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Frequency characteristics of a multiferroic Piezoelectric/LEMV/CFRP/Piezomagnetic composite hollow cylinder under the influence of rotation and hydrostatic stress

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Abstract. An analytical model is consider to scrutinize axisymmetric wave propagation in multiferroic hollow cylinder with rotating and initial stressed forces, where a piezomagnetic (PM) material layer is bonded to a piezoelectric (PE) cylinder together by Linear elastic materials with voids. Both distinct material combos are taken into account. Three displacement potential functions are introduced to uncouple the equations of motion, electric and magnetic induction. The numerical calculations are carried out for the non-dimensional frequency by fixing wave number and thickness. The arrived outputs are plotted as the dispersion curves for different layers. The results obtained in this paper can offer significance to the application of PE/PM composite hollow cylinder via LEMV and CFRP layers for the acoustic wave and microwave technologies.

Keywords: multiferroic material; LEMV/CFRP; initial stressed; rotating cylinders; dispersion relation

1. Introduction

Composite materials made out of piezoelectric (PE) and piezomagnetic (PM) stages are equipped for moving vitality among electrical and magnetic fields, called the impact of magnetoelectric (ME). In PE- PM Composites, ME impact is another item property emerging from the collaboration of two distinct stages. Because of the capacity of vitality change among electric and magnetic fields, PE-PM composites are potential contender for attractive sensors, transducers and microwave gadgets, for example, resonators, electric-field-tunable channels, stage shifters and postpone lines. These applications are engaged with vibrations and wave proliferations in PE-PM composites, and thus their dynamic conduct is of essential worry in configuration just as in execution. As of late, a few specialists have considered the spread attributes of enduring waves in PE-PM composites.

Micromechanics of magnetoelectroelastic composite materials: average field and effective done by Li and Dunn (1998). Closed-form solutions for the magnetoelectric coupling coefficients in fibrous composites with piezoelectric and piezomagnetic phases formulated by Wu and Huang

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(2000a). Li (2000b) investigate magnetoelectroelastic multi-inclusion and inhomogeneity problems and their applications in composite materials. Aboudi (2001) analyzed micromechanical analysis of fully coupled electromagneto- thermo-elastic multiphase composites. Ponnusamy and Selvamani (2013, 2012a) discussed about wave propagation in a magneto thermo elastic cylindrical panel and dispersion analysis of a generalized magneto thermo elastic cylindrical panel. Sharma and Mohinder (2004) studied Rayleigh-Lamb waves in magnetothermoelastic homogeneous isotropic Plate. Effect of rotation on Rayleigh-Lamb waves in magneto-thermoelastic media studied by Sharma and Thakur (2006a). Exact solution for simply supported and multilayered magneto-electro-elastic plates discussed by Pan (2001). Nan (1994) discovered magneto-electric effect in composites of piezoelectric and piezomagnetic phases. The general solution of three-dimensional problem in magneto-electro-elastic media derived by Wang and Shen (2002). Van Run *et al*. (1974) formulate in situ grown eutectic magnetoelectric composite material Physical-properties. Nan *et al*. (2008a) studied about multiferroic magnetoelectric composites: Historical perspective, status, and future directions. Soh and Liu (2006b) analyses interfacial shear horizontal waves in a piezoelectric-piezomagnetic bi-material. The effects of inhomogeneous initial stress on Love wave propagation in layered magneto-electro-elastic structure by Zhang *et al.* (2008b). Wang (2008c) discussed wave band gaps in two dimensional piezoelectric/piezomagnetic phononic crystals. Piliposian *et al.* (2012b) studied about shear wave propagation in periodic phononic/photonic piezoelectric medium. Du *et al*. (2009, 2007) examination about SH surface acoustic wave propagation in a cylindrically layered piezomagnetic/piezoelectric structure and Love wave propagation in layered magneto-electro-elastic structures with initial stress. Effect of initial stresses on dispersion relation of transverse waves in a piezoelectric layered cylinder studied by Abo-el-nour *et al.* (2007). Buchanan (2003) discussed free vibration of an infinite magneto-electro elastic cylinder. Ritz *et al*. (2019b) studied the heat and mass transfer in the Eyring-Powell model of fluid propagating peristaltically through a rectangular compliant channel. Bhatt *et al*. (2019c) numerically analysis the heat transfers and Hall current impact on peristaltic propulsion of particle-fluid suspension with compliant wall properties. Marin *et al*. (2015) discussed considerations on double porosity structure for micro polar bodies. Khan *et al*. (2019d) studied effects of chemical reaction on third-grade MHD fluid flow under the influence of heat and mass transfer with variable reactive index.

In this paper we form analytical laneway to scrutinize axisymmetric wave propagation in multiferroic hollow cylinder with rotating and initial stressed forces, where a piezomagnetic (PM) material layer is bonded to a piezoelectric (PE) cylinder together by Linear elastic materials with voids. Both distinct material combos are taken into account. Three displacement potential functions are introduced to uncouple the equations of motion, electric and magnetic induction. The numerical calculations are carried out for the non-dimensional frequency by fixing wave number and thickness and are figure out as the dispersion curves for different layers.

2. Modelling of problem

Consider a rotating multiferroic PE/LEMV/CFRP/PM transversely isotropic composite hollow circular cylinder. Geometry of problem shown in Fig. 1. The basic governing equations of motion, electrostatic displacement D_j and magnetic induction B_j in cylindrical co-ordinates (r, z) system, and the cylinder is assumed to be rotating with uniform angular velocity the absence of $\overline{\Omega}$ in the absence of volume force are, Ponnusamy and Selvamani (2013)

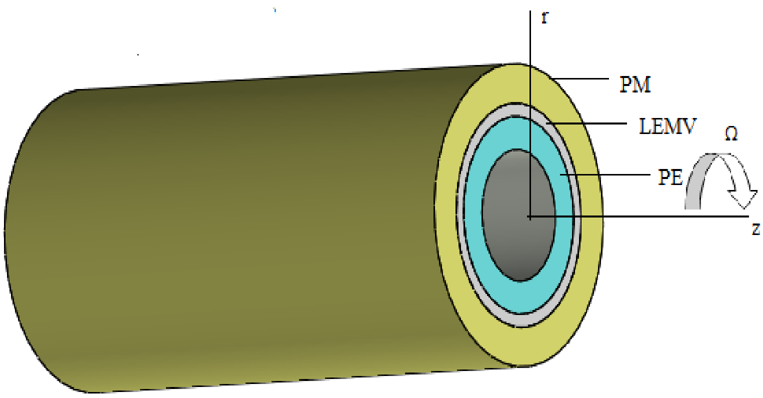

Fig. 1 Geometry of problem

$$\coprod^{l}_{rr,r} + \coprod^{l}_{rz,z} + r^{-1}\left(\coprod^{l}_{rr}\right) + \rho(\bar{\Omega} \times (\bar{\Omega} \times \bar{u}) + 2(\bar{\Omega} \times \bar{u}_{,t})) = \rho u_{,tt}. \tag{1}$$

$$\coprod^{l}_{rz,r} + \coprod^{l}_{zz,z} + r^{-1}\coprod^{l}_{rz} + \rho\left(\bar{\Omega} \times (\bar{\Omega} \times \bar{u}) + 2(\bar{\Omega} \times \bar{u}_{,t})\right) = \rho w_{,tt}. \tag{2}$$

The electric displacement equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rD^{l}{}_{r}\right) + \frac{\partial D^{l}{}_{z}}{\partial z} = 0. \tag{3}$$

The magnetic induction equation

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r^l) + \frac{\partial B_z^l}{\partial z} = 0. \tag{4}$$

The stress strain relations are given as follows, Ponnusamy and Selvamani [2013]

$$\begin{gathered}
\coprod^{l}_{rr} = c_{11}S^{l}{}_{rr} + c_{12}S^{l}{}_{\theta\theta} + c_{13}S^{l}{}_{zz} - e_{31}E_z{}^{l} - q_{31}H_z^l \ , \\
\coprod^{l}_{zz} = c_{13}S^{l}{}_{rr} + c_{13}S^{l}{}_{\theta\theta} + c_{33}S^{l}{}_{zz} - e_{31}E_z{}^{l} - q_{33}H_z^l \ , \\
\coprod^{l}_{rz} = c_{44}S^{l}{}_{rz} - e_{15}E_r - q_{15}H_r \ , \\
D_r = e_{15}S^{l}{}_{rz} + \varepsilon_{11}E^{l}{}_{r} + m_{11}H_r \ , \\
D_z = e_{31}\left(S^{l}{}_{rr} + S^{l}{}_{\theta\theta}\right) + e_{33}S^{l}{}_{zz} + \varepsilon_{33}E^{l}{}_{z} + m_{33}H_z \, , \\
B_r = q_{15}S_{rz} + m_{11}E_r + \mu_{11}H_r \ , \\
B_z + q_{31}(S_{rr} + S_{\theta\theta}) + q_{33}S_{zz} + m_{33}E_z + \mu_{33}H_z \ .
\end{gathered} \tag{5}$$

The strains $S^{l}{}_{ij}$ are related to the displacements given by

$$\begin{gathered}
S^{l}{}_{rr} = u^{l}{}_{,r}\, S^{l}{}_{\theta\theta} = r^{-1}\left(u^{l} + v^{l}{}_{,\theta}\right), S^{l}{}_{zz} = w^{l}{}_{,z}\, S^{l}{}_{r\theta} = v^{l}{}_{r} - r^{-1}\left(v^{l} - u^{l}{}_{,\theta}\right), \\
S^{l}{}_{z\theta} = v^{l}{}_{,z} + r^{-1}w^{l}{}_{,\theta}, S^{l}{}_{rz} = w^{l}{}_{,r} + u^{l}{}_{,z}.
\end{gathered} \tag{6}$$

Where u, v and w are the mechanical displacements corresponding to the cylindrical coordinate directions r, θ and z. The relation between the electric field vector E_i and the electric potential ϕ is given by

$$E_r = -\frac{\partial\phi}{\partial r}\ ,\ E_z = -\frac{\partial\phi}{\partial z}. \tag{7}$$

Similarly, the magnetic field H_i is related to the magnetic potential ψ as

$$H_r = -\frac{\partial\psi}{\partial r}\ ,\ E_z = -\frac{\partial\psi}{\partial z}. \tag{8}$$

Eqs. (5) to (8) substitute in Eqs. (1) to (4) results in the following three-dimensional equation of motion, magnetic and electric conduction. We note that the first two equations under the influence of hydrostatical stress become

$$c_{11}\left(u^l{}_{,rr} + r^{-1}u^l{}_{,r} + r^{-2}u^l\right) + c_{13}w^l{}_{,rz} + c_{44}\left(u^l{}_{,zz} + w^l{}_{,rz}\right) + (e_{15}+e_{31})\varphi^l_{,rz} - p_0\left(u^l{}_{,rr} + r^{-1}u^l{}_{,r} + r^{-2}u^l + u^l{}_{,zz}\right) + (q_{15}+q_{31})\psi^l{}_{,rz} + \rho\left(\Omega^2 u + 2\Omega w_{,t}\right) = \rho u_{,tt}. \tag{9a}$$

$$(c_{44}+c_{13})\left(u^l{}_{,rz} + r^{-1}u^l{}_{,z}\right) + c_{33}\left(w^l{}_{,zz}\right) + c_{44}\left(w^l{}_{,rr} + r^{-1}w^l{}_{,r}\right) + e_{15}\left(\varphi^l_{,rr} + r^{-1}\varphi_{,r}\right) + e_{33}\phi_{,zz} - p_0\left(w^l{}_{,rr} + r^{-1}w^l{}_{,r} + w^l{}_{,zz}\right) + q_{33}\psi_{,zz} + \rho\left(\Omega^2 w + 2\Omega u_{,t}\right) + q_{15}(\psi_{,rr} + r^{-1}\psi_{,r}) = \rho w_{,tt}. \tag{9b}$$

$$e_{15}\left(w^l_{,rr} + r^{-1}w^l_{,r}\right) + \varepsilon_{11}(\varphi^l_{,rr} + r^{-1}\varphi_{,r}) + (e_{31}+e_{15})(u^l_{,rz} + r^{-1}u^l_{,z}) + e_{33}w^l_{,zz} - \varepsilon_{33}\varphi^l_{,z} - m_{11}\left(\psi_{,rr} + r^{-1}\psi_{,r}\right) - m_{33}\psi_{,zz} = 0. \tag{9c}$$

$$q_{15}\left(w_{,rr} + r^{-1}w_{,r}\right) + (q_{31}+q_{15})\left(u_{,rz} + r^{-1}u_{,z}\right) + q_{33}w_{,zz} - \mu_{33}\psi_{,zz} - m_{33}\phi_{,zz} - \mu_{11}\left(\psi_{,rr} + r^{-1}\psi_{,r}\right) - m_{11}\left(\phi_{,rr} + r^{-1}\phi_{,r}\right) = 0. \tag{9d}$$

The solutions of Eq. (9) is considered in the form Nelson and Kathikeyan (2008d)

$$u^l = U^l{}_{,r}\exp\{i(kz+pt)\},$$
$$w^l = \left(\frac{i}{h}\right)W^l\exp\{i(kz+pt)\},$$
$$\varphi^l = \left(\frac{ic_{44}}{ae_{33}}\right)\phi^l\exp\{i(kz+pt)\},$$
$$\psi^l = \frac{i}{a}\left(\frac{c_{44}}{m_{33}}\right)\psi^l\exp\{i(kz+pt)\}.$$

Where, u^l, w^l, φ^l, T^l are displacement potentials, k is the wave number, p is the angular frequency and $i = \sqrt{-1}$. We introduce the non-dimensional quantities $x = \frac{r}{a}, \varepsilon = ka, c = \rho p$ 'a' is the geometrical parameter of the composite hollow cylinder.

$$\bar{c}_{11} = {}^{c_{11}}/_{c_{44}}, \bar{c}_{13} = {}^{c_{13}}/_{c_{44}}, \bar{c}_{33} = {}^{c_{33}}/_{c_{44}}, \bar{c}_{66} = {}^{c_{66}}/_{c_{44}}, \bar{e}_{ij} = \frac{e_{ij}}{e_{33}}\ ;\ \bar{q}_{ij} = \frac{q_{ij}}{q_{33}}\ ;$$
$$\bar{m}_{ij} = \frac{m_{ij}c_{44}}{e_{33}q_{33}}\ ;\ \bar{\varepsilon}_{ij} = \frac{\varepsilon_{ij}c_{44}}{e_{33}^2}\ ;\ \bar{\mu}_{ij} = \frac{\mu_{ij}c_{44}}{q_{33}^2}\ ;$$

The above Solutions substitute in Eq. (9) We obtain the following form

$$[(\bar{c}_{11} - p_0)\nabla^2 - (1-p_0)\varepsilon^2 + \zeta^2 + \chi^2 + ב]U^l - [\varepsilon(1+\bar{c}_{13})]W^l - \varepsilon(\bar{e}_{31}+\bar{e}_{15})\phi^l - \varepsilon(\bar{q}_{31}+\bar{q}_{15})\psi^l = 0, \tag{10a}$$

$$[\varepsilon(1+\bar{c}_{13})\nabla^2]U^l + [(1-p_0)\nabla^2 - (\bar{c}_{33}-p_0)\varepsilon^2 + \zeta^2 + \chi^2 + ב]W^l + (\bar{e}_{15}\nabla^2 - \varepsilon^2)\phi^l + (\bar{q}_{15}\nabla^2 - \varepsilon)\psi^l = 0, \tag{10b}$$

$$((\bar{e}_{31}+\bar{e}_{15})\nabla^2 U^l + (\bar{e}_{15}\nabla^2 + \varepsilon^2)W^l - (\bar{\varepsilon}_{11}\nabla^2 - \varepsilon^2\varepsilon_{33})\phi^l - (\bar{m}_{11}\nabla^2 - \varepsilon^2 m_{33})\psi^l = 0, \tag{10c}$$

$$\varepsilon(\bar{q}_{31}+\bar{q}_{15})\nabla^2 U^l + (\bar{q}_{15}\nabla^2 + \varepsilon)W^l - (\bar{m}_{11}\nabla^2 - \varepsilon^2\bar{m}_{33})\phi^l - (\bar{\mu}_{11}\nabla^2 - \varepsilon^2\bar{\mu}_{33})\psi^l = 0. \tag{10d}$$

For the existence of non-trivial solution of above Eq. (10) the determinant of the coefficient of the system is set to zero

$$\begin{vmatrix} (\bar{c}_{11}-p_0)\nabla^2 - s_1 + A_1 & -A_2 & A_3 & -A_4 \\ A_2\nabla^2 & (1-p_0)\nabla^2 - s_2 + A_1 & e_{15}^2\nabla^2 + A_5 & \bar{q}_{15}\nabla^2 - A_6 \\ A_3\nabla^2 & e_{15}^2\nabla^2 - A_5 & \varepsilon_{11}^2\nabla^2 + A_7 & \bar{m}_{11}\nabla^2 - A_8 \\ A_4\nabla^2 & A_6 & -A_8 & \bar{\mu}_{11}\nabla^2 - A_9 \end{vmatrix} \times (U^l, W^l, \phi^l, \psi^l) = 0 \tag{11}$$

Where $A_1 = \zeta^2 + \chi^2 + ב$, $A_2 = \varepsilon(1+\bar{c}_{13}), A_3 = \varepsilon(\bar{e}_{31} + \bar{e}_{15})$, $A_4 = \varepsilon(\bar{q}_{31} + \bar{q}_{15})$, $A_5 = \varepsilon^2$, $A_6 = \varepsilon$, $A_7 = \varepsilon_{33}^2\varepsilon^2$ $A_8 = \varepsilon^2\bar{m}_{33}$, $A_9 = \bar{\mu}_{33}\varepsilon^2$, $s_1 = (1-p_0)\varepsilon^2$, $s_2 = (\bar{c}_{33} - p_0)\varepsilon^2$.

Evaluating the determinant given in Eq. (11), we obtain a partial differential equation of the form

$$(A\nabla^8 + B\nabla^6 + C\nabla^4 + D\nabla^2 + E)\left(U^l\, W^l\, \phi^l\, \psi^l\right)^T = 0. \tag{12}$$

Factorizing the relation given in Eq. (12) into biquadratic equation for $\left(\alpha^l{}_j a\right)^2$, *i*=1, 2, 3, 4 the solutions for the symmetric modes are obtained as

$$U^l = \sum_{j=1}^{4}\left[A_j \Im_n(\alpha_j x) + B_j \Re_n(\alpha_j x)\right],$$

$$W^l = \sum_{j=1}^{4} a^l{}_j \left[A_j \Im_n(\alpha_j x) + B_j \Re_n(\alpha_j x)\right],$$

$$\phi^l = \sum_{j=1}^{4} b^l{}_j \left[A_j \Im_n(\alpha_j x) + B_j \Re_n(\alpha_j x)\right],$$

$$\psi^l = \textstyle\sum_{j=1}^{4} c^l{}_j \left[A_j \Im_n(\alpha_j x) + B_j \Re_n(\alpha_j x)\right], \tag{13}$$

Here $(\alpha_i^l a x) > 0$, for $(i = 1,2,3,4)$ are the roots of algebraic equation

$$A\left(\alpha^l{}_j a\right)^8 + B\left(\alpha^l{}_j a\right)^6 + C\left(\alpha^l{}_j a\right)^4 + D\left(\alpha^l{}_j a\right)^2 + E)\left(U^l, W^l, \phi^l, \psi^l\right) = 0.$$

The solutions corresponding to the root $(\alpha_i a)^2 = 0$ is not considered here, since $\Im_n(0)$ is zero, except for $n = 0$. The Bessel function $\Im_n$ is used when the roots $(\alpha_i a)^2$, $(i = 1,2,3,4)$ are real or complex and the modified Bessel function $\Re_n$ is used when the roots $(\alpha_i a)^2, (i = 1,2,3,4)$ are imaginary.

The constants $a^l{}_j, b^l{}_j$ and $c^l{}_j$ defined in Eq. (13) can be calculated from the following equations

$$[(\bar{c}_{11} - p_0) - (1-p_0)\varepsilon^2 + \zeta^2 + \chi^2 + ב] - [\varepsilon(1+\bar{c}_{13})]a_j - \varepsilon(\bar{e}_{31} + \bar{e}_{15})b_j - \varepsilon(\bar{q}_{31} + \bar{q}_{15})c_j = 0,$$

$$[\varepsilon(1+\bar{c}_{13})] + [(1-p_0)\nabla^2 - (\bar{c}_{33} - p_0)\varepsilon^2 + \zeta^2 + \chi^2 + ב]a_j + (\bar{e}_{15}\nabla^2 - \varepsilon^2)b_j + (\bar{q}_{15}\nabla^2 - \varepsilon)c_j = 0,$$

$$\varepsilon((\bar{e}_{31} + \bar{e}_{15})\nabla^2 + (\bar{e}_{15}\nabla^2 + \varepsilon^2)a_j - (\bar{\varepsilon}_{11}\nabla^2 - \varepsilon^2\varepsilon_{33})b_j - (\bar{m}_{11}\nabla^2 - \varepsilon^2 m_{33})c_j = 0,$$

$$\varepsilon(\bar{q}_{31} + \bar{q}_{15}) + (\bar{q}_{15}\nabla^2 + \varepsilon)a_j - (\bar{m}_{11}\nabla^2 - \varepsilon^2\bar{m}_{33})b_j - (\bar{\mu}_{11}\nabla^2 - \varepsilon^2\bar{\mu}_{33})c_j = 0.$$

3. Equation of motion for linear elastic materials with voids LEMV

The displacement equations of motion and equation of equilibrated inertia for an isotropic LEMV are

$$(\lambda+2\mu)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+\mu u_{,zz}+(\lambda+\mu)w_{,zz}+\beta E_{,r}=\rho u_{tt},$$
$$(\lambda+\mu)\left(u_{,rz}+r^{-1}u_{,z}\right)+\mu\left(w_{,rr}+r^{-1}w_{,r}\right)+(\lambda+2\mu)w_{,zz}+\beta E_{,z}=\rho w_{,tt},$$
$$-\beta\left(u_{,r}+r^{-1}u\right)-\beta w_{,z}+\alpha\left(E_{,rr}+r^{-1}E_{,r}+\phi_{,zz}\right)-\delta kE_{,tt}-\omega E_{,t}-\xi E=0. \tag{14}$$

The stress in the LEMV core materials are

$$\amalg^{1}_{rr}=(\lambda+2\mu)u_{,r}+\lambda r^{-1}u+\lambda w_{,z}+\beta E,$$
$$\amalg^{1}_{rz}=\mu\left(u_{,t}+w_{,r}\right).$$

The solution for Eq. (14) is taken as

$$u=U_{,r}expi(kz+pt),$$
$$w=\left(\frac{i}{h}\right)Wexpi(kz+pt),$$
$$E=\left(\frac{1}{h^2}\right)E\ expi(kz+pt). \tag{15}$$

The above solution in Eq. (14) and dimensionless variables x and ε, equation can be simplified as

$$\begin{vmatrix}(\lambda+2\mu)\nabla^2+M_1 & -M_2 & M_3\\ M_2\nabla^2 & \bar{\mu}\nabla^2+M_4 & M_5\\ -M_3\nabla^2 & M_5 & \alpha\nabla^2+M_6\end{vmatrix}\times(u,w,E)=0 \tag{16}$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x}$

$$M_1=\frac{\rho}{\rho^1}(ch)^2-\bar{\mu}\varepsilon^2\ ,\ M_2=(\bar{\lambda}+\bar{\mu})\varepsilon\ ,\ M_3=\bar{\beta},M_4=\frac{\rho}{\rho^1}(ch)^2-(\bar{\lambda}+\bar{\mu})\varepsilon^2\ ,M_5=\bar{\beta}\varepsilon$$
$$M_6=\frac{\rho}{\rho^1}(ch)^2\bar{k}-\bar{\alpha}\varepsilon^2-i\bar{\omega}(ch)-\bar{\xi}\,.$$

The Eq. (16) can be specified as

$$(\nabla^6+P\nabla^4+Q\nabla^2+R)(U,W,E)=0. \tag{17}$$

Thus the solution of Eq. (17) is as follows,

$$U=\sum_{j=1}^{3}\left[A_j\mathfrak{J}_0(\alpha_jx)+B_j\mathfrak{R}_0(\alpha_jx)\right],$$
$$W=\sum_{j=1}^{3}d_j\left[A_j\mathfrak{J}_0(\alpha_jx)+B_j\mathfrak{R}_0(\alpha_jx)\right],$$
$$E=\sum_{j=1}^{3}e_j\left[A_j\mathfrak{J}_0(\alpha_jx)+B_j\mathfrak{R}_0(\alpha_jx)\right],$$

$(\alpha_jx)^2$ are the roots of the equation when replacing $\nabla^2=-(\alpha_jx)^2$. The arbitrary constant d_j and e_j are obtained from

$$M_2\nabla^2+(\bar{\mu}\,\nabla^2+M_4)d_j+M_5e_j=0,$$

$$-M_3\nabla^2 + M_5 d_j + (\alpha\nabla^2 + M_6)e_j = 0.$$

By taking the void volume fraction E=0, and the lame's constants as $\lambda = c_{12}, \mu = \frac{c_{11}-c_{12}}{2}$ in the Eq. (14) we got the governing equation for CFRP core material.

4. Boundary conditions and frequency equations

The frequency equations can be obtained for the following boundary condition the electrical and magnetic boundary conditions for an infinite cylindrical bar are,

- On the traction free inner and outer surface $\coprod^{l}_{rr} = \coprod^{l}_{rz} = \phi_r{}^l = B_r{}^l = 0$ with $l = 1,3$.
- At the interface $\coprod^{l}_{rr} = \coprod_{rr}; \coprod^{l}_{rz} = \coprod_{rz}; \phi_r{}^l = 0; B_r{}^l = 0; D_r{}^l = 0.$

Substituting the above boundary condition, we obtained as a 22×22 determinant equation

$$\left|\left(Y_{ij}\right)\right| = 0\,, (i,j = 1,2,3, \dots .22) \qquad (18)$$

At $x = x_0$ Where $j = 1,2,3,4$

$$Y_{1j} = 2\bar{c}_{66}(\frac{\alpha_j^1}{x_0})\Im_1\left(\alpha^1{}_j x_0\right) - \left[\left(\alpha^1{}_j a\right)^2\bar{c}_{11} + \zeta\bar{c}_{13}a^l{}_j + \bar{e}_{31}\zeta b^l{}_j + \bar{q}_{31}c^l{}_j\right]\Im_0\left(\alpha^1{}_j a x_0\right).$$

$$Y_{2j} = \left(\zeta + a_j^1 + \bar{e}_{15}b_j^1\right)\left(\alpha_j^1\right)\Im_1\left(\alpha^1{}_j x_0\right).$$

$$Y_{3j} = b_j^1\Im_0\left(\alpha^1{}_j x_0\right).$$

$$Y_{4j} = \frac{c_j^1}{x_0}\Im_0\left(\alpha^1{}_j x_0\right) - \left(\alpha_j^1\right)\Im_1\left(\alpha^1{}_j x_0\right).$$

In addition, the other nonzero elements $Y_{1,j+4}$, $Y_{2,j+4}$ $Y_{3,j+4}$ and $Y_{4,j+4}$ are obtained by replacing $\Im_0$ by $\Im_1$ and $\Re_0$ by $\Re_1$.

At $x = x_1$

$$Y_{5j} = 2\bar{c}_{66}(\frac{\alpha_j^1}{x_1})\Im_1\left(\alpha^1{}_j x_1\right) - \left[\left(\alpha^1{}_j a\right)^2\bar{c}_{11} + \zeta\bar{c}_{13}a^l{}_j + \bar{e}_{31}\zeta b^l{}_j + \bar{q}_{31}c^l{}_j\right]\Im_0\left(\alpha^1{}_j a x_1\right).$$

$$Y_{5,j+8} = -[2\bar{\mu}\left(\frac{\alpha_j}{x_1}\right)\Im_1(\alpha x_1) + \left\{-\left(\bar{\lambda} + \bar{\mu}\right)\left(\alpha_j\right)^2 + \bar{q}_{31}b_j - \bar{\lambda}\zeta a_j\right\}\Im_0\left(\alpha_j x_1\right).$$

$$Y_{6j} = \left(\zeta + a_j^1 + \bar{e}_{15}b_j^1\right)\left(\alpha_j^1\right)\Im_1\left(\alpha^1{}_j a x_1\right).$$

$$Y_{6,j+8} = -\bar{\mu}\left(\zeta + a_j\right)\left(\alpha_j\right)\Im_1\left(\alpha_j x_1\right).$$

$$Y_{7j} = \left(\alpha_j^l\right)\Im_1\left(\alpha_j^l x_1\right).$$

$$Y_{7,j+8} = -\left(\alpha_j\right)\Im_1\left(\alpha^l{}_j x_1\right).$$

$$Y_{8j} = a_j^l\Im_0\left(\alpha_j^l x_1\right).$$

$$Y_{8,j+8} = -a_j^l\Im_0\left(\alpha_j^l x_1\right).$$

$$Y_{9j} = b_j^l\Im_0\left(\alpha_j^l x_0\right).$$

$$Y_{10j} = e_j\left(\alpha_j\right)\Im_0\left(\alpha_j^1 x_1\right).$$

$$Y_{11j} = \frac{c_j^l}{x_1}\Im_0\left(\alpha_j^l x_1\right) - \left(\alpha_j^l\right)\Im_1\left(\alpha_j^l x_1\right).$$

and the other nonzero element at the interfaces $x = x_1$ can be obtained on replacing $\Im_0$ by $\Im_1$ and $\Re_0$ by $\Re_1$ in the above elements. They are $Y_{i,j+4,}Y_{i,j+8,}Y_{i,j+11,}Y_{i,j+14,}(i = 5,6,7,8)$ and $Y_{9,j+4,}Y_{10,j+4,}Y_{11,j+4,}$. At the interface $x = x_2$, nonzero elements along the following rows Y_{ij} , $(i =$

$12,13,\ldots,18$ and $j = 8,9,\ldots,20$) are obtained on replacing x_1 by x_2 and superscript 1 by 2 in order.

Similarly, at the outer surface $x = x_3$, the nonzero elements Y_{ij}, ($i = 19,20,21,22$ and $j = 14,15,,\ldots,22$) can be had from the nonzero elements of first four rows by assigning x3 for x0 and superscript 2 for 1.

$$Y_{19j} = 2\bar{c}_{66}(\frac{\alpha_j^1}{x_0})\Im_1(\alpha^2{}_j x_3) - \left[(\alpha^2{}_j a)^2 \bar{c}_{11} + \zeta\bar{c}_{13}a^l{}_j + \bar{e}_{31}\zeta b^l{}_j + \bar{m}_{11}c^l{}_j\right]\Im_0\left(\alpha^1{}_j a x_3\right)$$

$$Y_{20j} = \left(\zeta + a_j^2 + \bar{e}_{15}b_j^2\right)(\alpha_j^1)\,\Im_1(\alpha^2{}_j x_3)$$

$$Y_{21j} = b_j^l \Im_0(\alpha^2{}_j x_3)$$

$$Y_{22j} = \frac{c_j^2}{x_0}\Im_0(\alpha^1{}_j x_3) - (\alpha_j^1)\,\Im_1(\alpha^1{}_j x_3)$$

In the case of without voids in the interface region, the frequency equation is obtained by taking $E = 0$ in Eq. (18) which reduces to a 20×20 determinant equation.

5. Numerical solution

In this problem, the free vibration of transversely isotropic multiferroic cylinder initially stressed and rotating motion is considered. The material properties of the electro-magnetic material $CoFe_2O_4$ are given bellow, Buchanan (2003)

$c_{11} = 218 \times 10^9\ N/m^2, c_{12} = 120 \times 10^9\ N/m^2\ c_{13} = 120 \times 10^9\ N/m^2, c_{33} = 215 \times 10^9\ N/m^2,\ c_{44} = 50 \times 10^9\ N/m^2,\ c_{66} = 49 \times 10^9\ N/m^2, e_{15} = 0,\ e_{31} = -2.5\ C/m^2, e_{33} = 7.5\ C/m^2,\ q_{15} = 200\ C/m^2,\ q_{31} = 265\ C/m^2, q_{35} = 345\ C/m^2, \varepsilon_{11} = 0.4 \times 10^{-9}\ C/Vm, \varepsilon_{33} = 5.8 \times 10^{-9}\ C/Vm, \mu_{11} = -200 \times 10^{-6}\ Ns^2/C^2, \mu_{33} = 95 \times 10^{-6}\ Ns^2/C^2, m_{11} = 2.82 \times 10^{-9}\ Ns/VC, m_{33} = 2.82 \times 10^{-9}\ Ns/VC$, and $\rho = 7500\ Kg\ m^{-2}$

The material properties of PZT-5A used for the numerical calculation given below, Selvamani and Mahesh (2019)

$C_{11} = 13.9 \times 10^{10}\ Nm^{-2};\ C_{12} = 7.78 \times 10^{10}\ Nm^{-2};\ C_{13} = 7.43 \times 10^{10}\ Nm^{-2}; C_{33} = 11.5 \times 10^{10}\ Nm^{-2};\ C_{44} = 2.56 \times 10^{10}\ Nm^{-2}; C_{66} = 3.06 \times 10^{10}\ Nm^{-2};\ T_0 = 298; \beta_1 = 1.52 \times 10^6 NK^{-1}m^{-2} \beta_3 = 1.53 \times 10^6 NK^{-1}m^{-2}, c_v = 420 Jkg^{-1}K^{-1}\ p_3 = -452 \times 10^{-6} CK^{-1}m^{-2}; e_{13} = -6.98 Cm^{-2}; K_1 = K_3 = 1.5 Wm^{-1}K^{-1}; e_{33} = 13.8 Cm^{-2} e_{15} = 13.4 Cm^{-2}; \rho = 7750 Kgm^{-2}; \varepsilon_{11} = 60.0 \times 10^{-10} C^2N^{-1}m^{-2}; \varepsilon_{33} = 5.47 \times 10^{-10} C^2N^{-1}m^{-2}$.

The above mentioned two different material constants are consider inner ($CoFe_2O_4$) and outer (PZT-5A) layers of multifonic cylinder. In Table.1 the non-dimensional frequency in adhesive layer of PE/LEMV/CFRP/PM hollow cylinder for different rotational parameter in tune with wavenumber is obtained. As observed, the influence of wave number is clearly noticed from the table values and the frequency can be raised by applying high intensity of wave number. Also, the rotational parameter softens the non-dimensional frequency quickly in LEMV than in CFRP. The nondimensional frequencies in adhesive layer of PE/LEMV/CFRP/PM hollow cylinder for different rotational parameter in tune thickness is revealed in Table 2. The rise in thickness values enlarge the stiffness of the composite but increasing rotational values again weakens the vibration mode. One more fact got out from these values is that the system can get larger dynamical response whenever the system is coupled with CFRP.

Table 1 Dimensionless frequency for different values of real wave numbers against rotation speed in the adhesive layer

Wave Number	Non dimensional Frequency in adhesive layer					
	LEMV			CFRP		
	Ω=0	Ω=0.5	Ω=1	Ω=0	Ω=0.5	Ω=1
0.2	0.008	0.0083	0.008	0.8186	0.8000	0.7437
0.4	0.0132	0.0128	0.0126	0.9586	0.6946	0.6321
0.6	0.0167	0.1380	0.0130	0.9722	0.7580	0.6710
0.8	0.0189	0.0154	0.0124	0.5414	0.5525	0.2909
	0.0169	0.0161	0.0101	0.5654	0.5879	0.6503

Table 2 Dimensionless frequencies for different values of thickness wave numbers against rotation speed in the adhesive layer

Thickness of cylinder	Non dimensional Frequency in adhesive layer					
	LEMV			CFRP		
	Ω=0	Ω=0.5	Ω=1	Ω=0	Ω=0.5	Ω=1
0.003	0.0061	0.0052	0.0044	0.8186	0.8000	0.7437
0.004	0.0072	0.0063	0.0054	0.8931	0.8735	0.8140
0.005	0.0082	0.0073	0.0063	0.9676	0.9470	0.8844
0.006	0.009	0.0081	0.0072	1.0422	1.0205	0.9547
0.007	0.0094	0.0084	0.0074	1.1167	1.0990	1.0250

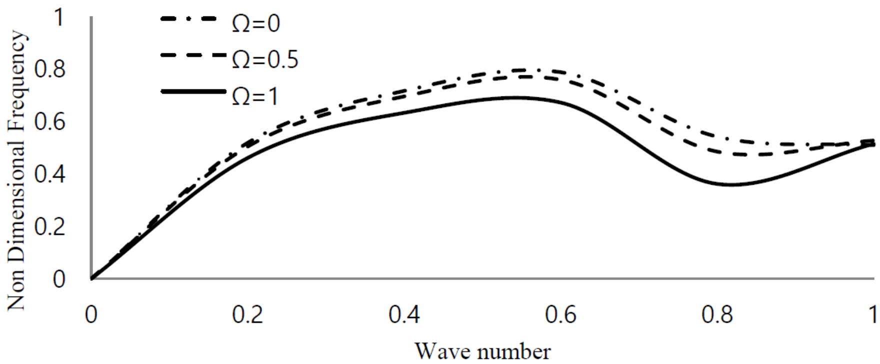

Fig. 2 Effect of non-dimensional Frequency over wave number for different values of rotation parameter **Ω** in PE

Figs. 2 to 4 are drawn to explore the influence of rotating parameters on the profile of non-dimensional frequency. In the following figures dimensionless wave number is represented by horizontal axis and dimensionless frequency is represented by vertical axis. The variation of dimensionless frequency gets decreases monotonically with the increment of rotation parameters. Furthermore, non-dimensional frequency follows unique nature for different rotation speeds in wave number close to origin otherwise for increasing values of wave number the non-dimensional frequency turns a slightly oscillating behavior in PE/PM layers but in LEMV layer try to maintain

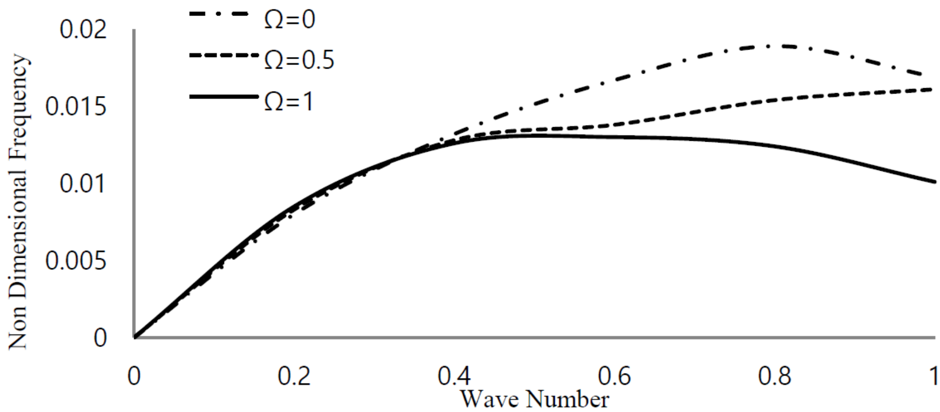

Fig. 3 Effect of non-dimensional Frequency over wave number for different values of rotation parameter **Ω** in LEMV

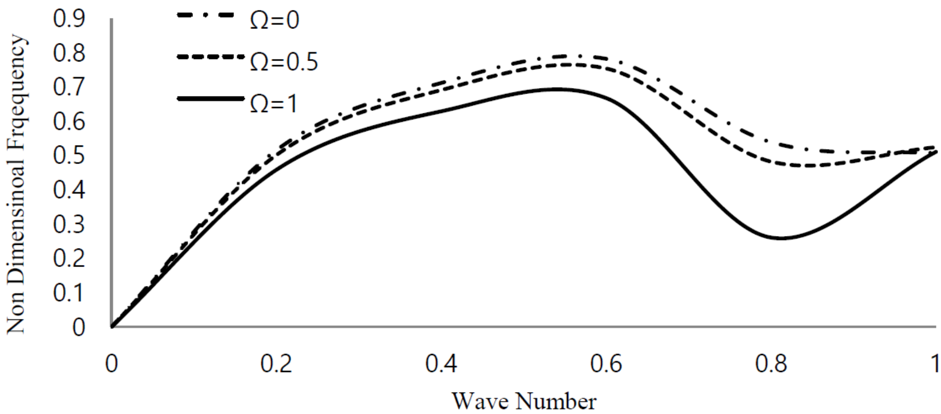

Fig. 4 Effect of non-dimensional Frequency over wave number for different values of rotation parameter **Ω** in PM layer

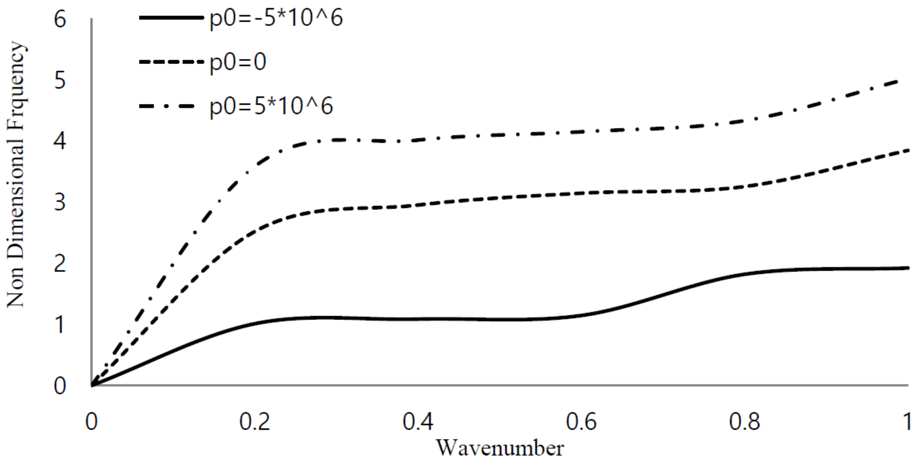

Fig. 5 Effect of non-dimensional Frequency over wave number for different values of initial stress parameter (p_0) in PE layer

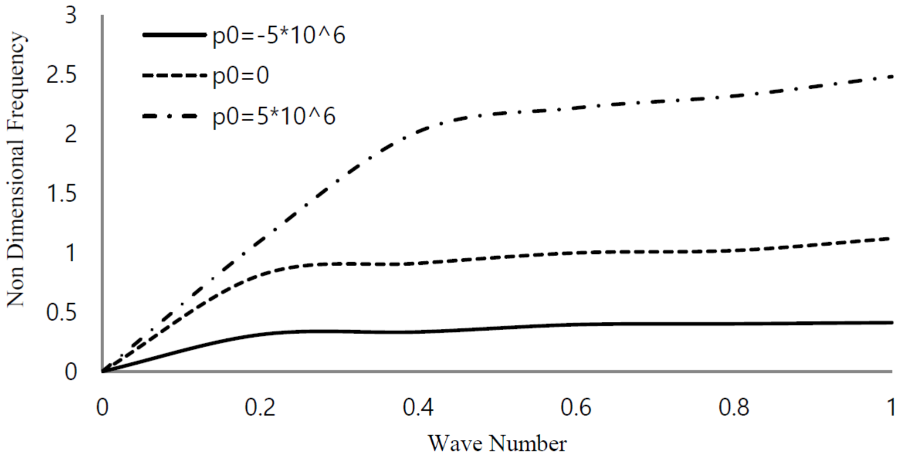

Fig. 6 Effect of non-dimensional Frequency over wave number for different values of initial stress parameter (p_0) in LEMV layer

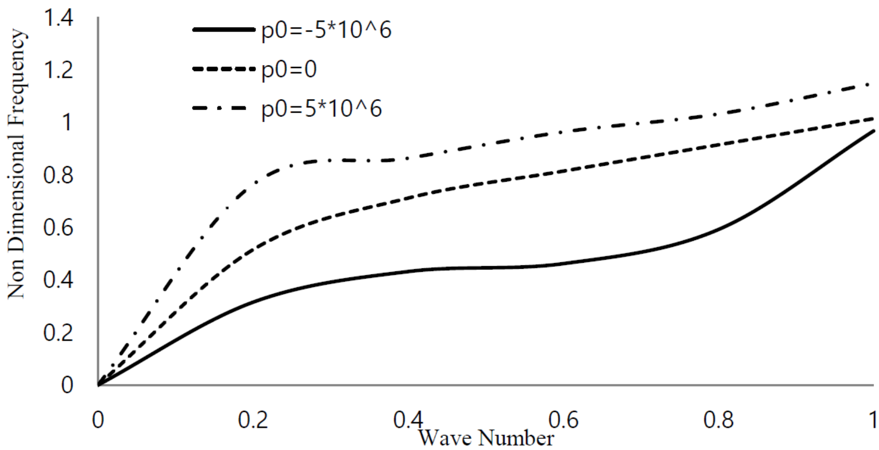

Fig. 7 Effect of non-dimensional Frequency over wave number for different values of initial stress parameter (p_0) in PM layer

symmetric nature. Finally we observe the influence of rotation in multifonic LEMV hollow cylinder is considerable.

Figs. 5 to 7 are drawn to explore the influence of initial stress on the profile of non-dimensional frequency. In the following figures dimensionless wave number is represented by horizontal axis and dimensionless frequency is represented by vertical axis. The variation of dimensionless frequency observed for different values of initial stress (p0) and without initial stress. Furthermore, non-dimensional frequency gradually increasing for different values of initial stress p_0 in wave number close to origin otherwise for increasing values of wave number the non-dimensional frequency monotonic increasing behavior in all layers of multifonic LEMV hollow cylinder is considerable.

Figs. 8 to 10 are drawn to explore the influence of rotating parameters on the profile of non-dimensional frequency. In the following figures dimensionless thickness of cylinder is represented

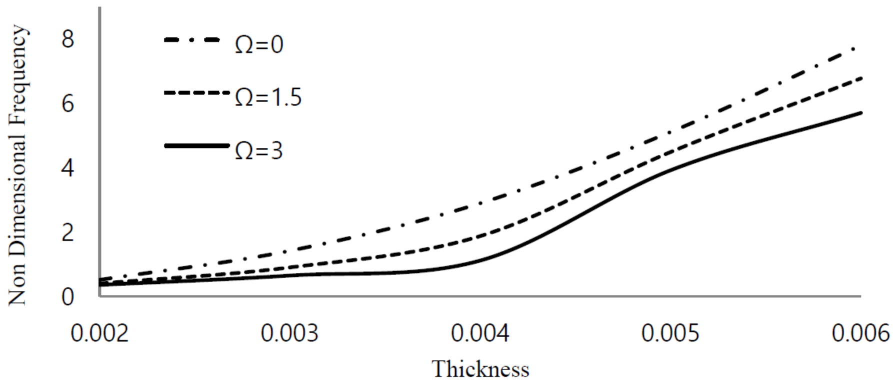

Fig. 8 Effect of non-dimensional Frequency over Thickness for different values of rotation parameter Ω in PE

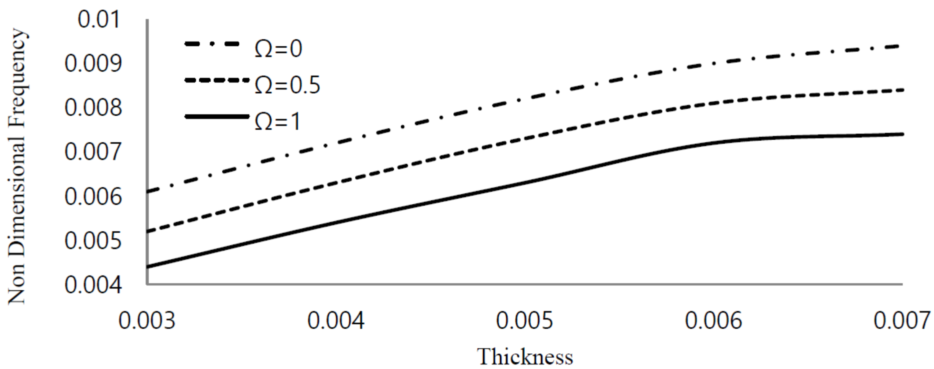

Fig. 9 Effect of non-dimensional Frequency over Thickness for different values of rotation parameter Ω in LEMV Layer

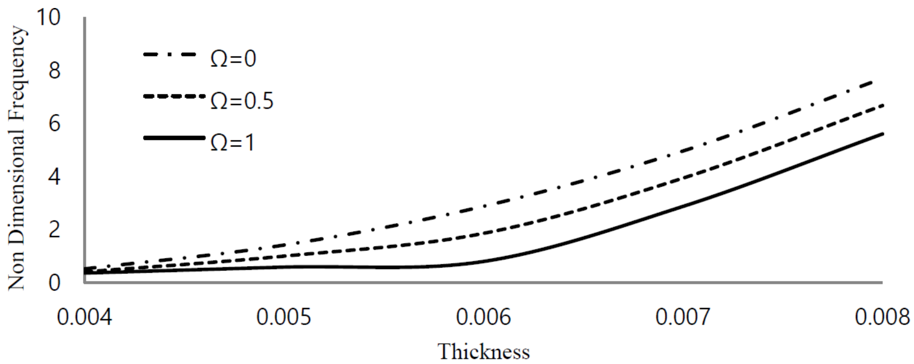

Fig. 10 Effect of non-dimensional Frequency over Thickness for different values of rotation parameter Ω in PM layer

by horizontal axis and dimensionless frequency is represented by vertical axis. The variation of dimensionless frequency gets decreases monotonically with the increment of rotation parameters. Furthermore, in the absences of rotation the thickness increases simultaneously the Non-dimensional frequency increases. In the presences of rotation, the non-dimensional frequency monotonically increases for the increasing values of thickness.

6. Conclusions

The present examination researches the wave spread through multiferroic (PE/PM) cylinder together by Linear Elastic materials with voids made out of ($CoFe_2O_4$) and (PZT-5A). Impacts of rotation and initial stress are explored. Some amazing results are as per the following:

- Wave spread examination in the present work following some numerical models proposes that rotation parameter unequivocally decreases the non-dimensional recurrence against the wavenumber.
- Simultaneously the non-dimensional recurrence continuously is expanding against the thickness of cylinder in the existences of rotation Parameters.
- The impacts of initial stress in recurrence against wave number additionally watched and its participation is commented.
- Likewise the glue layer CFRP renders amplified response against these parameters.

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