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Establishing non-linear convective heat transfer coefficient

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Abstract. The aim of the work presented in this paper is development of numerical model for prediction of temperature distribution in pavement according to the measured meteorological parameters, with introduction of non-linear heat transfer coefficient which is a function of temerature difference between the air and the pavement. Developed model calculates heat radiated from the pavement back in the air, which is an important part of the heat trasfer process in the open air surfaces. Temperature of the pavement surface, heat radiation together with many meteorological parameters were measured in series during two years in order to validate the model and calibrate model parameters. Special finite element method for temperature heat transfer towards the soil together with the time integration scheme are used to solve the governing equation. It is proved that non-linear heat transfer coefficient, which is a function of time and temperature difference between the air and the pavement, is required to decribe this phenomena. Proposed model includes heat tranfer coefficient callibration for specific climate region, through the iterative inverse procedure.

Keywords: convective heat transfer coefficient; non-linear heat transfer coefficient; pavement temperature; radiated heat; solar radiation; urban pavement

1. Introduction

The built environment has a profound impact on our natural environment, economy, health and productivity. Designers, builders, operators and owners are now seeking breakthroughs in building science, technology and operations to create sustainable built environment and maximize both economic and environmental performance (Council 2016).

Thermal environmental conditions, to which pavements are exposed, have significant impact on pavement performance and their contribution to the urban heat island effect. The temperature distribution within the layers of the pavement structure is one of the most influential environmental factors that affect the mechanical properties and load-bearing properties of asphalt pavements (Arangi and Jain 2015). The main task is to determine influencing parameters for pavement temperature predictions. Many efforts are given in pavement temperature modeling as a function of weather conditions. Extensive research has been conducted in several different climate regions on the topic of road temperature prediction models to determine the highest level of accuracy of the

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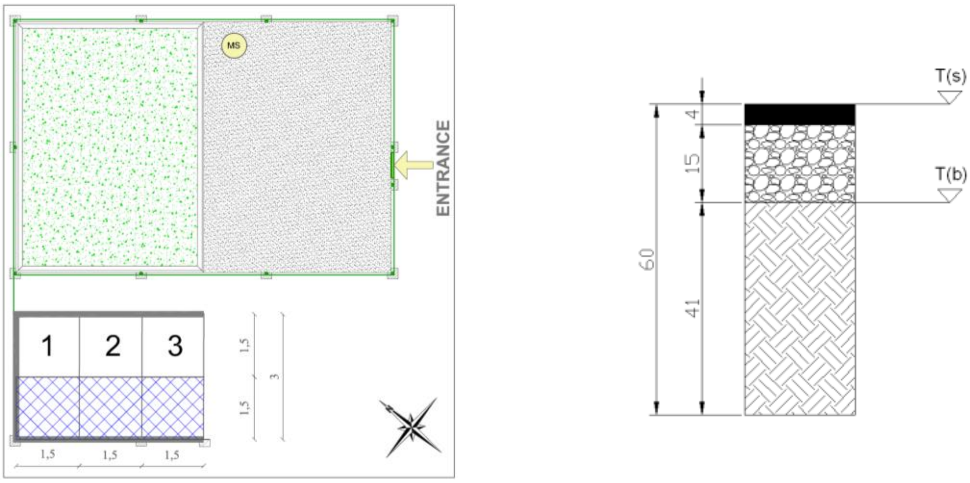


(a) Test field (b) Cross section of the pavement

Fig. 1 Test field position and cross section of the field paving structure

results (Nižetić and Papadopoulos 2018).

The models for temperature prediction of asphalt pavements can be divided in three categories: analytical models, numerical models and empirical models. Analytical models are simple and have many advantages but some results are unreliable due to complexity encountered in deriving the closed form analytical solution for multi-layer pavement system (Wang *et al.* 2009). In numerical analysis the complex heat transfer process is considered to enable pavement temperature prediction under different conditions (Chen *et al.* 2014). Empirical models are the simplest method for developing temperature prediction models, but such methods usually require a large amount of measured data (field and climatic) (Ovik *et al.* 1999).

Many different parameters, such as material properties, air temperature, solar radiation and wind speed, affect the accuracy of results in pavement temperature modeling. Heat exchange problems involving radiation lead to non-linear formulation (e.g., see Lozzi-Kožar and Kožar 2017,Jiji 2006) which can be simplified with careful parameterization (Kožar *et al*. 2018, Kožar *et al.* 2020, Kožar *et al* 2019). Preliminary research with introduction of non-linear convective heat transfer coefficient between pavement and open air has been performed in (Kožar *et al.* 2020)

The following section describes the measurement set-up and gives an overview of measured data. The third section presents the mathematical model followed by the section describing the finite element procedures used in the solution of governing equations (see Kožar and Lozzi-Kožar 2017) for use of finite elements in time derivative calculations). Section five is about calibration of the model describing various boundary conditions and their influence on the heat transfer coefficient. Section six presents some numerical examples followed by the conclusion.

The aim of this work is to prove that non-linear heat transfer coefficient is required and sufficient to describe the phenomena of heat transfer through multi-layer structure.

2. Test set-up and measurement

In order to get insight into the process of heating and cooling of pavement, series of measurements have been performed during the summer months (July and August) in year 2013 and 2015. Test field is set outside the city center where it was isolated from the urban traffic. Small area of 3×4,5 m with three types of paving materials (AC 8 surf, PA 8 and AC 11 surf) has been set up on the university campus near the Faculty of Civil Engineering in Rijeka (see Fig. 1(a)).

On the test field following measurements have been recorded: pavement top and bottom surface temperature, meteorological conditions such as air temperature and humidity, wind speed and direction, solar radiation etc. Thickness of the layer that was observed is 19 cm, consisting of two layers: 4 cm asphalt layer and 15 cm of granular base, Fig. 1(b). Upper surface of the asphalt is noted as *surface* (abbreviated S) and lower surface of gravel base is noted as *bottom* (abbreviated B). Measurements have been made with solarimeter, surface temperature sensors and meteorological station located on the test field.

2.1 Measured data

Sampling rate for air temperature and wind velocity was every 5 minutes, for solar radiation 1 minute and for surface temperature every 10 seconds.

For the purpose of analysis all recorded data were reduced on the same sampling rate, time increment of 1 minute, which means that some data had to be interpolated by linear spline interpolation curve, which appeared to be the best fit. Among many in-situ measurements that have been recorded, in this work four series have been chosen, two during 10 days in july and two during 7 days in august, for two years (2013 and 2015). Recorded data are presented in Fig. 2.

From the charts shown in Fig. 2 it can be seen that all measured data have quite same trend and the quantitative value, which means that observed quantities are constant for every year and observed climate region and materials. It also confirms that measurements are well performed and recorded data are reliable. Considering this fact, focus in this paper is on one-year measurements (2015) in the analyses.

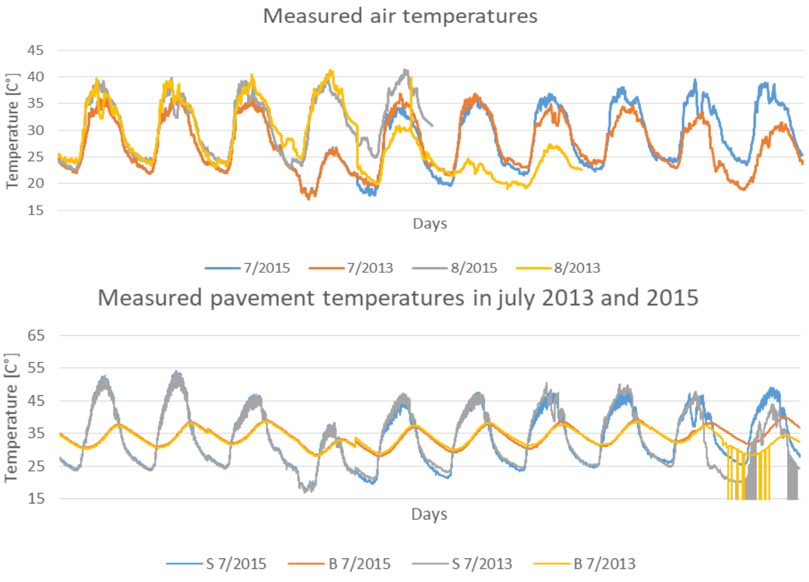


Fig. 2 Data measured in july and august 2013 and 2015

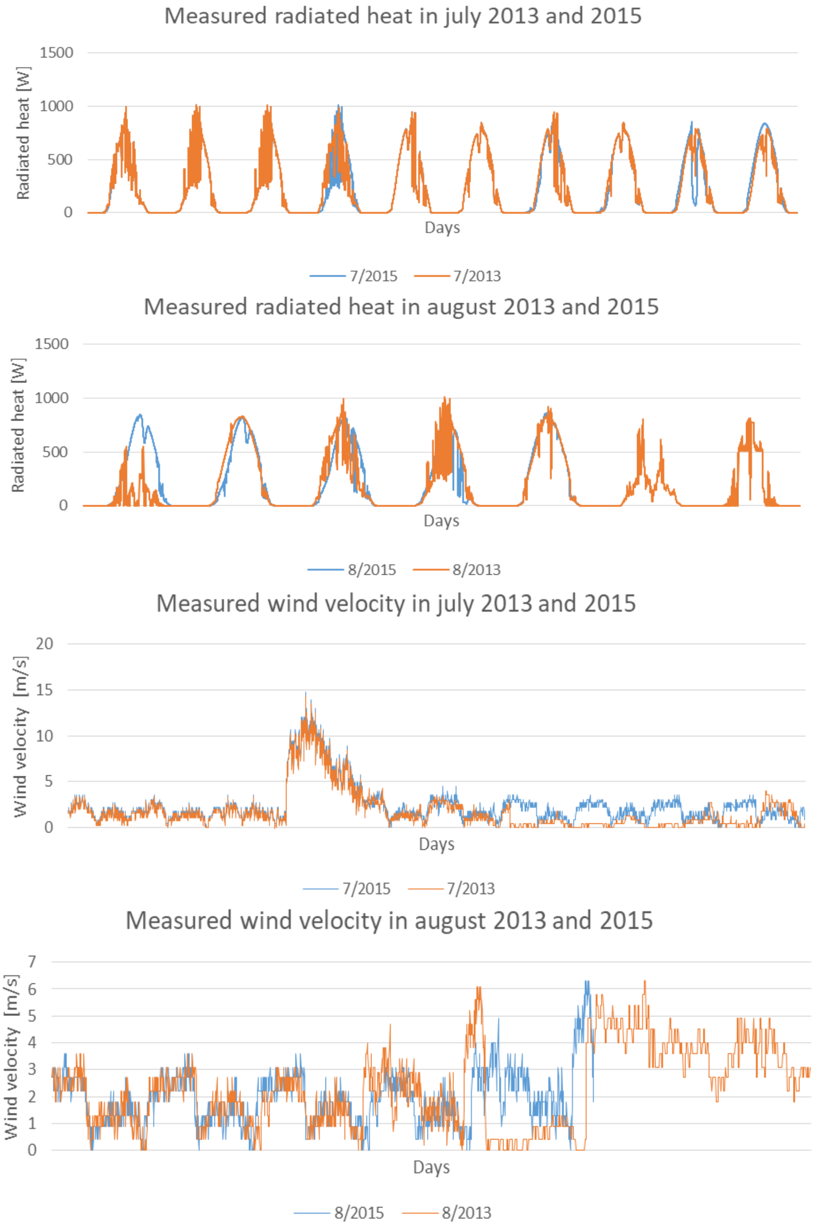


Fig. 2 Continued

3. Mathematical model

The mathematical formulation for the heat exchange between the surface and surrounding air and is based on non-stationary differential equation:

$$\rho c \frac{\partial T_{pav}}{\partial t} - \frac{\partial}{\partial z}\left(k(z)\frac{\partial T_{pav}}{\partial z}\right) = Q \qquad (1)$$

where T_{pav} is pavement temperature, $k(z)$ is diffusivity coefficient, z is pavement depth and Q is rate of heat (coming in or out), ρc is unit mass and heat capacity of the pavement. In our case, Q comprises of solar radiation as a source and convective part as a boundary condition.

In (Kožar and Lozzi-Kožar 2017), based on (Lewis 2004) we have shown that the heat capacity matrix is depth dependent and reads

$$C(c_\rho) = \int_0^{l_e} c_\rho N(z) \frac{dN(z)}{dz} dz \tag{2}$$

Also, the thermal conductivity matrix is written as

$$K(k) = \int_0^{l_e} N(z) \left[\frac{\partial}{\partial z} \left(k(z) \frac{\partial T}{\partial z} \right) \right] dz \tag{3}$$

4. Finite element model

Spatial domain is a one-dimensional vertical profile going through the pavement and the granular base, see Fig. 3. Considered depth is 19 cm (Fig. 1(b)), since it is supposed that bellow that point temperature is constant and does not influence on the surface and surrounding air temperature, which is confirmed with temperature distribution through depth, see Fig. 10. One-dimensional analysis is performed and heat transfer in vertical direction only is observed since the pavement horizontal dimensions are significantly greater then vertical one so the heat loss in that direction is negligible.

Fig. 3 Finite element mesh

Twenty finite elements are used for space discretization and time increment of 60 seconds in backward Euler time integration procedure. Discretized model equation in matrix notation reads

$$\mathrm{C}\frac{\Delta \mathrm{T}}{\Delta \mathrm{t}} + \mathrm{KT} = \mathrm{Q} \tag{4}$$

where C and K are heat capacity and heat conductance matrices respectively, and Dt is time increment. Heat conductance matrix consists of two parts: standard part and part with heat flux.

$$K = K_{CD} + K_{CV} \tag{5}$$

K_{CD} is a standard finite element stiffness matrix, while the local matrix, which accounts for the convective part is

$$\mathrm{K_{CV}} = \frac{\mathrm{hPl}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{6}$$

Right side of the equation consists of heat flux and convection influence

$$\mathrm{Q} = (\mathrm{q} + \mathrm{hPT})\mathrm{l}\begin{Bmatrix} 1/2 \\ 1/2 \end{Bmatrix} \tag{7}$$

Special finite element for heat flux towards the soil is used to avoid measuring temperature at a certain depth below the pavement. These elements could have boundary condition expressed in the terms of heat flux, which is usually not measured on the bottom, so the intermediate temperature measurement is used here for determination of the bottom heat flux and for validation purpose.

Equation (1) is solved using Backward Euler procedure for iteration in time domain with time increment *Dt*=60 seconds and 1440 iterations in total. Analyses are based on one day period.

Values needed for imposing the boundary conditions are obtained from the meteorological station on the test field. Time variable temperature (Dirichlet boundary condition) of the pavement surface and heat flux (Neumann) are imposed for initial and boundary conditions on the first node. Time variable temperature on the bottom of the pavement layer is set as a boundary condition on the last node, while the air temperature is incorporated in the convective heat transfer coefficient, or convective (Cauchy) boundary condition in the first element.

Thermal properties of the paving materials are taken from the literature and are given in the Table 1.

Table 1 Thermal properties of pavement materials

	Asphalt layer	Granular base	Subgrade
Thermal conductivity [W/mK]	0,75	0,25	2,00
Specific heat capacity [J/kg K]	920	800	1400

4.1 Heat convection coefficient

The relationship between the pavement surface temperature and radiated heat is obtained with the introduction of the convective heat transfer coefficient between the pavement and the open air. It is shown that this coefficient has to be nonlinear in order to include all phenomena that influence on the temperature exchange, during heating and especially cooling of the pavement. It is nonlinear because it depends on the temperature change and wind velocity. Or, it could be related to the temperature difference and in that case, we could suppose that the temperature depends on the wind velocity or that wind velocities are so small that their influence can be neglected. The second assumption is acceptable and true here, since the measurements have been made in the urban area and very close to the pavement surface, where the wind velocity is below 5 m/s.

It is obtained from thermal equilibrium which leads to the nonlinear equation

$$\alpha_{\mathrm{solar}} q_{\mathrm{solar}} + \epsilon_{\mathrm{IR}} \sigma T_{\mathrm{sky}}{}^{4} = \bar{h}\left(T_{\mathrm{pav}} - T_{\mathrm{air}}\right) + \epsilon_{\mathrm{IR}} \sigma T_{\mathrm{pav}}{}^{4} \tag{8}$$

where a_{solar} and e_{IR} are solar absorbtance and infrared emittance for the pavement, q_{solar} is solar radiation in [W/m^2] and $\bar{h}$ is convective heat transfer coefficient. The sky temperature could be assessed using empirical relation such as one given in (Berdahl 1984).

The convective heat transfer coefficient h is related to the eddy diffusivity principle (Lozzi-Kožar and Kožar 2017) and the thin film concept as in (Jiji 2006). The idea is that the constant in the Eq. (8) could be replaced with a functional value that comprises the influence of some other parameters which is not explicitly appearing in the equation. Eq. (8) can be rewritten in order to obtain heat flux from the radiation in a form similar to Newton heat exchange law, i.e., $q=h(DT)DT$, where $h(DT)$ is convective heat transfer coefficient as a function of temperature and $\Delta T = (T_{pav} - T_{air})$.

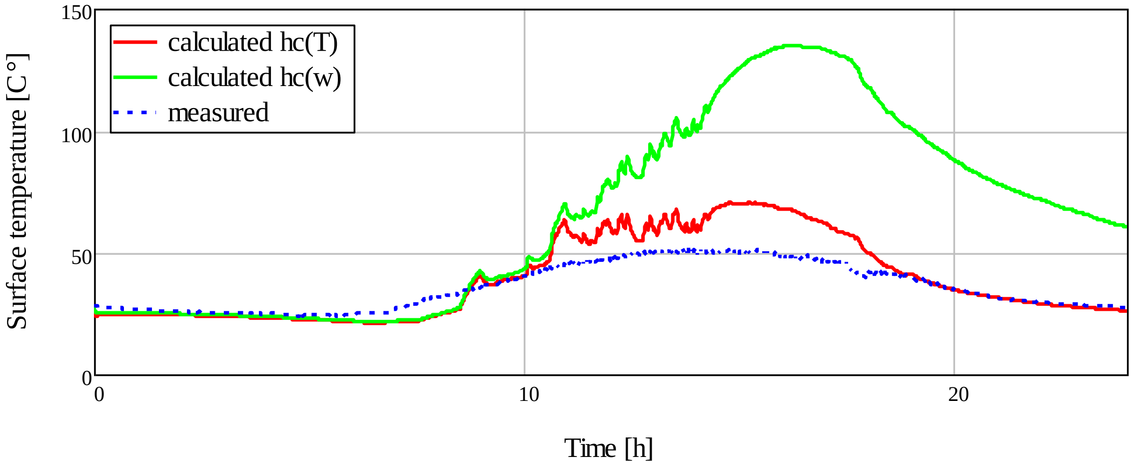


Fig. 4 Calculated pavement temperature for one day and convective heat transfer coefficient given in Eqs. (9)-(11)

Different convective heat transfer coefficients can be found in the literature, and here are considered and compared three types of them: related to wind velocity, related to temperature difference between the pavement and the air and related to both of them. They are given in the sequent.

Wind velocity dependent model adopted here is (Qin and Hiller 2014, Qin 2016)

$$h_c = 5{,}6 + 4{,}0\ U \quad for\ U \le 5\frac{m}{s}$$
$$h_c = 7{,}2U^{0{,}78} \quad for\ U > 5m/s \tag{9}$$

where U is wind velocity in m/s.

Group of nonlinear models based on meteorological conditions, which are used in EICM for pavement design, named Vehrecamps models are found in the literature to be most suitable for the temperature prediction, since it takes into account real heat convection caused by the temperature difference and wind velocity near surface.

$$h_c = 122{,}93\left[0{,}00144T_{mK}^{0{,}3}U^{d_c} + 0{,}00097(T_s - T_a)^{0{,}3}\right], d_c = 0{,}7 \tag{10}$$

Where U is wind velocity in [m/s], T_{mk} is temperature [K] obtained as average between the air and surface temperature, T_s [K] is upper surface temperature and T_a [K] is air temperature. But this model reduces temperature influence on the h, with multiplied constant and power value, so that this model could be compared to the one given in Eq. (9). Results obtained with these models are given in Fig. 4. with green curve. When compared to the blue curve which represents measured values discrepancy in the results is obvious.

Heat transfer coefficients which function of temperature is introduced in a form

$$h_c(T) = \bar{h}\left(T_{pav} - T_{air}\right)^n \tag{11}$$

Where n=1/2 is adopted, and $\bar{h} = 300$ W/(mK). Relation in Eq. (11) has given the best results for the predicted pavement temperature when compared to the measured ones, see red curve in Fig. 4. Heat transfer coefficients from Eqs. (9)-(11) are depicted in Fig. 5.

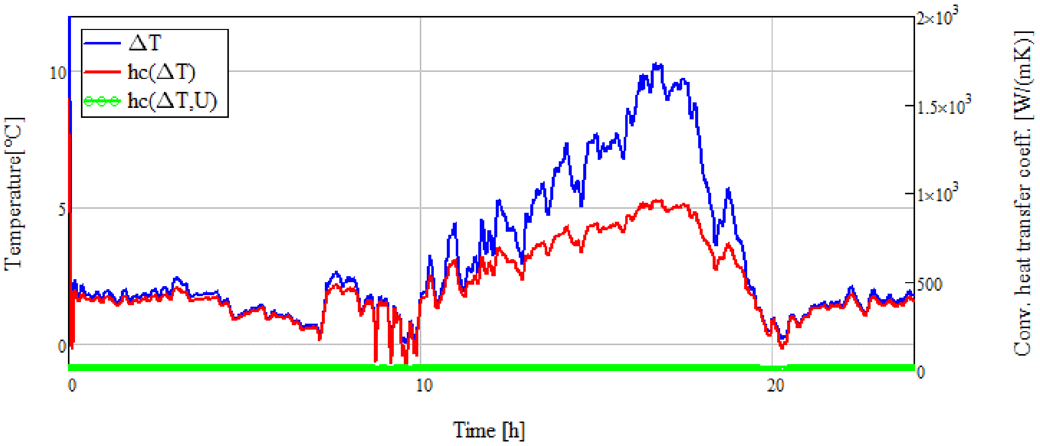


Fig. 5 Nonlinear convective heat transfer coefficient given in Eq. (8) compared to measured temperature difference

From results shown in Figs. 4-5 one can see that temperature difference (as a function of time) has dominant influence over the wind velocity, even in the case of higher wind velocities (over 5 m/s), Eq. (9). From the Fig. 5 and comparison of all heat transfer coefficient models introduced here we can confirm that heat transfer coefficient has to be nonlinear, but it has to be measure of the temperature difference as the red one from the Eq. (11) is in Fig. 5. This means that the green curve in Fig. 5 (showing h_c) from Eq. (10) has the same curve shape as DT, but significantly smaller measure, and it does not reflect real conditions, it underestimates temperature influence on heating and cooling process.

5. Calibration of the model

Although $h_c(DT)$ has given the best results, accuracy of the solution is not satisfactory, which means that some influencing parameters are still missing in the model. Separation and identification of all influencing parameters here is very demanding task. But in order to obtain all these parameters as one model calibration procedure is proposed here. It is based on the iterative inverse procedure for obtaining h_c such that contains all influencing parameters. Through iterative process using measured values of the temperature convective heat transfer coefficient is corrected and calibrated in order to reflect real conditions and to give acceptably accurate solution. The procedure is validated through comparison with measured data, and it is proved that even if we start iterative procedure with constant h_c after calibration process we obtain such h_c that produces solution confirmed with measured values, see red curve in Figs. 6-7. Through only three iteration steps calibrated heat transfer coefficient is obtained using this scheme

$$\overline{h_c}(i) = \overline{h_c}(i-1) + \Delta T(i-1)C \tag{12}$$

$$\overline{h_c}(1) = \bar{h}\left(T_{pav} - T_{air}\right)^n \tag{13}$$

$$\Delta T(i) = T_{pav}(i) - T_{air} \tag{14}$$

Where C is calibration parameter.

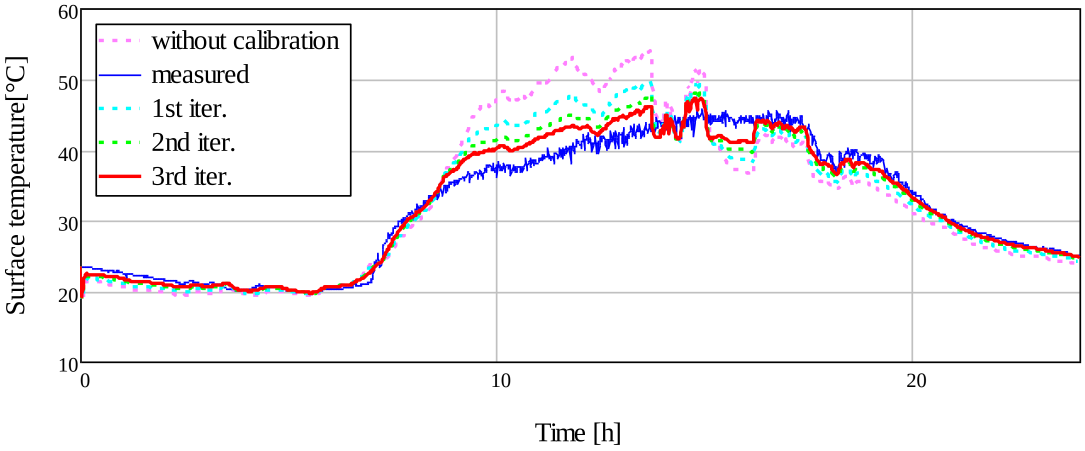
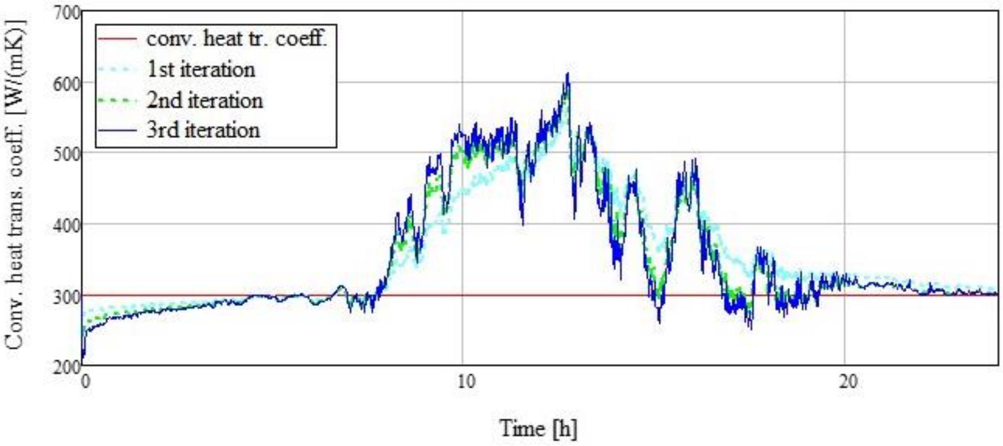
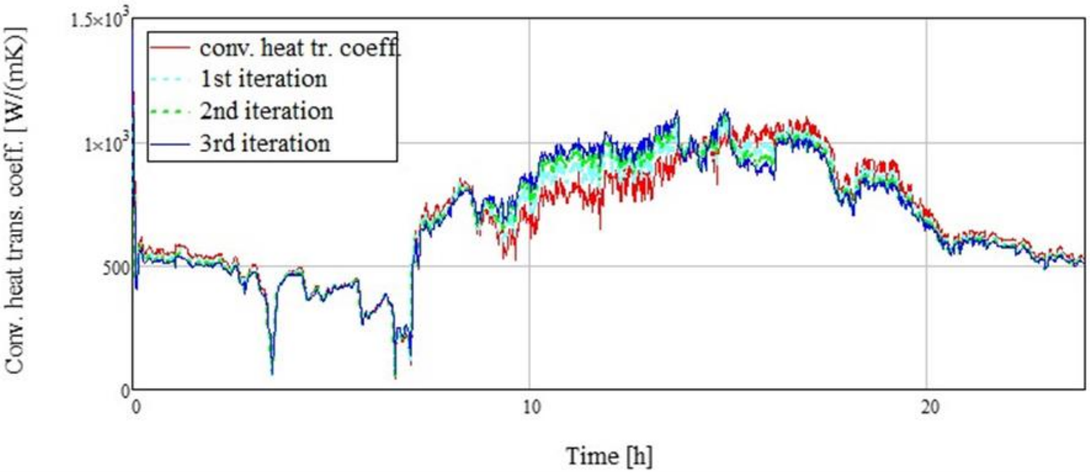


Fig. 6 Calculated pavement temperature for one day and convective heat transfer coefficient given in Eq. (8)

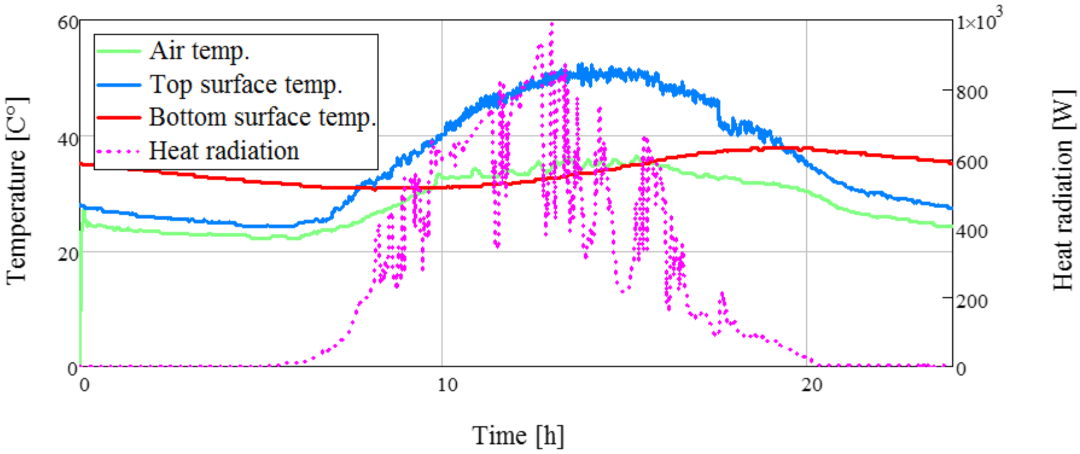


(a) constant heat transfer coefficient h_c=const.

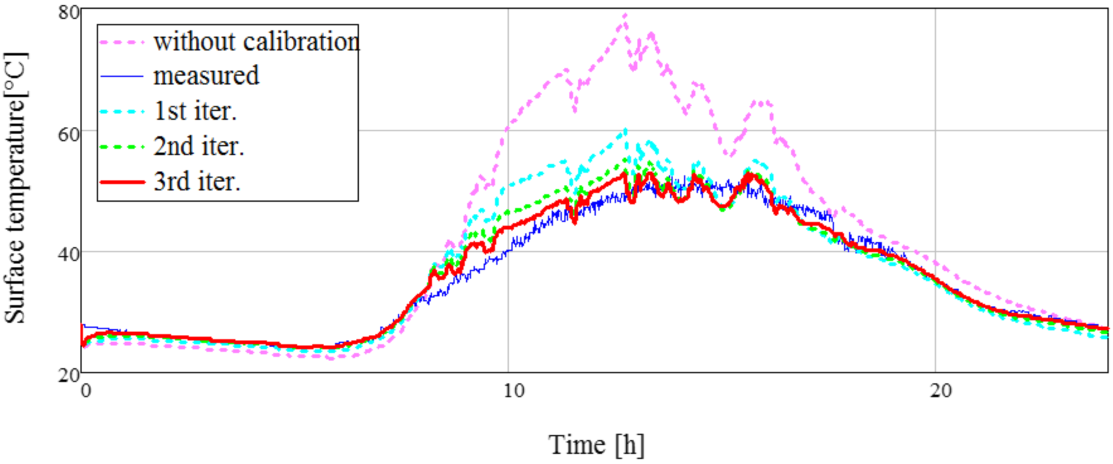


(b) Nonlinear convective heat transfer coefficient $h_c(DT)$

Fig. 7 Convective heat transfer parameter obtained through iterative inverse procedure



(a) Measured values of air and pavement temperature and solar radiation



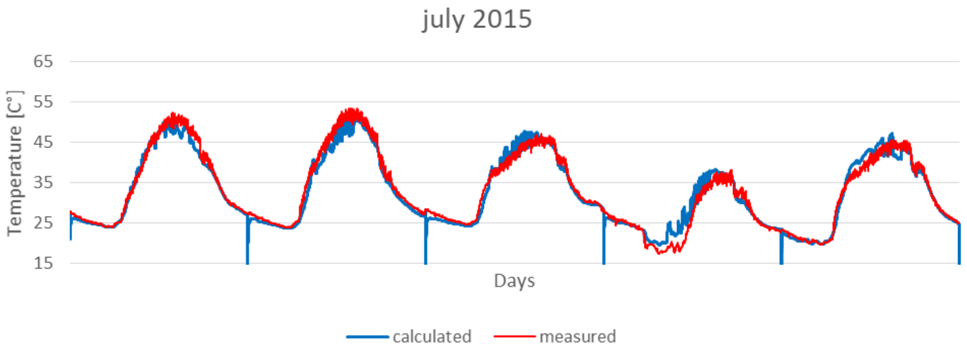
(b) Calculated values through the iterative procedure

Fig. 8 Measured and calculated values for one day sample

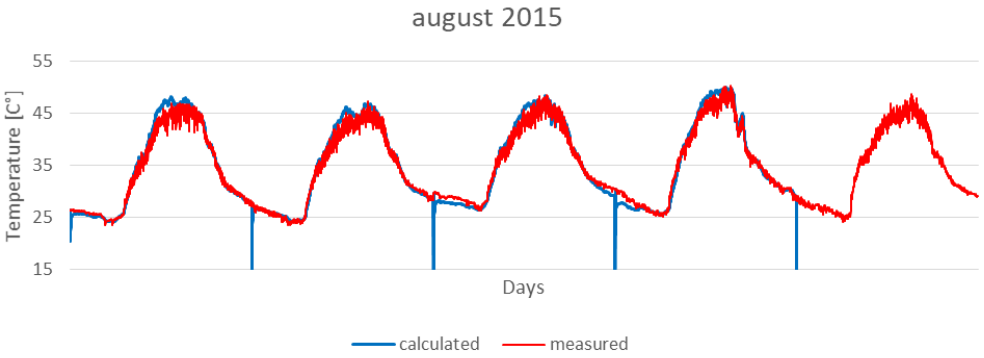
6. Numerical examples

Numerical examples use nonlinear convective heat transfer coefficient determined through the calibration procedure starting from the $h_c(DT)$ relation given in Eq. (11). Analyses results for five days in July and four days in August 2015 are given and compared with the measured values. All measured values are shown in Fig. 2, and for one day in Fig. 8.

Pavement temperature change in time for consecutive series of days is given in Fig. 9. It can be seen that for any weather conditions, air temperature and solar radiation, accurate results can be obtained with proposed model and iterative procedure, if convective heat transfer coefficient is calibrated for specific climate region. Obviously it includes some other influences beside temperature, radiation, humidity etc. such as soil and terrain configuration and it should be calibrated for specific region ones with use of measured values and further can be used for all necessary analyses and predictions.



(a) July 2015



(b) August 2015

Fig. 9 Calculated pavement temperature for series of days compared with measured values

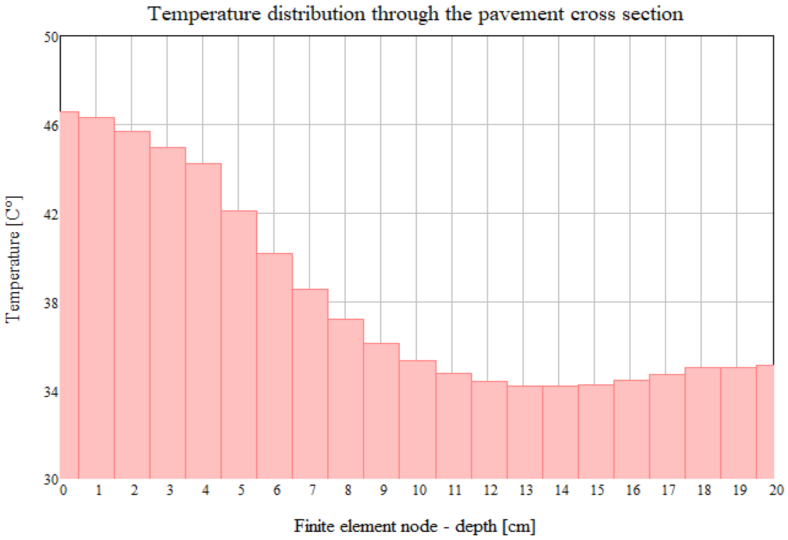


Fig. 10 Temperature distribution through vertical pavement profile

Beside pavement surface temperature change in time, temperature distribution through the vertical pavement cross section is also calculated in every analysis and one of them is shown in Fig.

10. Graphed distribution confirmes that one-dimensional analysis of depth of 19 cm paving material was a reasonable choice, since the temperature at the end of observed domain is pretty constant.

7. Conclusions

A numerical simulation procedure for prediction of radiated heat in urban pavements is presented in this work. It is based on the novel method for establishing nonlinear convective heat transfer coefficient which is a function of time and temperature difference between the air and the pavement and calibrated for the specific conditions. Heat radiated during the night from the pavement into surrounding air is taken into consideration in this model. Nonlinear convective heat transfer coefficient which includes all influencing parameters is obtained through the inverse procedure. Model is calibrated with use of series of measured values. The following conclusions have been made:

- temperature influence is dominant over the wind velocity, even in the case of higher wind velocities near the pavement surface
- radiated heat modeling requires introduction of nonlinear convective heat coefficient, which is a function of time and temperature difference between the air and the pavement surface
- additionally, it is shown that this coefficient should be calibrated for specific climate region or obtained with inverse procedure from the measured meteorological conditions
- further developments should include detailed inverse procedure for determination of h, similar to (Kožar, Torić Malić, & T., Inverse model for pullout determination of steel fibers, 2018.) which could provide explicit expression of convective heat transfer coefficient

Convective heat transfer coefficient is the crucial parameter in modeling radiated heat from urban pavement and should be carefully chosen and in addition calibrated for specific climate area.

Acknowledgments

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