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Torsional parameters importance in the structural response of multiscale asymmetric-plan buildings

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Abstract. The evaluation of torsional effects on multistory buildings remains an open issue, despite considerable research efforts and numerous publications. In this study, a large number of multiple test structures are considered with normally distributed topological attributes, in order to quantify the statistically derived relationships between the torsional criteria and response parameters. The linear regression analysis results, depict that the center of twist and the ratio of torsion (ROT) index proved numerically to be the most reliable criteria for the prediction of the modal rotation and displacements, however the residuals distribution and R-squared derived for the ductility demands prediction, was not constant and low respectively. Thus, the assessment of the torsional parameters' contribution to the nonlinear structural response was investigated using artificial neural networks. Utilizing the connection weights approach, the Center of Strength, Torsional Stiffness and the Base Shear Torque curves were found to exhibit the highest impact numerically, while all the other torsional indices' contribution was investigated and quantified.

Keywords: shear center; torsional radius; ratio of torsion; omega ratio; regression analysis; statistical inferences; artificial neural networks

1. Introduction

The coupling of floor torsional oscillations under purely transformational excitation of the base is the cause of structural overloading affected by torsion. This coupling occurs because of the non-symmetrical arrangement of the mass and the stiffness of the vertical structural elements of the building as the placement of external loading causes internal torque in the floor and subsequently torsional oscillation. In order to quantify the effect of torsion in buildings, many studies have been performed on the basic concept that the structure is oscillating around a specific point, while the distance of the center of mass from this point is directly related to torsional effects. In order to assess and design structures to withstand torsion, various approaches have been endeavored based

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on the centers of rigidity, shear and twist, or on more sophisticated indices such as omega, torsional radius, center of strength and Base Shear Torque (BST) curves (Llera and Chopra 1995, Mylimaj and Tso 2002, and Paulay 1997).

However, the evaluation of the torsional design criteria and their relationships to structural response remain open, as demonstrated in a critical review by Anagnostopoulos *et al*. (2015). This work aims to identify and measure the performance of the various torsional design criteria, the response parameters, utilizing a large database of structures, in order to estimate statistically reliable relationships. Two diverse earthquake analysis procedures have been applied in this work: a simplified demand spectrum based on the EC8 design code (modal spectrum) and also a pushover analysis for each test structure, where specific parameters are measured, such as displacements in the x and y axis, diaphragm rotation and story shears for the spectrum based procedures, while ductility demands and ultimate collapse load are additionally stored for the pushover analysis. This procedure is repeated via a generation of random structural designs with the assistance of a specific algorithm developed for this purpose. Additionally, for each random structure, the torsional design criteria are calculated and stored. This approach makes possible the calculation of correlations between torsional design criteria and response parameters, while several regression models are constructed in order to understand the relation of the independent variables (torsional design criteria) to the dependent variables (response parameters), and explore the nature of these relationships and the importance of each independent variable.

The purpose of regression analysis is to identify the most appropriate model to fit the actual data, and consequently estimate the parameters of such model by minimizing the sum of square errors between the predicted values by the model and the raw data, designated analytically by Makridakis *et al*. (2008). This is performed using the t-test and the adjusted multiple R-squared as described analytically by Glantz *et al*. (1990) and implemented in the subsequent sections of this work. The novelty of this work is the construction of a reliable regression model in order to evaluate the influence of the variation of each design criterion (independent variables) to the torsional response parameters, such as diaphragm displacements and base shear (dependent variables). For this purpose, the first stage of this work was the development of a large number of multiscale generated structures. This random generation procedure is implemented in a large group of structures, which are produced with a random number of floors, bays in X and Y dimension, and columns sizes.

Gaussian noise was introduced in the variables (regarding building geometry), since the computer algorithm, by default, routinely produces collinear independent random variables of uniform distribution. This assures the normal distribution of the values and that the independent variables are not collinear. This process establishes a reliable model showing that the torsional design criteria has a major influence on ductility demands at the vertical direction of the applied seismic forces and diaphragm rotation. The assessment of the criteria is achieved through multiple correlations (between criteria and response parameters) and via t-test values calculation (quantification of the probability of linear relationship of each independent variable with the dependent). The regression analysis is performed using the normalized data instead of the initial ones, using the general form

$$d_n^i = \frac{d_0^i - \mu}{\sigma}$$

Where: d_n^i stands for the normalized data of the series, μ and σ the mean value and standard deviation of the series, while d_0^i stands for the initial value. The normalized data allows the direct estimation of changes in independent variables to the dependent. Furthermore, within the structural designs examined, adequate confidence intervals (95%) for the limits (minimum and maximum) of the regression weights were defined. The aim is to extract reliable conclusions, based on accurate, not highly idealized models of eccentric one-story systems, contrary to most of the conducted research on this problem. As stated by Anagnostopoulos *et al.* (2015), most of the publications on torsion are limited to one story inelastic shear beam models. These models undergo many shortcomings-regarding the influence of the torsional design criteria on response parameters-so that the results obtained cannot be generalized to realistic buildings.

The various centers of single story systems coincide and for this reason several attempts have been made to investigate new, more accurate and better representative indices to assess the influence of torsional effects on buildings. Among them the center of strength, the base shear torque (BST) curves, and the Ω ratio, the classification to torsional stiff and flexible edges of a building plan as studied by Mylimaj and Tso (2002) and Paulay (1997). However, none of them has been proven to give a clear assessment of the strength and ductility demands amplification due to torsion. The present investigation is performed on a large number of structures, by varying the dimensions of the vertical structural elements and their topology and subsequently, all relevant stiffness and strength attributes, in order to obtain adequate and interpretable results for reliable conclusions. Regression analysis is stated as an efficient method for the assessment of the significance of a set of independent variables against a dependent one. In this work, the independent variables are the torsional design criteria and the dependent one the displacements due to torsion.

2. Building torsion: Definitions

2.1 Torsion

The definitions of the main characteristic quantities used in the present study are the following:

Center of stiffness, *CS*, (or rigidity) is the point in the floor plan, where if an external force is applied in any direction, the structure exhibits only translation without any torsion.

Centre of shear, *SC*, is the point in the floor plan, where the resultant of all internal shear forces and torsional moments passes.

Center of twist, *CT*, is the point in the floor plan, where if any external horizontal static torque is applied, it causes only rotation of the building diaphragm around it.

The principal axes, *I* and *II*, of the system are two orthogonal axes passing through the center

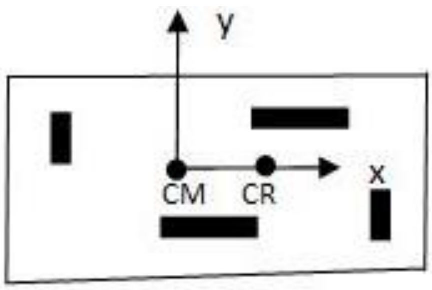

Fig. 1 General plan of a building

of rigidity, so that if a static horizontal force is applied along one of the principal axes of the system, the diaphragm translates only in the direction of the force without any twist.

The center of mass of the system is the point on the diaphragm through which the resultant of the inertia forces of the diaphragm is applied. If the masses of individual resisting elements are negligible, the center of mass of the diaphragm with uniform mass distribution coincides with its geometric center.

These definitions are based on the Berkeley report of Hejal and Chopra (1987).

2.2 Equations of motion

The linear equations of motion for the one-storey system shown in Fig. 1, subjected to earthquake ground motion written with respect to the reference point O, the center of mass and the center of rigidity, are written as follows:

Reference point O

$$\begin{bmatrix} m & 0 & -my_M \\ 0 & m & mx_M \\ -my_M & mx_M & J_0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{u} \\ \ddot{u} \end{Bmatrix} + \begin{bmatrix} K_x & K_{xy} & K_{x\theta} \\ K_{yx} & K_y & K_{y\theta} \\ K_{\theta x} & K_{\theta y} & K_\theta \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix} = -m \begin{Bmatrix} a_{gx}(t) \\ a_{gy}(t) \\ -y_M a_{gx}(t) + x_M a_{gy}(t) \end{Bmatrix} \tag{1}$$

Center of Mass

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & mr^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{u} \\ \ddot{u} \end{Bmatrix} + \begin{bmatrix} K_x & K_{xy} & K_{x\theta} \\ K_{yx} & K_y & K_{y\theta} \\ K_{\theta x} & K_{\theta y} & K_\theta \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix} = -m \begin{Bmatrix} a_{gx}(t) \\ a_{gy}(t) \\ 0 \end{Bmatrix} \tag{2}$$

Center of Rigidity

$$\begin{bmatrix} m & 0 & -me_y \\ 0 & m & me_x \\ -me_y & me_x & J_R \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{u} \\ \ddot{u} \end{Bmatrix} + \begin{bmatrix} \widetilde{K_x} & \widetilde{K_{xy}} & 0 \\ \widetilde{K_{yx}} & \widetilde{K_y} & 0 \\ 0 & 0 & \widetilde{K_\theta} \end{bmatrix} \begin{Bmatrix} \tilde{u} \\ \tilde{u} \\ \tilde{u} \end{Bmatrix} = -m \begin{Bmatrix} a_{gx}(t) \\ a_{gy}(t) \\ -e_y a_{gx}(t) + e_x a_{gy}(t) \end{Bmatrix} \tag{3}$$

where

$a_{gx}(t)$ *and* $a_{gy}(t)$ are the accelerations along the X- and Y- axes, while J_0 *is the polar moment of inertia* of the diaphragm with respect to point *O* given by

$$J_o = \mathrm{m}^*(r^2 + x_M^2 + y_M^2) \tag{4}$$

where r is the radius of gyration; x_M, y_M are the x and y coordinates of the center of mass

The polar moment of inertia J_R is about a vertical axis passing through the center of rigidity

$$J_R = m(e^2 + r^2) \tag{5}$$

The static eccentricity e of the single-storey building is defined as the distance between the CR and the CM of the floor.

$$\mathrm{e} = \frac{K_{y\theta}}{K_y} = \frac{\sum_i x_i K_{yi}}{\sum_i K_{yi}} \tag{6}$$

The static eccentricity is also defined as the distance between the center of mass and shear center of the building. For one-storey systems the two definitions are identical since the center of rigidity and the shear center of the system coincide. Moreover, for multi-storey buildings these two centers do not coincide.

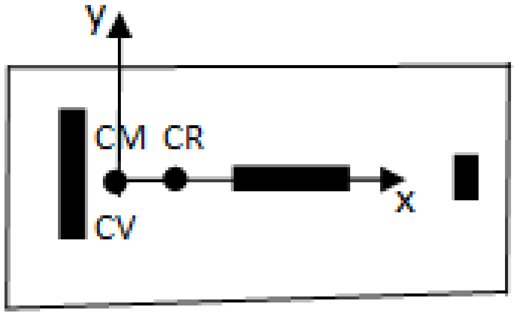

Fig. 2 Arrangement of lateral resisting elements in a torsionally unrestrained system

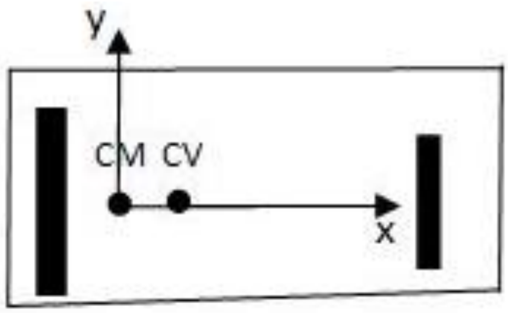

Fig. 3 Arrangement of lateral forces resisting elements in a torsionally restrained system

Ω index

The lateral vibration frequency of the corresponding uncoupled system is

$$\omega_y = \sqrt{\frac{K_y}{m}} \tag{7}$$

while the second uncoupled equation leads to the torsional vibrational frequency of the corresponding torsionally-uncoupled system

$$\omega_\theta = \sqrt{\frac{K_{\theta R}}{mr^2}} = \sqrt{\frac{K_\theta}{mr^2} - \left(\frac{e}{r}\right)\omega_y^2} \tag{8}$$

Thus, the uncoupled torsional to lateral frequency ratio Ω, is written as

$$\Omega = \frac{\omega_\theta}{\omega_y} \tag{9}$$

As presented by Hejal and Chopra (1987) for torsionally-stiff (Ω>1) systems, the fundamental mode is primarily lateral and the second mode is torsional.

Torsionally unrestrained systems are systems which cannot resist torsion in the post-yield range. Thus, no torsion is generated in the inelastic range, as it can be resisted only in the elastic range.

Torsionally restrained systems are able to resist earthquake-induced torque in the inelastic range also. In such systems, the earthquake-induced torque can be resisted when all vertical resisting elements respond in the plastic range.

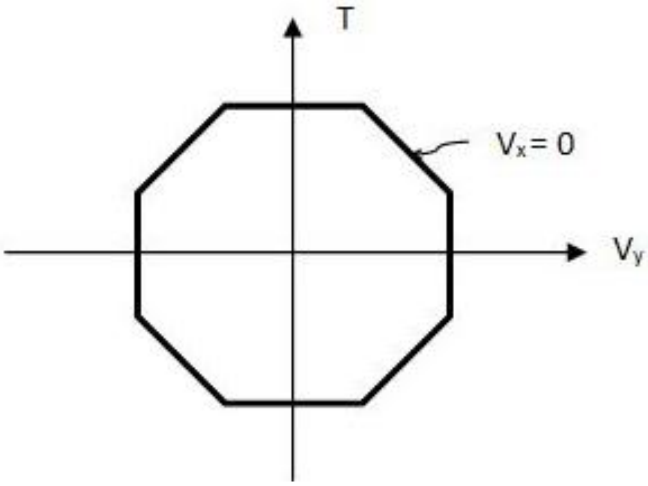

Fig. 4 Typical BST curve

Torsionally restrained systems are able to resist earthquake-induced torque in the inelastic range also. In such systems, the earthquake-induced torque can be resisted when all vertical resisting elements respond in the plastic range.

Strength eccentricity is the distance between CM and CV defined by the equation

$$e_{vx} = \frac{\Sigma x_i V_{ni}}{\Sigma V_{ni}} \tag{10}$$

where x_i is the distance of the element of the center of mass (CM) and V_{ni} is the column's shear resisting capacity. The arrangement of the nominal strength within the columns would lead a system with optimal torsional response, assuming that $\sum V_{ni} \geq V_E$. In this case the ROT is greater than unit. The position of CV is dominant in the inelastic range, as the center of rigidity cannot be defined when the vertical resisting elements have yield.

2.3 The base shear and torque surface (BST)

The base shear and torque surface (BST) is the graphical representation of the envelope of all combinations of base shear and torque that when applied statically to the horizontal diaphragm, cause collapse of the system. The shear (V_x) and torque (T) region is separated by the BST curve in two regions, the interior and the exterior. The interior contains combinations of base shear and torque causing elastic behavior of the structure, while the exterior contains statically inacceptable base shear and torque combinations causing inelastic performance of the resisting elements. The BST surface is convex and it is composed of linear segments as described by Llera and Chopra (1995).

It is recognized that using shear and twist centers, or other more sophisticated indices, such as omega, r RZ, CV or torsional radius, several problems may occur. Firstly, all these “centers” and criteria are well defined only in single floor structural systems, with absolutely rigid diaphragms. Secondly they do not offer a quantitative measurement of the negative effect of torsion. This is confronted with the utilization of ROT, as described by Stathi *et al.* (2015). The basic aspects of the concept of ROT is demonstrated in the following.

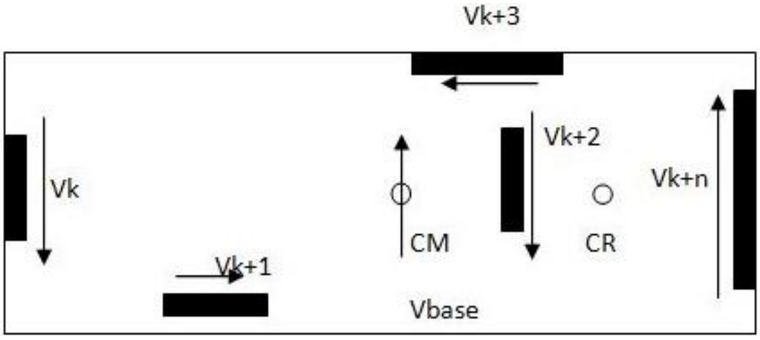

Fig. 5 vertical resisting elements internal forces

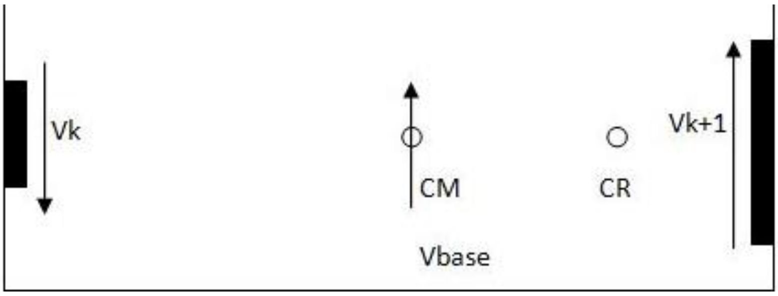

Fig. 6 vertical resisting elements for one axis symmetric systems

2.4 Ratio of torsion

In the Fig. 5, an incidental floor plan is shown where shear walls and the corresponding shear forces are demonstrated. The shear forces acting on the vertical resisting elements satisfy the subsequent expression

$$\sum_{k=1}^{n}|Vkij| \neq \sum_{k=1}^{n} Vkij \tag{11}$$

where:

n=the number of elements in a floor direction (*x* or *y*),

i=the corresponding shear force of the element,

and *j*=the direction of the earthquake motion

In the case of a seismic action along the y direction, Eq. (11) is written as

$$\sum_{k=1}^{n}|Vkxy| \neq \sum_{k=1}^{n} Vkxy = 0 \tag{12}$$

and

$$\sum_{k=1}^{n}|Vkyy| \neq \sum_{k=1}^{n} Vkyy = Vbase \tag{13}$$

Similar expressions can be written for a seismic action along X direction. The computation of the ROT value is demonstrated for the simple example of Fig. 6.

The total value of ROT for the above building of Fig. 6 is

$$\text{ROT} = \sum_{1}^{n} \sum_{i=x,j=y}^{y,x} ROTij \tag{14}$$

Where

$$ROTij = \frac{\sum_{k=1}^{n} |Vkij| - a * \sum_{k=1}^{n} Vkij}{\sum_{k=1}^{n} Vkij} \tag{15}$$

and

n=the number of elements in a floor direction (*x* or *y*)

i=the corresponding shear force of the element

j=the direction of the earthquake motion

and α=0 if $i \neq j$ or α=1 if $i=j$

3. Multivariate modelling

3.1 Linear regression

Regression is a widely used statistical technique to investigate the relationship between a dependent variable and one or more independent ones.

The estimated linear regression is of the form

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + \ \ldots\ + b_mX_{mi} + e_i \tag{16}$$

Where *Y*

$X_1, X_2, X_3, \ \ldots X_m$ are the independent ones

$a, b_1, b_2, b_3, \ldots b_m$ are the estimated regression coefficients, and e_i are independent error terms, with a mean of zero, a constant variance and a normal distribution.

The most important, practical advantage of linear regression is that the regression coefficients b_j indicates the amount of change in the dependent variable *Y* when X_j changes by one unit. This characteristic is true because the independent variables X_j are orthogonal.

As mentioned above, the regression method fits the linear Eq. (16) by estimating the regression coefficients *a* and b_j in such a way as to minimize the sum of square errors (Σe_i^2), *i*=1, 2, 3 … *n*, where *n* is the number of observations.

The procedure followed in this article generated more than two thousand data series by varying the columns' dimensions. This way, several hundreds of different structures and corresponding nonlinear pushover analysis were created. In order to produce a sample with normal distribution, a specific algorithm implementing Box-Muller (1958) transformation from randomly generated values was developed. The final aim was to produce a random sample of structural designs, each one corresponding to a particular structural set. Torsional eccentricity and other widely accepted design criteria related to the torsional behavior, were correlated to ROT in order to investigate the potential of ROT being used as a practical tool for structural design of new and existing buildings. Moreover, ROT quantifies the torsional introduced supplementary loading due to seismic actions and the floor plan's asymmetry.

Once the data was generated an appropriate regression model was identified and its parameters estimated in the form of Eq. (16). Such model is also written as

$$Y = a + \Sigma bi*xi \tag{17}$$

where a and b_i will be estimated using the least square error minimization between empirical data and a regression model Eq. (17).

The multiple correlation factor R^2 indicates that the independent variables X_j explain statistically, the R^2 (percentage) of the total fluctuations in Y. Thus, R^2 calculated value, is the assessment of the regression model fit to the raw data. However, in order for the regression to be valid, the following three additional assumptions to that of linearity must be satisfied:

1. The error terms e_i must be independent (before this assumption check, the dependent variable must have been sorted from the smallest to the largest value)
2. The variance of the e_i must be constant, and
3. The distribution of the e_i must be normal.

Once the parameters of the regression Eq. (16) have been estimated and the three assumptions mentioned above are satisfied, the regression Eq. (16) can subsequently be used to predict Y values for various values of the independent variables, thus simulating any desired inputs.

3.2 Neural networks

A neural network is a network of process nodes (neurons) that quantifies the interconnections between them. Each such node receives a collection of numerical inputs from diverse sources (either from alternative neurons or from the environment), performs calculations on these inputs and produces an output. Three kinds of neurons constitute the neural network: input neurons, output neurons and process (hidden) neurons. The process neurons multiply every entry with the corresponding weight and calculate the whole add of the product. Artificial Neural Networks (ANN), can be used to model numerically nonlinear relationships between a number of independent variables (predictors) and dependent ones (outputs). Thus, neural networks can be used as universal function approximators (Raul 2013).

The numerical model achieved using neural networks for nonlinear functions modelling can depict high R-squared between the actual and predicted variables and can be utilized without constraints, especially when the linear regression assumptions are not satisfied. However, an ANN model is characterized as a black box (Olden and Jackson 2002), because they cannot deliver explicit explanatory insight into the contribution of the independent variables to the predicted ones. Thus, a number of studies investigate equivalent methodologies to quantify the importance of the independent variables to the prediction (Olden *et al.* 2004, Gevrey *et al.* 2003). In this study, the connection weight approach is followed to investigate the influence of the torsional parameters to the structural response, as this approach exhibits the highest accuracy. The connection weight approach calculates the product of the connection weights between each input and output neuron and sums the products across all hidden neurons.

Additionally, special attention must be taken into account, in order to avoid overfitting and overtraining of the ANN model. Although neural networks can simulate computationally any nonlinear multivariate relationship with a high degree of accuracy, the prediction model derived may not be able to predict new outputs using raw data that is not included in the training data. This problem is called overfitting, meaning that the model fit well to the data used, but cannot generalize the approximation of the dependent variables. The effect of overfitting can be vast, as studied by Lawrence *et al*. (1997). To avoid this condition, the variables' database is divided into a

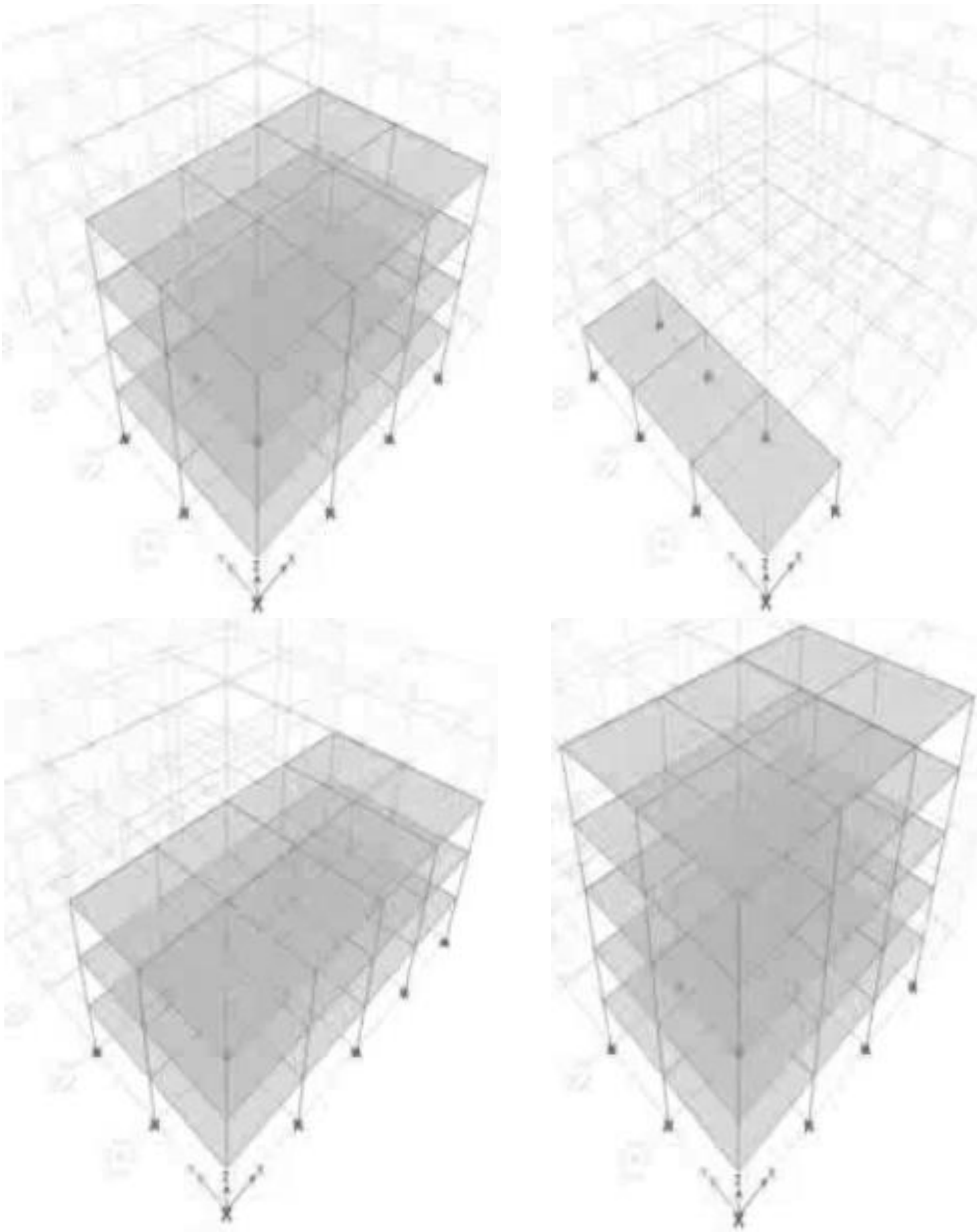

Fig. 13 Instances of random structure generation procedure

training set (used for ANN model constitution) and a training set to estimate the generalization ability of the network, as depicted later in Fig. 26. The ANN structure used in the current study, utilizes Bayesian Regularization, in order to improve generalization (Marquardt 1963).

ANNs are used in a number of research works in the field of mechanical behavior of structures, as demonstrated by Yavuz (2016) for shear strength approximation of RC beams, Hakim *et al.* (2013), for structural damage investigation, Mohammadhassani *et al.* (2013) for strain forecasting, Peng-hui *et al.* (2015) for identification of beams damage, Alapour *et al.* (2013), for strength of lightweight concrete, and Beycioğlu *et al.* (2015) for compressive strength of mortar. ANN exhibit high performance in the investigation of numerical patterns, due to the fact that they fit a nonlinear model (mapping) associating one or many independent variables with one or more dependent.

4. Database of random shape and topology test structures

This work's approach constitutes the creation and utilization of a large database of multiscale structures in order to derive statistically reliable numerical results for the various torsional criteria, structural response parameters and their modeled relationships. Thus, in the initial step of this study, the structure's database was created using a random number of floors, bays in X and Y direction, and columns sizes. This procedure is highlighted in Fig. 13. As the numerical algorithm routinely produces collinear independent random variables of uniform distribution, Gaussian noise was introduced in the variables so as to have normal uncorrelated distribution. In order to perform the structural analysis and design for the generated population of structures (several hundreds of instances) and to derive necessary response parameters such us eccentricity of each one design, a random structures generator algorithm was implemented and linked to the structural analysis and design software. For each step of the random geometry generation algorithm, the dimensions of the vertical elements were changed as well as the number of floors and bays, and the corresponding structural response parameters were calculated and saved. Afterwards, the derived results of the structural analysis were utilized to construct the regression analysis model.

Finally, in the random generation procedure (for cross section sizes, number of floors and number of bays), Gaussian noise was introduced in order to avoid multicollinearity of them. Gaussian noise or Gaussian distribution is a statistical noise which has a probability density function (PDF) equal to that of the normal distribution. Hence, this noise's values are Gaussian distributed. A Gaussian random variable z follows the probability density function p of Eq. (18)

$$p_G(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \tag{18}$$

where z represents the grey level, μ the mean value and σ the standard deviation.

5. Statistical inference of raw data

5.1 Numerical procedure

In this section, the outcome of statistical inference of the structural analysis results is demonstrated. Thus, an analytical regression analysis is performed, where in the regression model, only statistically significant independent variables were kept in a trial and error procedure. When excluding an independent variable from the model, the t-tests are recalculated and a prior

Table 1 Numerical procedure

1. Data generation
   - 1.1 Structural layout generation
   - 1.1.1 Random generation of bays in x and y direction
   - 1.1.2 Random generation of floors
   - 1.1.3 Random generation of cross sections
   - 1.2 Structural analysis using fem method
   - 1.3 Calculation of torsional response parameters (Table 2)
   - 1.4 Database storage of outputs
   - 1.5 Return to step: 1.1
2. Correlations of response parameters
3. Regression analysis of the results

Table 2 Outputs of the algorithm

ROT (Ratio of Torsion)	TwC (Center of twist)	Rkx (Torsional stiffness radius by x axis)
U2 (translation at y axis)	CMCV (distance between center of mass and center of strength)	Rky torsional stiffness radius by y axis)
R3 (rotation of the diaphragm)	CV (indicator depicting the angle of divergence when center of mass, center of rigidity and center of strength do not rely on a straight line)	M (total mass derived as the sum of axial forces)
Uyu (ultimate y displacement)	CR (stiffness center)	Omega
Uy0 (yield y displacement)	SC (shear center)	Vx0 (Yield Base Shear)
CM (center of Mass)	TS (torsional stiffness)	Vxu (Collapse Base Shear)

RZ modal participating mass of the first torsional eigenmode to modal participating mass of the first translational eigenmode

statistically significant variable (high value of t-test usually assumed greater than two) turns into an insignificant one. In addition, the residuals of the regression are examined and the outliers are excluded iteratively, until they are lower than three standard deviations. The aim of the regression was mainly to understand which, among the independent variables, are related to the dependent variable and to explore if the derived regression model depicts high values of R squared, meaning that the dependent variable (correspondent with torsional response) is explained accurately enough with the independent ones (torsional design criteria). Special attention was given to exclude collinear variables from the calculated regression model, aiming at the maximum R squared. This is a common procedure followed when modelling correlations between many independent variables and one dependent using a linear regression model. The reason is that collinear independent variables depict low t-test values due to arithmetical instability. Thus, they become insignificant and the adjusted R squared of the model decreases. Furthermore, the data was normalized in order to avoid computational errors, as the independent variables data range was different between the independent variables. Moreover, the intercept term a, was forced to be zero, so Eq. (17) turns into

$$Y = \Sigma bi*xi \qquad (19)$$

Table 2 Regression summary results

Regression Statistics	
Multiple R	0.9071
R Square	0.8229
Adjusted R Square	0.8209
Standard Deviation of Regression	0.3284
Observations	746
D.F.Numerator	8
D.F.Denominator	738

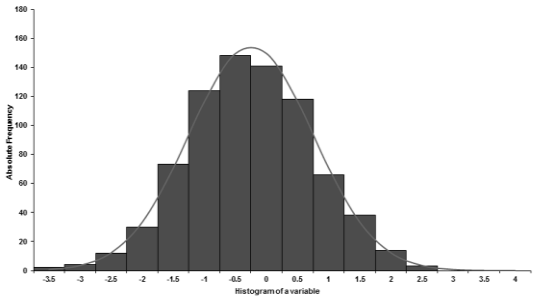

Fig. 14 Histogram of independent variable dimension

Table 3 Regression results

Analysis of Variance (ANOVA)

Source	Sum of Squares	d.f.	Mean Square	F	P-value
Regression	369.6714176	8	46.2089272	428.531615	0
Residual	79.57916535	738	0.107830847		
Total	449.2505829	746			

Dependent Variable U2

Independent Variable	Coefficient	Standard Error	t-stat	P-value	0.05 Significance?
Constant: a	0.0000				
ROT	0.7666	0.0155	49.41367838	0.0000	Y
CR	0.0294	0.0129	2.279882867	0.0229	Y
CM	-0.0977	0.0152	-6.414134991	0.0000	Y
TS	0.0979	0.0303	3.232131864	0.0013	Y
kx	-0.1778	0.0197	-9.029845441	0.0000	Y
ky	-0.0609	0.0205	-2.964744852	0.0031	Y
M	0.3179	0.0155	20.47726266	0.0000	Y
Rky	-0.1001	0.0264	-3.795117505	0.0002	Y

This is reasonable, as the torsional response (dependent variable Y) is equal to zero if the design criteria (independent variables) equals to zero. Namely, an in plan symmetric structure should respond zero diaphragm rotation for any direction and acting point of the horizontal loads.

After the random generation and structural analyses, the outputs of the algorithm are demonstrated in Table 2.

5.2 Linear regression results

In this section, ROT and center of twist are confirmed to be the torsional design criterion with the higher prediction capacity in terms of torsional response. For simplicity reasons, specific patterns of some of the families explored are demonstrated, while the others depict similar representations. The linear regression model, when structural shape and topology are randomly varying, takes a high value of multiple R equal to 0.9071, as shown in Table 2. The dependent variable is the displacement in direction Y while the earthquake is acting on direction X. This is a certain measure of torsional response, since for a symmetric building, the diaphragm

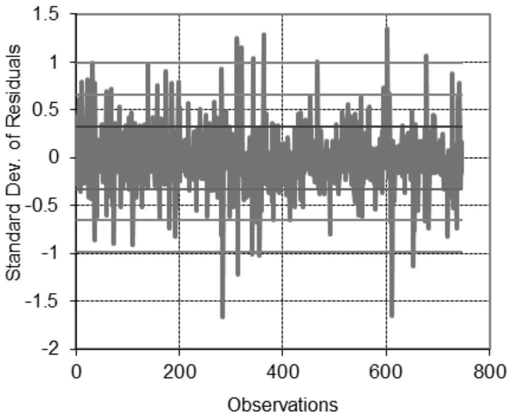

Fig. 15 Residuals plot

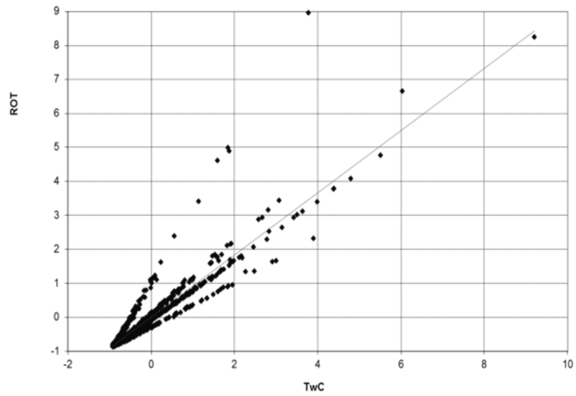

Fig.16 ROT vs TwC

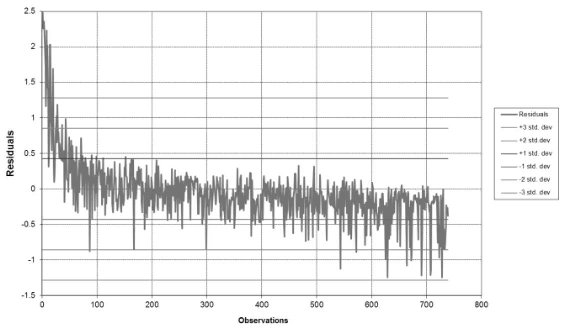

Fig. 17 Residuals against values

Table 4 Regression summary results

Regression Statistics	
Multiple R	0.6387
R Square	0.4079
Adjusted R Square	0.4055
Standard Deviation of Regression	0.4271
Observations	739
D.F.Numerator	3
D.F.Denominator	736

displacements in the vertical direction of the earthquake excitation-U2 should be equal to zero. Regression analysis is used, primarily to understand which among the independent variables are related to the dependent variable, and secondly to explore the forms of these relationships. After several trials to constitute a model with statistically significant independent variables and a high value of R, the optimum model found was the one using as predictors ROT, CR, SC, CM, TS, kx, ky, M and Rky. The data used are normalized, so the coefficients reflect the change in U2 when each independent variable changes for one unit. The constant term is forced to be zero, as the dependent variable equals to zero when the independent ones are also zero. The t-test values for ROT depict the higher value (49.41) as shown in Table 3. Values higher than 1.5 or 2.0, confirm empirically that there is a linear relationship between that particular independent variable and the dependent one.

The Twist Center was not included in the model, as it depicts high correlation with ROT, in order to avoid computational instabilities described above in section 6.1. However the four strong linear patterns demonstrated in Fig. 16 are of high interest. The residuals of the regression were lower than 2 standard deviations, exhibiting no pattern, meaning that the independent variables explain the majority of the U2 fluctuations.

6. Artificial neural network results

Table 5 Regression results

Analysis of Variance (ANOVA)

Source	Sum of Squares	d.f.	Mean Square	F	P-value
Regression	92.49932571	3	30.83310857	169.000753	0
Residual	134.2785018	736	0.182443617		
Total	226.7778275	739			

Dependent Variable	Vxu

Independent Variable	Coefficient	Standard Error	t-stat	P-value	0.05 Significance?
Constant: a	0.0000				
ROT	0.0939	0.0208	4.505423629	0.0000	Y
Omega	0.0403	0.0189	2.130097611	0.0335	Y
BST	0.3640	0.0168	21.69434152	0.0000	Y

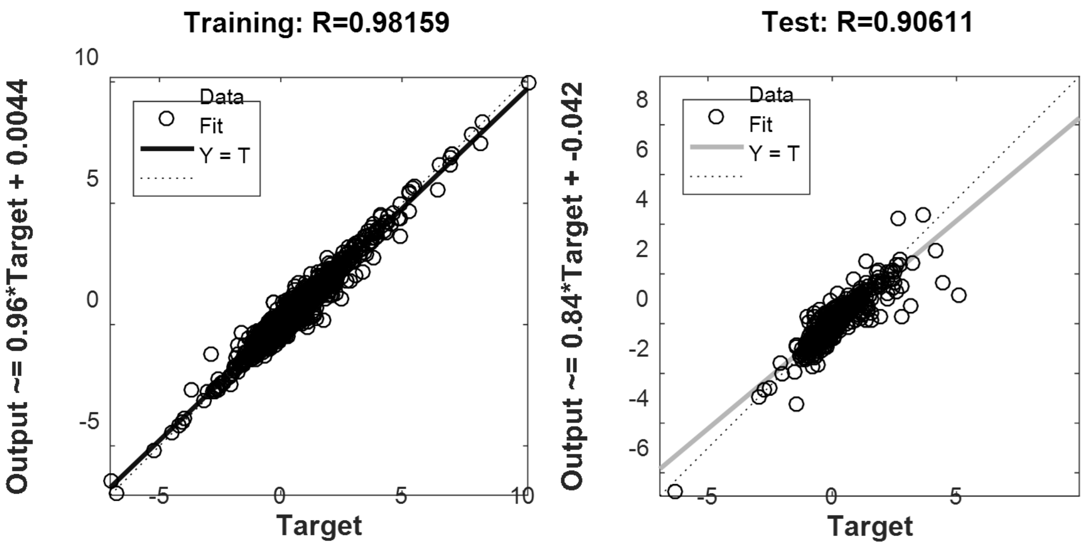

Fig. 18 ANN Regression

The regression analysis results for the prediction of the ultimate shear strength, gave poor results. In particular the residuals were not constant as demonstrated in Fig. 25, and the multiple R-squared was 0.6387 as depicted in Table 4. Therefore, the ANN model was used to investigate the torsional parameters influence on the structural response.

The ultimate collapse load is found to be strongly correlated with BST, with a t-test value of 21.69. However the multiple R found equal to 0.63 and the residuals indicate a highly nonlinear trend, as shown in Fig. 17. This means that a more complex non-linear relationship exists between independent variables and dependent, as demonstrated in the ANN modeling. Several ANN architectures were investigated and the most optimal was found to be the one with 150 hidden neurons. In Fig. 18 the R-squared for the test set calculated equal to 0.98159 for the training set and 0.90611 for the test set. Thus, the ANN model is assumed to be reliable enough to predict the structural response from the torsional parameters.

Early stopping was used so as to stop the training process when the test set error was minimum

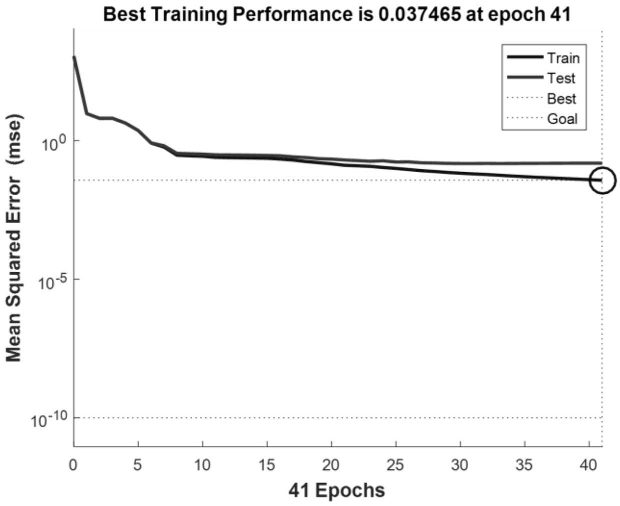

Fig. 19 ANN Training performance

Table 6 Regression summary results

	ROT	TwC	Omega	CMCV	CR	SC	TS	Rkx	Rky	BST
U2	0.0759	0.0260	0.0118	1.2516	-0.0664	0.0946	-0.2377	0.0683	0.0081	-0.1871
R3	0.0759	-0.0197	0.0510	0.6022	-0.0586	0.0699	-0.4935	0.0849	0.1193	-0.0153
Uyu	-0.0848	0.0959	0.0351	0.9488	-0.0064	-0.1807	1.4811	-0.3474	-0.1681	-0.6372
Uy0	-0.0007	0.0459	0.0414	0.8413	-0.0147	-0.4294	0.0382	-0.2089	-0.0969	-0.4624
Total	0.2372	0.1876	0.1393	3.6439	0.1460	0.7746	2.2504	0.7096	0.3925	1.3020

as shown in Fig. 19.

In the following Table 6, and Fig. 20 the sum of the product of the internal neuron weights are demonstrated. The Strength Center, the Torsional Stiffness and the BST curves were found to exhibit the highest impact on the structural response. In particular the Center of strength has a positive impact on all the torsional parameters while the others exhibit variant values and signs, according to Table 6 and Fig. 20. Additionally in Table 6, and Fig. 20 the particular contribution of each torsional parameter to the U2, R3, Uyu and Uy0 can be obtained. These outputs regarding a large, multiscale database of test structures can be considered as reliable due to the high R-squared of the test set (0.90611) and the overall approach, instead of using some specific structures to exclude contradictory conclusions.

In the following Table 7, the results of the ANN weight approach (Olden *et al*. 2002 and Gevrey *et al*. 2003) are depicted for the dynamic spectral analysis as well. In particular, an artificial neural network with ten internal nodes was trained, with input nodes the torsional criteria and output node the displacements U2, consistently with the linear regression analysis. The results obtained with the ANN are similar with the regression analysis, depicting ROT as the criterion with the highest influence to the output.

The BST surface as defined by Llera and Chopra (1995) "is assumed to divide the force space into two regions; the interior, containing combinations of the base shear and torque representing elastic behavior of the structure, and the exterior, containing statically inadmissible base shear and torque combinations. This surface is the boundary between these two regions and is where all the

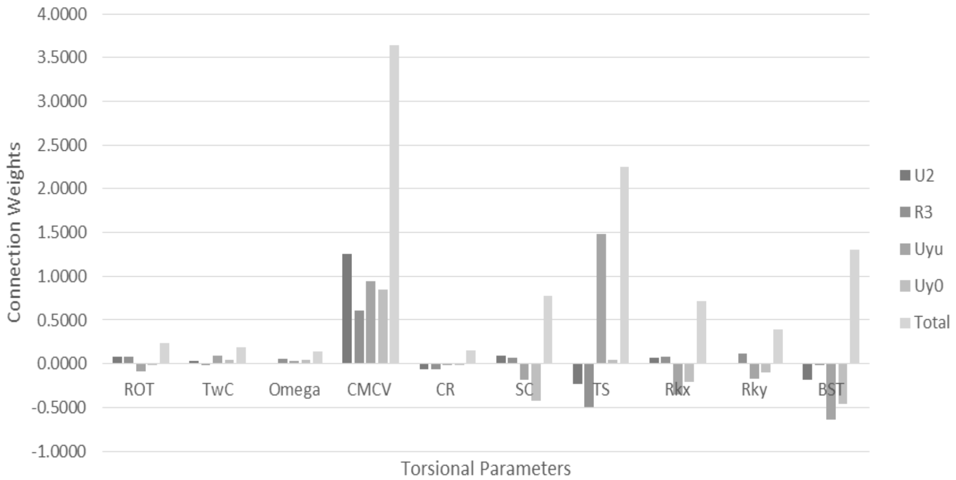

Fig. 20 Parameters importance in structural response

Table 7 Weights of independent variables in the ANN

ROT	M	TS	CR	ky	CM	Rky	kx
2.4654	1.5036	0.4898	0.0899	-0.4930	-0.2421	0.9881	-9.1460

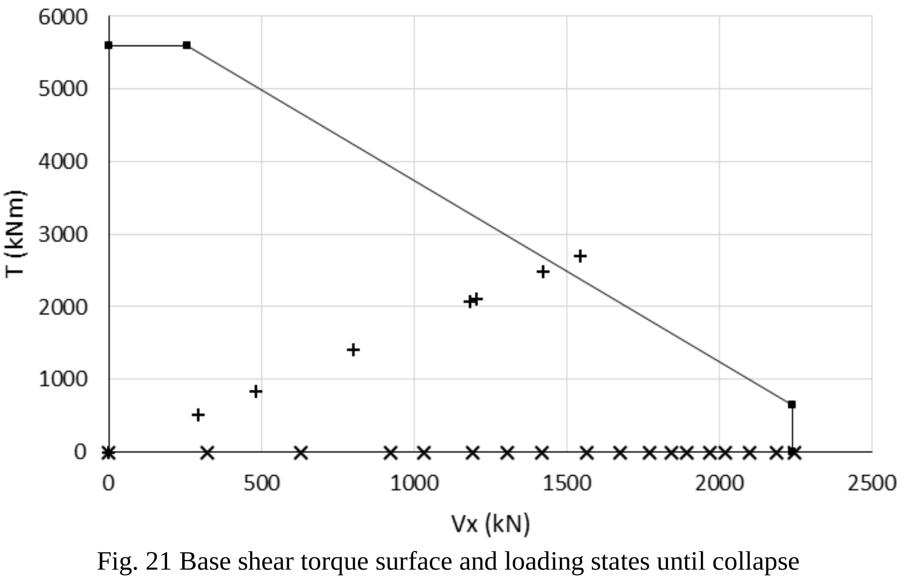

Fig. 21 Base shear torque surface and loading states until collapse

inelastic action of the system takes place". Thus, by definition can be derived, that a structure with initial combinations of base shear and torque near the BST boundary surface is more likely to pass the inelastic stage and eventually collapse with fewer increments of the initial elastic load during a pushover analysis. Furthermore, Lucchini *et al*. (2010) state that "the parameters governing the nonlinear response of the asymmetric-plan building are associated with the centers of resistances (CRs) of the system. These CRs correspond to the "base shear-torque" (BST) combinations

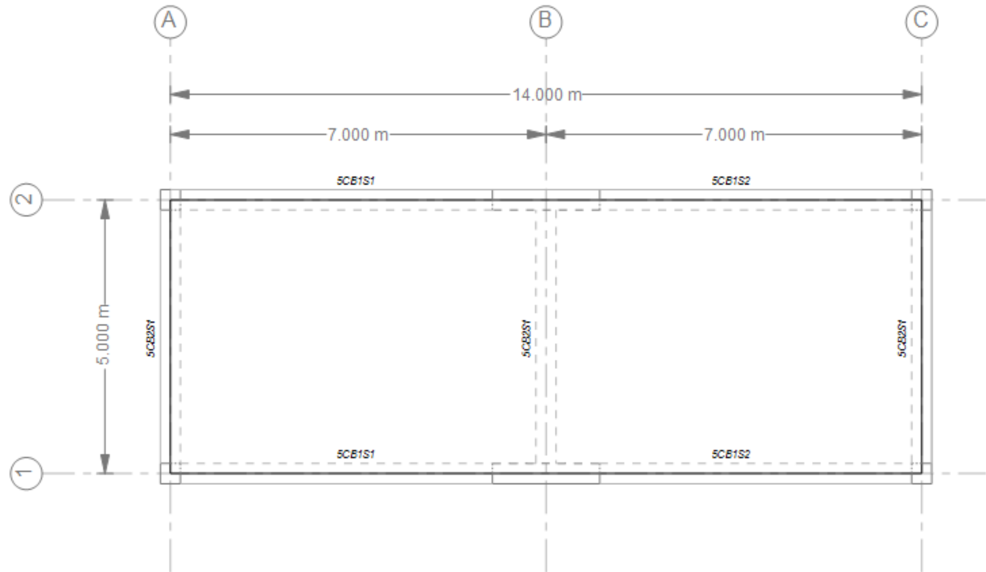

Fig. 22 Test example for the constitution of the BST surfaces

producing the plastic mechanisms". The correlation of BST surface and ultimate collapse capacity is also demonstrated by Humar *et al*. (2010). In this study it is specified that "with increasing earthquake intensity, which will push the model farther into the inelastic range…", while the positions of base shear and torque combinations inside and outside the BST boundary is graphically demonstrated, for various increasing intensities.

In the following Fig. 21, the BST surface for the example structure of Fig. 22 is demonstrated. Two loading cases are presented, one without torsion (with symbol *x*) and one with torsion (symbol +). These points represent the loading stages of the nonlinear analysis, from the beginning (0, 0) to the collapse (points near and outside the BST surface). It is apparent that as the loading factor increase, the structure's loading state is near the BST boundary and thus closer to the collapse loading conditions.

7. Conclusions

This work attempts to contribute to the literature on the reliability of structural design criteria considering torsional effects. In the majority of relative works, the conclusions reached are based on an insufficient amount of test examples and simplified assumptions. This lack of reliability is overcome via a statistical inference of various torsional design criteria, corresponding response parameters, for a numerically adequate, multiscale database of test structures.

The analysis of the derived data using linear regression and constructing a multivariate model with R-squared of 0.90 depict high t-test values for ROT and center of twist. Thus, these parameters are considered the most influential (to the displacements due to torsion). This is rational, as this approach is based on modal response spectrum analysis, in which these indices are defined, using elastic attributes of the structures.

The ANN analysis also derived a regression model with R-squared of 0.90 for the test set. The nonlinear indices Center of Strength and BST curves exhibit the highest impact on the ductility demands derived from the nonlinear pushover analysis. Additionally, this work quantifies the contribution of the other torsional indices to the ductility demands. The results are based on a large database of multiscale buildings, and the simulated numerical relationships exhibit high values of R-squared, thus can be considered as reliable, contributing to the explanation of the contradictory results of the relative literature.

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