\left(\frac{4\cdot f^2}{l^2}+1\right)^3}}\right) \tag{14}$$

and the value of force in support of that should achieve by prestressing makes the difference to the full value. Based on these charts, it is possible for the hybrid systems of different constellations to determine the required effective prestressing force (for a condition that is deflection in the typical cross section 0), or needed elongation of the cable, according to the Eq. (11).

[§§] Controls of the Eq. (11) performed on systems of suitable constellations for prestressing, or systems with a small geometric height of the system, provide 13% of error in result of the deflection value.



(a) dry wood (b) wet wood

Fig. 7 The relative values of the actual pressure force in support of a single strut by external load q and reading the effective prestressing force for a condition that deflection of prestressed system is ω(1/2)=0

4. Experimental research of the behaviour of reference prestressed hybrid system in real conditions

Experimental research were carried out at the Institute for Materials and Structures of Faculty of Civil Engineering in Sarajevo, within the framework of a research project funded by the Federal Ministry of Education and Science of Bosnia and Herzegovina. Girders of glued laminated timber are manufactured by "Krivaja – Tvornica montažnih kuća" Ltd. Zavidovići.

The subject of research is the examination of the global behavior of different girder’s constellations in real conditions, in precisely defined models, when introducing prior prestressing force, in order to increase their capacity and reduce deformabilities.

Theoretical estimations given in the previous discussions, as well as with its own analysis of the parameters, set the theoretical basis for the analysis of these systems and the ability to apply numerical methods in practice, and their approval in suitably selected models is the main goal of experimental research.

For the purpose of comparing simple and prestressed hybrid system there are used the same assumptions for the budget, including: a planar system behavior, and to determine the effective prestressing was used a condition of deflection’s cancellation in typical cross-sections, or achievement the effect of vertical stiffness of struts, from which is obtained a measure of deformable prestressing system.

Geometrical constellation of girder’s model for experimental research are selected in accordance with limit cases of numerical analysis. All components and additional equipment of samples for research of hybrid girders of different constellations and the quality of materials (divided into two series of tests according to quality class of glulam girder) are sized according to the characteristic phases of research (under the Protocol of testing).

Model testing was performed on a total of six samples - hybrid girder, composed of glued laminated beam (element with stiffness in bending), the required cross-section $b/h = 80/220$ mm and two-piece, external, symmetrically placed, steel cables (element with stiffness in strain of negligible own weight), a diameter of $\emptyset 16$ mm.

(a) The installation of cables on models of Series I (b) models of hybrid girders for testing

Fig. 8 Glulam girders of GL 24 (h), Series I and Series II

Girder's models, in each series, are shaped like three hybrid girders of different geometric rigidity of external cables. The first model (Girder 1) is a hybrid carrier with parallel cables $f/l=0$, externally mounted in the height of girder with the eccentricity of the girder's center of gravity of e=55 mm. The second model (Girder 2) is a single-type strut of hybrid girder, where $f/l=0{,}0375$, with a polygonal cables and infinitely rigid steel vertical in the middle of the range (f=150 mm). The third girder (Girder 3) is designed also as a single-type strut of hybrid girder, and in the limit of $f/l=0{,}02$.

Stages/phases of Testing are:

1. The load, without prestress (dry wood) - measuring deformations per increment of force at 20% of the total load of exploitation;
2. Relieving, without prestress (dry wood) - measuring residual deflections;
3. Prestress (dry wood) - measuring the resulting of finite deflections;
4. Load of prestressed system (dry wood) - measuring deformations per increment of force in steps of 20% of the total load of exploitation;
5. Relieving the prestressed system (dry wood) - measuring residual deflections;
6. Wetting wood of prestressed system - measurement of deflection's increase during the increase in the percentage of moisture carrier;
7. Load of prestressed systems (wet wood) - measurement the deformation step by step of exploitation load;
8. Relieving prestressed systems (wet wood) - measuring residual deflections;
9. Load to failure of prestressed system (wet wood).

The same measurement protocol has been implemented in all six samples, with different values of exploitation loads resulting computational analysis. Computational analysis was performed for the required material quality, the GL 24 (h) in dry state at nominal amounts for glulam and correctional coefficients kdef for wet wood I and III exploitation class by the EC 5.

Stages of testing the model are provided as cycles of loading and unloading the girders in simulated real conditions of ambient's humidity. Measurements during application of loading are direct indicators of model's behavior under load, while after each stage of load models are relieved and measurement results give backlog, viscous girder's deformation.

The prestressing force for each model is determined by the numerical analysis, and the conditions of cancellation the maximum deflection of exploitation load of each tested system in the dry state.

(a) deflection of glued laminated girder of prestressing force is increased due to an increase in the percentage of moisture content

(b) deflection of prestressed hybrid system of external loads on condition of moist wood beam

Fig. 9 Measuring of deflections

Measurements that are carried out referred to the measurement of deflection at load of electronic deflectometer, diletation of cables with dipstick that are over the Wheatson's bridge linked to the size converter and are recorded by increments of force of the external load. Buckling girders are controlled by deflectometers in critical sections of the possible losses of lateral stability due to the load or prestressing girder, but also for the needs of even the introduction of prestressing force in couples of cables.

5. Analysis of the obtained results

Analysis of the results of girder's laboratory tests were made by comparing the computational analysis of the reference constellation of hybrid system for dry and wet wood, comparing the individual tested constellations, as well as a series of the same constellation system.

Measuring the deformation in the typical cross-sections at the point of strut and in the field is established a little deviation in relation to the deformation calculation values in the dry state, indicating a slightly lower modulus of elasticity than the calculation of the nominal value for the required quality of material. For the load with prestressing in dry state the deformability remained the same, and the total deflection is increased by the value of residual deformation. In the wet state prestressed girder shows lower total deformation due to the additional deflection of prestressing force influence to wet wood before applying the external load.

Finally, all analyzes indicate the linear trend of the hybrid system behavior in the level of exploitation load, and by prestressing is achieved the improvement in the system's behavior in terms of linear behavior and in the state of the considerable increase in wood's moisture. Redistribution of moments and deformation of wet wood system is more favorable.

The girders are in the final phase, loaded to failure. The failure of typical cross-section $x=0,35\cdot l$ was created as a result of coating press-zone, after which, the rapid increase in tensile stress cancels the tensioned zone. The order of failure is desirable, because it can then be considered that the effect of prestressing is fully confirmed and in terms of capacity, where the limit stress of bending is reduced to the firmness on pressure parallel to fibers as a more reliable.

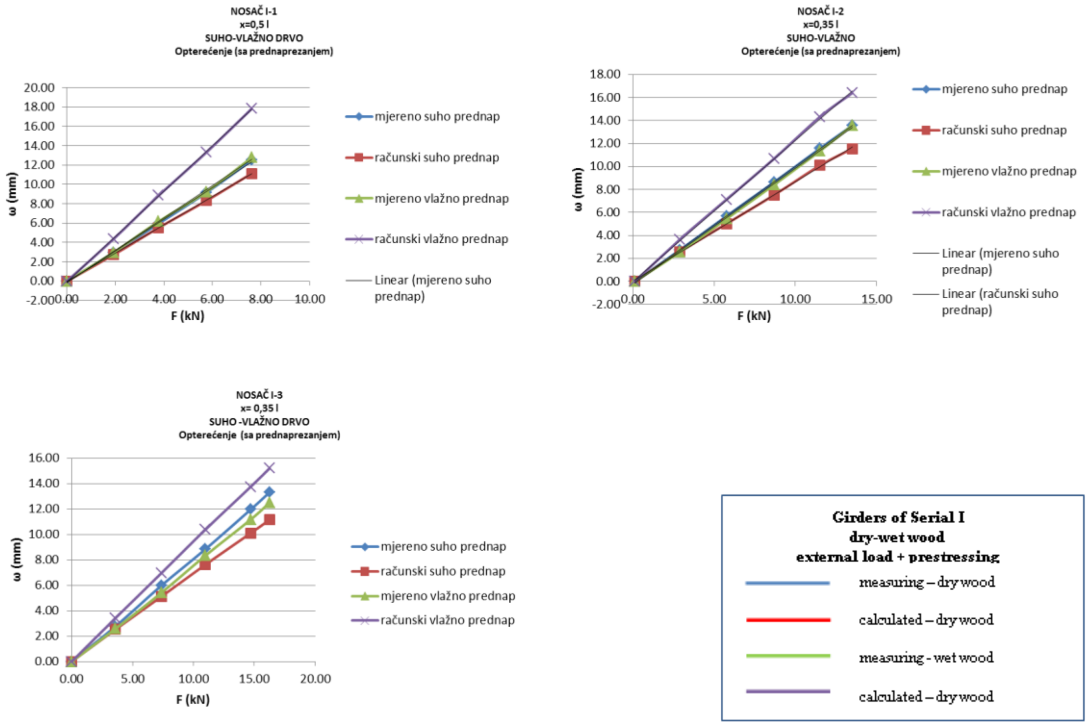


Fig. 10 Comparison of measured and calculated deformation values of a section in the system's field of Series I

(a) failure in compression zone (b) failure in tension zone

Fig. 11 By prestressing the contrived order of failure of hybrid girder's cross-section is a compression – tension

6. Conclusions

By external prestressing of glulam girder achieves the less deformability and increase of capacity while optimizing the base material, or girder acting like it is made of quality material and in real terms, which is confirmed by its own parametric analysis and laboratory testing of prestressed hybrid systems behaviour of different constellations.

The accompanying diagrams of parametric analysis provide sufficiently accurate orientation for finding the right constellations of hybrid systems wood-steel cable and the ability to assess the efficiency of hybrids in real conditions, as well as the necessary application of prestressing;

Derived terms of the approximate procedure and terms of finding effective prestressing force, assuming a linear system behavior, provide sufficiently accurate results for the design of prestressed hybrid systems in practice.

Experimental research on girders with prestressing and increased percentage of wood’s humidity was found that the elastic modulus does not change significantly compared to a timber with little humidity, which is in total contrast with the methodology given in EN 1995-1-1: 2009, whether there is the normal force of pressure (in the case of prestressing) states reduction coefficient kdef that nominal elastic modulus of wood reduces and the value of approximately 30% of the modulus of elasticity of dry wood, which basically takes the wrong calculation of hybrid system wood-steel cable with prestressing.

“..”

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