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Coupled temperature-displacement modeling to study the thermo-elastic instability in disc brakes

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Abstract. Macroscopic hot spots formed due to the large thermal gradients at the surface of the disc brake rotor, make the rotor to fail or wear out early. Thermo-elastic deformation results in contact concentration, leading to the non uniform distribution of temperature making the disc susceptible to hot spot formation. The formation of one hot spot event will predispose the system to future hot spotting at the same location. This leads to the complete thermo-elastic instability in the disc brakes; multitude parameters are responsible for the thermo elastic instability. The predominant factor is the sliding velocity and above a certain sliding velocity the instability of the brake system occurs and hot spots is formed in the surface of the disc brake. Commercial finite element package ABAQUS® is used to find the temperature distribution and the result is validated using Rowson's analytical model. A coupled analysis methodology is evolved for the automotive disc brake from the transient thermo-elastic contact analysis. Temperature variation is studied under different sliding speeds within the operation range.

Keywords: macroscopic hot spots; thermo elastic instability; Rowson's analytical model; automotive disc brake

1. Introduction

Friction brakes are required to transform large amounts of kinetic energy into heat energy, thus resulting in large thermal gradients from the contact surfaces. The temperature distribution at the friction interface generated in the braking process is a complex phenomenon, which directly affects the braking performance. These aspects have been investigated by many researchers over the years, yet defying a clear understanding (Ouyang *et al*. 2009, Day and Newcomb 1984, Choi and Lee 2003) specifically in disc brakes. Sliding speed plays a vital role in problems involving frictional heat generation. Hot spots are formed at the surface of disc brake rotor, at certain critical sliding speeds beyond which the Thermo Elastic Instability (TEI) sets-in. This work adopts a coupled approach and aims to propose a macro structural model of the thermo mechanical behavior of the disc brake, taking into account the real three-dimensional geometry of the disc pad couple. Such a model aims to give predictions of the critical sliding speed (V_{cr}) which precludes to the thermo elastic instability. Ideally, one will be able to design the brake system so that the operating conditions always lie below V_{cr}.

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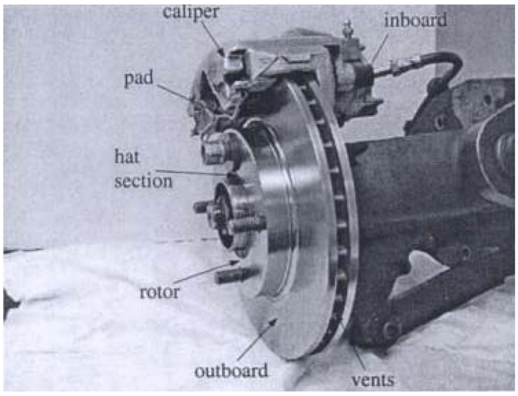


(Yi *et al*. 2001)
Fig. 1 A typical automotive disc brake

1.1 Automotive Disc Brakes

Fig. 1 shows the main features of a typical automotive disc brake. Two brake pads make contact with the two plane surfaces on both sides of the rotating disc, the surface facing out from the vehicle being known as the outboard surface and the other as the inboard surface. Most discs contain air vents at the mid plane for cooling purposes. It is also to be noted that the practical heat transfer coefficients are not sufficient to enable significant cooling made by air vents. The disc is connected to the axle by a hat section and the precise way in which this is connected to the disc can have a significant effect on thermo elastic distortion. The brake pads are pressed against the disc surfaces by hydraulic pressure through a caliper mechanism, designed to equalize the forces on the inboard and outboard sides. However, this equalization is usually achieved by a sliding mechanism and this may lack due to frictional effects during loading, preventing the system from responding to changes in pad loads during a single engagement. This can also have a significant effect on the stability of modes involving small numbers of hot spots.

2. Literature review

Lee *et al*. (1993) used the perturbation method to solve the thermal problem of the disc brake system wherein they first discussed the role of critical sliding speed. The Eigen value analysis is done to find the set of critical values at which the hot spots are formed on the surface of the disc brake rotor. Similarly Yi *et al*. (2000) conducted a Fourier reduction method to determine the growth rate of the temperature distribution and the hot spots. Joachim-Ajao *et al*. (1998) proposed a non-dimensional critical sliding speed in which the sliding speed fully depends on the thermal conductivity and geometric parameters.

Burton's perturbation method (Dow and Burton 1972) which is essentially equivalent to a solution of the transient thermo elastic contact problem of the system conforming the feedback mechanism shown in Fig. 2; the general transient problem can in principle be written down as an Eigen function expansion using the Eigen-modes of Burton's solution. In particular, it leads to that if two bodies of similar materials make contact at a plane boundary or one body makes contact with a rigid non conducting plane surface, the stability of a given steady-state or transient solution can be defined in terms of a single dimensionless critical speed V_{cr}. However, V_{cr} is significantly affected by the geometry of the system and hence it is generally necessary to use numerical methods (typically finite element method) to predict the critical sliding speed.

Finite element method has evolved as a more convenient and flexible method for the multiphysics problems especially nonlinear contact analysis where considerations of interaction between two bodies come into place. Availability of built-in algorithms in the commercial software packages provides added advantage. Thus it can capture more realistic representation of disc brake operation, including nonlinearities and flexibilities of the disc brake components. Temperature changes due to frictional heat generation cause axial and radial deformation of the rotor along with pad expansion. The resulting change in shape affects the contact between the rotor and pad surface area and thus will influence the load distribution at the friction interface. For the different sliding speeds hot spots formed at the surface of the disc brake rotor differs (Lam *et al*. 2001). The staggered approach is one of the most popular computation techniques used to solve highly coupled non-linear equations. Jung *et al*. (2012) generated the mechanical and thermal models for the disc brake separately, and solved iteratively using the staggered approach. Tang *et al*. (2011) solved a 3D thermo mechanical disc brake problem with water cooling system typically used in the offshore drilling systems by ABAQUS®. The sequentially coupled thermal stress model is utilized for the analysis of this disc brake model and the radial and circumferential stresses are plotted for the different instances of braking operation. The thermal analysis of the disc brake is conducted by the (Al Belhocine *et al*. 2011) where the numerical simulation for the transient thermal analysis alone conducted for the ventilated disc and the thermal behaviour was studied for the three different types of cast iron. Most of the work pertains to either a thermal analysis or sequentially coupled analysis by finite element method is available by the current work done on disc brake. A fully dedicated coupled temperature displacement model accounts for predicting the critical sliding speed and the onset of TEI is not noticed from the literature review done. The present work deals with a fully coupled thermo-elastic analysis of a disc brake.

3. Formulation of thermo elastic contact problem

The closed form feedback mechanism responsible for TEI is illustrated by the flow diagram in Fig. 2. Frictional heating during braking causes thermo-elastic distortion, which in turn modifies the

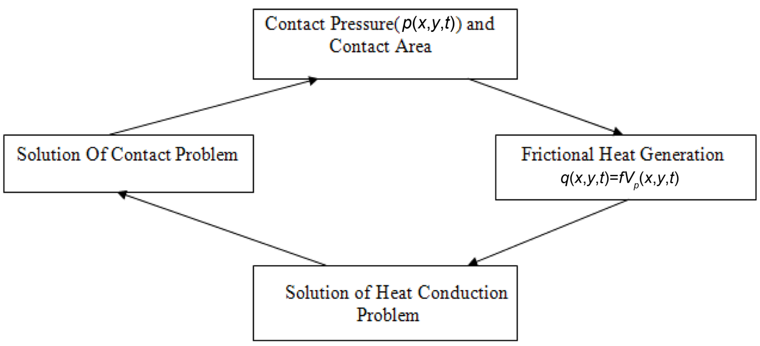


Fig. 2 Feedback mechanism of TEI

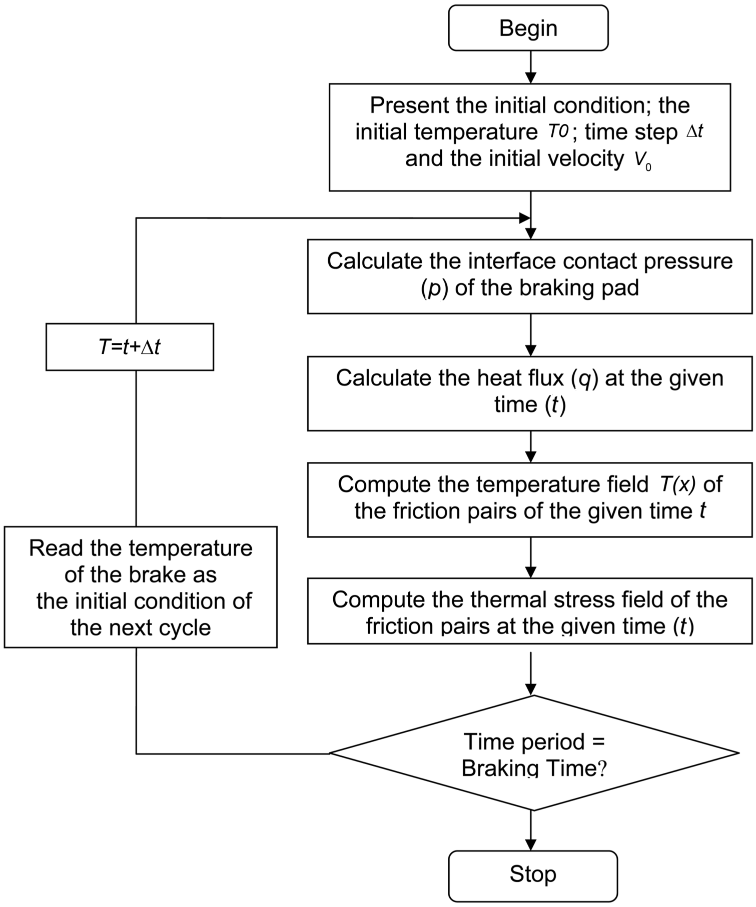


Fig. 3 Flow diagram of TEI

contact pressure distribution $p(x, y, t)$ and hence the distribution of frictional heating $q(x, y, t)$. Coupling between the mechanical and thermal problems is introduced by the heat flux from the energy balance relation.

The governing equation for the transient heat transfer problem is

$$q(x, y, t) = fVp(x, y, t) \tag{1}$$

Where f is the coefficient of friction and V is sliding speed. It is clear from Fig. 2 that the product fV functions as the gain in the feedback process and it follows that there will generally be a critical speed V_{cr} above which any given sliding system will be unstable (Yi *et al*. 2001) . Above the critical speed, non-uniform perturbations in the temperature field will grow, leading to a characteristic pattern of hot bands on the brake disc.

The step by step process of solving a fully coupled Thermo-mechanical problem is highlighted as a flow diagram in Fig. 3 (Gao *et al*. 2007). The initial condition of the temperature and angular velocity are given to the model as the input parameter and the contact parameter for the interface is calculated with the heat flux generated. The contact pressure is updated to the next cycle for the fixed time increment and the simulation is carried up to the full braking time.

This procedure is developed in commercial finite element package ABAQUS® with python command stream. A constant time step is followed for the uniform velocity and braking pressure values. The hydraulic pressure is applied which actuates the piston and develop contact pressure at the surface of the rotor. The contact pressure makes the heat flux at the surface and leads to the temperature generation at the surface of the disc brake.

4. Determination of critical sliding speed

Hot spotting is a substantial concern in disc brakes. The time of sliding is much longer in brakes. Brakes also operate in transient mode, this in turn, indicates that the sliding speed in the initial stage of the sliding if substantially higher than the critical speed, could result in hot spots.

An instantaneous operation of a brake above the critical speed seems to be a likely situation for TEI in many applications. This situation should be considered acceptable as long as the resulting extreme variations of contact pressure and temperature remain acceptable. An evaluation of the thermo elastic contact problem should therefore include estimation of the critical speed and, in case the operational speed exceeds it, an evaluation of an instantaneous unstable behavior. Thus the sliding speed plays vital role in problems involving frictional heat generation, where Thermo- elastic instability occurs at certain critical sliding speed beyond which the hot spots form at the surface of disc brake rotor. So it is a challenging task to determine the critical sliding speed for such a tribosystem. The presently available numerical solving techniques coupled with the high performance computing facilities makes way for solving the transient thermo elastic contact directly by an available commercial finite element packages. With the results of transient problem the range of the critical sliding speed can be determined by conducting the analysis in the specific range of sliding speed within the practical operation range of commercial disc brakes. The information can be utilized, while designing a disc brake for a vehicle which is to be designed. Initial sliding speed is taken by assuming the practical vehicle speed on road conditions.

Transient behavior of the disc brake certainly depends on the stability characteristics of the system and on the time course of the sliding speed. In addition, the magnitude of the factor that triggers the unstable behavior, e.g. contact pressure variation caused by geometric imperfections, plays a key role in the development of the transient process. The speed typically decreases from an initial high value to zero within a period of the order of 1 s or less.

5. Description of model and boundary conditions

As shown in Fig. 4 the analytical model presents a three-dimensional solid disc squeezed between two finite-width friction materials called pads which represent Newcomb model (Newcomb 1960) of disc brakes. The simulation is done for the drag brake condition with the braking time of 1 sec. The entire surface of the disc has two different regions S1 and S2. On S1 braking pressure is applied, which leads to the frictional heating between the pads and disc and S2 is defined for the convection boundary. The heat flux at back plate (i.e., top side of S1) is set at zero so that the surface is insulated. The disc is rotated with the angular velocity of ω rad/s. Note that the coordinate system origin is fixed with respect to the rotating disc.

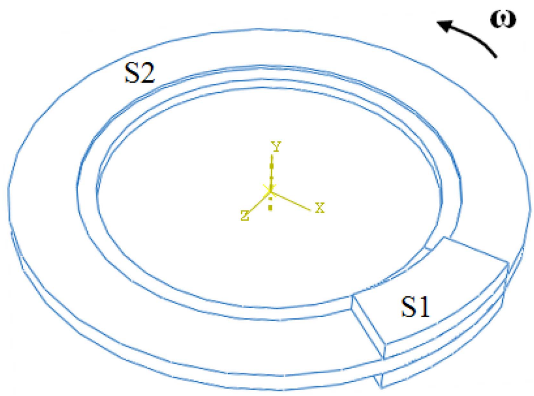


Fig. 4 A schematic of a disc brake system showing a disc and pads

A constant initial temperature value of 20ºC is assigned at the beginning of computation all over the surface, and a uniform pressure of 5.8e5 Pa is assigned initially over the contact side of each pad. Since the drag brake condition is used for this simulation the maximum pressure of 8e5 Pa is not applied at the surface of the rotor. The ratio of p_{av}/p_{max} is 0.8 for this disc brake system. The pad contact area on the disc in this study is chosen as 64 cm^2. The successive contact pressure distribution is automatically updated per thermo-elastic analysis at each step of the simulation.

6. Finite element model of disc brake

A simplified 3-dimensional model of automotive disc brake system has been constructed using the CAD capabilities in ABAQUS® (ABAQUS 2004). The disc brake model consists of the two pads and the disc rotor alone, whilst the caliper and piston is not explicitly defined, since the pressure is applied directly over the brake pad area. The back plates are not physically present but the adiabatic boundary condition is imposed in the two surfaces behind the pads. The full disc brake model shown in Fig. 5 consists of 13989 elements and 73007 degree of freedom. Three dimensional 8-noded solid

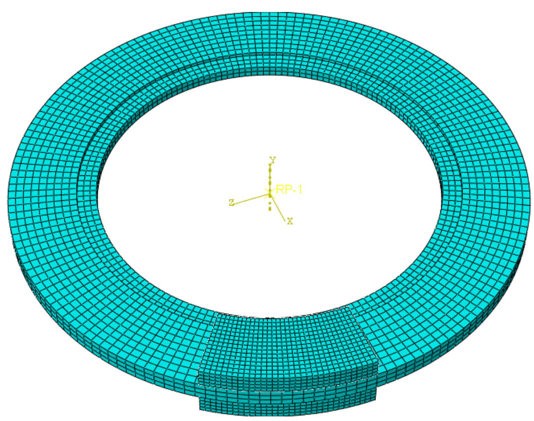


Fig. 5 FE model of disc brake system

Table 1 Geometrical and material properties of disc brake system

Parameters	Grey cast iron	
	Disc brake rotor	Pads
Diameter (mm)	270	--
Width (mm)	--	10
Thickness (mm)	10	--
Density (kg/m^3)	7071.0	2798
Young's modulus (GPa)	105	3
Poisson's ratio	.211	0.3
Conductivity (W/mK)	46.73	2.06
Specific heat (J/kg/K)	455	20
Coefficient of thermal expansion (/ ^{o}C)	6.6E-6	1.43E-5

brick elements with coupled thermal and stress capability and reduced integration (C3D8RT) was selected. Reduced integration leads to considerable savings in the analysis time and cost. This type of element also permits large scale sliding between contact surface (pad) and sliding target surface (disc). In order to simulate the axial movement of the pad, no constraint is applied in the direction normal to the disc surface. To simulate the hydraulic pressure, the two pads are allowed to move freely in the axial direction.

The centre of the disc is constrained in the normal direction only so that the disc is able to rotate. The rotational degree of freedom for the rotor (since solid brick elements do not have the rotational dof) is given by the rigid body connectors between the inner ring of the rotor and the axis of rotation. To simulate the contact scheme between the pad and disc surface, surface-to-surface discretization have been defined using the finite sliding contact algorithm. The master-slave approach is adopted where the disc surface is chosen as the master surface since it has a coarser mesh and is stiffer than the pad. The contact surface frictional behavior is simulated by the penalty method, rather than the Lagrange multipliers scheme. The material properties and the model dimensions are given in the Table 1.

7. Contact interaction schemes

7.1 Penalty versus Lagrange for friction constraints

By default, ABAQUS uses a penalty scheme to impose friction constraints for tangential interaction in the contact analysis. The penalty scheme allows some relative motion of the surfaces when it should be sticking. The magnitude of sliding is limited to an elastic slip, which is characterized by slip tolerance and contact surface length. Another method that can enforce the sticking and sliding constraints more precisely is Lagrange multipliers scheme. This scheme does not allow any relative motion between two closed surfaces until resultant shear stress is equivalent to the critical shear stress. Since Lagrange multipliers add more degrees of freedom to the model and often increase the number of iterations for a converged solution, this scheme leads to an increase in computational time. Furthermore, convergence is also an issue using the Lagrange multipliers compared to the penalty method. So penalty method is used throughout the analysis process.

7.2 Small sliding versus finite sliding

ABAQUS® provides three schemes in defining relative surface motions, namely small, finite and infinitesimal sliding. In infinitesimal sliding, the scheme assumes that both the relative motion of the surfaces and the absolute motion of the contacting bodies are small. Since large rotation is necessary for the disc brake problem, this type of scheme is not suitable. Small and finite sliding schemes allow large rotation and include geometric nonlinearities. Similarity and differences between small and finite sliding can be given as follows:

(1) Both formulations allow two bodies to undergo large motions. However limitation in the small sliding is that it assumes that there will be a relatively small amount of sliding of one surface along the other.

(2) The slave node can transfer load to any nodes on the master surface in the finite sliding while it can only transfer load to a limited number of nodes on the master surface for the small sliding.

(3) In small sliding analysis every slave node interacts with its own tangent plane on the master surface and consequently slave nodes are not monitored for possible contact along the entire master surface. Thus the small sliding is less expensive computationally than the finite sliding contact.

(4) The finite sliding contact requires that master surfaces to be smooth which it must has a unique surface normal at all points while the small sliding contact does not need master surfaces to be smooth.

Comparisons between the two sliding schemes are made in terms of simulation time and the finite sliding scheme is selected to proceed with the further analysis due to the computational advantage of less simulation time. For the entire analysis, a brake-line pressure of 5.8e5 Pa and a rotational speed of 5, 10, 15 rad/s are imposed to the model. For the contact interface between the pad and the disc a friction coefficient is related w.r.t the slip rate and temperature as shown in Table 2. The penalty friction constraint is chosen for comparison. The heat transfer coefficient h_r for the physics of frictional heat generation problem is calculated by adopting the Eq. (2) (Majcherczak *et al*. 2005), where Thermal contact conductance values are taken from the literature (Song *et al*. 2002) for the similar combination of the material and listed in Table 3.

$$h_r = \cdot 0.15 \frac{\lambda_{air}}{2\pi rm}\left(\frac{2\pi rm^2}{v^{air}}\right)^{0.8} \tag{2}$$

Table 2 Coefficient of friction *w.r.t* slip rate

S No.	Coefficient of Friction (μ)	Slip rate	Temperature (ºC)
1.	0.6	0	20
2.	0.4	100	100

Table 3 Contact thermal properties

S No.	Thermal contact conductance (W/m²K)	Pressure (Pa)	Heat Transfer convection coefficient (W/m²K) $T\infty = 20$ºC
1.	5e11	0	176
2.	5e11	2e5	176

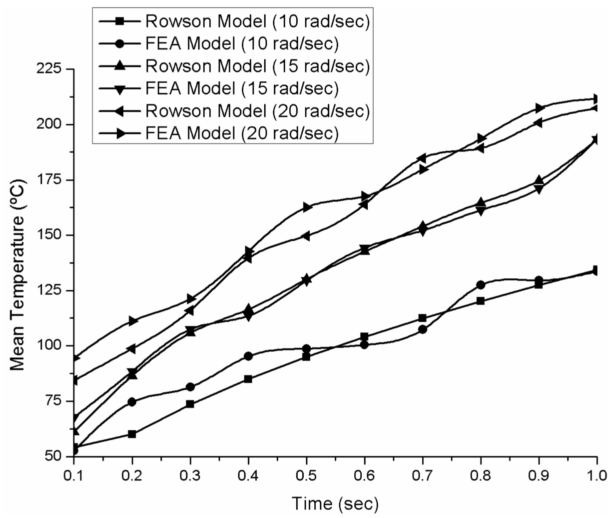


Fig. 6 Temperature Vs Time history plot for FE and Rowson's model

8. Validation of finite element model

The developed Finite element model is validated with the Rowson's (Rowson 1978) analytical model of disc brake in terms of the mean interface temperature generated due to the frictional heat. The analytical model derived by Rowson is already validated with the experimental observation of the mean heat flux density. The heat flux input for the model is assumed to be supplied continuously but linearly decreasing with respect to the time of braking applications.

The temperature rise at time t due to heat input at time t' is given by,

$$q(t') = Q(1 - t/t_s) \tag{3}$$

$$d\theta = \frac{2Rd\phi dRQ(1 - t/t_s)}{\nu C\{4\pi a(t - t')\}^{3/2}} e^{-R^2/(4\pi a(t - t'))} \tag{4}$$

Where $d\theta$ is the temperature gradient, R is the Radius of the disc, $d\phi$ is the change in angle for polar coordinate, Q is heat flux density, t is time at the instance, t_s is the stopping time, a is Prandtl number, ν is density of the surface, C specific heat capacity.

Triple Integration of the above equation gives

$$\theta = \frac{2Q\sqrt{a^*t}}{\lambda\sqrt{\pi}} \tag{5}$$

This is the solution for the surface temperature rise found by Newcomb and Bannister. The present finite element disc brake model is validated with this analytical model. The mean temperature generated in every instance is taken from the finite element results and is compared with this model for the same material properties and the dimension. Fig. 6 shows the validation plot for time and mean temperature at different sliding speeds. The mean temperature generated matches well at every instance for a disc speed of 15 rad/s and at other speed only slight variation is noted at some specific instances and hence the disc brake model validation is confirmed.

9. Results and discussion

First a uniform pressure and wear condition is imposed and the model is validated with the conventional disc clutch theory (Norton 2006). Then a parametric analysis is done with the different sliding speeds within the operational range of the vehicle. The simulation is carried out at specific sliding speeds 5, 10, 15 rad/s.

9.1 Uniform Pressure and wear condition

The pressure between the disc and pad surfaces can approach a uniform distribution over the surface if the discs are flexible enough. In such cases, the wear will be greater at the outer diameters because wear is proportional to $p.V$ factor (pressure time's velocity) and the velocity increases linearly with radius. However, as the discs wear preferentially towards the outside, the loss of material will change the pressure distribution to a non uniform one and the disc will approach a uniform wear condition of $pV =$ constant. Thus the two extremes are a uniform-pressure and a uniform-wear condition. A disc may be close to a uniform pressure condition when new, but will tend toward a uniform wear condition with use. A rigid disc will more rapidly approach the uniform wear condition with use. The effect of both uniform pressure and wear is simulated and compared for temperature distribution at the maximum temperature point.
The applied pressure vary for uniform pressure and wear as shown below relations.
(1) uniform pressure

$$p = p_{max} \tag{6}$$

(2) uniform wear

$$\delta = kpr = const \tag{7}$$

$$p = p_{max}\frac{r_1}{r} \tag{8}$$

where δ is wear, p_{max} is the maximum braking pressure
Fig. 8 shows the contour plot of temperature for the uniform pressure and wear condition. In

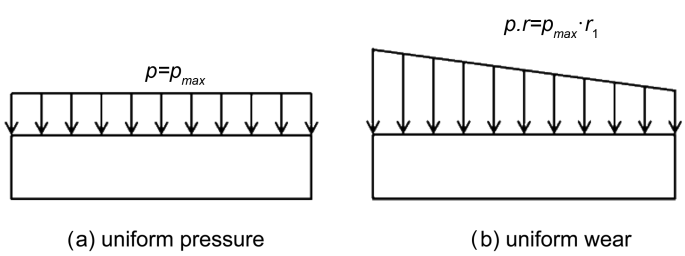


Fig. 7 (a) and (b) shows the uniform presure and the wear condition

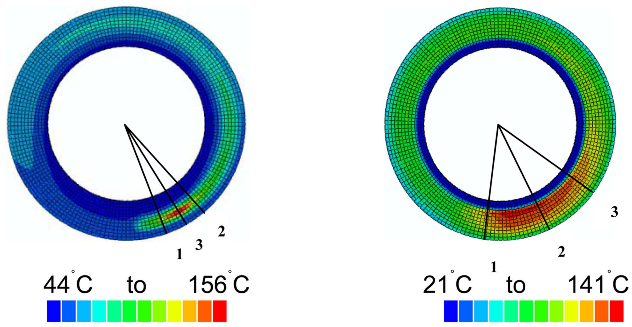


Fig. 8 (a) uniform pressure (b) uniform wear

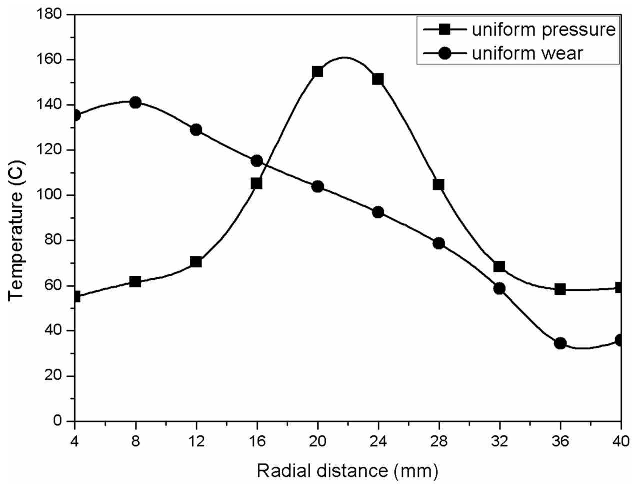


Fig. 9 Nodal Temperature Vs Radial distance along the rotor ($\omega = 10$ rad/sec)

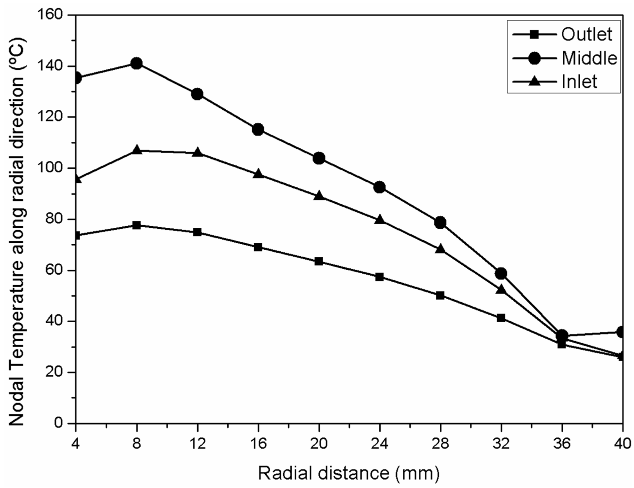


Fig. 10 Nodal Temperature Vs Radial distance along circumferential direction of rotor for uniform wear ($\omega = 10$ rad/sec)

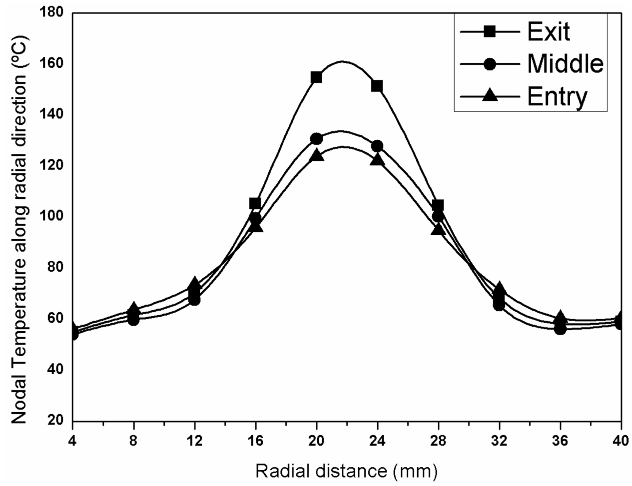


Fig. 11 Nodal Temperature Vs Radial distance along the rotor for uniform pressure condition (ω = 10 rad/sec)

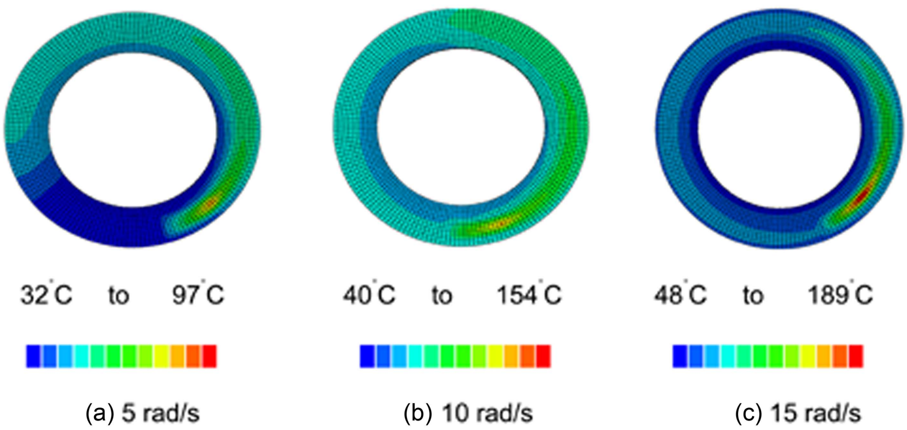


Fig. 12 Contour plots for nodal temperature in the surface of rotor for 1 sec braking

uniform pressure condition the temperature is more concentrated and is high compared to the uniform wear condition where the temperature is distributed in the surface and is low. Nodal temperatures observed along the radial direction at the centre point, where temperature concentration is noticed, after 1 second of brake (pressure) application is shown in Fig. 9. For these two conditions of analysis; more severe temperature concentration in the case of uniform pressure can be clearly observed. Fig. 10 shows the variation of temperature at three specific locations marked, in the circumferential direction along the rotational direction for entry, peak and exit regions of pad contact to the rotor.

9.2 Parametric analysis with sliding speed

Different sliding speeds within the operational range of the vehicle are taken and the simulation is carried out under uniform pressure condition at sliding speeds 5, 10, 15 rad/s.

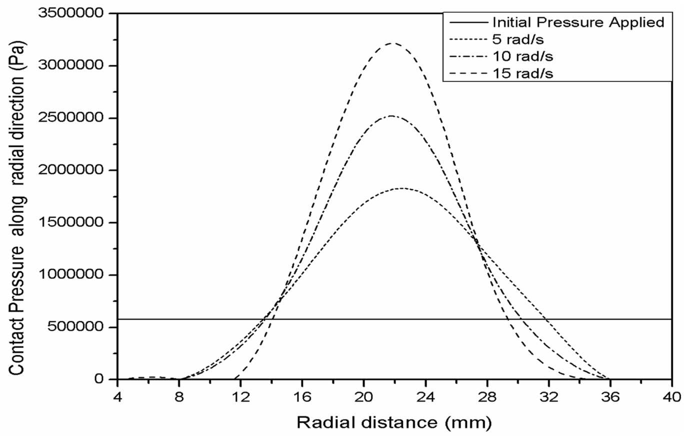


Fig. 13 Contact Pressure Vs Radial distance along the rotor

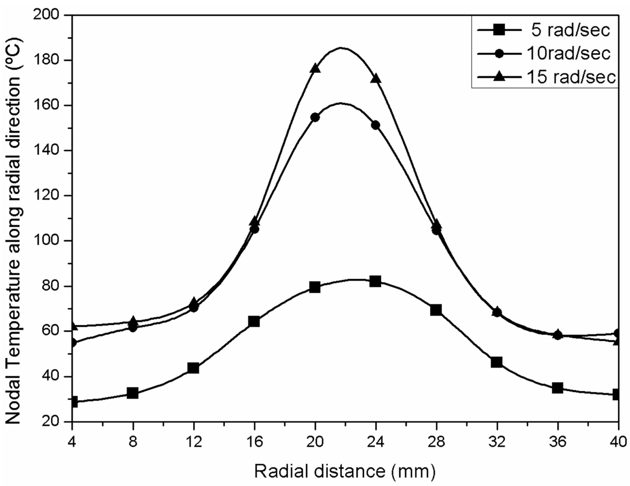


Fig. 14 Nodal Temperature Vs Radial distance along the rotor

Non-uniform distribution and localized concentration of temperature in particular region and ultimately in one spot can be clearly visualized from the disc temperature contour plot shown in Fig. 12. As the speed increases the concentration becomes more severe. Fig. 13 shows the variation of contact pressure in the radial direction which follows the same trend clearly indicating that the coupled analysis is able to bring out the thermo-elastic deformation and this leads to the non uniform distribution of temperature. As the sliding speed increases the maximum contact pressure point show a tendency to shift from the outer radius to the inner radius.

The deformation in the surface of the disc brake rotor along the radial distance is plotted in Fig. 15. The displacement in the y direction of pad is substantially high compared to that of the x and z direction due to contact concentration on the surface of the rotor. Displacement of material in that particular spots causes high thermal gradients at particular location. The deformation increase in this direction with increase in the sliding speed, with high frictional heat generation. The 3D deformation contour plot for the different sliding speeds is shown in the Fig. 16. The deformation further

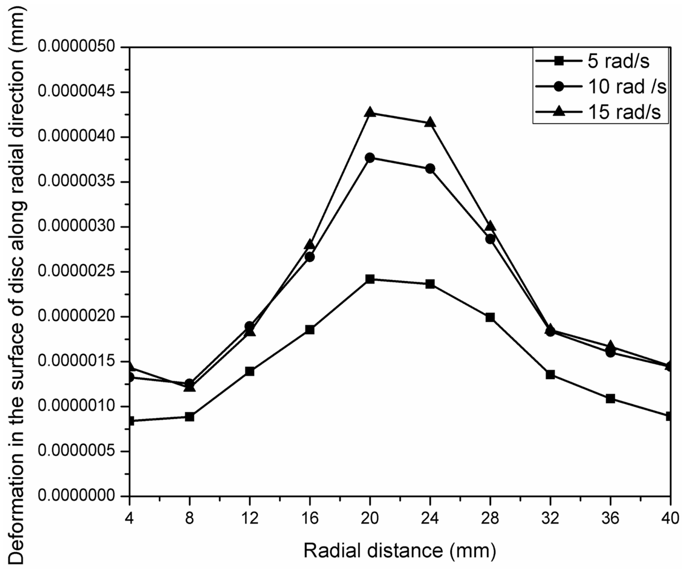
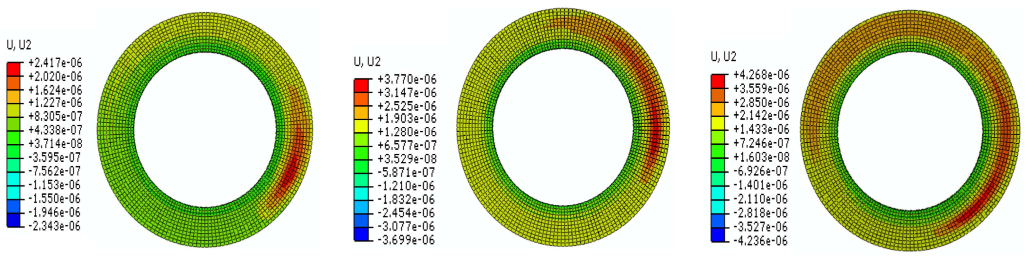


Fig. 15 Deformation in the surface Vs Radial distance along the rotor



(a) 5 rad/s (b) 10 rad/s (c) 15 rad/s

Fig. 16 Contour plots for deformation in the surface of rotor for 1 sec braking

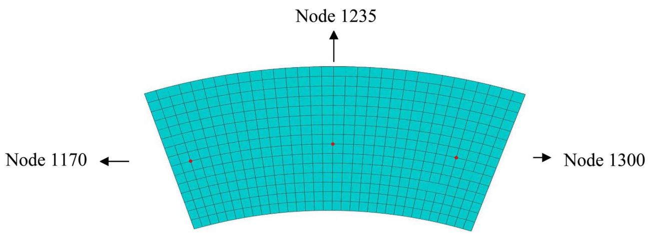


Fig. 17 Location of nodes in pads

propagates in the circumferential region, with the increase in the sliding speed.

As the sliding speed increases the contact pressure and the corresponding nodal temperature increases. The contact pressure increases by about 1.8 times whereas the temperature increases 2.3 times, for increase of 3 times in the sliding speed.

Exact instance of the instability is a complex phenomenon and cannot be determined directly,

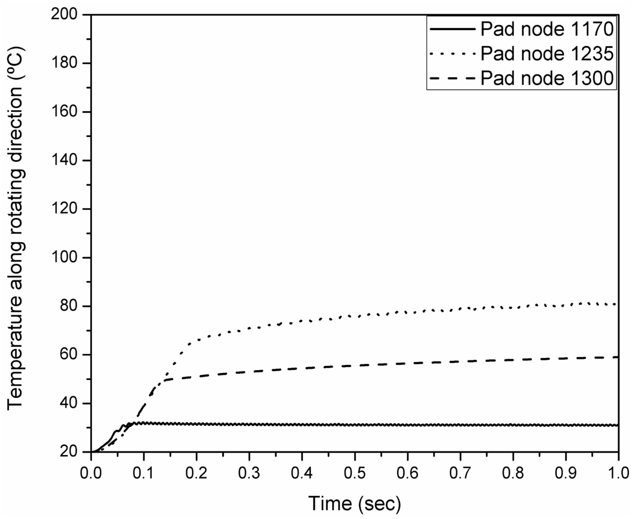


Fig. 18 Temperature Vs Time history plot for different pad nodes for 5 rad/s

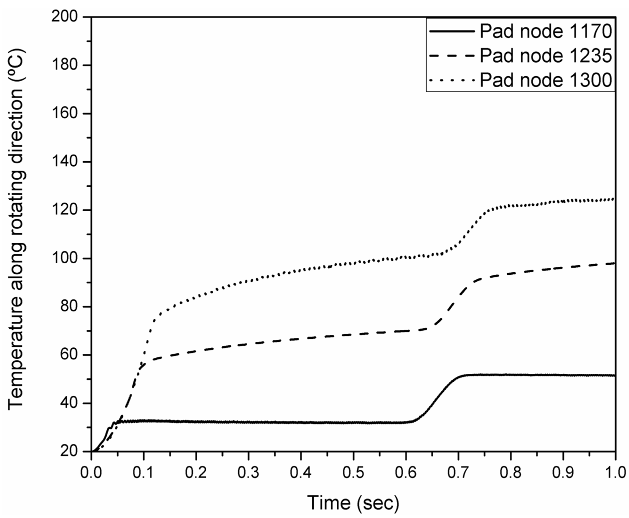


Fig. 19 Temperature Vs Time history plot for different pad nodes for 10 rad/s

which depends on the geometry, material and the contact conditions. Temperature perturbation gives an idea of the onset of the instability. The particular nodes in the pad are selected for the study and the locations are selected as from the literature (Lee and Barber 1994) to get exact perturbation along the contact surface between the disc and pad. The nodal temperature in the surface of the pads during the braking is shown in Figs. 18, 19 and 20 for different sliding speeds.

After a sudden increase in the temperature due to the heating effect, the nodal temperature is almost constant till the next cycle of exposure and the process continues. As the sliding speed increases the localized region/spot get exposed to frictional heating more frequently and the temperature concentration becomes more severe. This could be more clearly observed from the plot of the temperature Vs time history plot at the sliding speed of 15 rad/sec shown in Fig. 21 where in

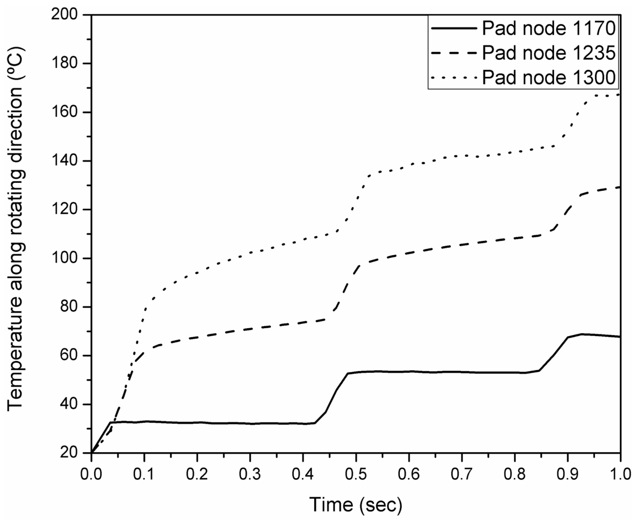


Fig. 20 Temperature Vs Time history plot for different pad for sliding speed 15 rad/s

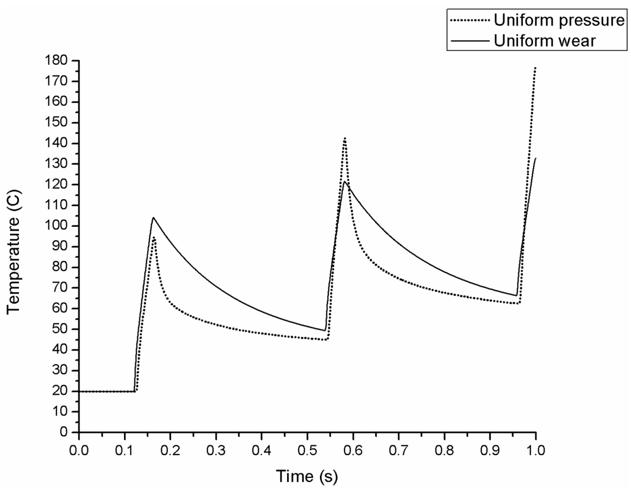


Fig. 21 Temperature Vs Time history plot for uniform pressure and wear condition in rotor (ω = 15 rad/sec)

the heating cooling effect is reflected with progressive increase in the disc mean temperature in both the cases, more drastic in the case of uniform pressure than in the case of uniform wear. Thus the mean temperature generated due to the frictional heat generation between the master (disc) and slave (pad) surface increase.

As the sliding speed increases temperature variation and local hot spot region becomes perceptibly non-linear which is an indication of the onset of Thermo-elastic instability TEI. The contact area varies due to the change in the material properties and hence the contact pressure and the temperature vary according to it. When the variation in the contact area becomes random the change in temperature also becomes random and unstable. This variation is defined as the TEI leads to the formation of the macroscopic hot spots at the surface of the disc brakes. The temperature contour plot shown in Fig. 12 of the disc brake rotor provides such evidence. Hence there exist a correlation between the increase in sliding speed and the hot spot formation. These hot spots as mentioned earlier occur repetitively in the same location and change the mechanical and thermal properties of the surface. This leads to the numerous problems in disc brakes like fading, judder squeal etc.

Exact prediction of the critical sliding speed needs a clear understanding of the contact conditions, material and geometrical constraints involved. Continuing the analysis till temperature perturbation results in will give the critical sliding speed for the onset of TEI.

10. Conclusions

The objective of this work is to develop a coupled temperature displacement model and advocate a technique for finding the range of critical sliding speed for a three-dimensional disc brake system. Due to its tremendous time and cost requirements, there are no known works on instability simulation evolving via a stable region, but excellent efforts on determining instability conditions using Eigen value formulation have been reported by Barber and co-workers. For the purpose of design and analysis, understanding both transient behaviors and post-instability is of paramount importance in the industry. Thus, in this study using the commercial finite element package ABAQUS® the transient thermo elastic contact problem is solved for different sliding speed within the working operational range. The braking time considered for this analysis s 1 second which is quantitatively low but still gives the implication of the long term braking for drag brake condition. At particular sliding speed the temperature-time history plot turns non-linear and perturb-ate, whereby the Thermo Elastic Instability is clearly established. So the approximate range of the region for critical sliding speed is defined and the future computational work is to be done so that the exact value of critical speed can be identified. The inter relation between the critical sliding speed and the number of hot spots is also need to be sorted out.

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List of symbols

C	$\rightarrow$	Specific heat capacity, J/kgK
p	$\rightarrow$	Contact Pressure, Pa
p_{max}	$\rightarrow$	Maximum Braking pressure, Pa
k	$\rightarrow$	Constant of proportionality for wear coefficient
r	$\rightarrow$	radius of the rotor, m
δ	$\rightarrow$	wear coefficient
h_r	$\rightarrow$	Convective heat transfer co-efficient, W/m^2K
λ_{air}	$\rightarrow$	Thermal conductivity of air, W/mK
ν^{air}	$\rightarrow$	Viscosity of air, kg/ms
m	$\rightarrow$	mass of the body, kg
q	$\rightarrow$	Heat flux, W/m^2
μ	$\rightarrow$	Coefficient of friction
v	$\rightarrow$	Sliding speed, m/s
V_{cr}	$\rightarrow$	Critical sliding speed, m/s
$d\theta$	$\rightarrow$	Temperature gradient for the Rowson model, C
$d\phi$	$\rightarrow$	Change in angle for polar coordinate
t	$\rightarrow$	Time of braking, s
t_s	$\rightarrow$	Stopping time, s
a	$\rightarrow$	Prandtl number
Q	$\rightarrow$	Heat flux density, W