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Parametric study of piled raft for three load-patterns

V.A. Sawant*, S.V. Pawar and K.B. Ladhane

Indian Institute of Technology Roorkee, India

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Abstract. Paper presents an improved solution algorithm based on Finite Element Method to analyse piled raft foundation. Piles are modelled as beam elements with soil springs. Finite element analysis of raft is based on the classical theory of thick plates resting on Winkler foundation that accounts for the transverse shear deformation of the plate. Four node, isoparametric rectangular elements with three degrees of freedom per node are considered in the development of finite element formulation. Independent bilinear shape functions are assumed for displacement and rotational degrees of freedom. Effect of raft thickness, soil modulus and load pattern on the response is considered. Significant improvement in the settlements and moments in the raft is observed.

Keywords: pile; raft; thick plate; winkler foundation; load pattern

1. Introduction

As the use of piled raft foundations as an alternative to conventional piled foundation for tall buildings has been increasing, different technique have been developed for performing analyses over the last decade. Piled rafts are composite structures comprised of the piles, raft and soil. Such foundation will be subjected to the vertical loadings transferred directly from the structure and horizontal loading mostly due to wind loads. These loads are transferred to the soil through the raft and the piles. Unlike the conventional piled foundation design in which the piles are designed to carry the majority of the load, the design of a piled-raft foundation allows the load to be shared between the raft and piles and it is necessary to take the complex soil-structure interaction effects into account. Methods developed for analysis of piled raft foundation incorporate algoritm based on boundary element method, finite element method and combined boundary element and finite element method.

Kakurai *et al*. (1987) examined the settlement behaviour of a piled raft foundation on soft ground. The raft was modelled by beam and bending elements. The piles and soil were modelled as vertical springs supporting the raft at selected nodal points. Kuwabara (1989), and Poulos (1993) described a boundary element analysis based on elastic theory to examine the behaviour of a piled raft foundation in a homogeneous elastic soil mass. Mendonça and de Paiva (2000) presented a boundary element method for the analysis of piled rafts in which full interaction between the raft, piles and the soil is considered. A coupled boundary element and finite element formulation was

* Corresponding author, Assistant Professor, E-mail: sawntfce@iitr.ernet.in

described by Mendonça and de Paiva (2003) in which full interaction of the structure has been incorporated into the analysis.

Hooper (1973) studied the behavior of piled raft foundation supporting a tower block in central London. The field measurements taken during several years are presented, together with the results of a detailed finite element analysis. The analysis is carried out assuming uniformly distributed load on the raft. Based on the field measurements the estimated proportions of load taken by piles and the raft at the end of construction were 60% and 40%. The long-term effect of consolidation was found to increase the load carried by piles and to decrease raft contact pressure. Franke (1991) discussed design of 4 buildings supported on piled raft in Germany. The analysis shows that compared to a raft foundation, piled raft reduces the settlement by about 50%. The actual measurements of pile head forces, contact pressure between raft and soil, and the settlements of piled raft for some of these buildings also reported. Noorzaei *et al*. (1991) used beam element, plate element and brick element to model space frame, raft and soil in the analysis. A detailed parametric study of the effect of variation in raft thickness on space frame-raft soil system had presented. Wiesner (1991) presented a method for the analysis of a circular piled raft that was constructed in Cairns. Clancy and Randolph (1993) employed a hybrid method which combined finite elements and analytical solutions. The raft was modelled by 2-D thin plate elements, the piles were modelled by 1-D bar elements and the soil response was calculated by using an analytical solution.

Yamashita *et al*. (1994) reported a five story building on piled raft foundation of size 24 m × 23 m with 20 piles of length 16 m and diameter 0.75 m. The results of field observations during construction and analytical study of the same building have been compared. Gandhi and Maharaj (1996) have reported the load sharing between pile and raft based on three-dimensional linear finite element method. The effects of spacing, soil modulus and length of pile on load sharing between pile and raft have been discussed. Smith and Wang (1998) proposed the use of iterative techniques with the finite element method to examine the behaviour of a non-uniformly loaded piled raft. Prakoso and Kulhawy (2001) analysed piled raft foundations by the use of linear elastic and non-linear plane strain finite element models which involved the analysis of a three-dimensional piled raft as a two-dimensional strip piled raft.

Poulos (2001) discussed the philosophy of using piles as settlement reducers and the conditions under which such an approach may be successful. The design process for a piled raft was considered as a three-stage process. In the preliminary stage the effects of the number of piles on load capacity and settlement were assessed via an approximate analysis. A more detailed examination, to assess where piles are required and to obtain some indication of the piling requirements in the second stage. The third is a detailed design phase in which a more refined analysis is employed to confirm the optimum number and location of the piles, and to obtain essential information for the structural design of the foundation system. Some typical applications of piled rafts were described, including comparisons between computed and measured foundation behaviour. Cunha *et al*. (2001) extended the design philosophy for piled rafts by exploring the factors that control the design of a published case history where the piled raft was instrumented. An extensive series of backanalyses was initially carried out with this case history, in order to calibrate the numerical program adopted. These analyses were followed by a parametric analysis for the evaluation of different design alternatives. These alternatives adopted distinctive pile characteristics (number, location, and length), which were varied for different raft thicknesses, yielding 26 different cases for cross comparison and comparison with the "reference" case history. Each of the solutions was also assessed in terms of relative costs against the reference case, allowing the establishment of conclusions of practical interest for those involved in the design of piled raft structures.

Maharaj (2003) presents the results based on three dimensional nonlinear finite element analysis of piled raft foundation. It has been found that the ultimate load carrying capacity of flexible raft increases with increase in soil modulus and length of pile. It has also been found that although the increase in soil modulus reduces the overall settlement, and the differential settlement increases with increase in soil modulus for the same overall settlement. Reul and Randolph (2003) presented a three-dimensional elasto-plastic finite element method for the analyses of piled raft foundations in overconsolidated clay - Frankfurt clay. Reul and Randolph (2004) analyzed, 259 different piled raft configurations by means of three-dimensional elastoplastic finite element analyses. In the study, the pile positions, the pile number, the pile length, and the raft-soil stiffness ratio as well as the load distribution on the raft had been varied. The results of the parametric study were presented and design strategies for an optimized design of piled rafts subjected to nonuniform vertical loading were discussed.

Maharaj and Gandhi (2004) presented a non-linear finite element method for the analysis of a piled raft subjected to a uniformly distributed load. The non-linear behaviour of the soil was modelled by the Drucker-Prager yield criterion. Hasen and Buhan (2005) proposed a two-dimensional multi-phase model for the analysis of soil structures reinforced by stiff inclusions in the context of an elastoplastic behaviour for both the soil and the reinforcements. A finite element numerical tool incorporating a plasticity algorithm was developed and illustrated for the example of a piled raft foundation. Seo *et al*. (2006) investigated a piled-raft system with disconnection gap, sand cushion, between the pile and raft to compare the influence of ultimate bearing capacity and settlement. Load-settlement relation curves were used to evaluate the ultimate bearing capacity. In the numerical analyses, a plane strain elasto-plastic finite element model (Mohr-Coulmb model) was used to present the response of the piled-raft foundation.

de Sanctis and Russo (2008) reported the main criteria adopted for the design and some aspects of the observed behavior of the piled foundations of a cluster of circular steel tanks. The piles were designed to reduce the settlement and improve the overall performance of the foundations. While conventional capacity based design approach led to a total of 160 piles to support the five tanks the settlement based design approach led to a total of 65 piles achieving significant savings on the cost of the project. Effect of pile configuration, pile number, pile length and raft thickness on piled raft foundation behaviour were considered by Rabiei (2009) in the parametric study. It has been found that the maximum bending moment in raft increases with increase in raft thickness, decrease in number of piles and pile length. Central and differential settlement decreases with increase raft thickness and uniform increase in pile length. It has also been found that pile configuration is very important in pile raft design. In the scope of this paper, the results of the parametric study are presented and design strategies for piled rafts are discussed.

Kitiyodom *et al*. (2009) carried out post-analysis of the deformation of a large piled raft foundation using a three-dimensional analysis program PRAB. The soil parameters used in the analysis were obtained from the back analysis of the results of the pile load test that was conducted at the construction site. In the deformation analysis of the whole foundation, the concept of the equivalent pier was employed. The results of the analysis match well with the measured distribution of the foundation settlements. Sonoda *et al*. (2009) A building and its foundation were constructed in sandy ground using a reverse construction method. The measured settlements were scompared with those predicted in the design stage, satisfying the design requirements for the building. Post-analysis of the deformation of the foundation was carried out using the results of the pile load test at the construction site, and the results of the analysis are compared with the observed settlements of the foundation, aiming at an improvement in pile foundation design.

Based on literature review it is found that few studies were carried out on piled raft system with piles of different dimensions. In the present study finite element analysis of piled-raft is presented by modeling raft as thick plate resting on Winkler foundation. Piles are modeled as beam elements with soil springs. The pile dimensions are kept constant as our main emphasis is to compare the response between two raft foundations (with and without pile) of different thicknesses and soil conditions. Effect of raft thickness, soil modulus and load pattern on the response is considered. Results of the piled-raft foundation are compared with only raft foundation system. Significant improvement in the settlements and moments in the raft is observed.

2. Finite element formulation

2.1 Raft

The finite element method transforms the problem of plates on elastic foundation into a computer-oriented procedure of matrix structural analysis. The plate (raft) is idealized as a mesh of finite elements interconnected only at the nodes (corners), and the soil is modeled as a set of isolated springs (Winkler foundation).

The finite element analysis adopted for raft is based on the classical theory of thick plates resting on Winkler foundation (Fig. 1) that accounts for the transverse shear deformation of the plate. The formulation is based on the assumptions that deflections are small compared with the thickness of plate, and that a normal to the middle surface of the undeformed plate remains straight, but not necessarily normal to the middle surface of deformed plate. The stresses normal to the middle surface are considered negligible.

Four node, isoparametric rectangular elements (Fig. 2) with three degree of freedom per node (the transverse displacement w, rotation about x-axis θ_x, and rotation about y-axis θ_y) are considered in the development of finite element formulation. Independent bilinear shape functions are assumed for displacement and rotational degrees of freedom.

The strain energy of an isotropic, linear elastic plate including transverse shear deformation effects can be expressed as

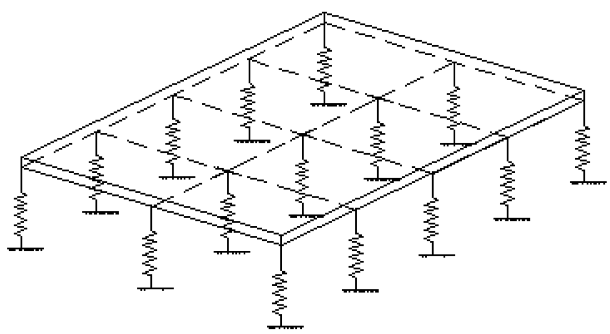


Fig. 1 Structural idealization of raft and supporting soil

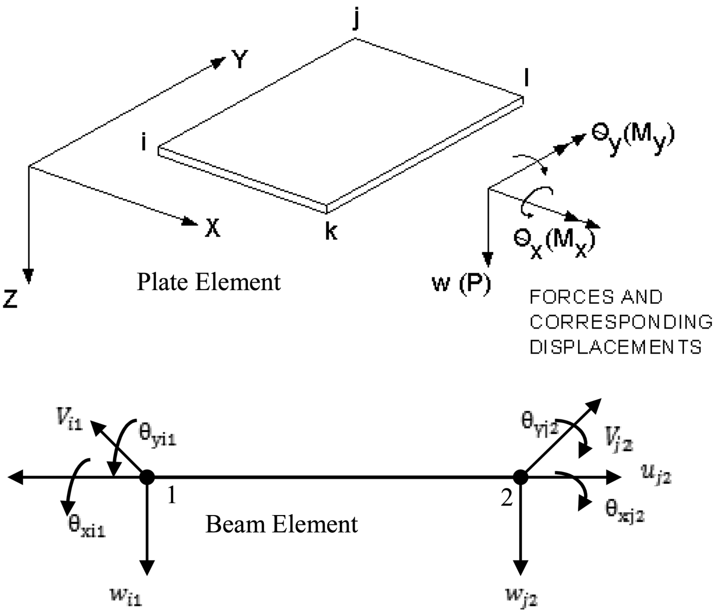


Fig. 2 Rectangular plate element and Beam element

$$U = \frac{1}{2}\int_A [H^T][D_b][H]dA + \frac{1}{2}\int_A [\gamma^T][D_s][\gamma]dA - \int_A w\ q(x,y)\ dA \tag{1}$$

$$[H] = \begin{Bmatrix} \dfrac{\partial\theta_x}{\partial x} \\ -\dfrac{\partial\theta_y}{\partial y} \\ \dfrac{\partial\theta_x}{\partial y} - \dfrac{\partial\theta_y}{\partial x} \end{Bmatrix}, \quad [D_b] = \frac{Et^3}{12(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} \tag{2}$$

$$[\gamma] = \begin{Bmatrix} \dfrac{\partial w}{\partial y} - \theta_y \\ \dfrac{\partial w}{\partial x} + \theta_x \end{Bmatrix}, \quad [D_s] = \frac{Et\kappa}{2(1+\nu)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

In the above equations, q is the intensity of load per unit area, E is the Young's modulus, ν is the Poisson's ratio, κ is the shear correction factor, t is the plate thickness and A is the element area. Using independent shape functions, the nodal variables w, θ_x, and θ_y can be written in matrix form as

$$\begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix}\{d\} = [N]\{d\}$$

$$with\ \{d\}^T = [w_1\ \theta_{x1}\ \theta_{y1}\ w_2\ \theta_{x2}\ \theta_{y2}\ w_3\ \theta_{x3}\ \theta_{y3}\ w_4\ \theta_{x4}\ \theta_{y4}] \tag{4}$$

$$[H] = [B_b]\{d\}\ and\ [\gamma] = [B_s]\{d\}\ in\ which$$

$$[B_b]_i = \begin{bmatrix} 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial N_i}{\partial y} \\ 0 & \frac{\partial N_i}{\partial y} & -\frac{\partial N_i}{\partial x} \end{bmatrix}\ and\ [B_s]_i = \begin{bmatrix} \frac{\partial N_i}{\partial y} & 0 & -N_i \\ \frac{\partial N_i}{\partial x} & N_i & 0 \end{bmatrix} \quad i = 1, 4 \tag{5}$$

From which the equation of strain energy can be expressed in a simplified form as follows.

$$U = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\{d\}^T[B_b]^T[D_b][B_b]\{d\}|J|d\xi d\eta$$

$$+\frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\{d\}^T[B_s]^T[D_s][B_s]\{d\}|J|d\xi d\eta - \int_{-1}^{1}\int_{-1}^{1}\{d\}^T[N_w]^T q|J|d\xi d\eta \tag{6}$$

$$where\ [N_w] = [N_1\ 0\ 0\ N_2\ 0\ 0\ N_3\ 0\ 0\ N_4\ 0\ 0]$$

The above equation represents the strain energy due to bending and transverse shear deformation of plate and potential energy due to applied external load. The total strain energy of the raft-foundation system can be obtained by adding the strain energy U_f of the foundation. By assuming full contact between raft and soil-subgrade, the strain energy of foundation with modulus of subgrade reaction E_s can be written as

$$U_f = \frac{1}{2}\int_A w^T k_s w\ dA = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\{d\}^T[N_w]^T E_s[N_w]\{d\}|J|d\xi d\eta \tag{7}$$

The total strain energy U_t of the plate foundation system is given by addition of two.

$$U_t = U + U_f \tag{8}$$

By equating first variation of total strain energy to zero, the force-deflection equation for plate-foundation element can be expressed as

$$[[k_b] + [k_s] + [k_f]]\{d\} = \{Q\} \tag{9}$$

$$where\ [k_b] = \int_{-1}^{1}\int_{-1}^{1}[B_b]^T[D_b][B_b]\ |J|d\xi d\eta\ ;\ [k_s] = \int_{-1}^{1}\int_{-1}^{1}[B_s]^T[D_s][B_s]\ |J|d\xi d\eta$$

$$[k_f] = \int_{-1}^{1}\int_{-1}^{1}[N_w]^T E_s\ [N_w]\ |J|d\xi d\eta\ ;\ \{Q\} = \int_{-1}^{1}\int_{-1}^{1}[N_w]^T\ q\ |J|d\xi d\eta \tag{10}$$

2.1.1 Membrane effect

Since five degrees of freedom $\{u, v, w, \theta_x, \theta_y\}$ are considered in the formulation of beam element, there is difference between the degrees of freedom considered in raft elements and pile elements. To make raft element compatible with beam element, lateral displacements u and v, in X and Y directions are considered in membrane effect for a plate element. For the inplane or membrane loading the plane stress idealization is considered. The nodal displacement vector, $\{\delta\}^T$, given by

$$\{\delta\}^T = \{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\}$$

The coordinates (x,y) of a point within the elements are expressed in terms of local coordinates (ξ,η) of the elements as

$$x = \sum_{i=1}^{4} N_i x_i \qquad and \qquad y = \sum_{i=1}^{4} N_i y_i \tag{11}$$

where, (x_i, y_i) are nodal coordinates of the element and N_i, defines shape functions

For the node numbering shown in Fig. 3 with the element of size $2a \times 2b$, shape functions are

$$N_i = \frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta) \quad where \quad \xi = \frac{x-x_c}{a} \quad and \quad \eta = \frac{y-y_c}{b} \tag{12}$$

where, (ξ_i, η_i) are local coordinates of i^{th} node.

Same shape functions are also used to define displacements u and v, within the element

$$u = \sum_{i=1}^{4} N_i u_i \quad and \quad v = \sum_{i=1}^{4} N_i v_i \tag{13}$$

The strain components ε_x, ε_y and γ_{xy} are related to displacements u and v as follows

$$\{\varepsilon\} = [\varepsilon_x \ \varepsilon_y \ \gamma_{xy}]^T = \left[\left(\frac{\partial u}{\partial x}\right) \left(\frac{\partial v}{\partial y}\right) \left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]^T = [B]\{\delta\} \tag{14}$$

where, $[B]$, is strain-displacement transformation matrix, $\{\varepsilon\}$, is strain vector and, $\{\delta\}$, is vector of unknown displacements.

Stress strain relationship is given as

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [D]\{\varepsilon\} \tag{15}$$

in which, $[D]$, is the constitutive relation matrix, $\{\sigma\}$, is stress vector, and ν is Poisson's ratio.

Element stiffness matrix, $[K]_{in}$, is given by

$$[k]_{in} = abh\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]d\xi d\eta \tag{16}$$

Here, h represents thickness of the element. Numerical integration is carried out with respect to ξ and η, using Gauss quadrature.

Stiffness of soil spring supporting plate element can be found out using principle of virtual work. A virtual displacement $\{\Delta\delta\}$ is applied to the spring system, and by equating internal work done to external work, soil stiffness is given by

$$[K]_s = \int [N]^T \begin{bmatrix} E_{sx} & \\ & E_{sy} \end{bmatrix} [N] \, dA \tag{17}$$

2.2 Finite element formulation of pile

Piles are placed under the columns. Pile is considered beam element. Surrounding soil is modelled by Winkler's hypothesis in which soil is replaced by infinitely closely spaced independent elastic springs. Pile is divided in number of beam elements.

2.2.1 Beam element

Beam element has six degrees of freedom at each node, which includes lateral displacement u and v, axial displacement w, and rotation about three axes. If rotation about z-axis is not considered the degree of freedom are reduced to 5 at each node. The displacement u, v, and w can be expressed as,

$$\begin{aligned} u &= \alpha_1 + \alpha_2 z + \alpha_3 z^2 + \alpha_4 z^3 \\ v &= \alpha_5 + \alpha_6 z + \alpha_7 z^2 + \alpha_8 z^3 \\ w &= \alpha_9 + \alpha_{10} z \\ \theta_x &= \frac{\partial u}{\partial z} \quad and \quad \theta_y = \frac{\partial v}{\partial z} \end{aligned} \tag{18}$$

Nodal displacement vector, $\{\delta\}_e$

$$\{\delta\}_e^T = \{u_1 v_1 w_1 \theta_{x1} \theta_{y1} u_2 v_2 w_2 \theta_{x2} \theta_{y2}\}$$

For beam bending, relevant strain-displacement relation is

$$\{\varepsilon\} = \begin{bmatrix} \frac{\partial^2 u}{\partial z^2} & \frac{\partial^2 v}{\partial z^2} & \frac{\partial w}{\partial z} \end{bmatrix}^T = [B]\{\delta\} \tag{19}$$

where $[B]$, is strain-displacement transformation matrix, $\{\varepsilon\}$ is strain Vector.

Stiffness matrix of the element $[k]_e$, is given by the expression

$$[k]_e = \int_0^L [B]^T [D][B] dz \tag{20}$$

Where, $[D]$ is constitutive relation matrix for beam element given by

$$[D] = \begin{bmatrix} EI_y & 0 & 0 \\ 0 & EI_x & 0 \\ 0 & 0 & EA \end{bmatrix}$$

Final stiffness matrix is rearranged for degrees of freedom, $\{u, v, w, \theta_x, \theta_y\}$ and final form of the stiffness matrix is as follows

$$[K]_e = \begin{bmatrix} \frac{12EI_y}{L^3} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & -\frac{12EI_y}{L^3} & 0 & 0 & 0 & \frac{6EI_y}{L^2} \\ 0 & \frac{12EI_x}{L^3} & 0 & \frac{6EI_x}{L^2} & 0 & 0 & -\frac{12EI_x}{L^3} & 0 & \frac{6EI_x}{L^2} & 0 \\ 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{6EI_x}{L^2} & 0 & \frac{4EI_x}{L} & 0 & 0 & -\frac{6EI_x}{L^2} & 0 & \frac{2EI_x}{L} & 0 \\ \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{4EI_y}{L} & -\frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{2EI_y}{L} \\ -\frac{12EI_y}{L^3} & 0 & 0 & 0 & -\frac{6EI_y}{L^2} & \frac{12EI_y}{L^3} & 0 & 0 & 0 & -\frac{6EI_y}{L^2} \\ 0 & -\frac{12EI_x}{L^3} & 0 & -\frac{6EI_x}{L^2} & 0 & 0 & \frac{12EI_x}{L^3} & 0 & -\frac{6EI_x}{L^2} & 0 \\ 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{6EI_x}{L^2} & 0 & \frac{2EI_x}{L} & 0 & 0 & -\frac{6EI_x}{L^2} & 0 & \frac{4EI_x}{L} & 0 \\ \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{2EI_y}{L} & -\frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{4EI_y}{L} \end{bmatrix} \tag{21}$$

2.2.2 Spring element

Soil support at various nodes of beam element, is simulated by using a series of equivalent and independent elastic springs in three directions (x, y, z). Soil stiffness can be found out using principle of virtual work. A virtual displacement $\{\Delta\delta\}$ is applied to the spring system, and by equating internal work done to external work, soil stiffness can be worked out.

Pile displacements are given by

$$\{\delta\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} 1\ z\ z^2\ z^3 & & \\ & 1\ z\ z^2\ z^3 & \\ & & 1\ \mathrm{z} \end{bmatrix} \{\alpha\} = [R]\{\alpha\}$$

$$and\ \{\alpha\}^T = \{\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5\ \alpha_6\ \alpha_7\ \alpha_8\ \alpha_9\ \alpha_{10}\} \tag{22}$$

From Eq. (22), displacements u, v and w are expressed as

$$\{\delta\}_e = [A]\{\alpha\}\ and\ \{\alpha\} = [A]^{-1}\{\delta\}$$

This can be further simplified as

$$\{\delta\} = [u\ v\ w]^T = [R]\{\alpha\} = [R][A]^{-1}\{\delta\}_e = [N]\{\delta\}_e\ \ where\ \ [N] = [R][A]^{-1} \tag{23}$$

Soil reactions at any point $\{p_x, p_y, p_z\}$ within the element are given by

$$\begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{bmatrix} E_{sx} & 0 & 0 \\ 0 & E_{sy} & 0 \\ 0 & 0 & E_{sz} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{24}$$

$\{E_{sx}, E_{sy}, E_{sz}\}$ are soil subgrade reaction modulus at depth z and they can be written in terms of soil modulus at i^{th} and j^{th} node $\{E_{sxi}, E_{syi}, E_{szi}$ and $E_{sxj}, E_{syj}, E_{szj}\}$ as

$$E_{sx} = E_{sxi} + \frac{E_{sxj} - E_{sxj}}{L}z \qquad E_{sy} = E_{syi} + \frac{E_{syj} - E_{syj}}{L}z \qquad E_{sz} = E_{szi} + \frac{E_{szj} - E_{szj}}{L}z$$

Soil support element stiffness matrix, $[K]_s$, can be obtained as

$$[K]_s = \int_0^L [N]^T \begin{bmatrix} E_{sx} & 0 & 0 \\ 0 & E_{sy} & 0 \\ 0 & 0 & E_{sz} \end{bmatrix} [N]\, dz \tag{25}$$

After integration total soil stiffness $[K]_s$, is simplified as individual stiffness in (x, y, z) directions as ($[k]_x$, $[k]_y$, $[k]_z$) given by

$$[K]_s = \begin{bmatrix} [K]_x & & \\ & [K]_y & \\ & & [K]_z \end{bmatrix} \tag{26}$$

In which, individual stiffnesses ($[k]_x$, $[k]_y$, $[k]_z$) are given as

$$[k]_x = [k]_y = \begin{bmatrix} 156B + 72C & L(22B + 14C) & 54B + 54C & L(-13B - 12C) \\ & L^2(4B + 3C) & L(13B + 14C) & L^2(-3B - 3C) \\ & & 156B + 240C & L(-22B - 30C) \\ & & & L^2(4B + 5C) \end{bmatrix}$$

$$[k]_z = \frac{E_{szi}L}{12}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} + \frac{E_{szj}L}{12}\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \tag{27}$$

where, $B = E_{si}L / 420$ and $C = (E_{sj} - E_{si})L / 840$

Stiffness matrices of the raft and pile elements are assembled in to global stiffness matrix. The loads coming from columns are assembled in global load vectors. These set of simultaneous equations are solved for unknown nodal displacements using Gauss elimination method.

3. Validation

For validation of the computer code, simply supported square plate with dimension $a \times a$ subjected to uniformly distributed load q is considered. Central deflections w_{max} and maximum moments obtained from the finite element analysis are compared with the analytical solutions available for thin plates, which are converted in non-dimensional form w_n and M_n, as follows

$$w_n = \frac{w_{\max}Et^3}{12(1 - \nu^2)qa^4} \quad and \quad M_n = \frac{M_{\max}}{qa^3} \tag{28}$$

Table 1 Non-dimensional deflection of simply supported square plate

Plate thickness t (m)	t/a	Non-Dimensional Deflection w_n	Non-Dimensional Moment M_n
0.4572	0.06	0.00344	0.04063
0.5334	0.07	0.00366	0.04301
0.6096	0.08	0.00383	0.04485
0.6858	0.09	0.00397	0.04634
0.7620	0.10	0.00410	0.04760
Timoshenko and Krieger		0.00406	0.04790

Non- dimensional deflections and moments for square plates with dimensions 7.62 m × 7.62 m and thickness ranging from 0.1524 m to 0.7620 m are reported in Table 1 along with analytical values reported by Timoshenko and Krieger (1959) for thin square plate. Computed values of non-dimensional deflection and moment for thickness ratio 0.06 are found to be in close agreement with standard solution. This provides necessary validation check for the developed computer program.

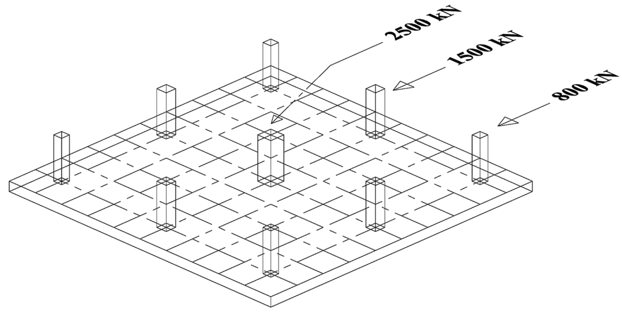
4. Parametric study

In the present investigations it is aimed to study the effect of following parameters on the rectangular piled-raft foundation using finite element analysis.

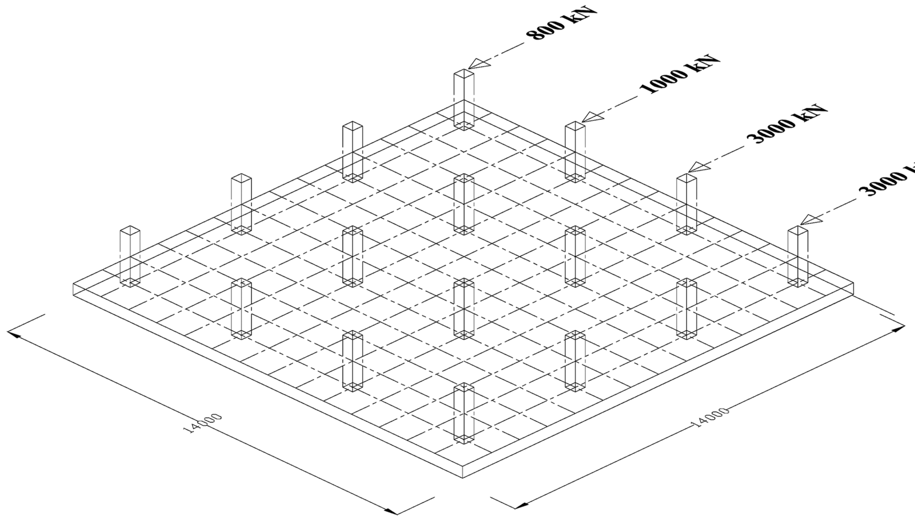
1. Young's modulus $E = 2.48 \times 10^7$ kN/m^2, Poisson's ratio 0.3
2. Thickness of the raft (0.45 m, 0.9 m, 1.5 m)
3. Piles with length 3 m and diameter 0.3 m
4. Modulus of subgrade reaction (40000, 100000, 200000, 400000) kN/m^3
5. Column loads (three load patterns LP-I, LP-II, LP-III)
   - Load-Pattern-I 10 m × 10 m raft Loads 800 kN on corner columns, 1500 kN middle columns at edges, 2500 kN cental column as indicated in Fig. 3
   - Load-Pattern-II 10 m × 10 m raft Loads 1000 kN on all 9 columns
   - Load Pattern-III 14 m × 14 m raft Loads 800 kN, 1000 kN, 3000 kN, 3000 kN as indicated in Fig. 3

For specified three load patterns, raft thickness and soil modulus are varied to study their effect on the response. Maximum deflection and bending moments in the raft are devised to compare the response.

Fig. 4 to 9 presents variations in the maximum displacement with raft thickness and soil modulus for three load patterns. In general, maximum deflections are found to be decreasing with increase in raft thickness and soil modulus. For both the configurations, reduction in the maximum deflection is observed with increase in the raft thickness. For raft foundations, maximum deflections are decreasing with increasing soil modulus. For piled raft foundation, if soil modulus is doubled, deflection of raft reduced by half in all load pattern cases. In case of piled-raft configuration the values of maximum deflections are substantially reduced as compared to raft foundation. Maximum moments in the case of piled raft foundation are increasing with increase in the raft thickness. An increase in the thickness from 0.45 to 0.9 resulted in a percentage increase of 100.62 in the bending



Load Pattern-I



Load Pattern-III

Fig. 3 Load Patterns considered in the analysis

moment and an increase from 0.9 to 1.5 resulted in increase of 30.67% for soil modulus of 20000

Comparisons of maximum moments for two configurations are illustrated in Figs. 10 to 12. In the case of raft foundations, maximum moments are increasing with raft thickness. With increase in the soil modulus, the reduction in moments is observed. However, for load pattern-I and load pattern-III the reduction is marginal, but for load pattern-II moments are increasing with increase in soil modulus for raft thickness of 0.45 m. For other thicknesses (0.9 m and 1.5 m) moments are decreasing with increase in soil modulus. In case of piled-raft foundations, maximum moments are increasing with raft thickness. For load-pattern-II, maximum moments developed are considerably lower as compare to other two patterns. This may be attributed to uniform nature of loading. This effect is more pronounced for piled raft configurations. In piled raft foundation, reductions in moment are marginal for all load cases. Percentage decrease in the moments of piled raft configurations as compared to the raft foundations is observed to be in the range of 2% to 15%.

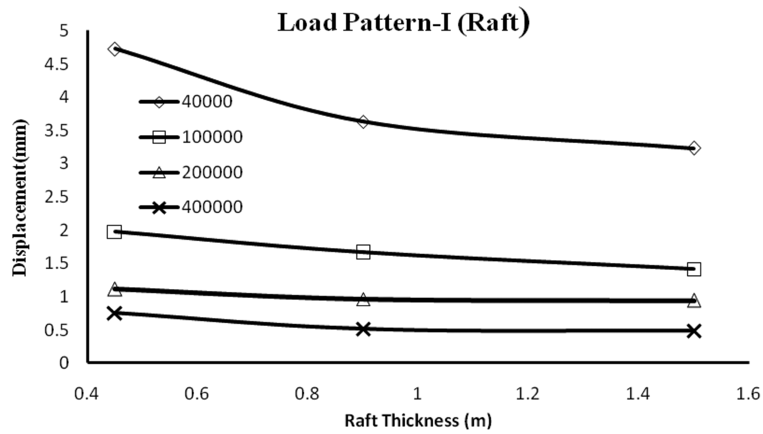


Fig. 4 Variations in maximum deflection for raft foundation (LP-I)

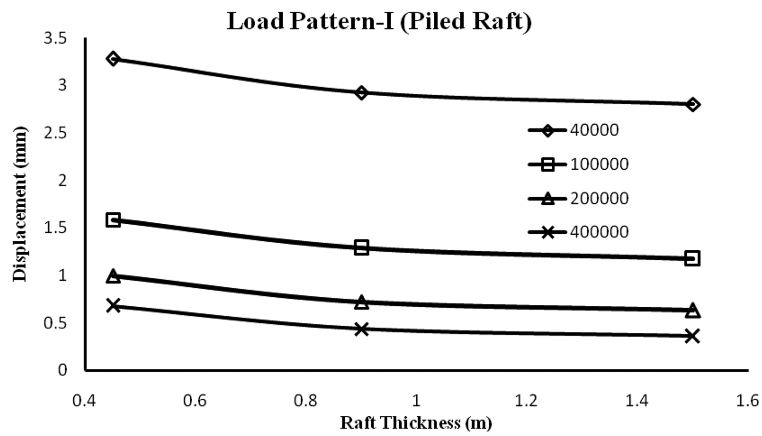


Fig. 5 Variations in maximum deflection for piled-raft foundation (LP-I)

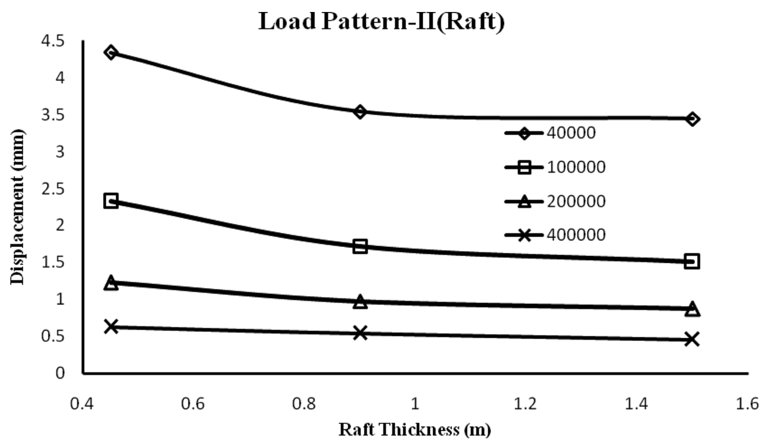


Fig. 6 Variations in maximum deflection for raft foundation (LP-II)

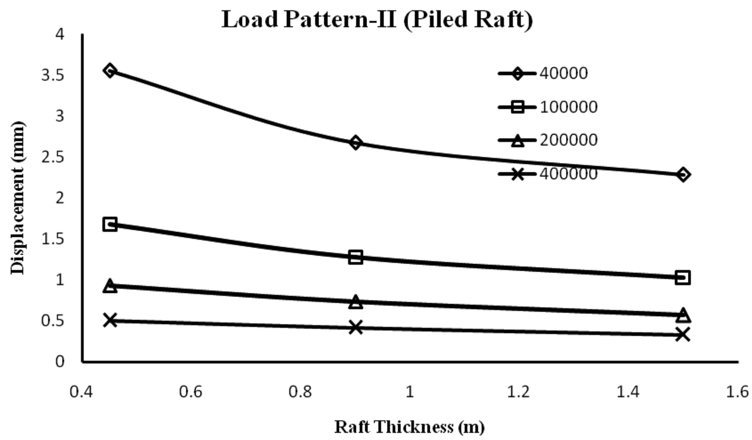


Fig. 7 Variations in maximum deflection for piled-raft foundation (LP-II)

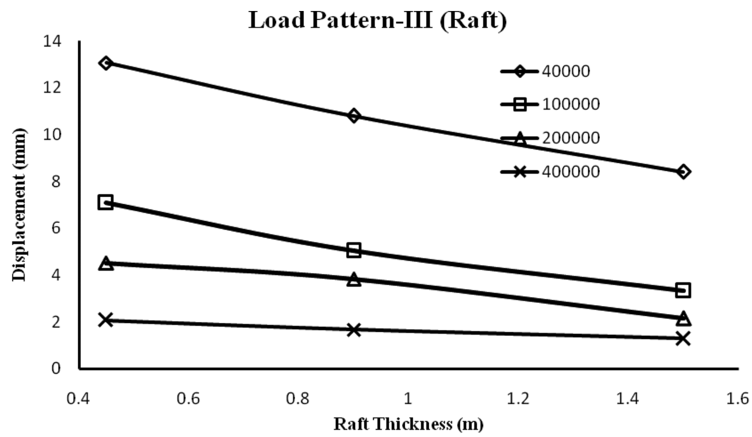


Fig. 8 Variations in maximum deflection for raft foundation (LP-III)

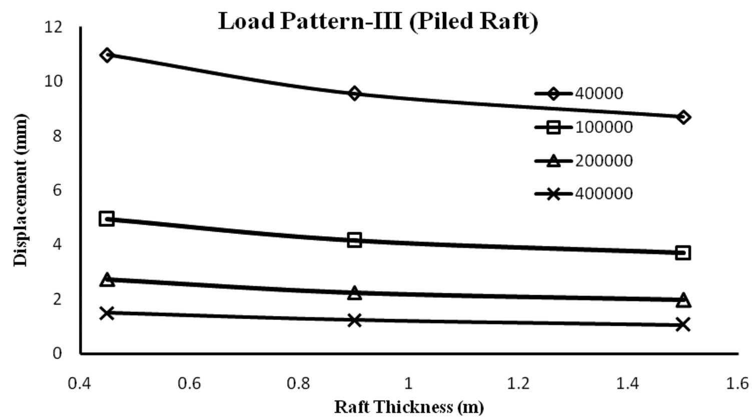
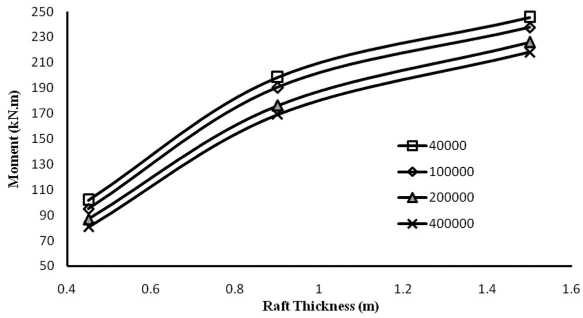
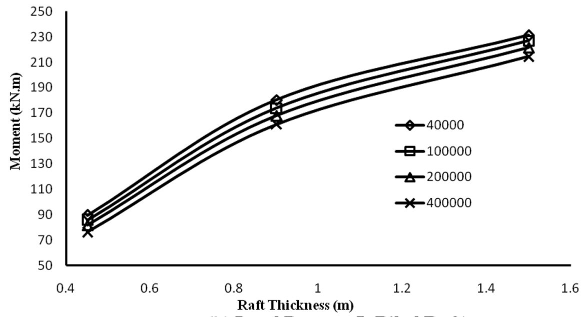


Fig. 9 Variations in maximum deflection for piled-raft foundation (LP-III)

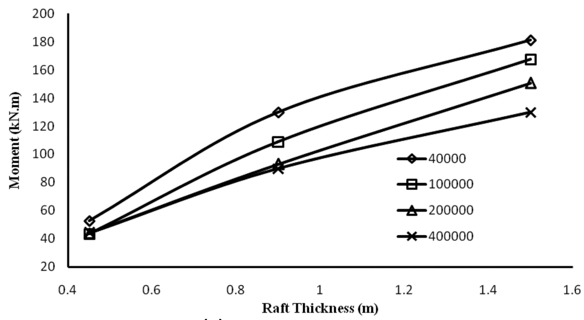


(a) **Load Pattern-I (Raft)**

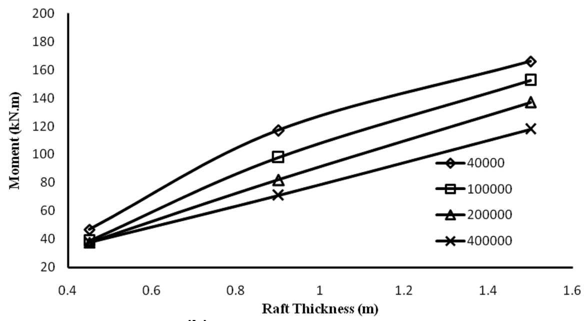


(b) **Load Pattern-I (Piled Raft)**

Fig. 10 Comparison of maximum moment for load pattern-I



(a) **Load Pattern-II (Raft)**



(b) **Load Pattern -II (Piled Raft)**

Fig. 11 Comparison of maximum moment for load pattern-II

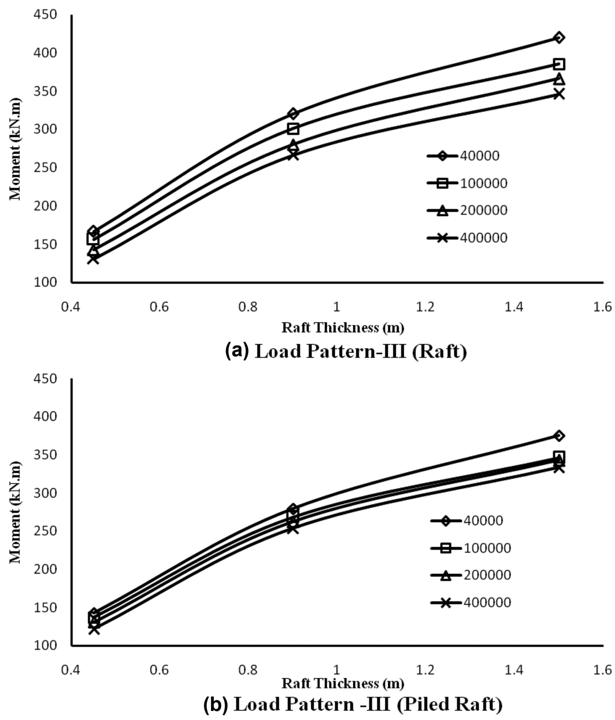


Fig. 12 Comparison of maximum moment for load pattern-III

5. Conclusions

A parametric study on piled-raft foundations is presented wherein effect of raft thickness, soil modulus and load pattern on the response is considered. Substantial reduction in maximum deflections and maximum moments are observed in case of piled-raft configurations compared with the response of raft foundation. For both the configurations the reduction in maximum deflections are observed with increase in raft thickness. Also for both configurations, maximum deflections are decreasing with increase in soil modulus. Maximum moments are decreasing with increase in soil modulus for both configurations. Percentages of decrease in moments of piled raft foundation compared with raft foundation go on increasing with increase in soil modulus for load case in which all columns are subjected to same loading. Range of decreasing percentage of deflection in case of piled raft foundation compared to raft foundation is between 10% and 30%.

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