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Parametric study on flexible footing resting on partially saturated soil

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Abstract. Coupled finite element analysis is carried out to study the effect of degree of saturation on the vertical displacements and pore water pressures simultaneously by developing a FORTRAN90 code. The finite element formulation adopted in the present study is based upon Biot's consolidation theory to include partially saturated soils. Numerical methods are applied to a two-dimensional plane strain strip footing (flexible) problem and the effect of variable degree of saturation on the response of excess pore water pressure dissipation and settlement of the footing is studied. The immediate settlement in the case of partly saturated soils is larger than that of a fully saturated soil, the reason being the presence of pore air in partially saturated soils. On the other hand, the excess pore water pressure for partially saturated soil are smaller than those for fully saturated soil.

Keywords: partially saturated soils; suction; coupled analysis; degree of saturation

1. Introduction

Conventional principles of geomechanics treat soils as a two-phased medium, namely: (i) fully saturated soil consisting of soil solids and water (ii) completely dry soil consisting of soil solids and air. Partially saturated soils constitute a three-phase medium (namely soil solids, water and air), which at higher degrees of saturation have comparable pore water pressure and the pore air pressure. The classical theories of soil mechanics have been developed by considering soil as a two-phase medium which might not be applicable to all soils. The presence of capillary fringe is an example of unsaturated zones that are located above the ground water table. Tamped fills are rendered unsaturated by the definition of compacted soils.

Several researchers have focussed on developing new theories and constitutive models to study and understand the behaviour of partially saturated soils. A simplified coupled formulation based on Biot's general theory of three-dimensional consolidation, the virtual work principle and the continuity equation for the fluid phase has been presented (Biot 1941). Dakshanamurthy *et al.* (1984) extended Biot's theory and presented a coupled transient flow model. However, because of the difficulty in incorporating the proposed principles for unsaturated soil mechanics,

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implementation of the same into engineering practice has proven to be a challenge.

Generalized effective stress expressions were proposed in order to include partially saturated soils into the conventional soil mechanics framework, the best known being that proposed by Bishop *et al*. (1960). Sandhu and Wilson (1969) were the first to implement finite element formulation of Biot's three-dimensional consolidation theory. Since then, extensive research began by formulating the governing finite element equations for elastic materials. The study contributing to this field included the works of Christian and Boehmer (1970), Hwang *et al*. (1971), Borja (1986). Extension of the research to account for non-linearity into the formulation has been discussed by Small *et al*. (1976), Borja (1989), Schrefler and Zhan (1993). In all of these linear and non-linear expressions, a set of coupled differential equations form the governing finite element relationships.

Ng and Small (2000) extended Biot's consolidation theory and illustrated its application to geotechnical problems. Sheng *et al*. (2003) proposed an adaptive-time stepping scheme and used an explicit stress integration scheme by treating suction as a strain component. Georgiadis *et al*. (2005) assumed a four-dimensional stress space with two yield curves and proposed a three-dimensional constitutive model with twenty-two parameters. Laloui and Nuth (2009) explained the use of generalized effective stress equations and showed that suction is not to be treated as a hardening variable but rather as a shape parameter for the yield surface expressed in the matric suction versus mean effective stress space. Coupled numerical modelling of excavations has been studied by Nogueira *et al*. (2009) and simulated the results under plane strain conditions. Sheng (2011) considered the performance of shear strength equations and coupled the hydraulic component with the mechanical component. Li *et al*. (2011) proposed a state-parameter based generalized plasticity model for unsaturated soils. Maheswari and Kumar (2011) reported probabilistic analysis of strip footing on layered soils. Kumari and Sawant (2013) suggested the soil behaviour must be analysed by incorporating the effects of the transient flow of the pore-fluid through the voids, and highlighted requirement of two-phase continuum formulation for saturated porous media.

From the above cited review of relevant literature, it is clear that not much of work has been reported on analysis of footing resting on partially saturated soil. Present study is aimed to carry out to develop a coupled formulation based on Biot's theory for the analysis of a footing resting on partially saturated soil. Variations in excess pore pressure and vertical settlement with time have been observed. Results for partially saturated case are compared with the case of fully saturated soil to understand the effect of degree of saturation.

2. Biot's theory of three-dimensional consolidation

Biot (1941) defined soil consolidation as gradual adaptation of the soil to the load variation. Of the many assumptions made in this theory, the linear behaviour of stress-strain curve and the reversibility of the stress-strain relations under final equilibrium are considered to be crude for many practical scenarios. The fluid flow through the pores of the soil skeleton is assumed to be governed by Darcy's law. The governing equations for consolidation of a saturated porous medium may be written in the following form:

Mechanical equilibrium

$$\frac{\partial \sigma_{ij}}{\partial x_j} = F_j \tag{1a}$$

Effective Stress Relationship

$$\sigma_{ij}^T = \sigma'_{ij} - p\delta_{ij} \tag{1b}$$

Constitutive Relationship

$$\sigma'_{ij} = D_{ijkl}\varepsilon_{kl} \tag{1c}$$

Strain-Displacement Relationships

$$\varepsilon_{kl} = \frac{1}{2}(q_{l,k} + q_{k,l}) \tag{1d}$$

Velocity-Strain Relationship

$$\frac{\partial v_i}{\partial x_i} = \dot{\varepsilon}_v \tag{1e}$$

Darcy’s Flow Rule

$$v_i = -\frac{k}{\gamma_w}\frac{\partial p}{\partial x_i} \tag{1f}$$

where i and j denote the directions in Cartesian space, σ_{ij}^T and σ'_{ij} are the total and effective stress tensors, F_i are components of body force, p is the pore water pressure, δ_{ij} is the Kronecker delta, D_{ijkl} is the elastic constitutive matrix (for linear stress-strain relationship), ε_{kl} is the strain tensor, $q_{k,l}$ is the derivative of components of displacement in k^{th} direction with respect to l^{th} direction, v_i denotes the components of the superficial velocity, $\dot{\varepsilon}_v$ is the volumetric strain rate, k_{ij} are the Darcy's coefficients of permeability, and γ_w is the unit weight of water.

The total stress (σ^T) in the soil skeleton is defined as the effective stress (σ') over the entire cross-sectional area, thus the relationship between the total stress and effective stress would be as given by Eq. (1(b)). The negative sign is introduced in accordance with the general sign convention to take tensile components of stress as positive.

3. Coupled formulation

The behaviour of unsaturated soil is governed by using two independent stress state variables (Fredlund *et al*. 1977). In the present study, the two stress state variables used to define the behaviour of partially saturated soils are the effective stress and the pore water pressure. Bishop and Henkel (1962) have stated that the principle of effective stress concept cannot be extended to soils having partial degree of saturation. Also, at higher degrees of saturation (above about 80%) the pressure exerted by the water and the air phase of the partially saturated soils are comparable. Therefore the effective stress equation as stated by above Eq. (1(b)) is supposed to be applicable in this case.

A comprehensive model would include several aspects of the soil and would require a lot of soil parameters as input data, which would be cumbersome as extensive laboratory experimentations are to be conducted. In order to make the theory more viable, it is necessary to ignore certain processes and to make a justifiable approximations. Thermal stresses, affecting pore fluid changes, due to change in temperatures is ignored in the following research. The liquid-moisture phase can be effectively modelled by applying suitable boundary conditions. The simplest of coupled formulations which encapsulate the main aspects of partially saturated soil behaviour is based on mass conservation of water and mechanical equilibrium of the total soil volume. The development of such a hydro-mechanical model can also provide a good platform to tackle more general problems.

3.1 Mechanical equilibrium

The Eq. (1(a)) can be expressed in the following manner

$$\nabla\sigma - F_b = 0 \tag{2}$$

where ∇ is the differential operator and F_b denotes the body force vector. Applying the Green–Gauss theorem and Galerkin weighted residual method to Eq. (2) leads to

$$\int_V B^T \sigma dV - \int_S N^T T dS - \int_V N^T F_b dV = 0 \tag{3}$$

where V is the volume of interest, S is the surface area over which tractions are applied, T is the external surface traction vector, and B and N are strain–displacement and displacement shape function matrices defined as

$$q = Nq_e; \qquad \varepsilon = Bq_e; \qquad B = \nabla N \tag{4}$$

where q is the unknown displacements, q_e denoted the nodal displacements, N is the shape function matrix, ε is the strain vector, B is the strain-displacement matrix which is the obtained by using the differential operator on the shape function matrix.

$$p = N_P p_e \quad and \quad B_P = \nabla N_P \tag{5}$$

where p_e is the nodal pore pressure vector and N_p is the pore pressure shape function matrix.

Therefore, on substitution final form of the equilibrium equation is as follows

$$Kq_e - L^T p_e = F_u \tag{6}$$

with,

$K = \int_V B^T DBdV$, is the stiffness matrix

$L^T = \int B^T \delta_{ij} N_P dV$, is the coupling matrix

$F_u = \int_S N^T T dS + \int_V N^T F_b dV$, is the external force vector

3.2 Continuity equation

To develop the equation, an infinitesimal element of an unsaturated soil is considered such that its sides are parallel to Cartesian co-ordinate axes. The continuity equation of the element in terms of water flow can be written by considering the rate of change of the volumetric moisture content.

$$\nabla v = \frac{\partial \theta_m}{\partial t} \tag{7}$$

where v is the superficial velocities of pore water in vectorial notation, ∇ is the differential operator vector for Cartesian directions, θ_m represents the volumetric moisture content. Volumetric moisture content is defined as the ratio of volume of water to the volume of soil. It can be rearranged as the product of porosity and the degree of saturation. Assuming the soil grains to be incompressible and the small-strain theory to be applicable, the rate of change of porosity is equal to the rate of change of volumetric strain. The above Eq. (7) results in the following equation

$$\nabla v - S_r \frac{\partial \theta}{\partial t} = n \frac{\partial S_r}{\partial t} \tag{8}$$

The variation of the degree of saturation with pore-water pressure can be used to account the compressibility of the pore air fluid. The relationship for degree of saturation versus negative pore water pressure has been studied in the past by applying calculated amount of back pressure to saturate the test samples in a triaxial cell. Lowe and Johnson (1960) proposed a theoretical relationship to describe the variation in S_r with positive pore water pressure.

$$S_r = \frac{0.0099 p + 0.98 S_{r_0}}{0.98 + 0.0097 p} \tag{9}$$

Discretizing pore water pressure in space and applying the Green–Gauss theorem and Galerkin weighted residual method along with Biot's theory, to Eq. (8) leads to

$$L\dot{q} + S\dot{p} + Hp = F_P \tag{10}$$

where

$$S = \int \frac{n}{S_r} \frac{\partial S_r}{\partial p} N_P^T N_P dV \quad and \quad H = \int \frac{1}{S_r} B_P^T \frac{k}{\gamma_w} B_P dV$$

$\{F_P\}$ is the force vector that deals with the initial pore-water pressure at the nodes and the superior dot denotes the derivative with time.

Applying a suitable time-marching scheme to Eq. (10)

$$Lq_e^i + (S + \alpha \Delta t H) p_e^i = \Delta t F_P + Lq^{i-1} + \left(S - (1-\alpha)\Delta t H\right) p^{i-1} \tag{11}$$

Combining Eqs. (6) and (11), a system of coupled equation can be written as

$$\begin{bmatrix} K^{i-1} & -L^T \\ L & S^{i-1}+\alpha\Delta t H^{i-1} \end{bmatrix} \begin{Bmatrix} q_e^i \\ p_e^i \end{Bmatrix} = \begin{Bmatrix} F_u \\ \Delta t F_P + L q^{i-1} + \left(S-(1-\alpha)\Delta t H\right) p^{i-1} \end{Bmatrix} \tag{12}$$

A non-dimensional time factor, T is devised to express the excess pore-water pressure dissipation. This is also known as the adjusted time factor.

$$T = \frac{\bar{c}t}{a^2} \tag{13}$$

In which, the modified coefficient of consolidation, $\bar{c}$ is given by

$$\bar{c} = \frac{2Gk}{\gamma_w} \tag{14}$$

where G is the elastic shear modulus, k is the coefficient of permeability, γ_w is the bulk unit weight of the water, and t denotes time.

A FORTRAN90 code is developed for the above derived Eq. (12) is validated for the data from Schiffman *et al*. (1969). The data obtained from Schiffman *et al*. (1969) is for a non-dimensional time factor, T=0.1 and μ =0.0 is plotted (Fig. 1) against the data obtained from the FORTRAN90 code. The mesh comprised of 225 elements and the domain extended for 7.5 m by 7.5 m (Fig. 2).

The discrepancy between the two results may be due to the reasoning that the close-form solution obtained (Schiffman *et al*. 1969) was based on the assumption of a half infinite space which has to be curtailed during generating a finite element mesh. Further, the finite element mesh had close proximity at the bottom and/or the lateral boundaries.

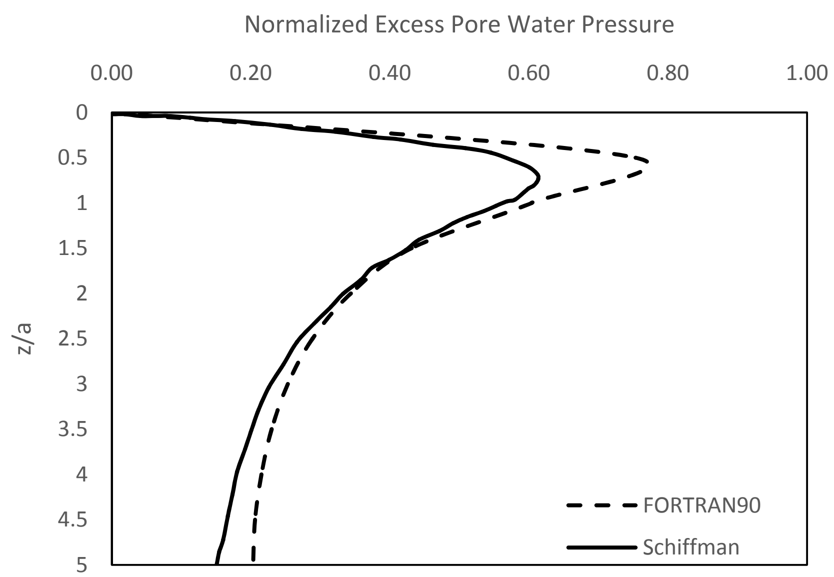

Fig. 1 Validation of the FORTRAN90 code with the data from Schiffman *et al*. (1969)

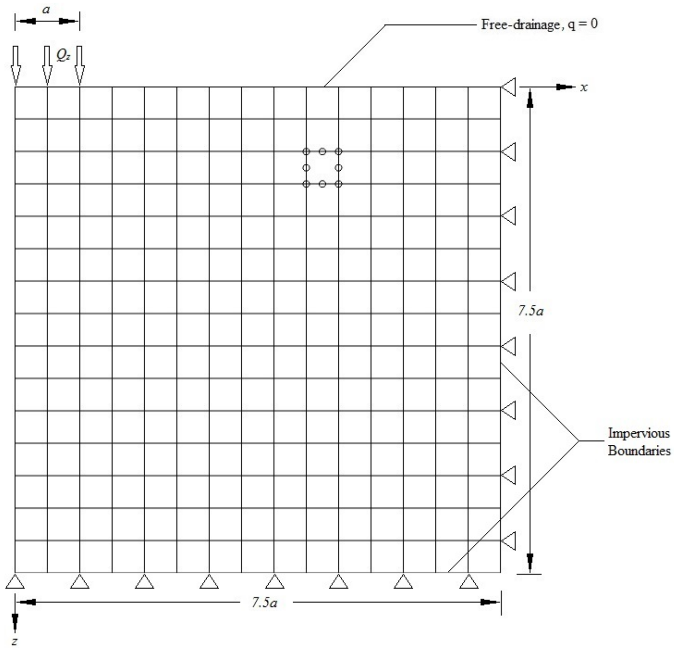

Fig. 2 Coupled Finite Element Mesh for the data from Schiffman *et al.* (1969)

4. Results

The finite element mesh comprises of 120 coupled finite eight noded quadrilateral elements, with fine spacing near the loaded area (Figs. 3 and 4). The analysis is carried out in 60 time steps of uniform increment. The iterations on the variation of degree of saturation is carried out by iterative scheme. It is assumed that the footing is flexible and entire load intensity is applied instantaneously. A forward time marching scheme has been used. The number of iterations for each time step is based upon the difference between the current degree of saturation and the initial degree of saturation. All the results converge in utmost 5 iterations.

In order to represent the results, a non-dimensional time factor as given by Eq. (13) is chosen. By varying initial degree of saturation, the dissipation of normalized excess pore water pressure (p/Q_z) with respect to the non-dimensional time (T) at a point B (located centrally at $z/a = 0.3$) is shown in Fig. 5. Similarly the normalized vertical displacements at the centre (point A) of the footing (with respect to the half-width of the footing) is plotted in Fig. 6. The immediate displacements occurring at full saturation are lesser than that of the immediate displacements occurring at a lesser degree of saturation, the reason being that at full saturation the load is transferred to the soil particles and the water present in the pores of the soil. In the unsaturated case, the immediate settlement is higher because of the presence of air along with the water and soil solids. On the other hand, excess pore water pressure for partially saturated soil are smaller than those for fully saturated soil.

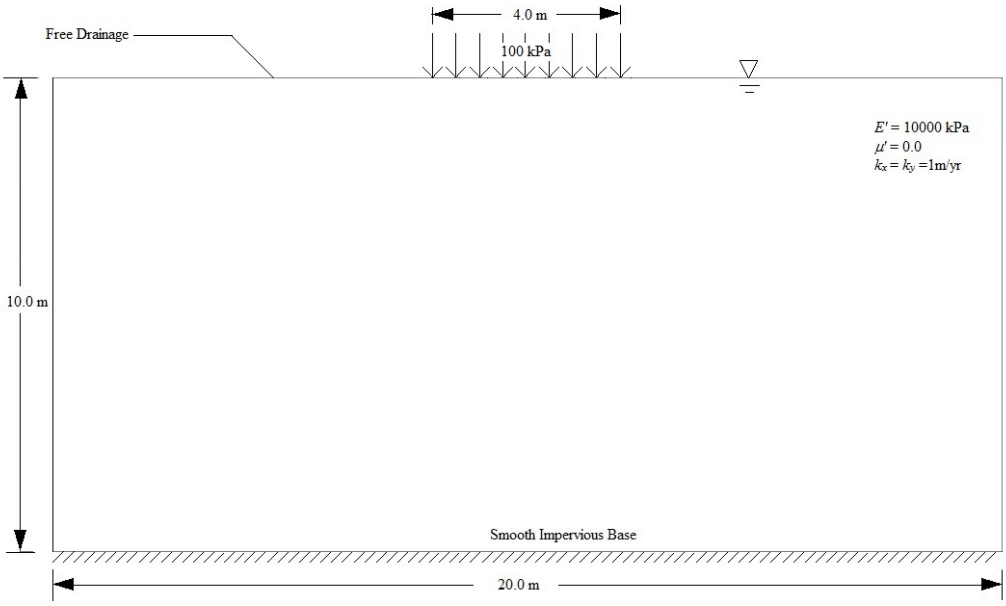

Fig. 3 Problem Statement

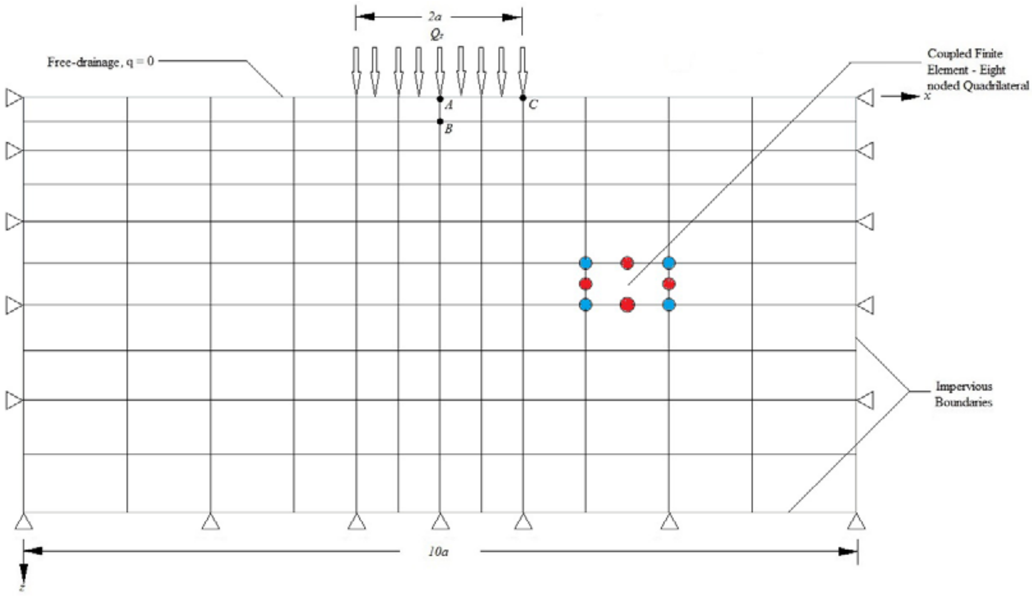

Fig. 4 Finite Element Mesh for the Problem comprising of Coupled Finite Elements

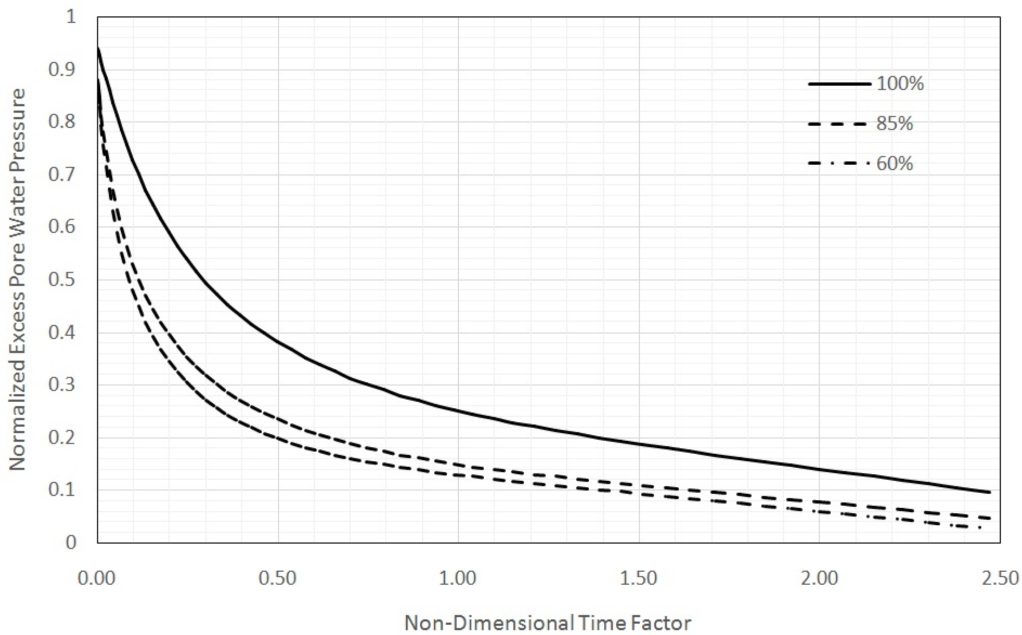

Fig. 5 The dissipation of Normalized Excess Pore-water Pressure (p/Q_z) with respect to the Non-Dimensional Time Factor (T) at a point B ($z/a = 0.3$)

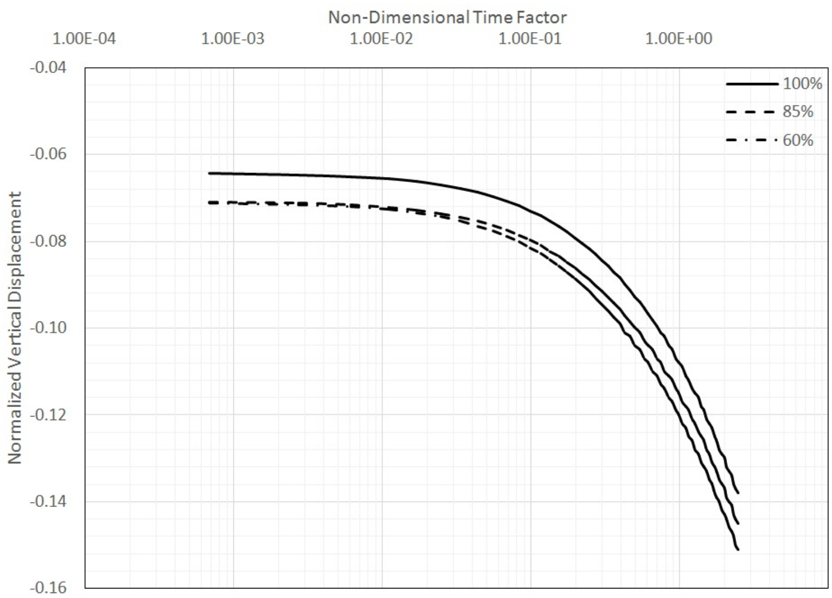

Fig. 6 Normalized vertical displacements at the point A

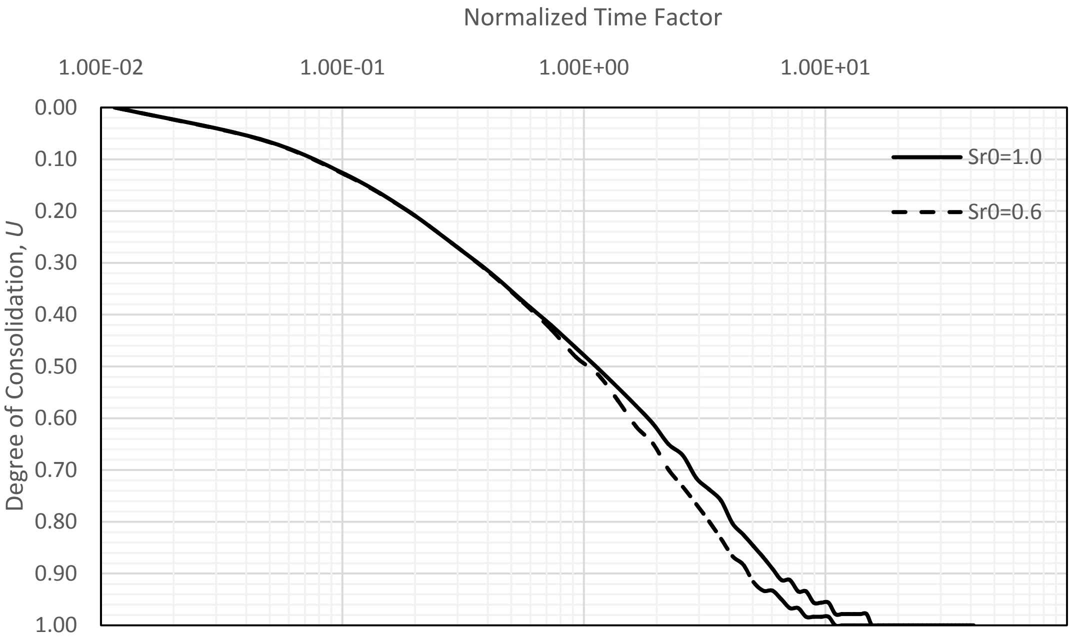

Fig. 7 Variation of Degree of Consolidation for unsaturated soil

Gibson *et al*. (1970) have proposed a relationship for the degree of consolidation, U as a function of vertical displacements (settlements) to study the effect on the degree of consolidation when the soil is in a state of partial saturation.

$$U = \frac{v_t - v_i}{v_{ult} - v_i} \tag{15}$$

where v_t denotes the vertical displacement at any time, v_i is the immediate settlement and v_{ult} is the ultimate settlement.

The time span of the analysis has been increased in order to evaluate the effect of partial saturation on the degree of consolidation, U. Fig. 7 shows the variation of the degree of consolidation with normalized time factor for the case when the soil has an 85% initial degree of saturation.

4.1 Variation of poisson ratio

The value of Poisson ratio is varied and the vertical displacement at the center of the footing is plotted in Fig. 8. It is seen that as the Poisson ratio is increased from 0.0 to 0.3, the effect of immediate settlement on the application of load is quite significant. There is a change of 31.7% in the numerical value for the settlements at the center of the footing at the extreme values. As lateral strain increases with increase in Poisson ratio, which may cause to reduce deformations in the vertical direction. The effect of Poisson ratio on the degree of consolidation can be seen from Fig.

9. The values indicated in the graph are for point A (Fig. 4). As the coefficient of volume compressibility m_v, decreases with increase in Poisson ratio, the degree of consolidation *U*, also reduces with increase in Poisson ratio and same is reflected in Fig. 9.

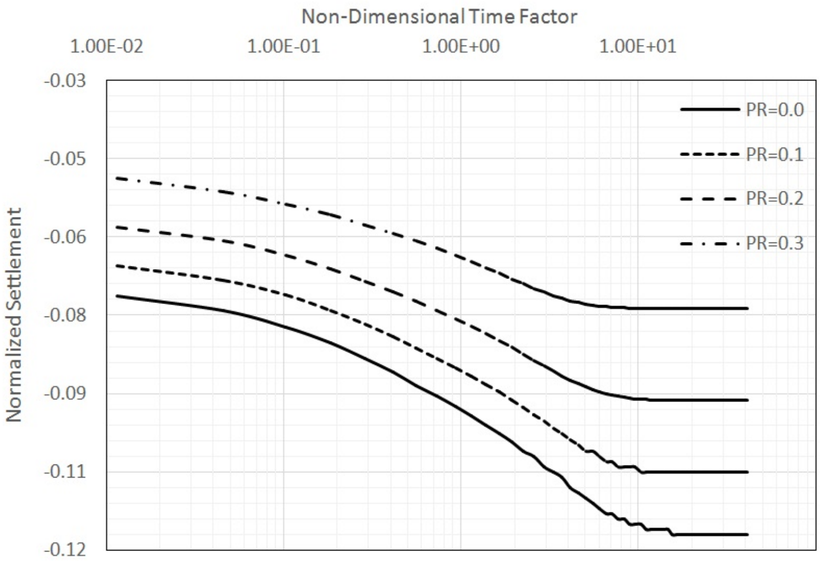

Fig. 8 Variation of Settlement at the Centre of Footing with Poisson ratio

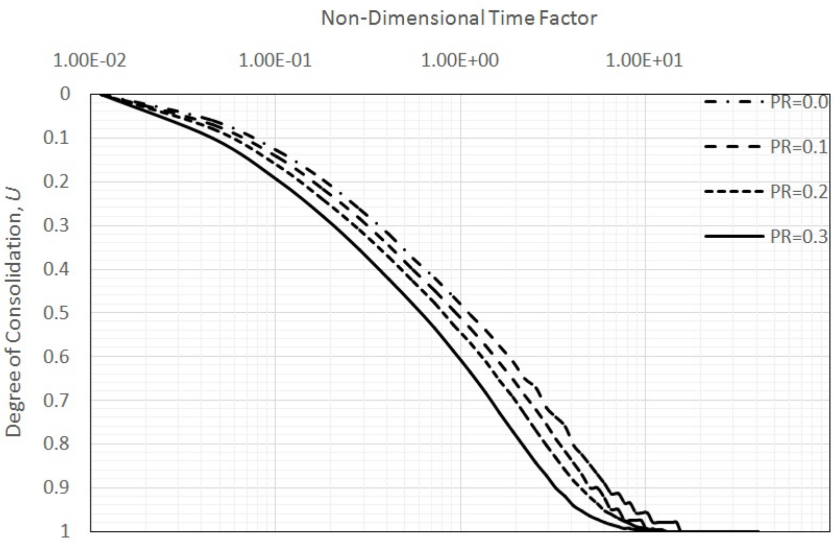

Fig. 9 Variation of Degree of Consolidation with different Poisson ratio

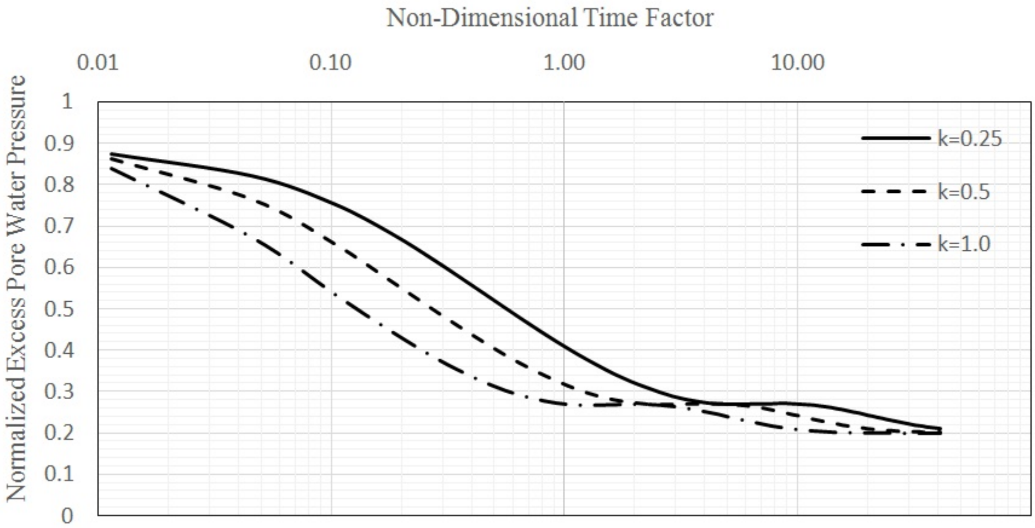

Fig. 10 Variation of Normalized Excess Pore Pressure for Three Values of *k*

4.2 Variation of coefficient of permeability

By varying the values of the coefficient of permeability, the dissipation of excess pore water pressure at point B ($z / a = 0.3$, Fig. 4) is evaluated and the effect can be seen from Figs. 6-10. It can be noted that as the value of the coefficient of permeability reduces the dissipation of pore water pressure takes further time. The units of the coefficient of permeability being in m/yr. With increase in permeability, the dissipation of pore water pressure is faster at initial stage due to larger void area available for drainage.

5. Conclusions

The work presented in this paper presents the procedure to extend the Biot's three dimensional consolidation theory to describe the partially saturated soil behaviour. Coupled finite element formulation has been implemented by developing a FORTRAN90 code to study the effects of partial degree of saturation on the settlement and the pore-water dissipation behaviour of soils subjected to a strip loading. The program is validated against existing literature and it gives a satisfactory outcome. The immediate settlements in the case of partly saturated soils is larger than that of a fully saturated soil, the reason being the presence of pore air in partially saturated soils. On the other hand, excess pore water pressure for partially saturated soil are smaller than those for fully saturated soil. The degree of consolidation, *U* reduces with increase in Poisson ratio, as the coefficient of volume compressibility m_v, decreases with increase in Poisson ratio. The dissipation of excess pore water pressure is quicker for the soils with higher coefficient of permeability due to the presence of larger void area for drainage.

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